SPECIAL PROJECT FINAL REPORT

All the following mandatory information needs to be provided.

Project Title:	**New kinetic equations and their modelling for wind wave forecasting**
Computer Project Account:	**spgbvssa**
Start Year - End Year :	**2012-2014**
Principal Investigator(s)	**Prof V.I. Shrira**
Affiliation/Address:	**Keele University, UK**
Other Researchers (Name/Affiliation):	**Dr Sergei Annenkov**

The following should cover the entire project duration.

Summary of project objectives

(10 lines max)

This project aims at creating a new conceptual and numerical framework for the study of wind wave evolution, by developing a new way of wind wave modelling based on novel kinetic equations derived from first principles. The target is to find an effective way of numerical evaluation of the generalised kinetic equations and develop a robust parallel code. The developed code will first be applied to basic prototype situations, which are beyond the limits of applicability of the classical kinetic equation, such as instantly changing or gusty wind forcing. The results will be validated by comparing with direct numerical simulations (DNS). We will study the evolution of kurtosis and other higher momenta of wave field for a number of model situations, both within and outside the limits of applicability of the classical kinetic theory.

Summary of problems encountered

(If you encountered any problems of a more technical nature, please describe them here.)

No problems encountered ………………………………………………………………………………

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Experience with the Special Project framework

(Please let us know about your experience with administrative aspects like the application procedure, progress reporting etc.)

No administrative issues to report…………………………………………………………………

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Summary of results

(This section should comprise up to 10 pages and can be replaced by a short summary plus an existing scientific report on the project.)

Report is attached ……………………………………………………………………………………

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List of publications/reports from the project with complete references

Shira V.I. & Annenkov S.Y. (2013) Towards a new picture of wave turbulence. In: Advances in Wave Turbulence, World Scientific Series on Nonlinear Science, vol. 83, pp.239-281.………………

Annenkov S.Y., Shrira V.I. Evaluation of kurtosis of JONSWAP spectra. Geophysical Research abstracts Vol. 15, EGU2013-9331 (2013) …………………………………………………………

Shrira V. I. Towards probability distribution of wave heights in the ocean from first principles. Workshop on Wave Interactions and Turbulence, Fields Institute, Toronto, 20-24 May 2013.

Annenkov, S. Y. & Shrira, V. I. (2013). Large-time evolution of statistical moments of wind–wave fields. J. Fluid Mech. **726**, 517-546…………………………………………………………………

Annenkov, S. Y. & Shrira, V. I. (2014). Evaluation of skewness and kurtosis of wind waves parameterized by JONSWAP Spectra. J. Phys. Oceanogr. **44**, 1582-1594.……………………………

Shrira, V. I. & Annenkov, S. Y. (2014) Modeling fast evolution of wave turbulence with DNS and the 'Generalized Kinetic Equation' (GKE). In: Book of Abstracts of the international conference on Wave Interaction, 22-26 April 2014, J.Kepler University Linz, Austria, pp. 56-57.…………………………

Annenkov, S. Y. & Shrira, V. I. (2014). Modelling of squall with the generalised kinetic equation. Geophysical Research Abstracts Vol. 16, EGU2014-4615, 2014, EGU General Assembly 2014……

Annenkov, S. Y. & Shrira, V. I. (2014). Modelling transient sea states with the generalized kinetic equation. WISE Meeting 2014, 8-12 June 2014, ECMWF, Reading, UK………………………………

Annenkov S.Y. & Shrira V.I. (2015) Modelling the impact of squall on wind waves with the generalized kinetic equation. Maths Foresees Workshop, University of Leeds, 18-20 May 2015……

Annenkov S.Y. & Shrira V.I. (2015) Modelling the impact of squall on wind waves with the generalized kinetic equation. J. Phys. Oceanogr. **45**, 807–812. …………………………………………

Future plans

(Please let us know of any imminent plans regarding a continuation of this research activity, in particular if they are linked to another/new Special Project.)

We plan to continue the research with a new Special Project, **Direct numerical simulation of wind wave fields in a rapidly changing environment** ……………………………………………………

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All the existing approaches to the modelling of longtime evolution of wind waves are based upon the key concept of wave turbulence. Within the framework of the wave or weak turbulence paradigm, the wave field statistical description is provided by the so-called kinetic equation (KE) for the second statistical moments of the field with appropriate input and dissipation terms.

In his pioneering papers K. Hasselmann (*Hasselmann*, 1962) developed this approach for wind waves and derived the equation for evolution of wave spatial spectra which is often referred to as the Hasselmann equation. The equation takes into account wind input, dissipation and interaction between waves of different scales and directions and describes the slow evolution of wind wave spectra in time and space (*Komen et al*, 1994; *Janssen*, 2004). In terms of wave action density $n(\mathbf{k}, \mathbf{x}, t)$, where $\mathbf{k}$ is the wavevector, while $\mathbf{x}$ and t are "slow" spatial and temporal variables, it can be written as

$$\frac{dn(\mathbf{k}, \mathbf{x}, t)}{dt} = S_{input} + S_{diss} + S_{nl},$$

where S_{input}, S_{diss} and S_{nl} describe contributions to dn/dt due to a variety of physical processes grouped as input, dissipation and nonlinear interactions respectively. The interaction term S_{nl} is dominant for energy carrying waves (*Zakharov and Badulin*, 2011). The expression for S_{nl} is derived under the assumption of quasi-stationarity of the random wave field in hand, and the resulting equation has a $O(\varepsilon^{-4})$ timescale of evolution. Therefore, strictly speaking, the Hasselmann equation is not applicable to situations with rapid changes of the environment, such as wind gusts. Due to the lack of alternatives, this difficulty is usually ignored and the Hasselmann theory (*Hasselmann*, 1962) is currently being used beyond the domain of its applicability. In the detailed numerical study by *Young and van Agthoven* (1998), the Hasselmann equation was used to model the response to an instant and sharp increase or decrease of wind. It is not clear to what extent these results can be trusted, since such an increase clearly violates the condition of validity of the equation. Thus, there is no adequate tool to provide the description of transient sea states with characteristic life spans of hundreds of dominant wave periods, resulting from the intrinsic variability of winds over sea at the timescales of tens of minutes. Wind squalls over sea fall into this range of timescales and are an important natural hazard, with poorly understood implications for wind wave fields. Experimental study of such short-lived states in the sea is extremely difficult, since the events are rare and hence not reproducible. One of the motivations for this project is to quantify applicability of the Hasselmann equation to changing winds. We specifically address the spectral response of a wind wave field to a squall, defined as a sharp increase of wind speed, which lasts for a short time period before the wind speed returns to near its previous value. Our aim is to perform the modelling of wave field under squall with the generalised kinetic equation (GKE), which was derived without the quasi-stationarity assumption (*Annenkov and Shrira*, 2006). The GKE differs from the Hasselmann equation by the form of S_{nl}, which for the GKE is non-local in time, i.e. S_{nl} at time t_1 depends on the values of $n(\mathbf{k}, \mathbf{x}, t)$ not just at $t = t_1$, but on all $t \leq t_1$. In contrast to the Hasselmann equation, the GKE includes not only resonant wave-wave interactions, but all non-resonant interactions as well, although only those not too far from resonance contribute significantly to spectral evolution. The first attempt at the numerical solution of the GKE was made by *Gramstad and Stiassnie* (2013), who have obtained the evolution of model spectra without wind forcing or dissipation and demonstrated the conservation of the invariants. *Gramstad and Babanin* (2014) performed preliminary numerical simulations that compared the GKE with the Hasselmann equation in the absence of dissipation and wind. Here we present a new efficient and highly parallelized algorithm for the numerical simulation of the GKE and perform simulations of a wave field development under constant, instantly increasing and squall-like (instantly increasing, by a factor of 2–4, and then returning back to the initial value after $O(10^2)$ characteristic wave periods) wind forcing. We show that, as expected, under steady wind the wave field evolution predicted by the GKE practically coincides with the evolution obtained with the Hasselmann equation. The GKE, however, allows modelling of rapidly changing wind conditions, for which the Hasselmann equation is not applicable, and also provides, along with the spectra, information on the evolution of higher statistical moments. This allows one to build the probability density function (p.d.f.) of surface elevation, and to consider its evolution. We show that after an instant wind increase, and after a squall as well, the probability of extreme wave events increases, and quantify the increase in terms of $h_{0.001}$ (the height of the highest wave in a thousand waves).

Theoretical background. We consider gravity waves on the surface of deep ideal fluid, described in Fourier space by the physical variables $\zeta(\mathbf{k},t)$ and $\varphi(\mathbf{k},t)$ (position of the free surface and the velocity potential at the surface respectively), where $\mathbf{k}$ is the wavevector and t is time. Statistical description of the wave field is sought in terms of correlators of complex amplitude $b(\mathbf{k},t)$, linked to $\zeta(\mathbf{k},t)$ and $\varphi(\mathbf{k},t)$ through an integral-power series (*Krasitskii*, 1994), assuming that wave slopes are $O(\varepsilon)$ small. The classical derivation (e.g., *Zakharov et al.*, 1992) leads to the kinetic (Hasselmann) equation for the second statistical moment (spectrum) $n(\mathbf{k},t)$

$$\frac{\partial n_0}{\partial t} = 4\pi \int T_{0123}^2 f_{0123} \delta_{0+1-2-3} \Delta\omega \, \mathrm{d}\mathbf{k}_{123} + S_f, \tag{1}$$

where $n_0 = \langle b_0^* b_1 \rangle = n_0 \delta_{0-1}$, angular brackets mean ensemble averaging, $f_{0123} = n_2 n_3 (n_0 + n_1) - n_0 n_1 (n_2 + n_3)$, $\omega(\mathbf{k}) = (gk)^{1/2}$ is the linear dispersion relation, g is gravity, $k = |\mathbf{k}|$, $\Delta\omega = \delta(\omega_0 + \omega_1 - \omega_2 - \omega_3)$, δ is Dirac delta function, S_f is the forcing/dissipation term, and integration is performed over the entire $\mathbf{k}$-plane. The compact notation used designates the arguments by indices, e.g., $n_0 = n(\mathbf{k},t)$, $T_{0123} = T(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$, $\delta_{0+1-2-3} = \delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)$, etc. The derivation of (1) assumes a random wave field proximity to Gaussianity and stationarity, since its derivation includes taking a large-time limit. Without the latter approximation, the *generalized kinetic equation* (GKE) derived in *Annenkov and Shrira* (2006) reads

$$\frac{\partial n_0}{\partial t} = 4\mathrm{Re} \int \Big\{ T_{0123}^2 \Big[\int_0^t \mathrm{e}^{-\mathrm{i}\Delta\omega(\tau - t)} f_{0123} \, \mathrm{d}\tau \Big] - \frac{\mathrm{i}}{2} T_{0123} J_{0123}^{(1)}(0) \mathrm{e}^{\mathrm{i}\Delta\omega t} \Big\} \delta_{0+1-2-3} \, \mathrm{d}\mathbf{k}_{123} \, . \tag{2}$$

Here $J_{0123}^{(1)}(0)$ is the initial value of the fourth-order cumulant, usually assumed to be zero at the start of the evolution. During the evolution, small but nonzero correlators emerge due to nonlinear interactions, so that

$$J_{0123}^{(1)}(t) = 2\mathrm{i} T_{0123} \int_0^t e^{-\mathrm{i}\Delta\omega(\tau - t)} f_{0123} \, \mathrm{d}\tau + J_{0123}^{(1)}(0) e^{\mathrm{i}\Delta\omega t} \tag{3}$$

Unlike the Hasselmann equation (1), which takes into account exact resonances only, the GKE (2) includes all interactions, although only those not too far from resonance contribute to spectral evolution. The GKE reduces to the Hasselmann equation in the large time limit $t \to \infty$.

Non-Gaussianity. Linear random wave fields are Gaussian and are always described by the Rayleigh distribution for the envelope, if the envelope is appropriately chosen. For weakly nonlinear waves there are two primary causes of departure from Gaussianity: quartet wave-wave resonant interactions of free modes, and bound harmonics, understood as all combinations of slave modes up to third order in nonlinearity. Thus, kurtosis chosen as a measure of field departure from Gaussianity and denoted as C_4, can be presented as a sum

$$C_4 = C_4^{(d)} + C_4^{(b)}$$

where $C_4^{(d)}$ we refer to as 'dynamic' kurtosis and $C_4^{(b)}$ as 'bound harmonics' one. These constituents of kurtosis have been expressed in terms of wave spectra by P.Janssen (2003).

Let us first consider the non-Gaussianity of the canonically transformed wave field $b(\boldsymbol{k}, t)$, denoting its statistical moments as m_j, $j = 2, 3, 4$. The second moment has the form

$$m_2 = \int \omega_0 b_0 b_0^* \, \mathrm{d}\boldsymbol{k}_0.$$

For deep water gravity waves the third moment m_3 and, hence, the skewness equals zero.

The fourth moment m_4 is readily obtained from the real part of $J^{(1)}_{0123}(t)$ as

$$m_4 = 3m_2^2 + \frac{3}{2}\mathrm{Re} \int (\omega_0\omega_1\omega_2\omega_3)^{1/2} J^{(1)}_{0123} \delta_{0+1-2-3} \, \mathrm{d}\mathbf{k}_{0123},$$

assuming that the initial value $J^{(1)}_{0123}(0)$ is zero. Since the value of $J^{(1)}_{0123}(t)$ is obtained during the numerical solution of the GKE, simulations of the GKE provide not only the evolution of spectra, but of the fourth moment m_4 as well, and, hence, of the kurtosis

$$C_4^{(d)} = m_4/m_2^2 - 3. \tag{4}$$

If the previous evolution of a wave field is not known (only the information about the instantaneous spectrum is available), the kurtosis can be evaluated approximately in terms of the spectrum as a principal value singular integral

$$C_4^{(d)} \approx -\frac{3}{2m_2^2} \int T_{0123} \, (\omega_0\omega_1\omega_2\omega_3)^{1/2} \, \frac{f_{0123}}{\Delta\omega} \delta_{0+1-2-3} \, \mathrm{d}\boldsymbol{k}_{0123}. \tag{5}$$

However, even if the wave field in canonical variables is Gaussian (for example, if there are no nonlinear interactions close to resonance), in the physical space non-Gaussianity is non-zero due to the presence of bound harmonics. This bound harmonics non-Gaussianity is described in the physical space in terms of the surface elevation, and can be calculated from the canonical transformation, provided that the dynamic component is small; otherwise the separation of the two components is not possible.

The statistical moments of the surface elevation have the form

$$\mu_j = \langle \zeta^j \rangle.$$

Janssen (2009) derived expressions for μ_j, $j = 2, 3, 4$ in terms of energy density defined as $E(\boldsymbol{k}) = \omega n(\boldsymbol{k})/g$. Note that

$$m_2 = \int \omega_0 b_0 b_0^* \, \mathrm{d}\boldsymbol{k}_0 = g \int E_0 \, \mathrm{d}\boldsymbol{k}_0.$$

For the second statistical moment in physical space, Janssen (2009) obtained

$$\mu_2 = \langle \zeta^2 \rangle = \int E_1 \, \mathrm{d}\boldsymbol{k}_1 + \int \left(\mathcal{A}_{1,2}^2 + \mathcal{B}_{1,2}^2 + 2\mathcal{C}_{1,1,2,2} \right) E_1 E_2 \, \mathrm{d}\boldsymbol{k}_{12}, \tag{6}$$

where expressions for coefficients $\mathcal{A}_{1,2}$, $\mathcal{B}_{1,2}$, $\mathcal{C}_{1,1,2,2}$ are given in *Annenkov and Shrira* (2013). Janssen (2009) showed that in the case of one-dimensional wave-vectors the second integral is equal to zero due to symmetry, and made a conjecture that this property also holds in the general two-dimensional geometry. The proof of this conjecture is given in *Annenkov and Shrira* (2013). Thus, we can write

$$\mu_2 = \langle \zeta^2 \rangle = \int E_1 \, \mathrm{d}\boldsymbol{k}_1 = \frac{m_2}{g}. \tag{7}$$

The third and fourth moments have the form (*Janssen*, 2009)

$$\mu_3 = \langle \zeta^3 \rangle = 3 \int \left(\mathcal{A}_{1,2} + \mathcal{B}_{1,2} \right) E_1 E_2 \, \mathrm{d}\boldsymbol{k}_{12}, \tag{8}$$

$$\mu_4 = \langle \zeta^4 \rangle = 3 \int E_1 E_2 \, \mathrm{d}\boldsymbol{k}_{12} + 12 \int \mathcal{J}_{1,2,3} E_1 E_2 E_3 \, \mathrm{d}\boldsymbol{k}_{123}, \tag{9}$$

where

$$\mathcal{J}_{123} = \mathcal{A}_{1,3}\mathcal{A}_{2,3} + \mathcal{B}_{1,3}\mathcal{B}_{2,3} + 2\mathcal{A}_{1,3}\mathcal{B}_{2,3} + \frac{1}{2}\mathcal{C}_{1+2-3,1,2,3} + \frac{1}{2}\mathcal{D}_{1+2+3,1,2,3}.$$

The coefficients are given in *Annenkov and Shrira* (2013). Then, we can write out the expressions for the bound harmonics components of skewness and kurtosis as

$$C_3^{(b)} = \frac{\mu_3}{\mu_2^{3/2}}, \qquad C_4^{(b)} = \frac{\mu_4}{\mu_2^2} - 3. \tag{10}$$

Computations of the higher statistical moments for wave fields paramaterized by JONSWAP spectra were performed by *Annenkov and Shrira* (2014), and simple formulas for skewness and kurtosis valid for a broad range of parameters were derived (see also Progress report for 2013).

Numerical algorithm. Although the GKE (2) is nonlocal in time, it can be solved iteratively by specifying the current value of $J_{0123}^{(1)}$ as the new initial condition, so the time integration is performed over the current time step only. The right hand side is computed over all interacting quartets $\mathbf{k}_0 + \mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3$, where $\mathbf{k}_j$, $j = 0, 1, 2$ are chosen at the grid points, and the value of amplitude for k_3 is found by bilinear interpolation. For the simulations reported below the computational grid has 101 logarithmically spaced points in the range $0.5 \leq \omega \leq 3$ and 31 uniformly spaced angles $-7\pi/9 \leq \theta \leq 7\pi/9$. All interactions satisfying $\Delta\omega/\omega_{min} \leq 0.25$, where ω_{min} is the minimum frequency of waves within the interacting quartet, were taken into account. Thus, the algorithm accounts for all resonant and approximately resonant interactions allowing a

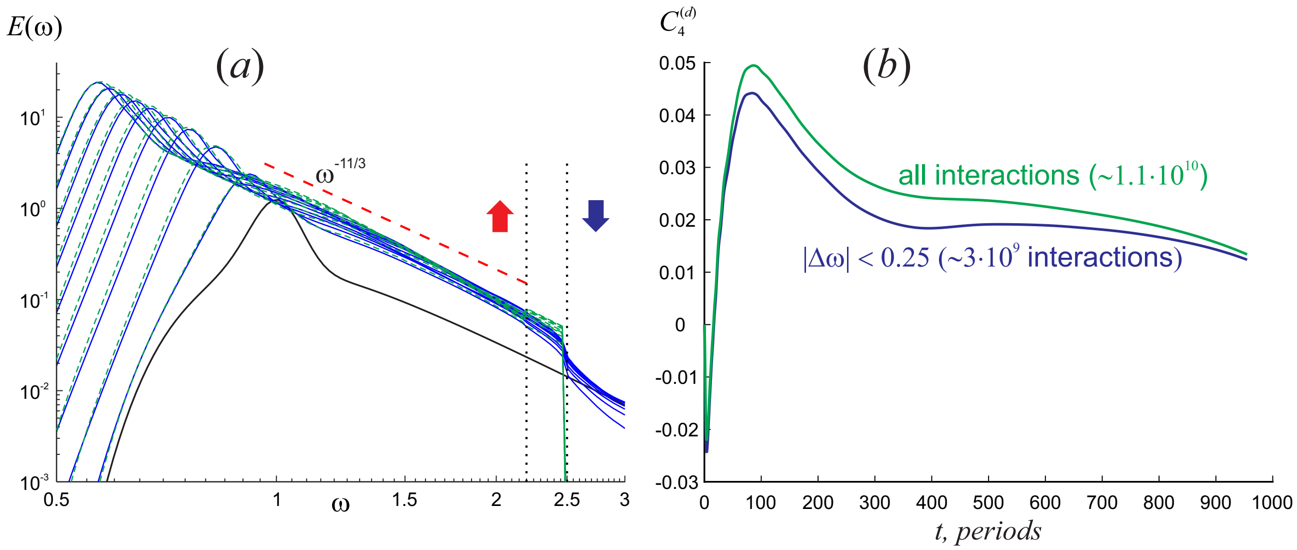

Figure 1: (a) Comparison of the energy spectra $E(\omega)$ obtained by the numerical solution of the GKE (blue curves) and the Hasselmann equation (dashed green curves) under constant wind forcing. Initial condition is taken as an empirical spectrum by Donelan et al (1985) for $U_{10}/c = 5$, wind forcing corresponds to $U_{10}/c = 5$ at the initial moment, initial peak is at $\omega = 1$. Spectra are plotted every 100 characteristic periods. (b) Evolution of the dynamical kurtosis $C_4^{(d)}$ for $U_{10}/c = 5$: value obtained along with simulations (blue) and using all resonant and non-resonant interactions (green)

large mismatch. The total number of interactions exceeds $3 \cdot 10^9$. Time stepping is performed by Runge-Kutta-Fehlberg algorithm with absolute tolerance 10^{-10} and timestep limited from above by approximately 1/3 characteristic wave period. A typical computation takes 1–3 days on 64 computational cores. Initial conditions were specified as the *Donelan et al* (1985) spectra for $2 \leq U_{10}/c \leq 5$, where c is the phase speed of the spectral peak, and wind forcing is according to *Hsiao and Shemdin* (1983). The spectral evolution was traced with for various initial values of U_{10}/c, either kept constant throughout the evolution, or instantly changing to a higher value. To simulate the squall, in a series of numerical experiments the wind forcing was set back at the initial value after $O(10^2)$ periods.

Simulations of the Hasselmann equation (1) were performed using the Gurbo Quad 5 set of subroutines based on the Webb-Resio-Tracy (WRT) algorithm (*van Vledder*, 2006), kindly provided by Gerbrant van Vledder. The same computational grid was used, with the same maximum timestep and absolute tolerance 10^{-6}, and the same initial conditions. To perform a more detailed and precise comparison of the simulations with the GKE and the Hasselmann equation, in a number of runs the spectra obtained with the GKE were used as initial conditions for the Hasselmann equation.

Results. First, the new algorithm for the numerical integration of the GKE was thoroughly validated on a number of model situations without forcing or dissipation, with the analysis of the invariants conservation properties and the large time asymptotics of the spectral evolution.

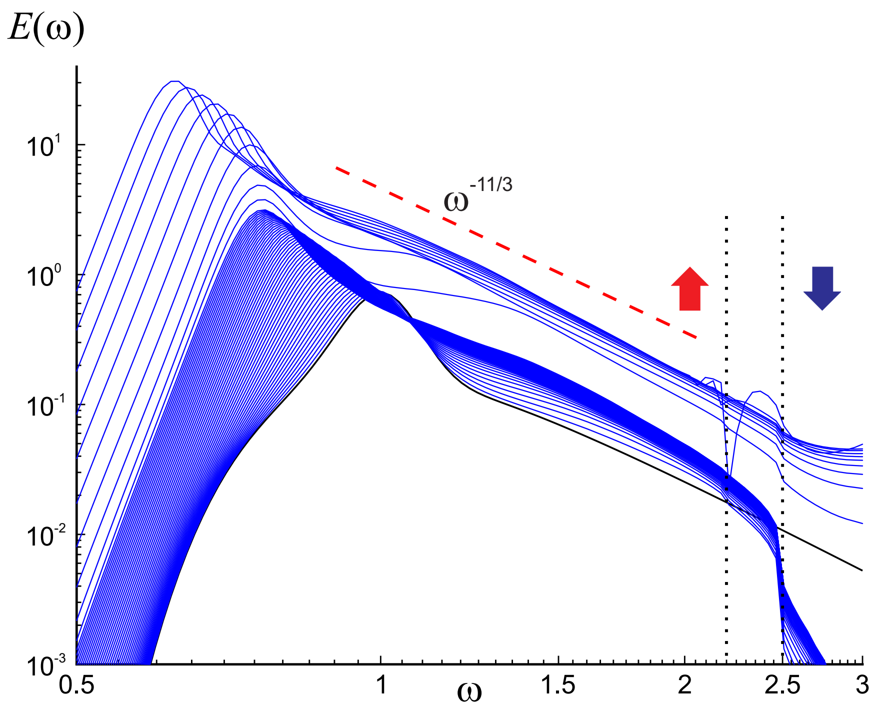

Figure 2: Evolution of the energy spectrum $E(\omega)$ with time, under constant wind forcing with initial $U_{10}/c = 3$, after about 800 periods instantly increasing to 7.5. Spectra are plotted every 22 characteristic periods.

The results of these preliminary simulations are not reported here. For the algorithm validation, we consider the development of wave spectrum under the action of constant wind, and compare the results with the simulations of the Hasselmann equation for the same forcing and dissipation and the same initial conditions. In figure 1a, results of the simulations for the GKE and the Hasselmann equation are shown, for a rather high wind (initial $U_{10}/c = 5$). Evolution is traced for approximately 900 characteritic wave periods. The results of the numerical simulation of the two equations are in close agreement. In both cases the evolution tends to the theoretical asymptotic of the spectral peak $k_p = t^{-6/11}$. Figure 1b shows the corresponding evolution of the dynamical kurtosis $C_4^{(d)}$ (blue curve). Here we take into account that the value of $C_4^{(d)}$ depends on all (resonant and non-resonant) interactions, although non-resonant interactions do not contribute to the evolution of the spectrum. Therefore, the kurtosis $C_4^{(d)}$, recomputed using (3) with the account of all interactions (the total number exceeds 10^{10}), is shown by in figure 1b by green curve.

Our primary interest is in the effects of changing wind, since a fast change of forcing formally invalidates the Hasselmann equation, and the gKE is the only tool able to describe the evolution under such conditions. In this work, we have considered only changes of wind speed, without changing wind direction. Since the GKE is nonlocal in time, its numerical modelling is performed as one continuous computation under changing wind conditions.

Figure 2 shows the evolution of the spectrum of a wave field that is allowed to evolve under

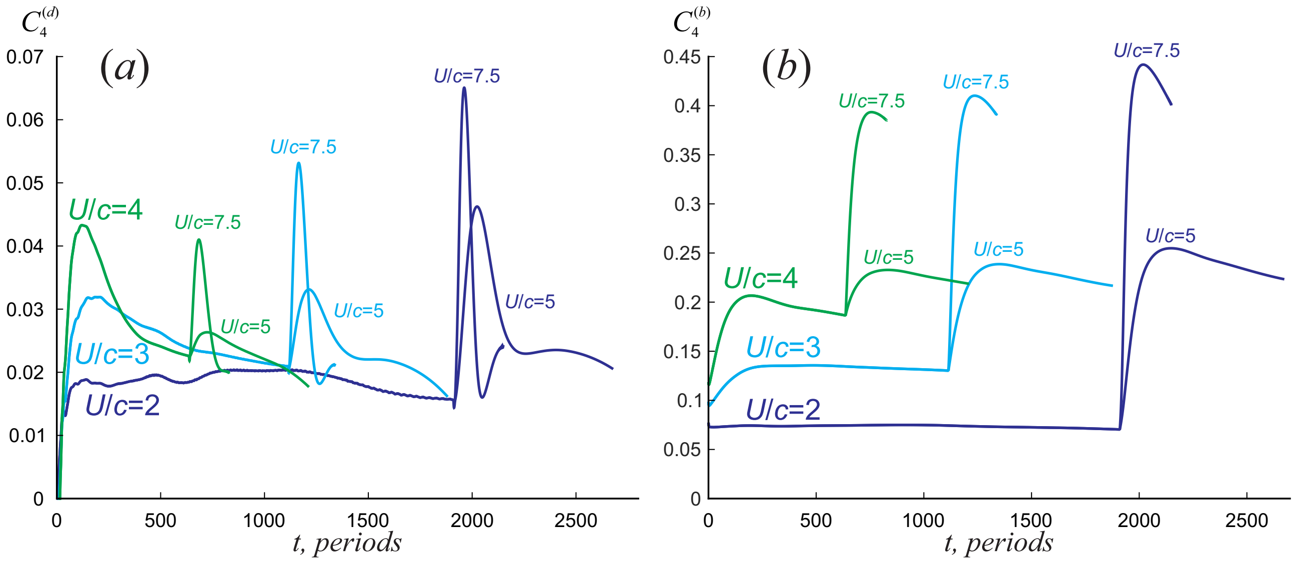

Figure 3: Evolution of the (a) dynamical and (b) bound harmonics kurtosis $C_4^{(d)}$, initially under constant wind forcing with $U_{10}/c = 2, 3, 4$, then instantly increasing to 5 or 7.5

constant wind forcing (with initial $U_{10}/c = 3$) for approximately 800 periods, then the forcing instantly increases by a factor of 2.5. The wave spectrum quickly (after about a hundred periods) adapts to the new conditions, approaching the new self-similar evolution with increased energy and peakedness, and faster downshift.

In a series of numerical experiments, wind forcing initially corresponding to $U_{10}/c = 2, 3, 4$ was instantly increased to a value corresponding to $U_{10}/c = 5$ or 7.5. Evolution of the spectrum, obtained with the GKE, was used to calculate the dynamical kurtosis $C_4^{(d)}$ (by (4), with the account of all resonant and non-resonant interactions). Bound harmonics skewness and kurtosis were calculated using (10). Results for the kurtosis are shown in figure 3a,b. The dynamical kurtosis is found to be an order of magnitude smaller than the bound harmonics one. For a constant wind and developed wave field, the dynamical kurtosis shows very weak dependence on wind. In other words, the increase of the kurtosis with the increase of steepness (on which it has the quadratic dependence) is nearly compensated by its decrease due to the increased peakedness of the spectrum under higher wind (see *Annenkov and Shrira* (2014) for a discussion of the dependence of the dynamical kurtosis on the parameters of JONSWAP spectra). An increase of wind leads to a sharp and short-lived increase of the kurtosis, which then, after a few dozen wave periods, returns to its value before the increase. The bound harmonics kurtosis and skewness (not shown here) follow the evolution of steepness (figure 3b).

In this work, we have performed an analysis of the effects of the squall, which for the purposes of this study is understood as an instant and sharp increase of wind (start of squall), followed by its decrease back to the initial value after $O(10^2)$ characteristic wave periods (end of squall). A wave field is developed under a constant wind forcing initially corresponding to $U_{10}/c = 2, 3, 4$, which is then instantly increased to $U_{10}/c = 5$ or 7.5 and then decreased back to the initial value after approximately 270 periods for $U_{10}/c = 5$ and 100 periods for $U_{10}/c = 7.5$. The

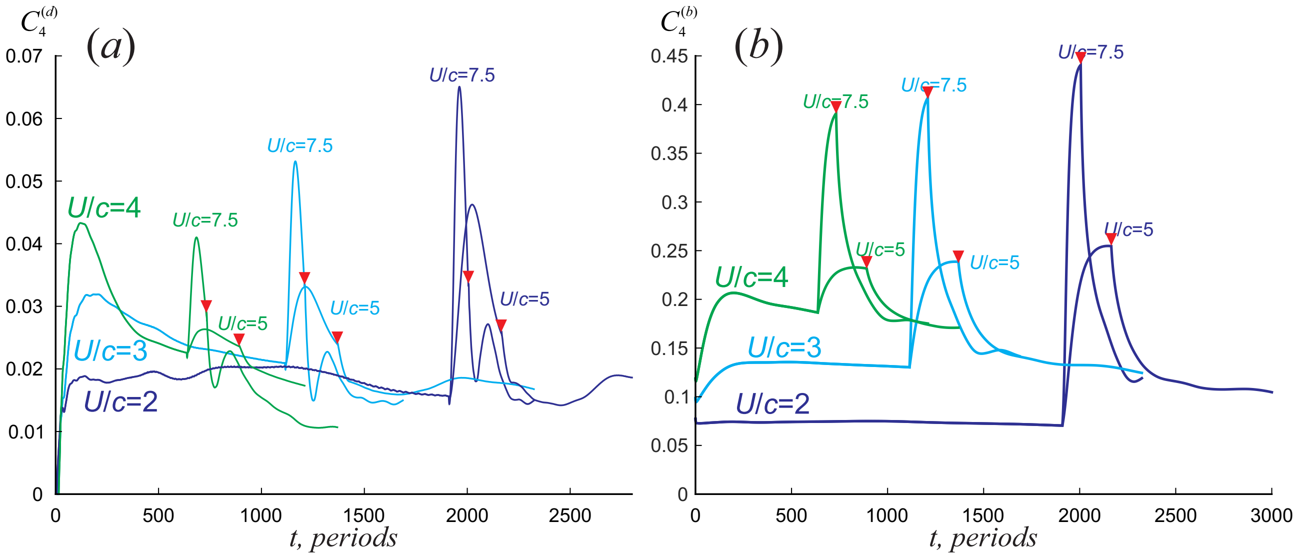

Figure 4: Evolution of the (a) dynamical and (b) bound harmonics kurtosis $C_4^{(d)}$, initially under constant wind forcing with $U_{10}/c = 2, 3, 4$, then instantly increasing to $U_{10}/c = 5$ (7.5) and decreasing back to the initial value after approximately 270 (100) periods. Red triangles mark end of squall

detailed analysis of the impact of the squall on the evolution of wave spectra was presented in the Progress report for 2014 (see also *Annenkov and Shrira*, 2015) and is not repeated here. In figure 4a,b, we show the dynamical and bound harmonics kurtosis for the case of a squall. While the dynamic kurtosis returns back to its undisturbed value soon after the start of the squall, the bound harmonics component in the case of initially weak forcing after the end of the squall is higher than before the squall.

The obtained values of the two components of kurtosis and the skewness were used to calculate the p.d.f. of surface elevations, using the approach by *Janssen* (2014), and the probability of extreme waves. In figure 5a,b we show these results in the form of the wave height corresponding to the probability of 0.001 ("highest wave in a thousand") for the case of constant wind of various speeds, and for the squall. The wave height is shown in terms of significant wave height. For comparison, the corresponding values for a linear wave field (Rayleigh distribution) are also plotted. The squall leads to a significant increase of the height of extreme waves.

Discussion. We report the breakthrough in modelling of evolution of wind wave fields under rapidly changing forcing. An efficient highly parallel numerical algorithm, based on the generalized kinetic equation (GKE) has been created and applied to the study of transient sea states, caused by changing forcing. The significance of the new numerical approach is twofold. First, it is based on the generalized statistical theory, which, unlike the standard approach based on the Hasselmann equation, is free of the stationarity assumption and allows to study the evolution of wave fields in rapidly changing environment. Second, along with the evolution of wave spectra, the new approach allows to obtain the evolution of higher statistical moments as well, and to build the p.d.f. of surface elevations. These advantages of the approach have been

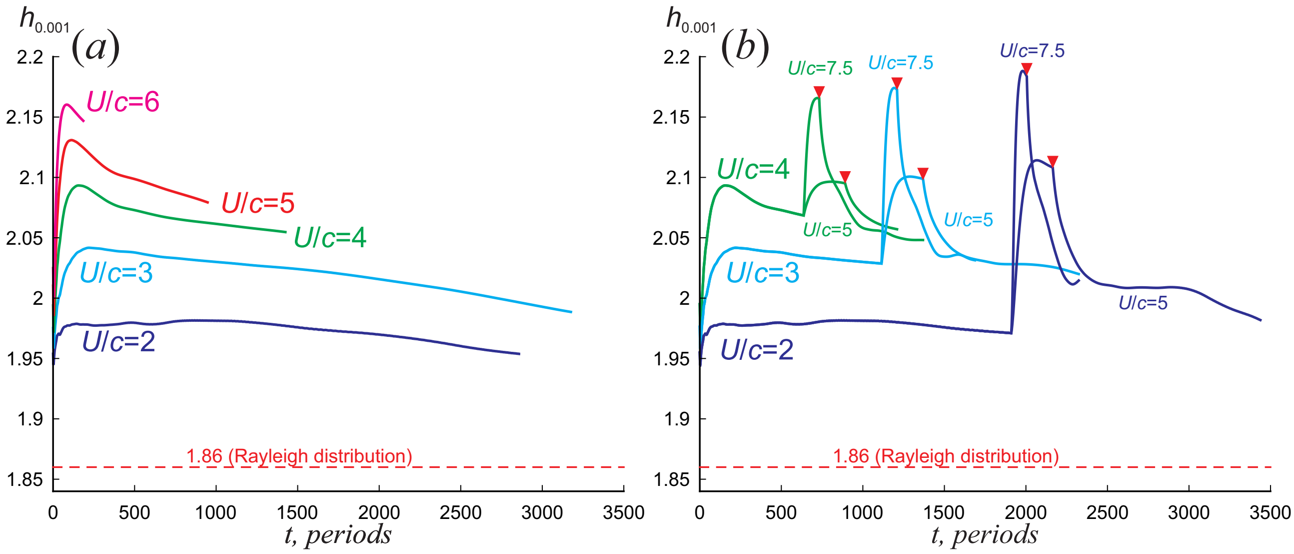

Figure 5: Maximum individual wave height with a probability of 0.001 ("highest wave in a thousand"), normalized by the significant wave height, under (a) constant wind with initial $U_{10}/c = 2 - 6$, (b) constant wind forcing with initial $U_{10}/c = 2 - 4$, then instantly increasing to 5 or 7.5 and decreasing back to the initial value. Red triangles mark end of squall

demonstrated for model cases with an instant increase of wave forcing and for squall. Thus, we have presented an approach that allows, for the first time, to get the complete description of the evolution of a wind wave field in a fast changing environment.

Probably the most far reaching implications stem from the demonstrated efficiency of the proposed highly parallel algorithm for solving the GKE, we have got the tool not only for studying a variety of short-lived transient processes, in the long term, it has a potential to replace the Hasselmann equation as the basis of wave modelling. Simulating the GKE could be faster and with a sufficient number of parallel processors could be even made much faster that the existing codes for the Hasselmann equation.

References

Annenkov, S. Y., and V. I. Shrira (2006), Role of non-resonant interactions in the evolution of nonlinear random water wave fields, *J. Fluid Mech.*, *561*, 181–207.

Annenkov, S. Y. and V. I. Shrira (2009), "Fast" nonlinear evolution in wave turbulence, *Phys. Rev. Letters*, *102*, 024502.

Annenkov S. Y., and V. I. Shrira (2013), Large-time evolution of statistical moments of a wind wave field, *J. Fluid Mech.*, *726*, 517–546.

Annenkov S. Y., and V. I. Shrira (2014), Evaluation of skewness and kurtosis of wind waves parameterized by JONSWAP spectra. *J. Phys. Oceanogr.*, *44*, 1582–1594.

Annenkov S. Y., and V. I. Shrira (2015), Modelling the impact of squall on wind waves with the generalized kinetic equation, *J. Phys. Oceanogr.*, *45*, 807-812.

Donelan, M. A., J. Hamilton, and W. H. Hui (1985), Directional spectra of wind-generated waves, *Phil. Trans. R. Soc. London*, *A315*, 509–562.

Gramstad, O., and A. Babanin (2014), Implementing new nonlinear term in third-generation wave models, In: *Proceedings of the ASME 2014 33rd International Conference on Ocean, Offshore and Arctic Engineering*, OMAE2014-24677.

Gramstad, O., and M. Stiassnie (2013), Phase-averaged equation for water waves, *J. Fluid Mech.*, *718*, 280–303.

Hasselmann, K. (1962), On the non-linear energy transfer in a gravity-wave spectrum. Part 1: General theory, *J. Fluid Mech.*, *12*, 481–500.

Hsiao, S. V. and O. H. Shemdin (1983), Measurements of wind velocity and pressure with a wave follower during MARSEN, *J. Geophys. Res.*, *88*, 9841–9849.

Janssen, P. A. E. M. (2004), *The Interaction of Ocean Waves and Wind*, Cambridge University Press.

Janssen, P. A. E. M. (2009), On some consequences of the canonical transformation in the Hamiltonian theory of water waves. *J. Fluid Mech.*, *637*, 1–44.

Janssen, P. A. E. M. (2014), On a random time series analysis valid for arbitrary spectral shape, *J. Fluid Mech.*, *759*, 236–256.

Komen, G. J., Cavaleri, L., Donelan, M., Hasselmann, K., Hasselmann, S. and P. A. E. M. Janssen (1994) *Dynamics and Modelling of Ocean Waves*, Cambridge University Press.

Krasitskii, V. P. (1994), On reduced Hamiltonian equations in the nonlinear theory of water surface waves, *J. Fluid Mech.*, *272*, 1–20.

van Vledder, G. Ph. (2006), The WRT method for the computation of non-linear four-wave interactions in discrete spectral wave models, *Coastal Engineering*, *53*, 223–242.

Young, I. R., and A. van Agthoven (1998), The response of waves to a sudden change in wind speed, *Advances in Fluid Mechanics*, *17*, 133–162.

Zakharov, V. E., and S. I. Badulin (2011), On energy balance in wind-driven seas, *Doklady Earth Sciences*, *440*, 1440–1444.

Zakharov, V. E., V. S. L'vov, and G. Falkovich (1992), *Kolmogorov Spectra of Turbulence I: Wave Turbulence*, Springer, Berlin.