Fast Vector Radiative Transfer Method Based on the Small-angle Approximation

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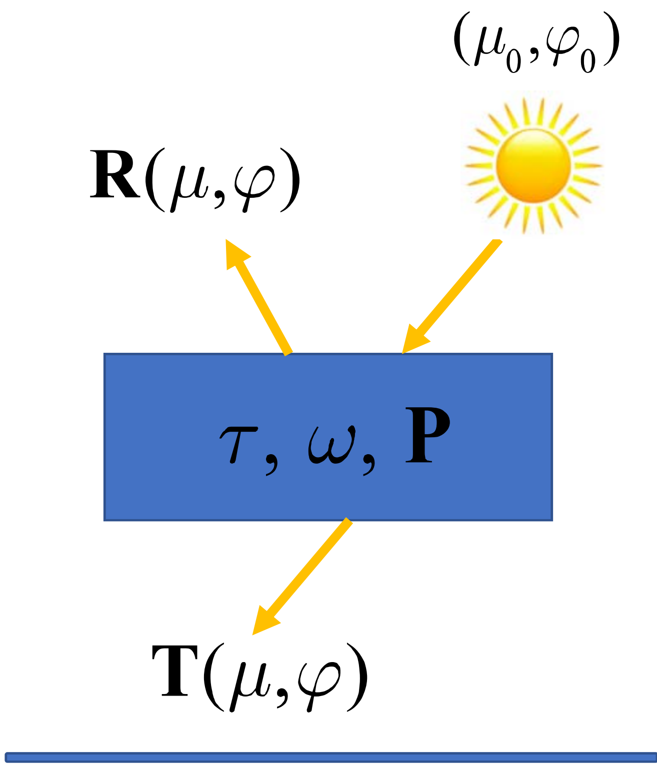
Background

- Stokes vector (I,Q,U,V)
- Vector Radiative Transfer Equation:

$$\mu \frac{d\mathbf{I}(\mu,\varphi)}{d\tau} = \mathbf{I}(\mu,\varphi) - \frac{\omega}{4\pi} \iint \mathbf{P}(\mu,\mu',\varphi-\varphi')\mathbf{I}(\mu',\varphi')\, d\mu'\, d\varphi'.$$



Outline

- Introduction
- Method
- Validation
- Application
- Summary

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Introduction

- Radiative transfer equation (RTE) is extensively discussed by many textbooks (Chandrasekhar 1960; Preisendorfer 1965; van de Hulst 1980; Liou 2002; …)
- Methods:
  - Discrete-ordinate radiative transfer method, Adding-doubling method, Monte Carlo method, Successive order of scattering method, Matrix operator method, invariant imbedding method, …
- Computational time:
  - Numerically accurate simulation of RTE in a strongly forward-scattering medium requires high computational effort.
- Approximations:
  - Van de Hulst and Grossman (1968) scale the optical thickness by a factor (1-ωg) and replace the original phase function with an isotropic counterpart.
  - Coakley and Chylek (1975), Liou (1974), Meador and Weaver (1980) and Liou et al. (1988) use the two-stream or four-stream approximations.
  - Potter (1970) uses the delta-function to replace the forward peak.
  - Joseph et al. (1976) and Wiscombe (1977a) uses the delta-Eddington approximation.
  - Wiscombe (1977b) introduces the δ-M approximation and Hu et al. (2000) introduces the δ-Fit approximation.

Introduction

- The invariance of scaling of optical thickness and phase function is introduced by McKellar and Box (1981):

$$P \approx 2f\delta(1-\cos\Theta) + (1-f)P_s,$$

$$\tau_s = \tau(1-\omega f),\ \omega_s = \frac{1-f}{1-\omega f}\omega.$$

Introduction

- Small-angle approximation has been extensively used in multiple scattering of charged particles and radiative transfer (e.g. Scott 1963; Ishimaru 1978; Sobolev 1975…).

- Nakajima et al. (1983), Nakajima and Tanaka (1988), and Weinman et al. (1975) uses the summation of Gaussian functions to replace the forward peak and the small-angle approximation is realized by the successive order of scattering method and the Hankel transform technique.

- Semi-analytical small-angle approximation is introduced by Sobolev (1975), Dolin (1973), Remizovich et al. (1982), Zege et al. (1987)…

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Method

Vector Radiative Transfer Equation (VRTM):

$$\mu\frac{d\mathbf{I}(\mu,\varphi)}{d\tau}=\mathbf{I}(\mu,\varphi)-\frac{\omega}{4\pi}\iint\mathbf{P}(\mu,\mu',\varphi-\varphi')\mathbf{I}(\mu,\varphi')\,d\mu'\,d\varphi'.\qquad \mathbf{I}(\tau=0,\mu,\mu_0,\varphi-\varphi_0)=\delta(\mu-\mu_0)\delta(\varphi-\varphi_0)\begin{pmatrix}F_0 & 0 & 0 & 0\end{pmatrix}^T.$$

Decomposition:

$$\mathbf{P}(\mu,\mu',\varphi-\varphi)=f\mathbf{P}_f(\mu,\mu',\varphi-\varphi)+(1-f)\mathbf{P}_r(\mu,\mu',\varphi-\varphi),$$

$$\mathbf{I}(\mu,\varphi)=\mathbf{I}_f(\mu,\varphi)+\mathbf{I}_r(\mu,\varphi)+\mathbf{I}_e(\mu,\varphi).$$

Three-Component VRTM:

Forward Equation:

$$\mu\frac{d\mathbf{I}_f(\mu,\varphi)}{d\tau}=\mathbf{I}_f(\mu,\varphi)-\frac{\omega_f}{4\pi}\iint\mathbf{P}_f(\mu,\mu',\varphi-\varphi')\mathbf{I}_f(\mu',\varphi')\,d\mu'\,d\varphi',\ \omega_f=\omega f;$$

Regular Equation:

$$\mu\frac{d\mathbf{I}_r(\mu,\varphi)}{d\tau}=\mathbf{I}_r(\mu,\varphi)-\frac{\omega_r}{4\pi}\iint\mathbf{P}_r(\mu,\mu',\varphi-\varphi')\mathbf{I}_r(\mu',\varphi')\,d\mu'\,d\varphi'-\frac{\omega_r}{4\pi}\exp(-\frac{\tau_r}{\mu_0})\mathbf{P}_r(\mu,\mu_0,\varphi-\varphi_0)\mathbf{I}_r(\mu_0,\varphi_0).$$

$$\omega_r=\omega\frac{1-f}{1-\omega f},\ \tau_r=(1-\omega f)\tau;$$

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Method

Error component:

$$\mu \frac{d\mathbf{I}_e(\mu,\varphi)}{d\tau} = \mathbf{I}_e(\mu,\varphi) - \frac{\omega}{4\pi}\int_{4\pi} \mathbf{P}(\mu,\mu',\varphi-\varphi')\mathbf{I}_e(\mu',\varphi')\,d\mu'\,d\varphi' - \mathbf{J}_1(\mu,\varphi) - \mathbf{J}_2(\mu,\varphi),$$

$$\mathbf{J}_1(\mu,\varphi) = \omega f\left[\frac{1}{4\pi}\int_{4\pi} \mathbf{P}_f(\mu,\mu',\varphi-\varphi')\mathbf{I}_r(\mu',\varphi')\,d\mu'\,d\varphi' - \mathbf{I}_r(\mu,\varphi)\right],$$

$$\mathbf{J}_2(\mu,\varphi) = \frac{\omega(1-f)}{4\pi}\left[\int_{4\pi} \mathbf{P}_r(\mu,\mu',\varphi-\varphi')\mathbf{I}_f(\mu',\varphi')\,d\mu'\,d\varphi' - \exp\left(-\frac{(1-\omega f)\tau}{\mu_0}\right)\mathbf{P}_r(\mu,\mu_0,\varphi-\varphi_0)\mathbf{E}_0\right].$$

Forward Component:

$$\mu \frac{d\mathbf{I}_f(\mu,\varphi)}{d\tau} = \mathbf{I}_f(\mu,\varphi) - \frac{\omega_f}{4\pi}\iint \mathbf{P}_f(\mu,\mu',\varphi-\varphi')\mathbf{I}_f(\mu',\varphi')\,d\mu'\,d\varphi',$$

$$\mathbf{I}^f \approx \mathbf{I}_0^f + \mathbf{I}_1^f + \mathbf{I}_2^f$$

$$\mathbf{I}^f \approx \frac{F}{2\pi V} e^{-\frac{\theta^2}{2V}}.$$

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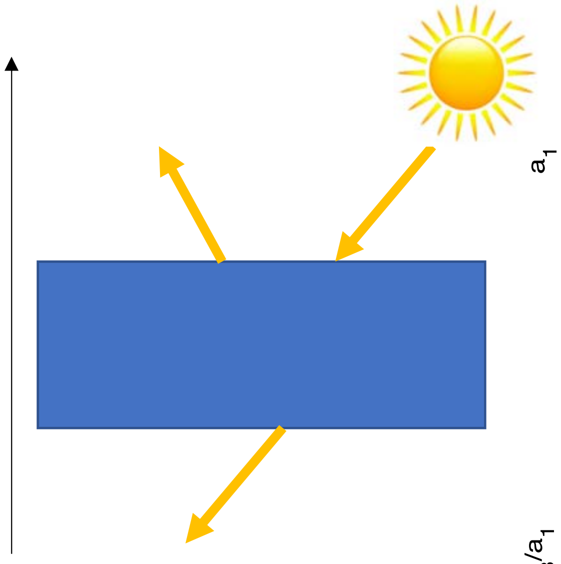
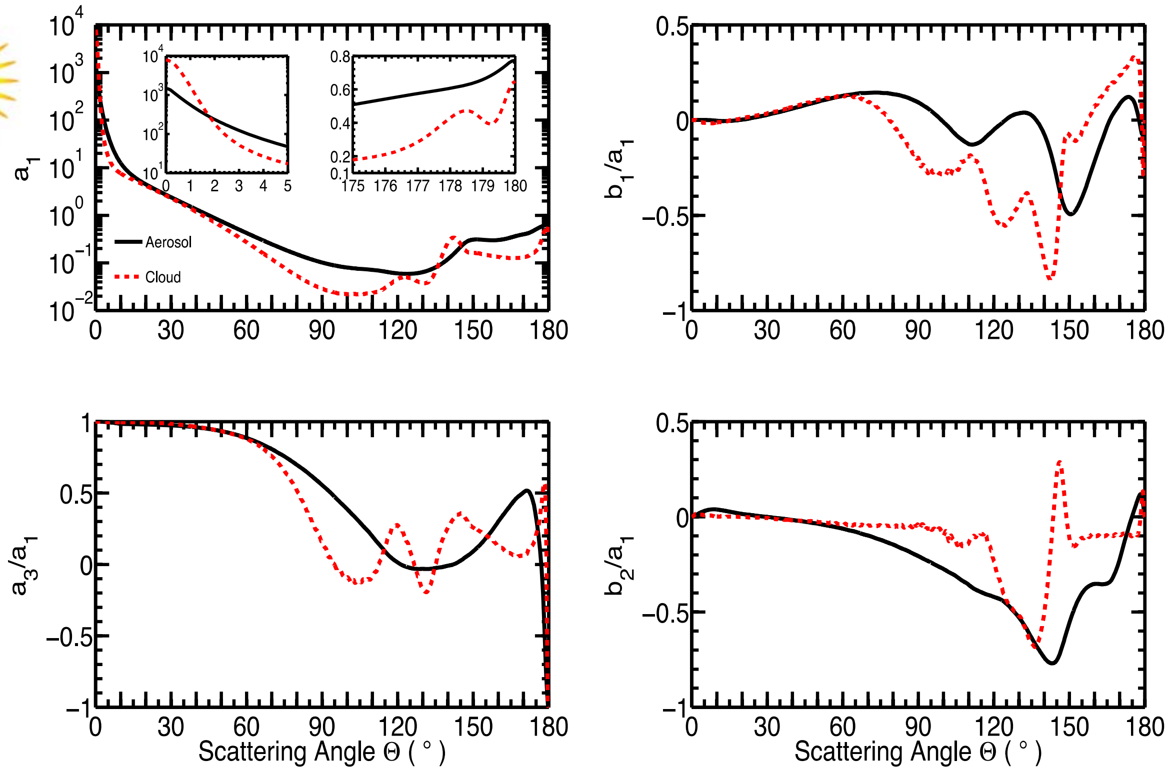
Validation





$\mu_0 = 0.5,\ \omega = 1,$

$\tau_{\text{aerosol}} = 0.3262,$

$\tau_{\text{cloud}} = 5.$

*Figure 1. Phase matrix elements of predefined aerosol and cloud layers

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*Kokhanovsky et al., 2010: Benchmark results in vector atmospheric radiative transfer. *J. Quant. Spectrosc. Radiat. Transfer*, **111,** 1931-1946.

Validation

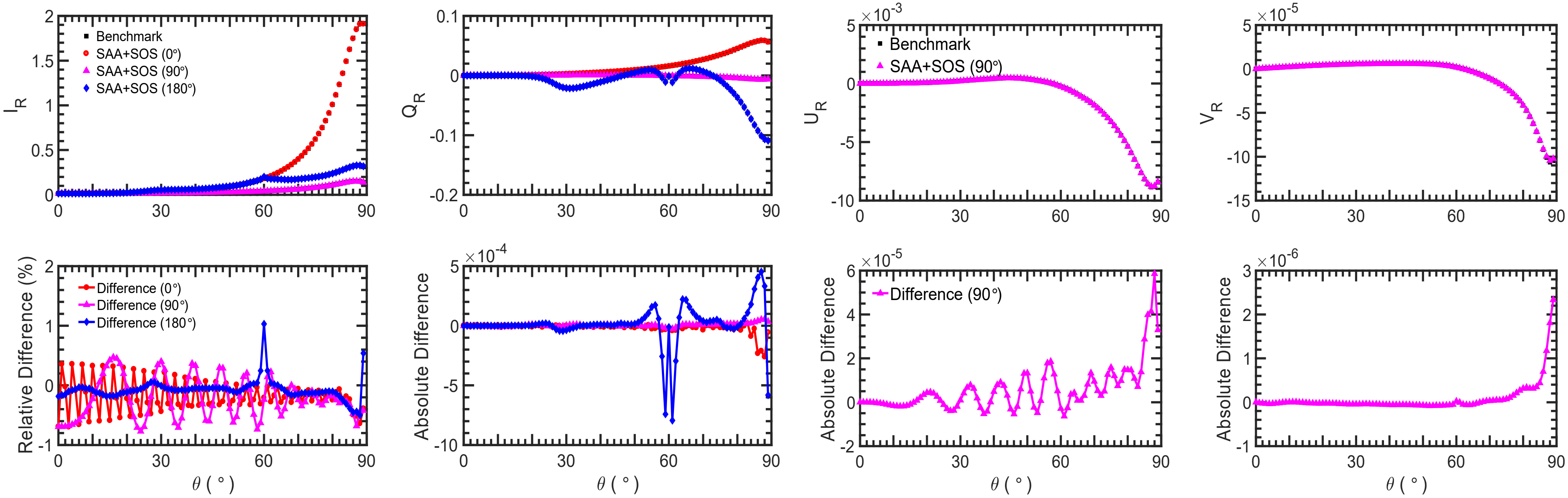

Figure 2. Stokes vector elements related to the reflected radiation and the differences in the case of aerosol.

Validation

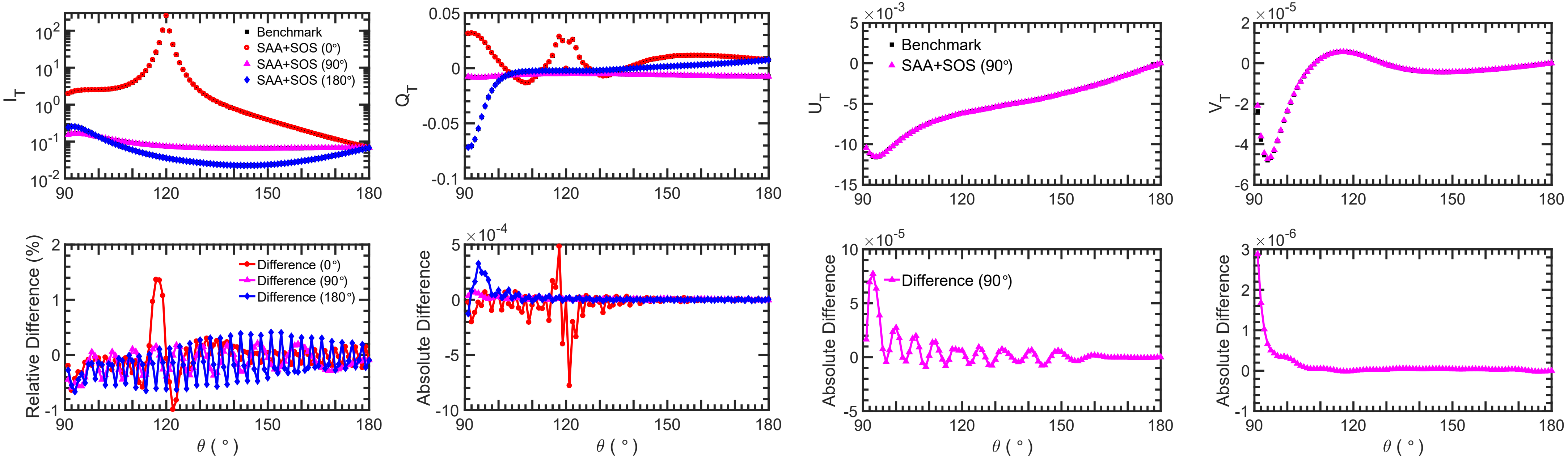

Figure 3. Stokes vector elements related to the transmitted radiation and the differences in the case of aerosol.

Validation

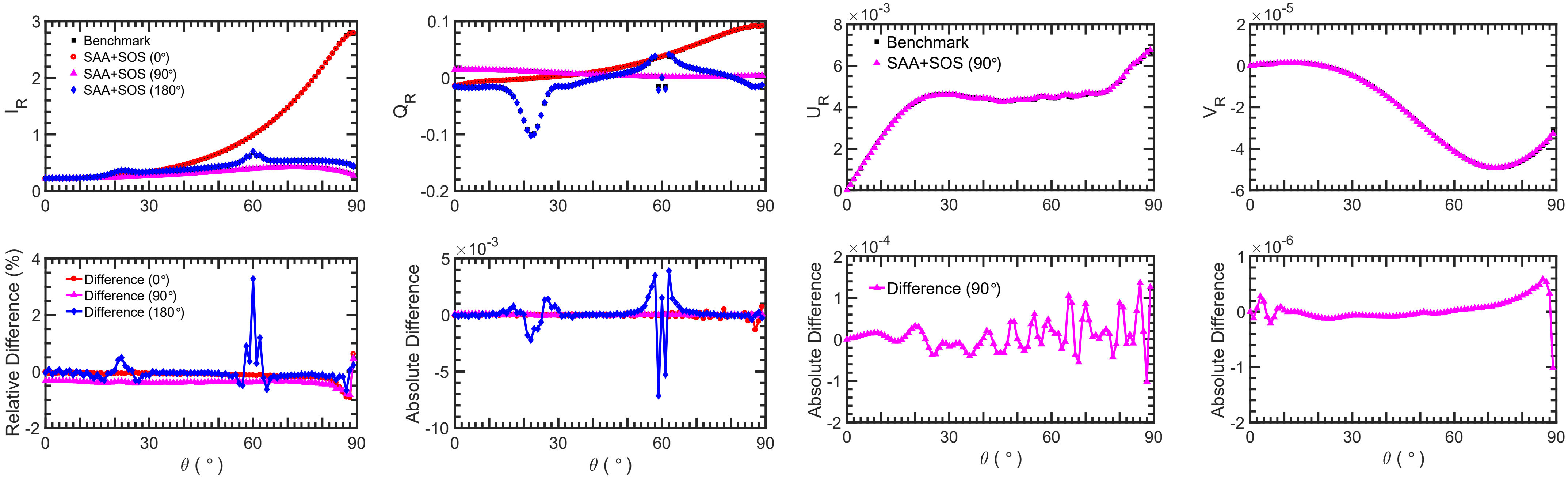

Figure 4. Stokes vector elements related to the reflected radiation and the differences in the case of cloud.

Validation

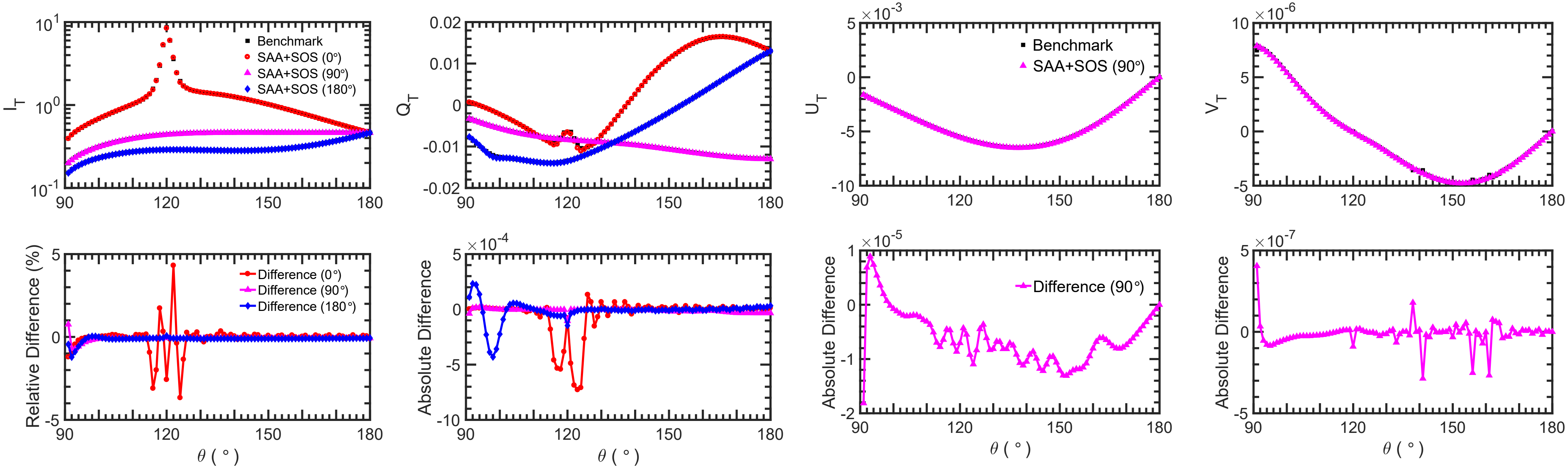

Figure 5. Stokes vector elements related to the transmitted radiation and the differences in the case of cloud.

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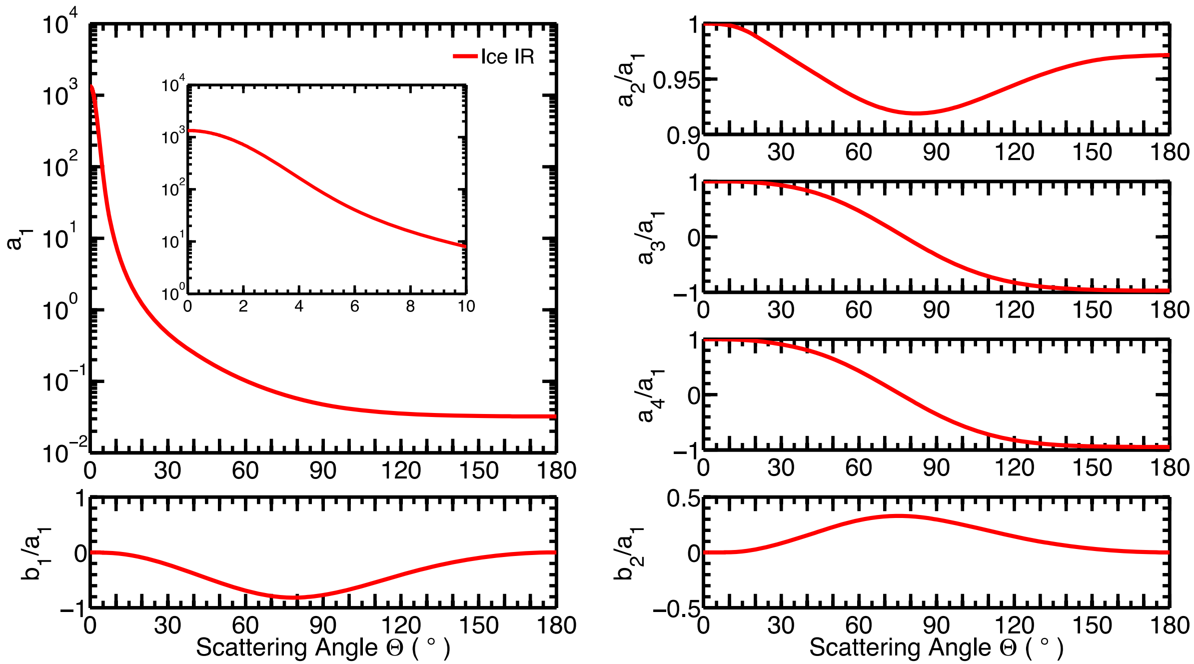
Application



$\mu_0 = 0.5,$
$\omega = 0.523801,$
$\tau = 5.$

Figure 6. Phase matrix elements of ice cloud

Application

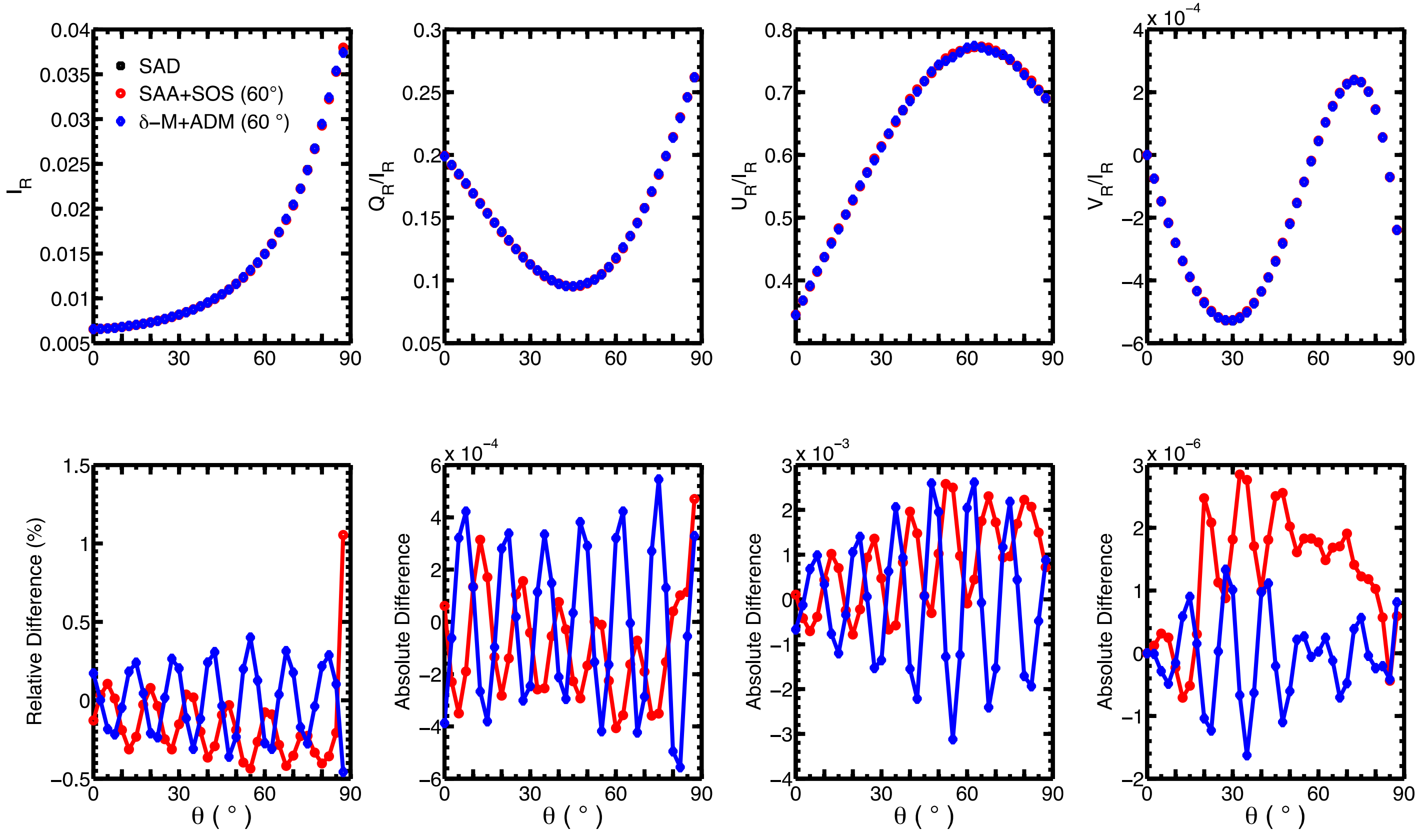


Figure 7. Stokes vector elements related to the reflected radiation

Application

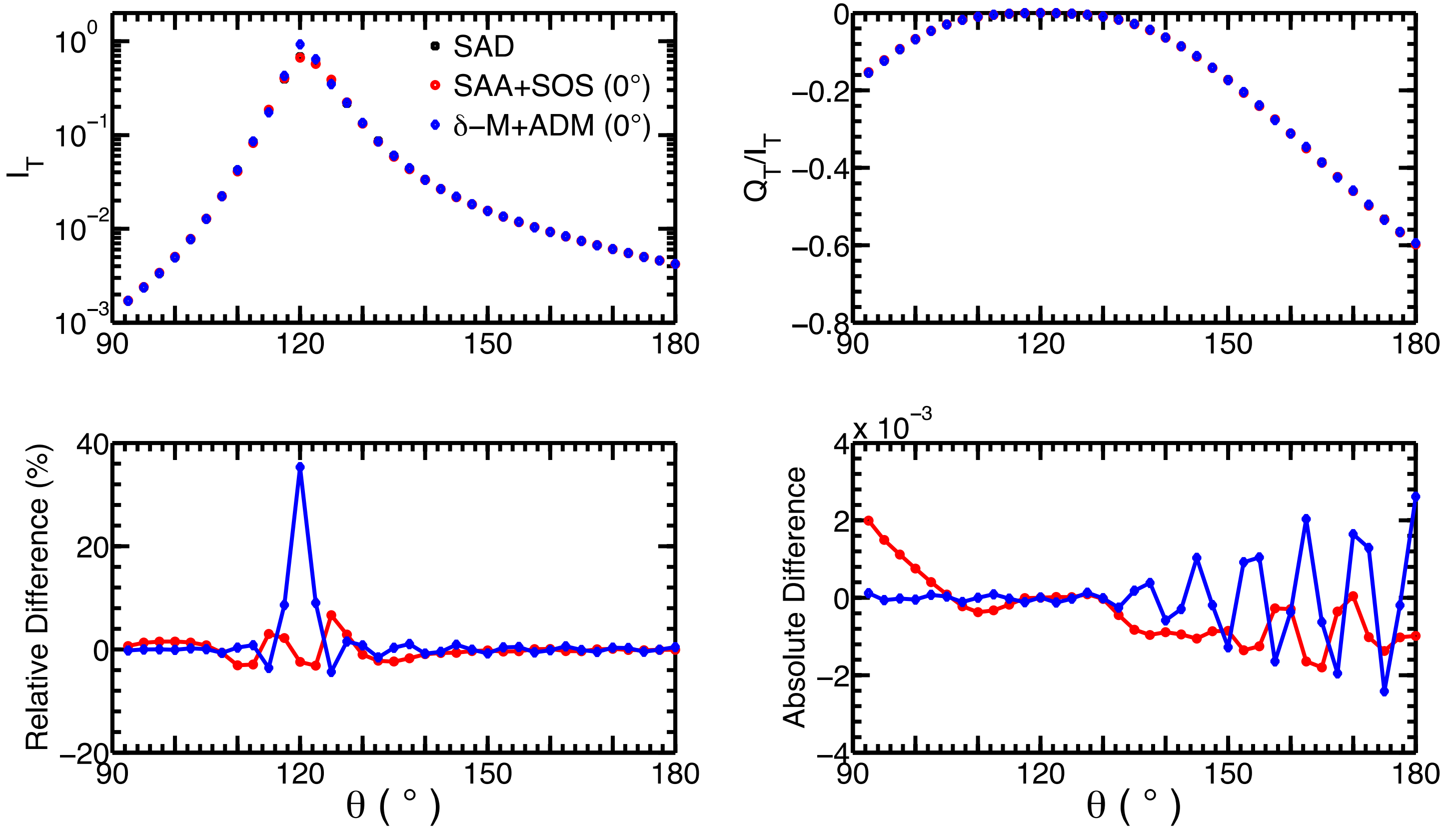


Figure 8. Stokes vector elements related to the transmitted radiation

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Application

	SAA+SOS			δ-M+ADM		
	M	GSFs	Time (s)	M	GSFs	Time (s)
Reflection	30	40	3.9	30	40	15.2
Transmission	30	40	7.2	30	40	14.8

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- The methods combining the small-angle approximation and the successive order of scattering are validated using the benchmark results.

- The improvement and a forward model about radiative transfer are in progress.

Sun B., Kattawar G. W., Yang P., and Mlawer E. J. (2017). An Improved Small-Angle Approximation for Forward Scattering and Its Use in a Fast Two-Component Radiative Transfer Method. *J. Atmo. Sci.*, **74**(6), 1959-1987.

		SAA+SOS							
		n_f	Δτ	Θ_c (°)	N_{sca}	N_{src}	M	GSFs	Time (s)
Aerosol Ref.	0°	1.0	0.1631	6	30	N/A	100	100	26.9
	90°	1.0	0.1631	6	30	N/A	100	100	26.8
	180°	1.0	0.1631	6	20	N/A	120	120	27.8
Aerosol Trans.	0°	1.0	0.1631	6	30	N/A	100	120	27.8
	90°	1.0	0.1631	6	30	N/A	120	120	28.9
	180°	1.0	0.1631	6	30	N/A	120	120	28.9
Cloud Ref.	0°	1.4	0.05	6	20	N/A	60	132	31.5
	90°	1.4	0.05	6	N/A	N/A	100	132	40.1
	180°	1.4	0.05	6	130	N/A	100	132	53.8
Cloud Trans.	0°	0.72	0.05	3	35	10	60	100	29.7
	90°	5	0.05	10	20	N/A	50	100	22.2
	180°	5	0.05	10	20	N/A	30	100	16.8