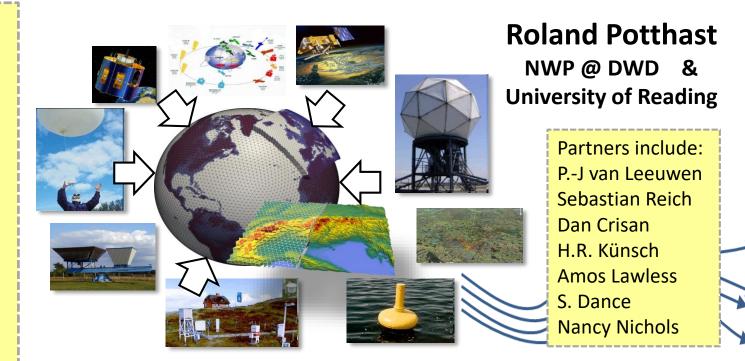


DWC

# Ensemble Data Assimilation and Particle Filters for NWP

With the help of many people, in particular:

<u>Anne Walter,</u> <u>Andreas Rhodin</u> Harald Anlauf, Christina Köpken, Robin Faulwetter, Olaf Stiller, Alexander Cress, Martin Lange, Stefanie Hollborn, E. Bauernschubert, Christoph Schraff, Hendrik Reich, Klaus Stephan Ulrich Blahak



Heinz-Werner Bitzer, Annika Schomburg, Silke May, Marc Pondrom, Kristin Raykova, Thomas Rösch, Michael Bender, Christian Welzbacher, Lilo Bach, Lisa Neef, Zoi Paschalidi, Walter Acevedo, Axel Hutt, Daniel Egerer, Gerhard Paul, Ana Fernandez, Stefan Declair







- Why and Where Distributions, Risk and Uncertainty?
- 2. Discussion of **Ensemble (+Particle) Methods**
- 3. Framework Global+LAM+LES Model: ICON and

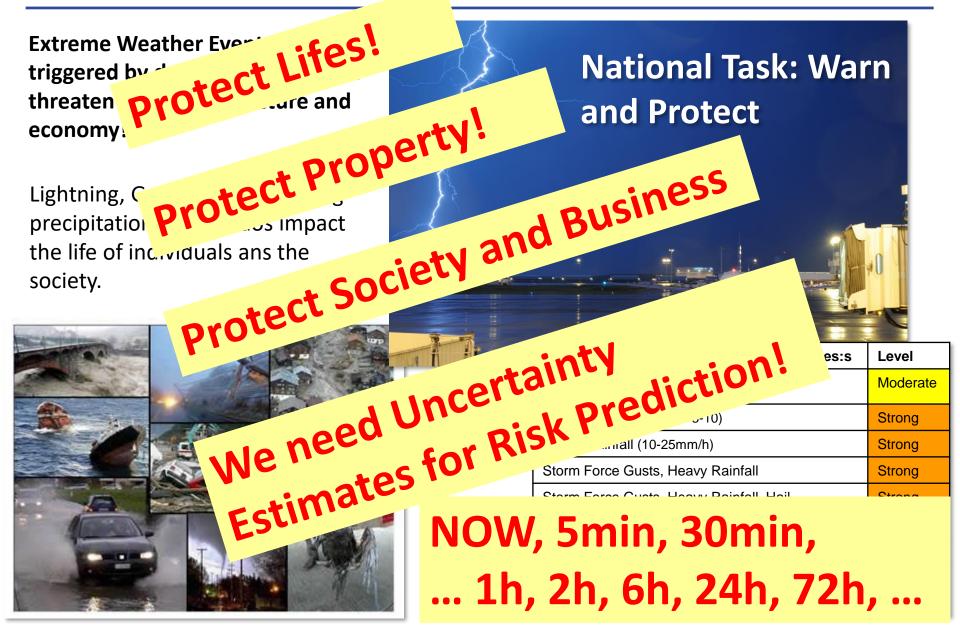
ICON-EPS and the LEKTF+EnVAR/KENDA System

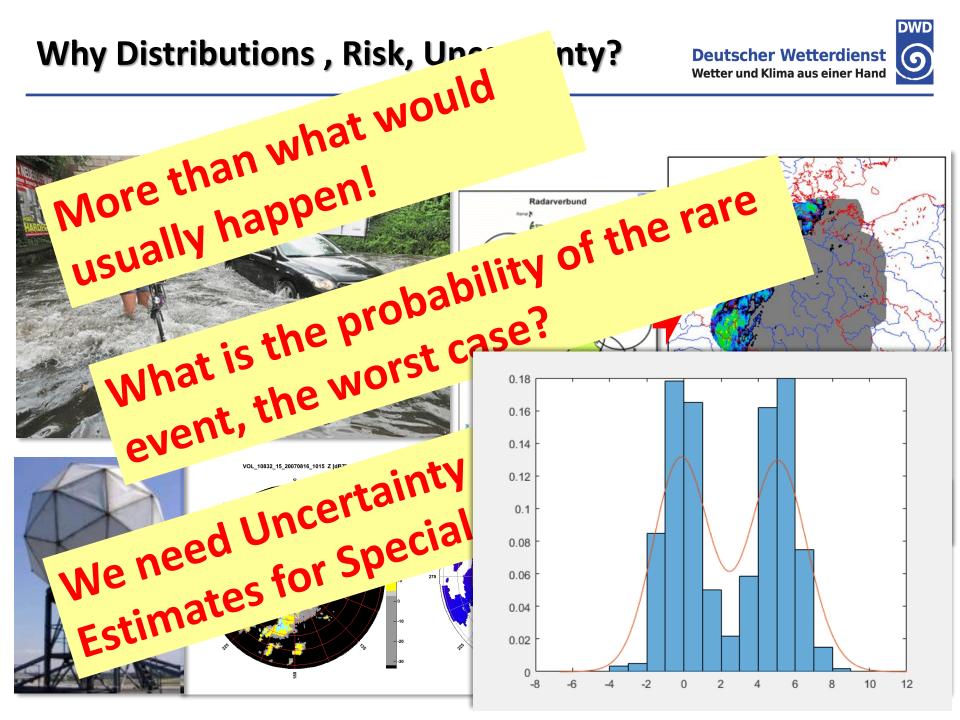
**4. LAPF & LMCPF Particle Filters** for Non-Gaussian Distributions – Details and Results

#### Why Distributions, Risk, Uncertainty?

**Deutscher Wetterdienst** Wetter und Klima aus einer Hand







#### Why Distributions, Risk, Uncertainty?

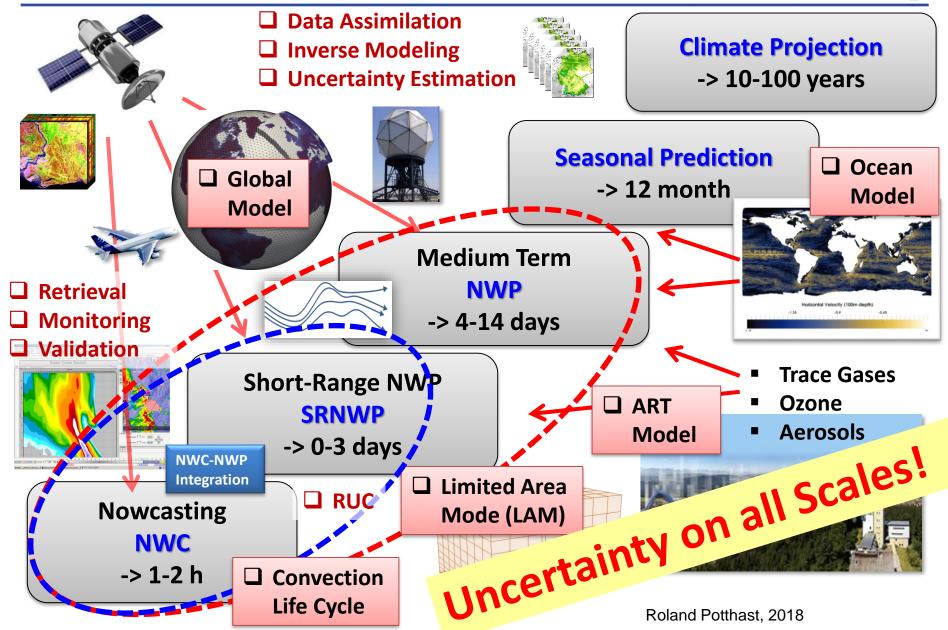
**Deutscher Wetterdienst** Wetter und Klima aus einer Hand

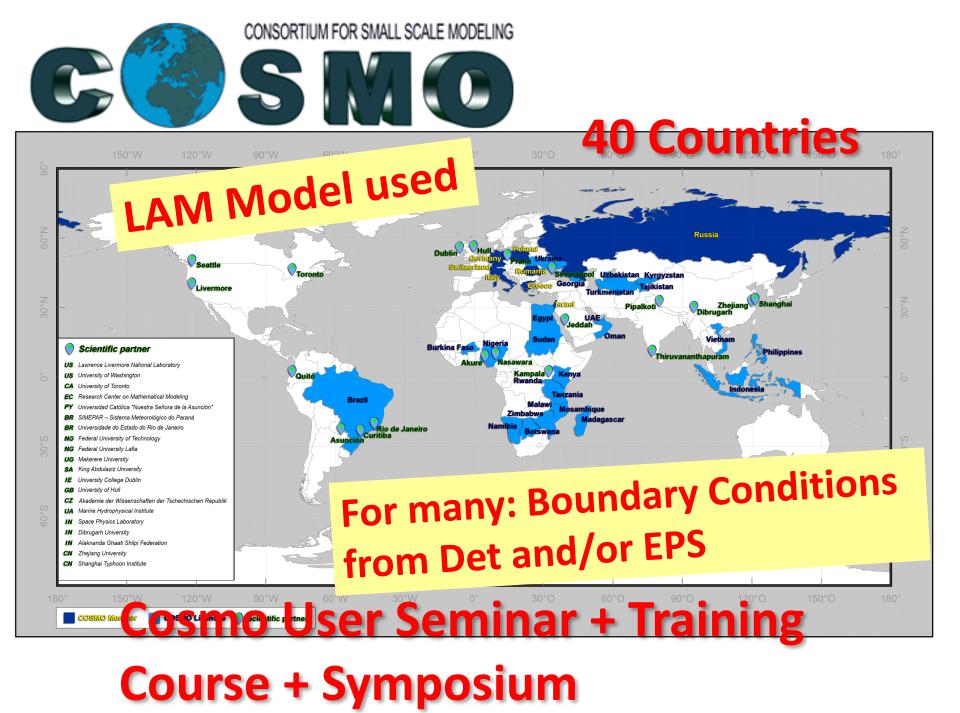




#### **Framework Numerical Weather Prediction**











Deutscher Wetterdienst Wetter und Klima aus einer Hand



1. Why and Where **Distributions, Risk and Uncertainty?** 

# 2. Discussion of **Ensemble (+Particle) Methods**

3. Framework Global+LAM+LES Model: ICON and

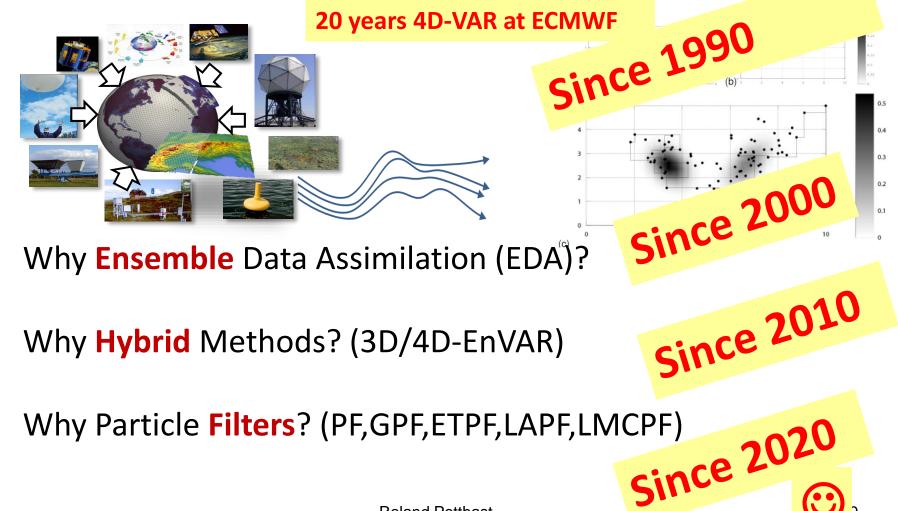
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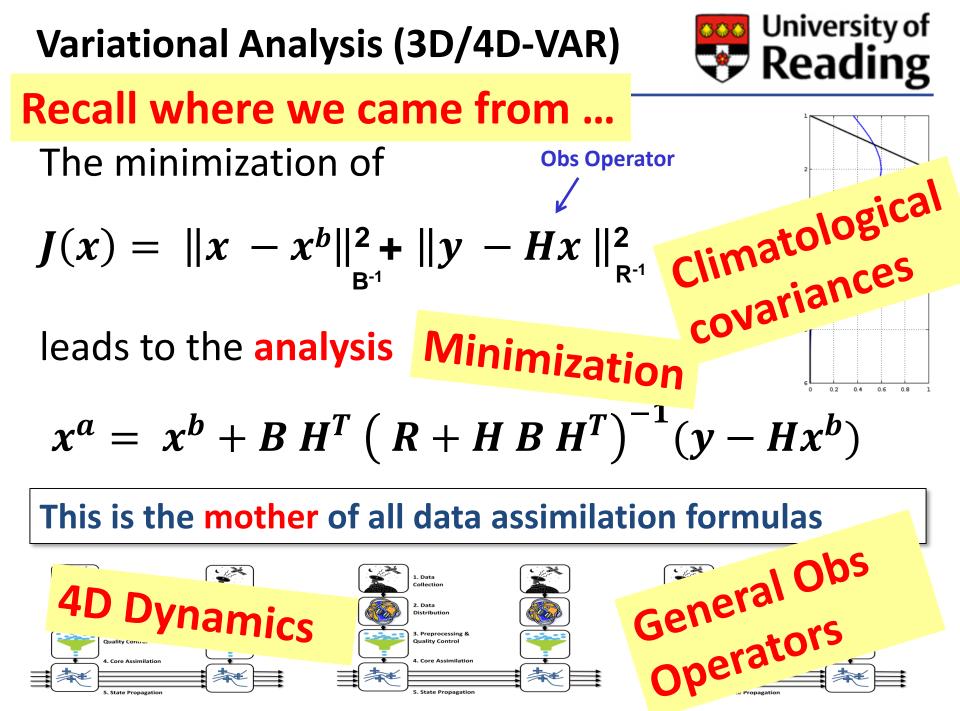
# **Data Assimilation Methods**



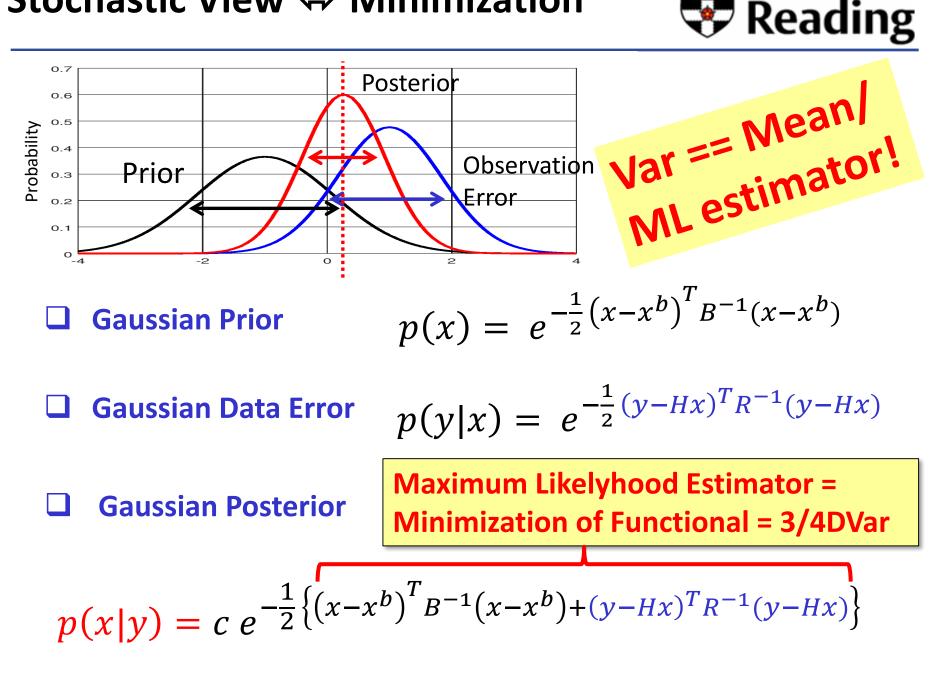
Why variational Data Assimilation (3D/4D-VAR)?



**Roland Potthast** 

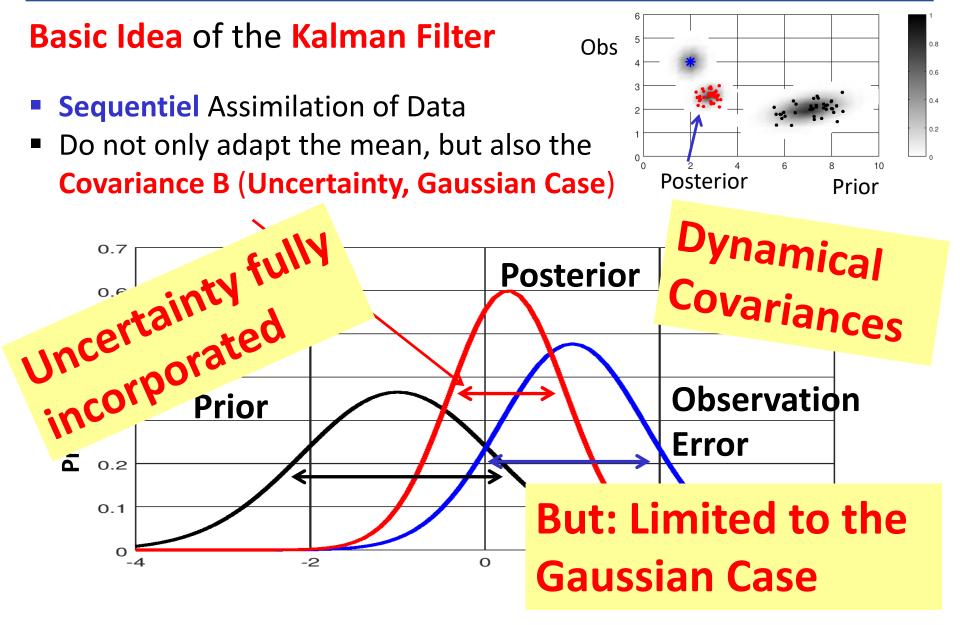


## Stochastic View 🗇 Minimization



University of

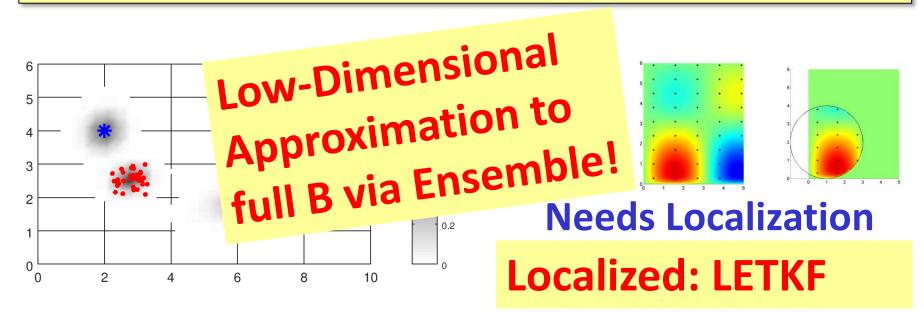


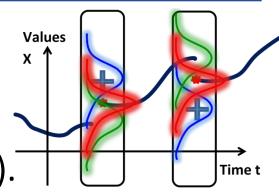


# EDA: Ensemble Kalman Filter (EnKF)

- Kalman Filter needs B update => expensive!
- Estimate B based on an ensemble of forecasted states (stochastic estimator).

B will be **flow-dependent** and variable, depending on the **model dynamics** and on the **observations** 















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#### **Global NWP Modelling: Det + EPS**

**Deutscher Wetterdienst** Wetter und Klima aus einer Hand

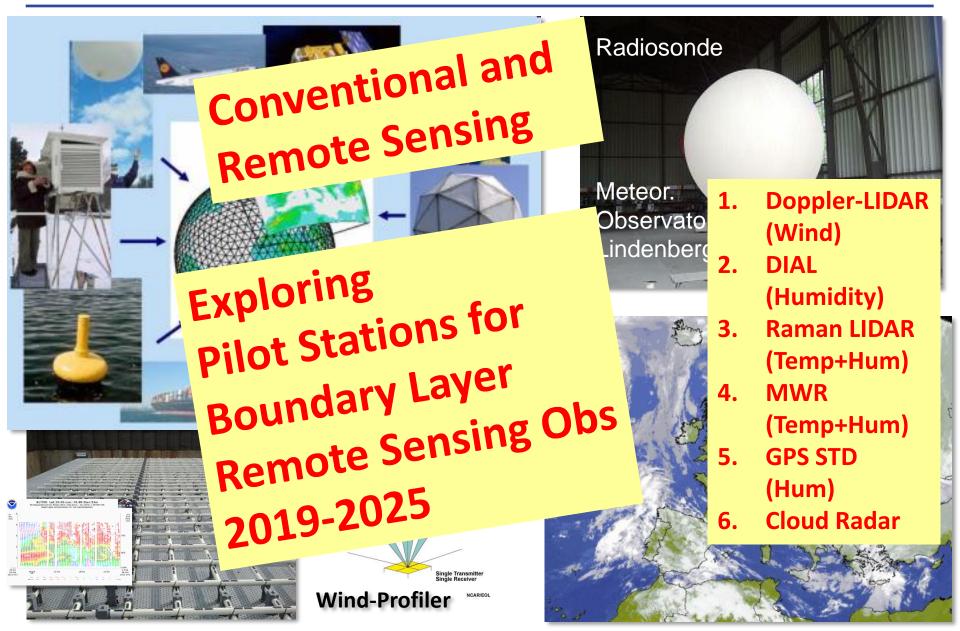


ICON<sup>®</sup> Model 13km 1 Nest<sup>53878</sup>over Europe<sup>10</sup> 2 (6.5km; 2-way) Nest over Germany (1km; 2-way) D1 **NWC Ensemble 3h cycles global+Europe: 180h fc**  $\frac{\partial v_n}{\partial t}$ 1h ana cycle LAM, 3h fc cycle: 27h fc  $\frac{\partial w}{\partial t}$  $\frac{\partial \rho}{\partial t}$ RUC cycle 1h: 6-12h fc  $+\nabla \cdot (\vec{v} \rho \theta_{v}) = 0$ 

# **Full Observation System**

**Deutscher Wetterdienst** Wetter und Klima aus einer Hand



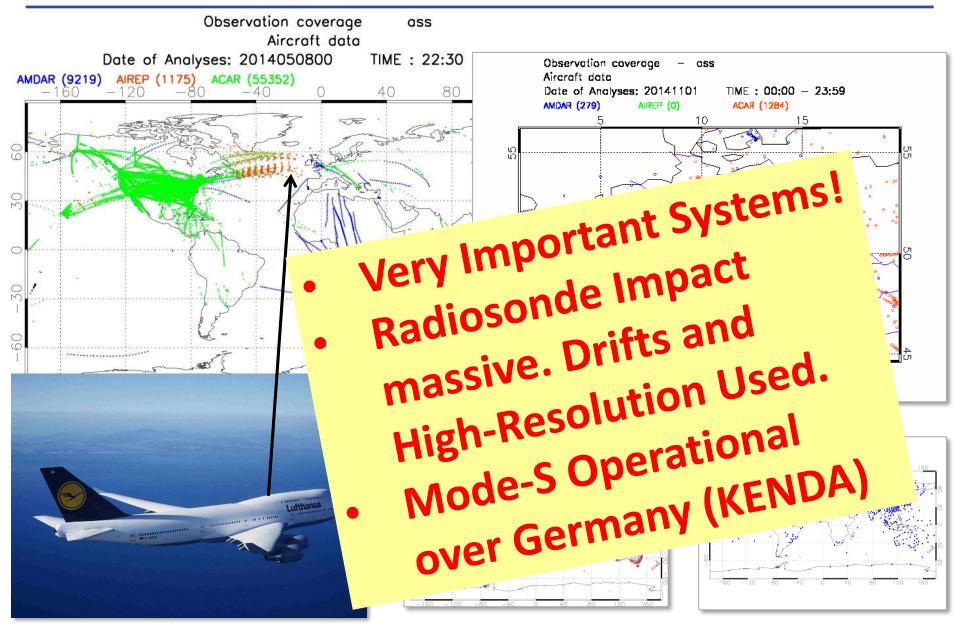


# **Conventional Synop + Airplanes**

**Deutscher Wetterdienst** 





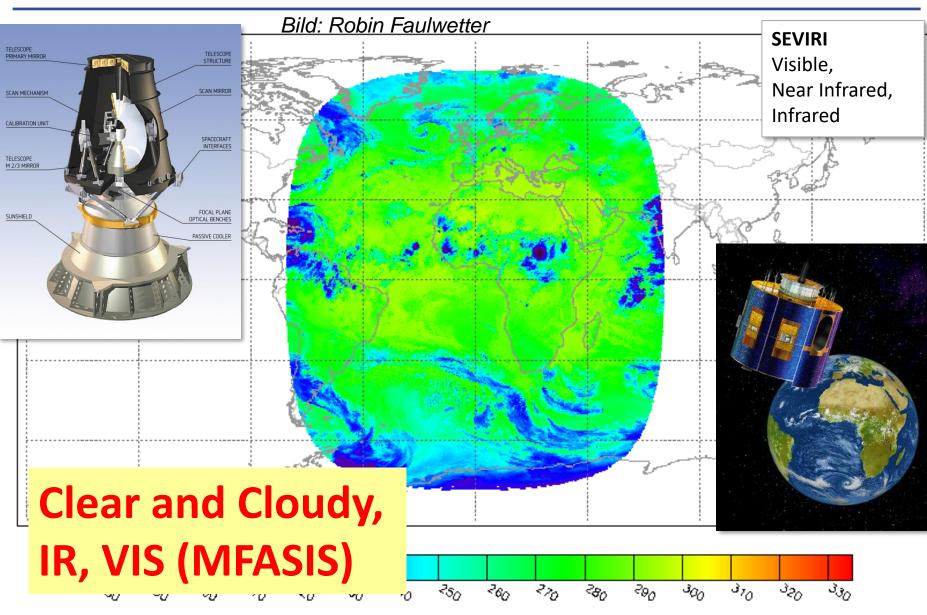


#### **Observations: Geostationary Satellites**

**Deutscher Wetterdienst** 



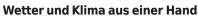
Wetter und Klima aus einer Hand

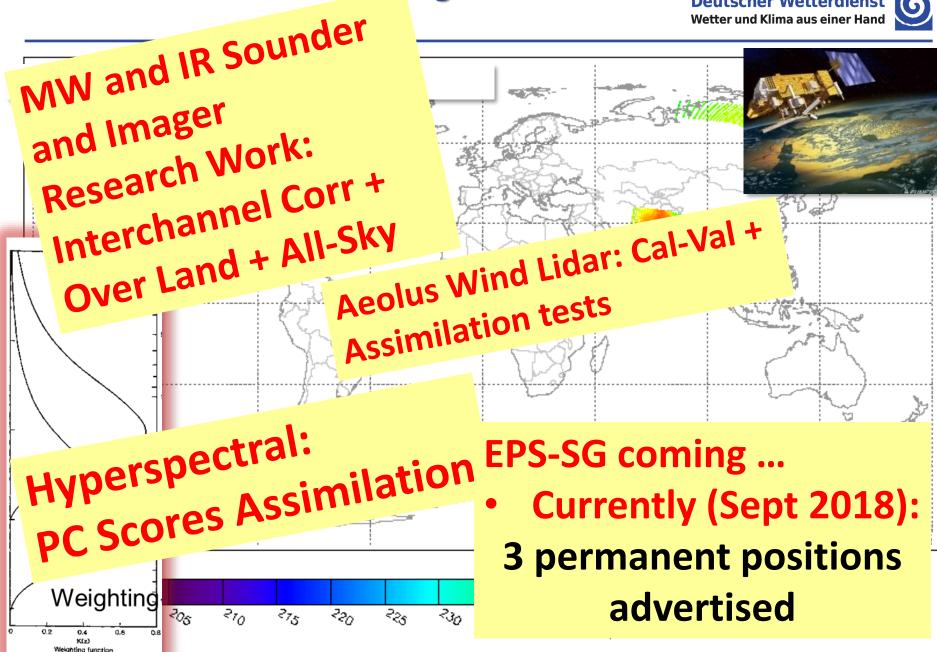


#### **Observations: Polar Orbiting Satellites**



DWD





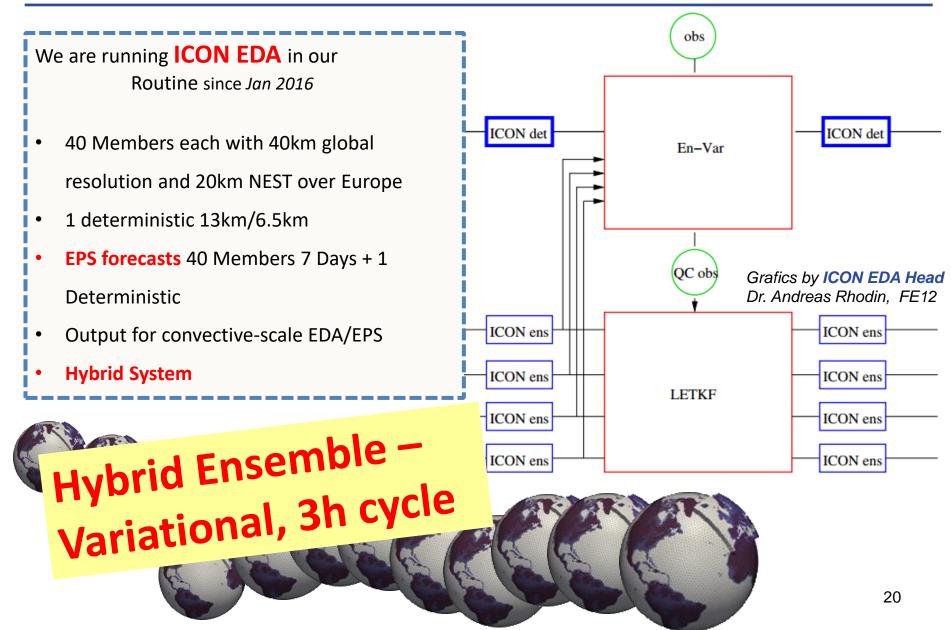
## **Ensemble Datenassimilation EnVar**

**Deutscher Wetterdienst** 

Wetter und Klima aus einer Hand



**Operational since January 2016** 

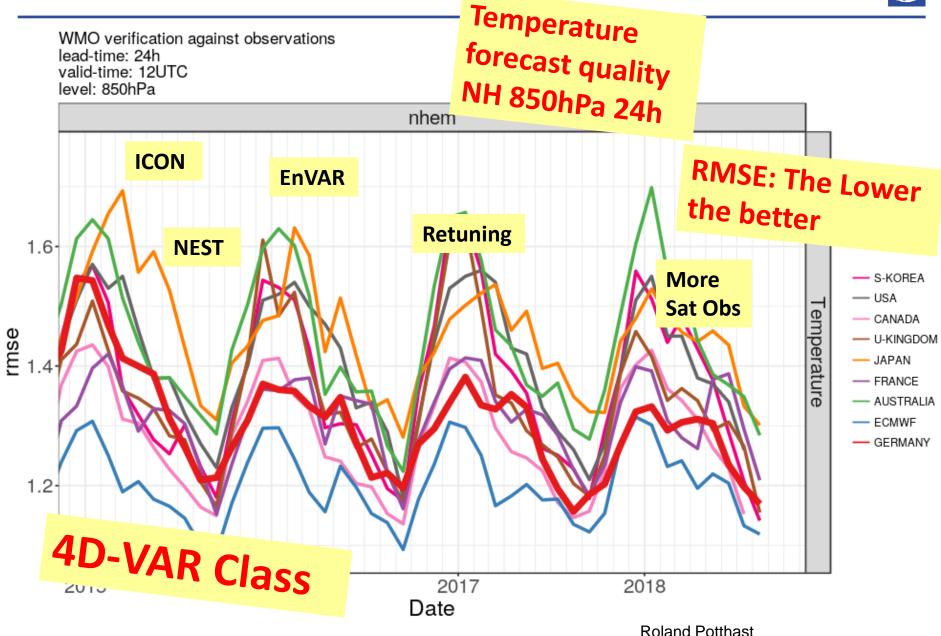


Deutscher Wetterdienst

Wetter und Klima aus einer Hand

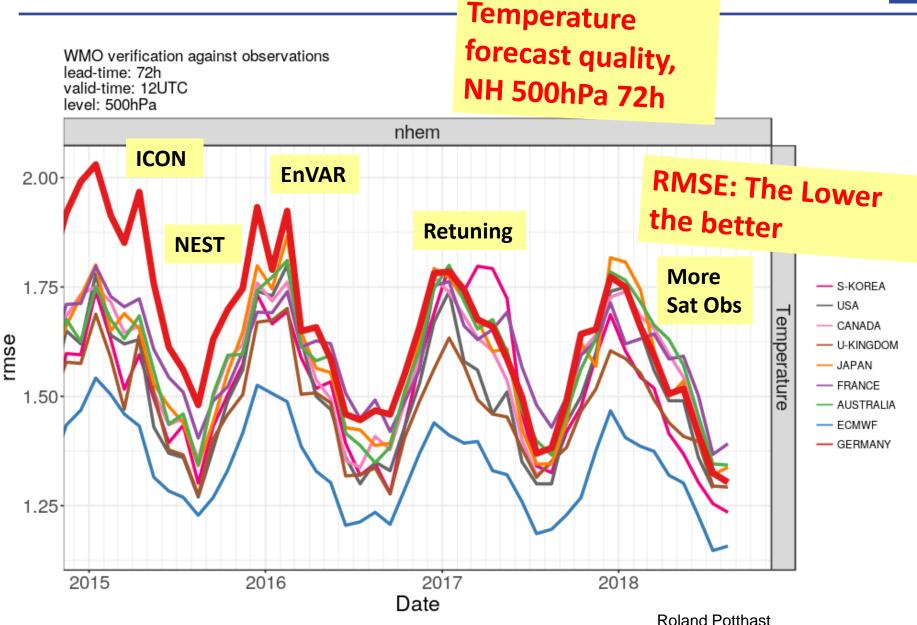
DWD

6



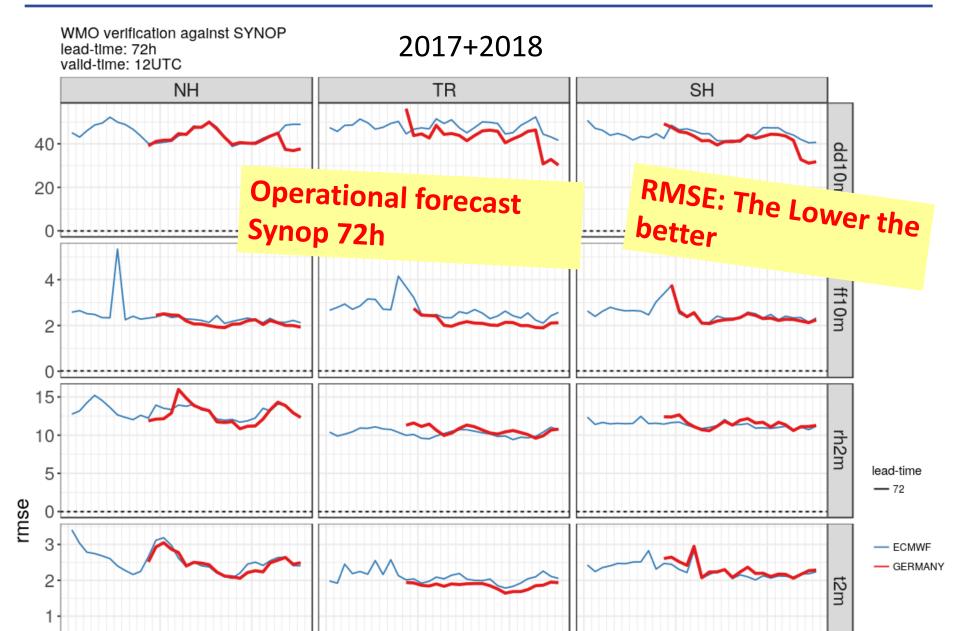
**Deutscher Wetterdienst** Wetter und Klima aus einer Hand



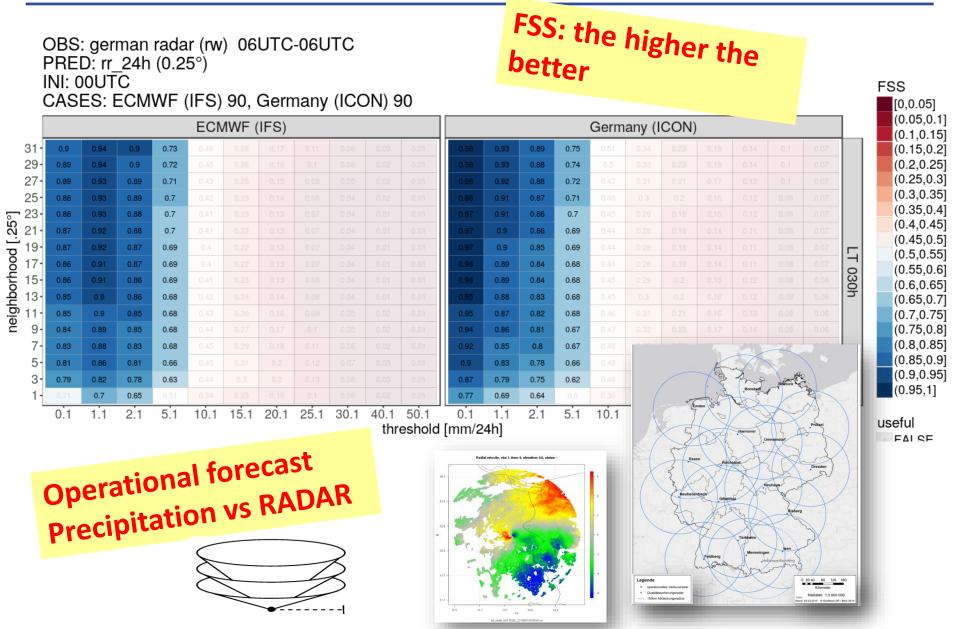


Deutscher Wetterdienst Wetter und Klima aus einer Hand













Deutscher Wetterdienst Wetter und Klima aus einer Hand



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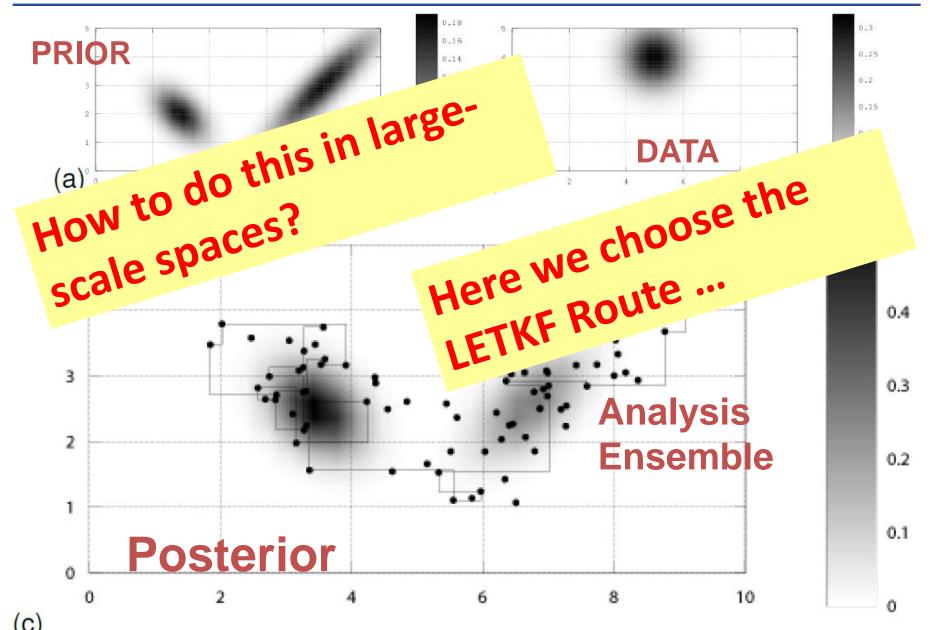
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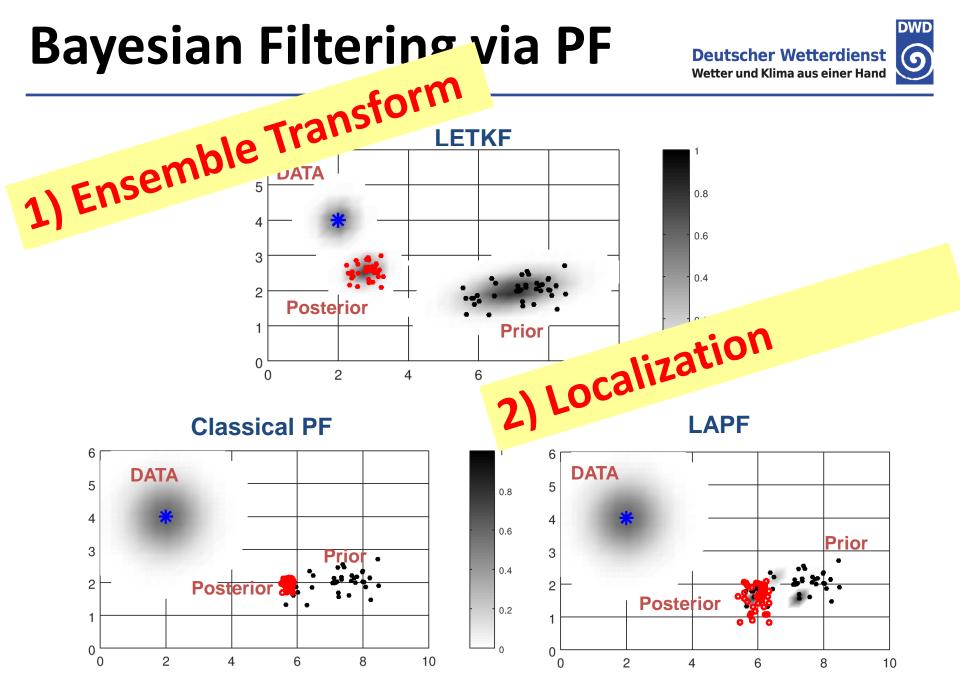
#### **BAYES Data Assimilation**

**Deutscher Wetterdienst** 

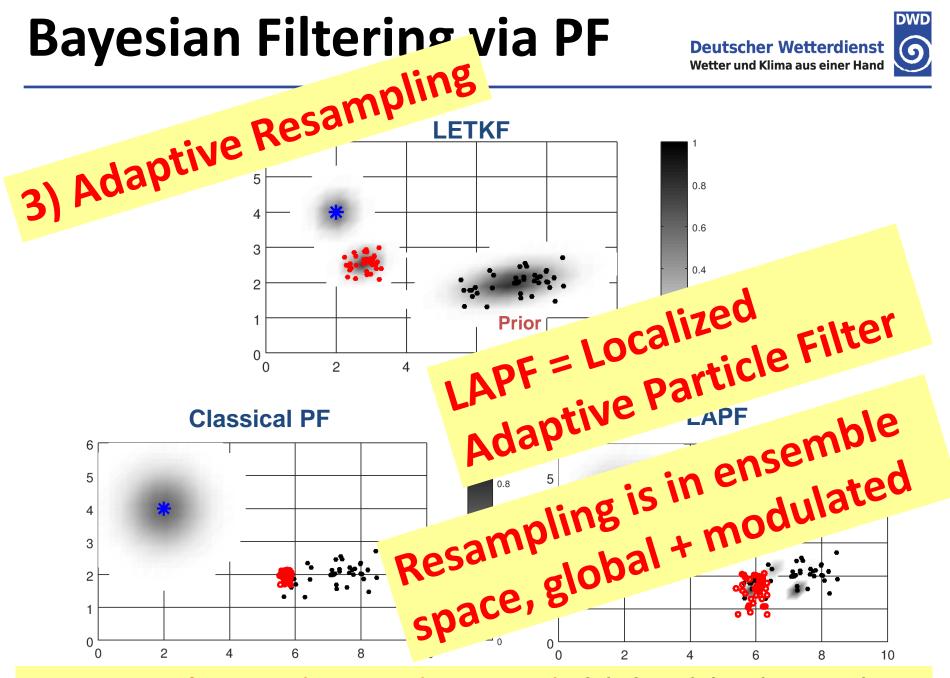
Wetter und Klima aus einer Hand







April 2018



LAPF = Transform, Localization, Adaptivity with global modulated Resampling



 Bayes formula to calculate new analysis distribution  $p_k^{(a)}(x) := p(x|y_k) = c \ p(y_k|x) p_k^{(b)}(x),$  $x \in \mathbb{R}^n$ 

c is a normalization factor:  $\int_{x} p_{k}^{(a)}(x) dx = 1$ 

# **Classical PF Approach**

• To carry out the analysis step at time  $t_k$ **aposteriori weights**  $p_k^{(a)}$  are calculated

$$p_{k,l}^{(a)} = c \ e^{-\frac{1}{2}(y - Hx^{(l)})^{T}R^{-1}(y - Hx^{(l)})}$$

c is chosen such that  $\sum_{l=1}^{L} p_{k,l}^{(a)} = L$ 

DWD

• Accumulated weights  $w_{ac}$  are defined:

$$w_{ac_0} = 0$$
  
 $w_{ac_i} = w_{ac_{i-1}} + p_i^a$ ,  $i = 1, ..., L$ 

where L denotes the ensemble size

• Drawing  $r_i \sim U([0,1]), j = 1, ..., L$ , set  $R_i = j - 1 + r_i$  and define transform matrix W for the particles by:

$$W_{i,j} = \begin{cases} 1 & if \ R_j \in (w_{ac_{i-1}}, w_{ac_i}], \\ 0 & otherwise, \end{cases}$$

i, j = 1, ..., L with  $W \in \mathbb{R}^{L \times L}$ , (s, t] denotes the interval of values  $s < \eta \leq t$ . Resampling



# Adaptivity based on o-b statistics

 Based on the adaptive multiplicative inflation **factor** *p* determined by the LETKF

$$\rho = \frac{\mathrm{E}\left[\boldsymbol{d}_{o-b}^{T}\boldsymbol{d}_{o-b}\right] - \mathrm{Tr}(\mathbf{R})}{Tr(\boldsymbol{H}\boldsymbol{P}^{b}\boldsymbol{H}^{T})}$$

• Weighting factor  $\alpha$  has been chosen, due to the small ensemble size (L = 40)

$$\rho_k = \alpha \tilde{\rho}_k + (1 - \alpha) \rho_{k-1}$$



• Pertubation factor  $\sigma$  is used to add spread to the system

$$\sigma = \begin{cases} c_0, & \rho < \rho^{(0)} \\ c_0 + (c_1 - c_0) * \frac{\rho - \rho^{(0)}}{\rho^{(1)} - \rho^{(0)}}, & \rho^{(0)} \le \rho \le \rho^{(1)} \\ c_1, & \rho > \rho^{(1)} \end{cases}$$

where  $c_0 = 0.02$ ,  $c_1 = 0.2$ ,

$$\rho^{(0)} = 1.0 \text{ and } \rho^{(1)} = 1.4, \text{ with}$$
  

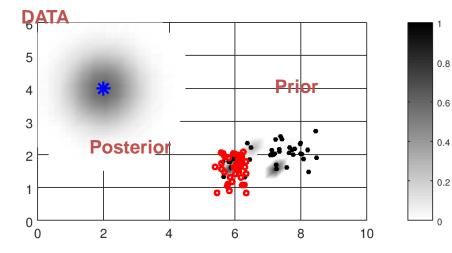
$$\sigma = c_1 \text{ if } \rho \ge \rho^{(1)} \text{ and}$$
  

$$\sigma = c_0 \text{ if } \rho \le \rho^{(0)}$$
  
Enforce the desired spread!

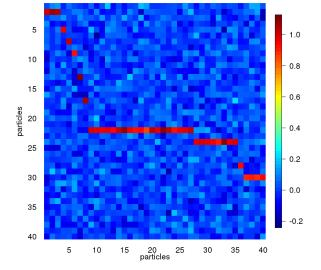
# Weights W are modified by applying the pertubation factor σ

$$W = W + R_{nd} * \sigma$$

with  $R_{nd}$  normally distributed random numbers



#### **Fourth Step: Gaussian Resampling**



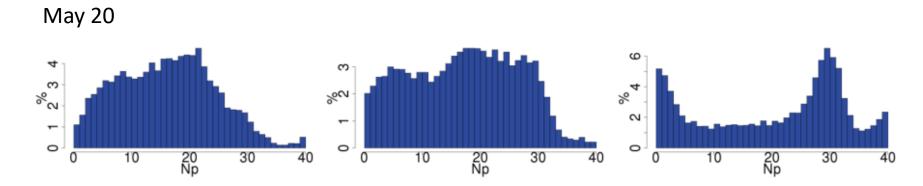
An example for a W-Matrix after applying  $\sigma$ determined with for 60% **Enforce the Enforce the a**re chosen

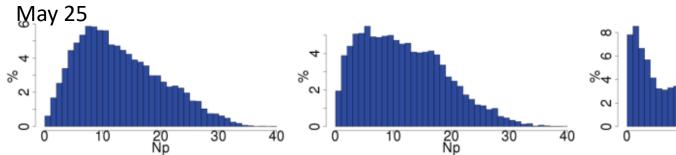


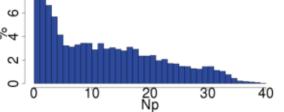
## Effective Ensemble Size Distributions Deutscher Wetterdienst

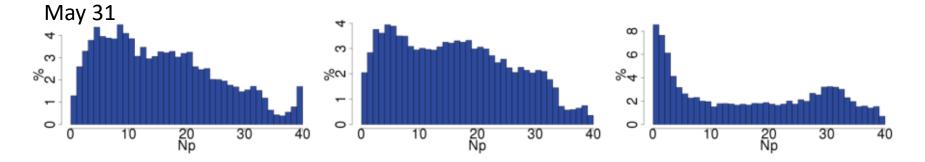
Wetter und Klima aus einer Hand







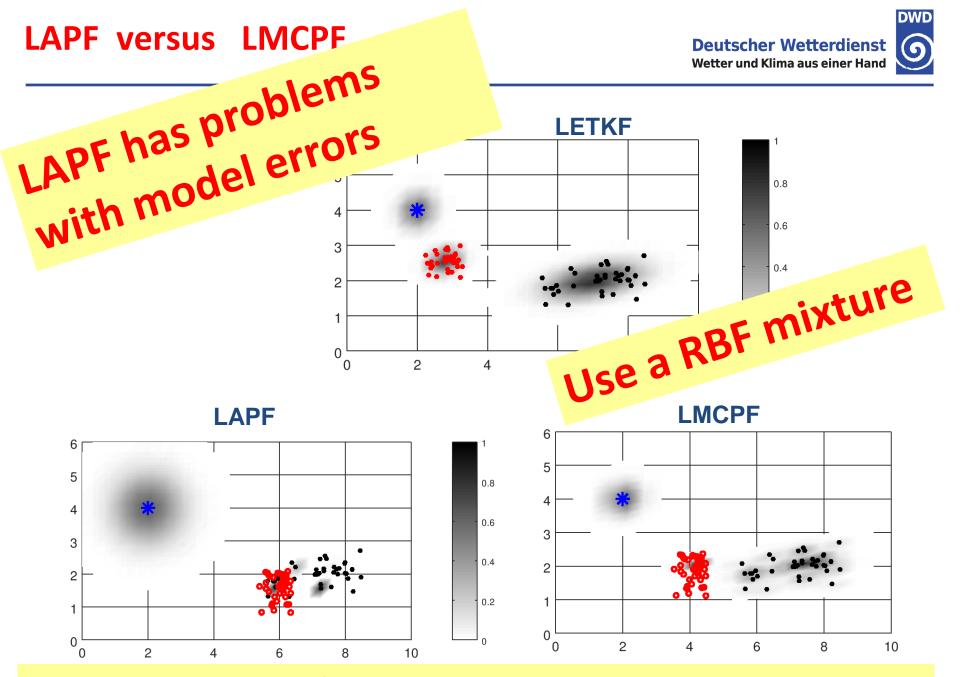




100 hPa

500 hPa

1000 hPa



LMCPF = Transform, Localization, RBF mixture, Adaptivity

**LMCPF** Basics

6 **Deutscher Wetterdienst** Wetter und Klima aus einer Hand



**Kalman Filter** 

$$x^{(a)} = x^{(b)} + BH^T (R + HBH^T)^{-1} (y - Hx^{(b)})$$

 $K = BH^T (R + HBH^T)^{-1} \qquad \tilde{B} = (I - KH)B$ 

**Ensemble B Estimator** 

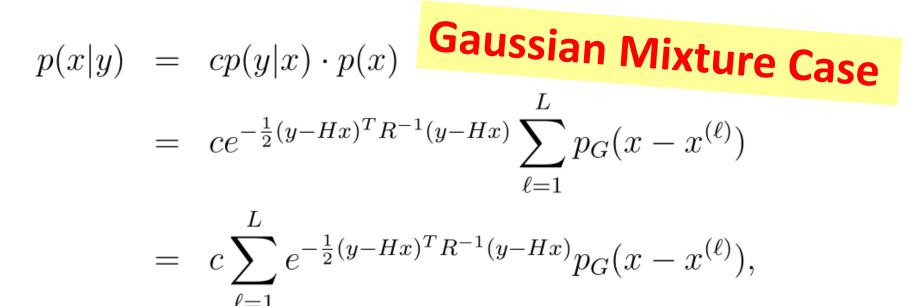
$$\bar{x} := \frac{1}{L} \sum_{\ell=1}^{L} x^{(\ell)}$$

$$B = \frac{1}{L-1}XX^T$$

 $X = (x^{(1)} - \bar{x}, ..., x^{(L)} - \bar{x}) \in \mathbb{R}^{n \times L}.$ 



$$\begin{split} \tilde{B} &= (I - KH)B \qquad \qquad Y := HX \\ &= (I - BH^{T}(R + HBH^{T})^{-1}H)B \\ &= (I - \gamma X X^{T}H^{T}(R + \gamma HX X^{T}H^{T})^{-1}H)\gamma XX^{T} \\ &= X (I - \gamma Y^{T}(R + \gamma YY^{T})^{-1}Y)\gamma X^{T} \\ &= X (I - \gamma (I + \gamma Y^{T}R^{-1}Y)^{-1}Y^{T}R^{-1}Y)\gamma X^{T} \\ &= X ((I + \gamma Y^{T}R^{-1}Y)^{-1}(I + \gamma Y^{T}R^{-1}Y) - \gamma Y^{T}R^{-1}Y)\gamma X^{T} \\ &= X (I + \gamma Y^{T}R^{-1}Y)^{-1}\gamma X^{T} \\ &= X (I + \gamma Y^{T}R^{-1}Y)^{-1}\gamma X^{T} \\ &= X (\frac{1}{\gamma}I + Y^{T}R^{-1}Y)^{-1}X^{T} \\ &= X (\frac{1}{\gamma}I + Y^{T}R^{-1}Y)^{-1}Y^{T}R^{-1} \end{split}$$
**RBF Basis Function in Ensemble Space**



$$p_G(x - x^{(\ell)}) = \tilde{c}e^{-\frac{1}{2}(x - x^{(\ell)})^T G^{-1}(x - x^{(\ell)})}$$

Explicit Calculations possible for each term We need a selection based on relative weights!



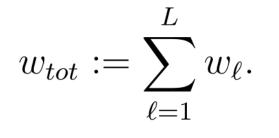
$$w_{\ell} := e^{-\frac{1}{2}(y - Hx^{(\ell)})^T R^{-1}(y - Hx^{(\ell)})}, \quad \ell = 1, \dots, L$$

0.8

0.6

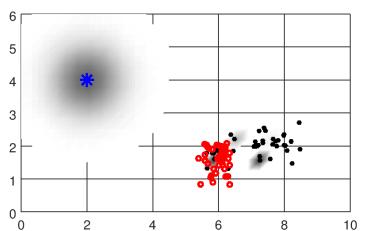
0.4

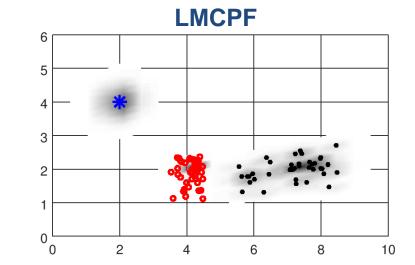
0.2













### **Projection onto Ensemble Space**

Abbreviating  $A := \mathbf{Y}^T \mathbf{R}^{-1} \mathbf{Y}$  and  $C := A^{-1} \mathbf{Y}^T \mathbf{R}^{-1} (\mathbf{y}^o - \overline{\mathbf{y}}^b)$ 

**Projection Operator** 

$$P(\mathbf{y}^o - \overline{\mathbf{y}}^b) = \mathbf{Y}(\mathbf{Y}^T \mathbf{R}^{-1} \mathbf{Y})^{-1} \mathbf{Y}^T \mathbf{R}^{-1} (\mathbf{y}^o - \overline{\mathbf{y}}^b),$$

**Projected discrepancy** 

Exponent

$$P(\mathbf{y}^o - H\mathbf{x}^{(\ell)}) = \mathbf{Y}A^{-1}\mathbf{Y}^T\mathbf{R}^{-1}((\mathbf{y}^o - \overline{\mathbf{y}}^b) - \mathbf{Y}e_\ell)$$

$$=$$
 **Y**(*C* - *e* <sub>$\ell$</sub> ),  $\ell = 1, ..., L$ .

$$P(\mathbf{y}^{o} - H\mathbf{x}^{(\ell)})]^{T}\mathbf{R}^{-1}P(\mathbf{y}^{o} - H\mathbf{x}^{(\ell)}) = [C - e_{\ell}]^{T}A[C - e_{\ell}], \ \ell = 1, ..., L,$$

# Weight $w_{k,\ell} = c e^{-\frac{1}{2}[C-e_\ell]^T A[C-e_\ell]}, \ \ell = 1,...,L.$



### **Classical versus projected weights**

$$\begin{split} w_{k,\ell}^{classical} &= e^{-\frac{1}{2}[(\mathbf{y}^{o}-H\mathbf{x}^{(\ell)})]^{T}\mathbf{R}^{-1}[(\mathbf{y}^{o}-H\mathbf{x}^{(\ell)})]} \\ &= e^{-\frac{1}{2}[\left(P+(I-P)\right)(\mathbf{y}^{o}-H\mathbf{x}^{(\ell)})]^{T}\mathbf{R}^{-1}[\left(P+(I-P)\right)(\mathbf{y}^{o}-H\mathbf{x}^{(\ell)})]} \\ &= e^{-\frac{1}{2}[P(\mathbf{y}^{o}-H\mathbf{x}^{(\ell)})]^{T}\mathbf{R}^{-1}[P(\mathbf{y}^{o}-H\mathbf{x}^{(\ell)})]} \cdot \underbrace{e^{-\frac{1}{2}[(I-P)(\mathbf{y}^{o}-H\mathbf{x}^{(\ell)})]^{T}\mathbf{R}^{-1}[(I-P)(\mathbf{y}^{o}-H\mathbf{x}^{(\ell)})]}_{=\tilde{c}}}_{=\tilde{c}}^{\tilde{c}} \end{split}$$
Factor is a constant term, since we have

$$(I-P)(\mathbf{y}^o - Hx^{(\ell)}) = (I-P)(\mathbf{y}^o - \overline{\mathbf{y}}^b + \mathbf{Y}e_\ell)$$
$$= (I-P)(\mathbf{y}^o - \overline{\mathbf{y}}^b) - \underbrace{(I-P)\mathbf{Y}e_\ell}_{-0}.$$

Projected particle filter weights and classical particle filter weights are <u>equivalent theoretically</u>, but

numerically remove a very small common factor

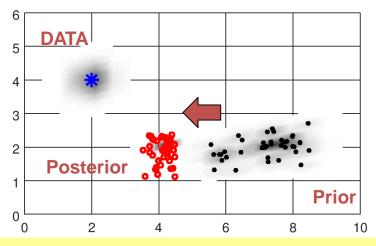
### LMCPF = Local Markov Chain Particle Filter

 Weights W are calculated by drawing from the posterior

 $W = W + A_{shift} * W + B_{post} * R_{nd} * \sigma$ 

with  $R_{nd}$  normally distributed random numbers,

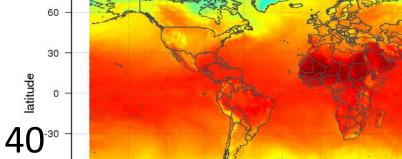
 $A_{shift}$  and  $B_{post}$  calculated with Gaussian radial basis function (rbf) Approximation for prior density and observation error



 It is an explicit calculation of the Bayes posterior based on radial basis function approximation of the prior, with subsequent draws from that distribution in the MCMC sense.

### Large-Scale Experimental Set-up

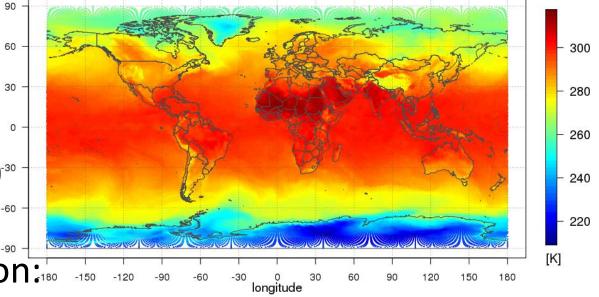




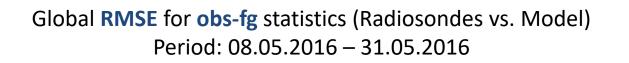
- Full ensemble: 40<sup>-30</sup> members
- Reduced resolution:
  - 26km deterministic
  - 52km ensembles
- Period: 01.05.2016 -31.05.2016

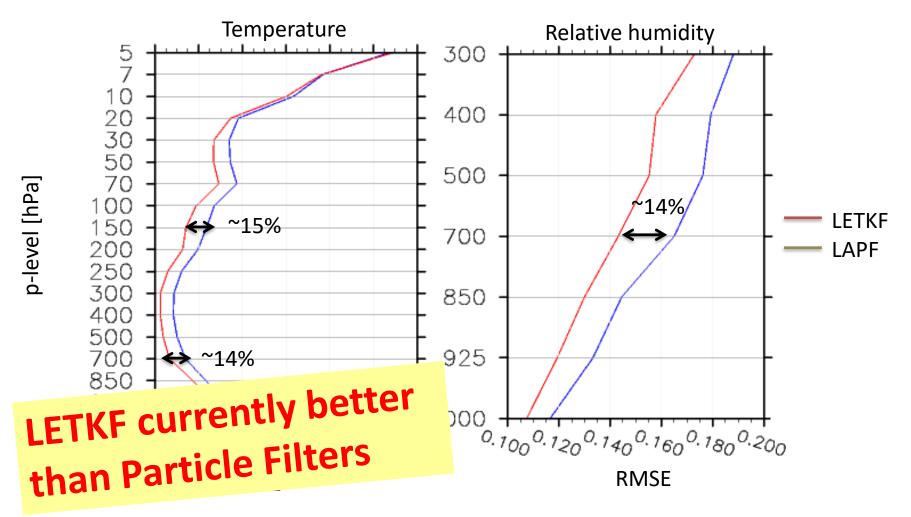
**Experiments programmed and carried out** by Anne Walter, DWD& Uni Reading, and **Roland Potthast, DWD& Uni Reading** 

In Cooperation with Peter-Jan van Leeuwen, Uni Reading



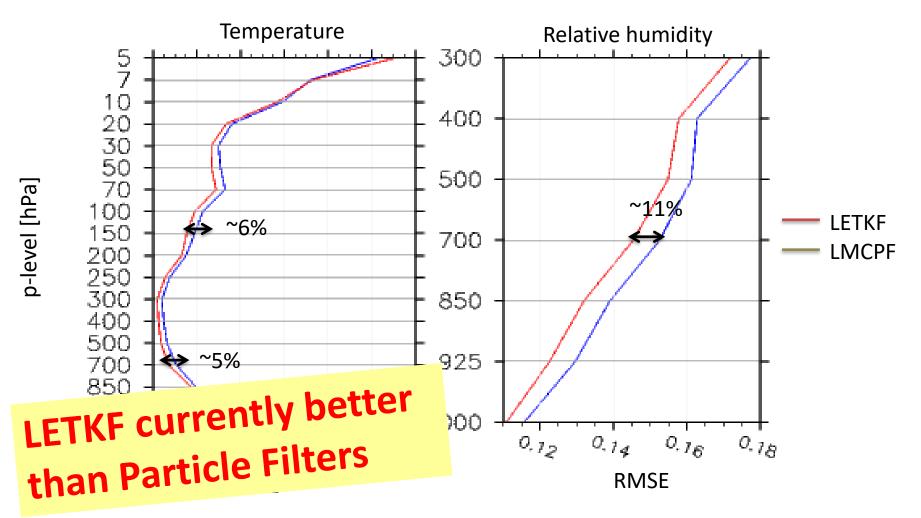






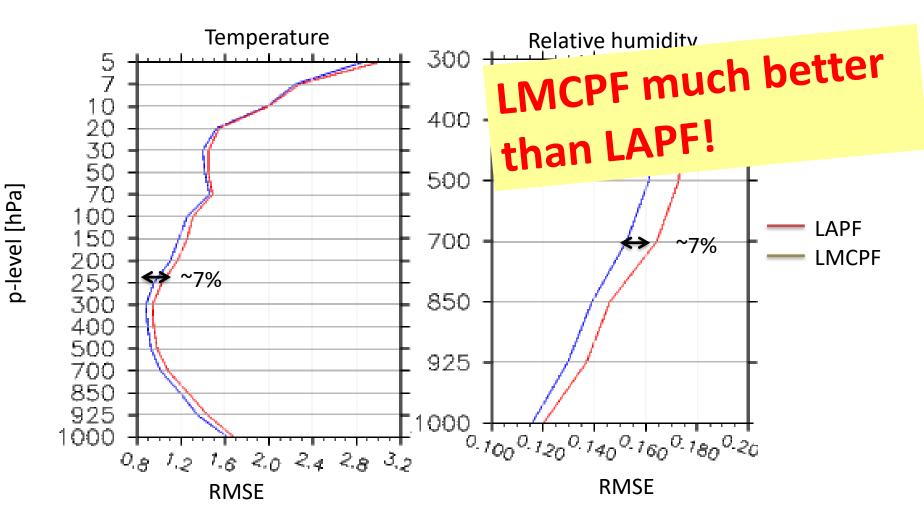


Global **RMSE** for **obs-fg** statistics (Radiosondes vs. Model) Period: 08.05.2016 – 22.05.2016





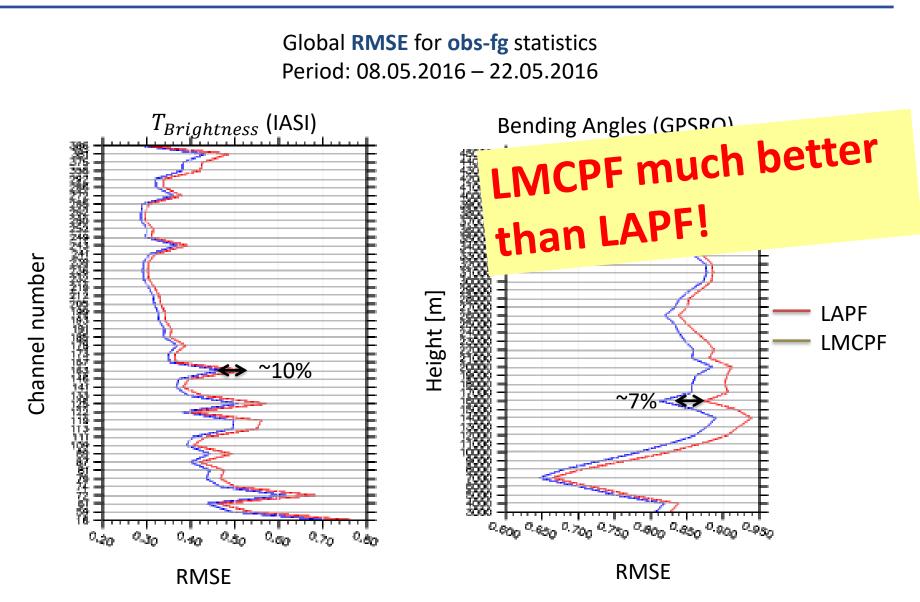
Global **RMSE** for **obs-fg** statistics (Radiosondes vs. Model) Period: 08.05.2016 – 22.05.2016



### **LMCPF Scores vs LAPF**

Deutscher Wetterdienst Wetter und Klima aus einer Hand



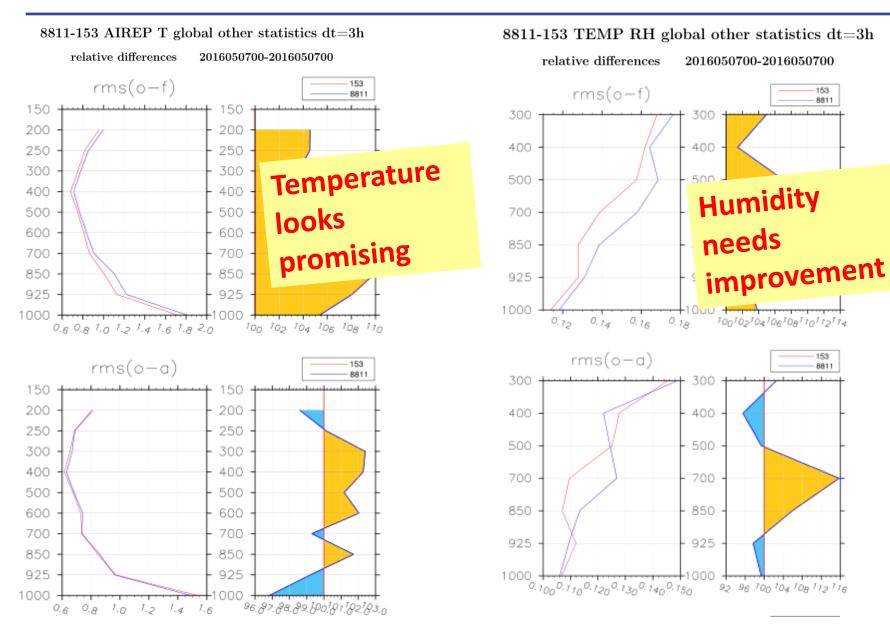


Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni Reading

### **New LAMCPF Scores vs LETKF**



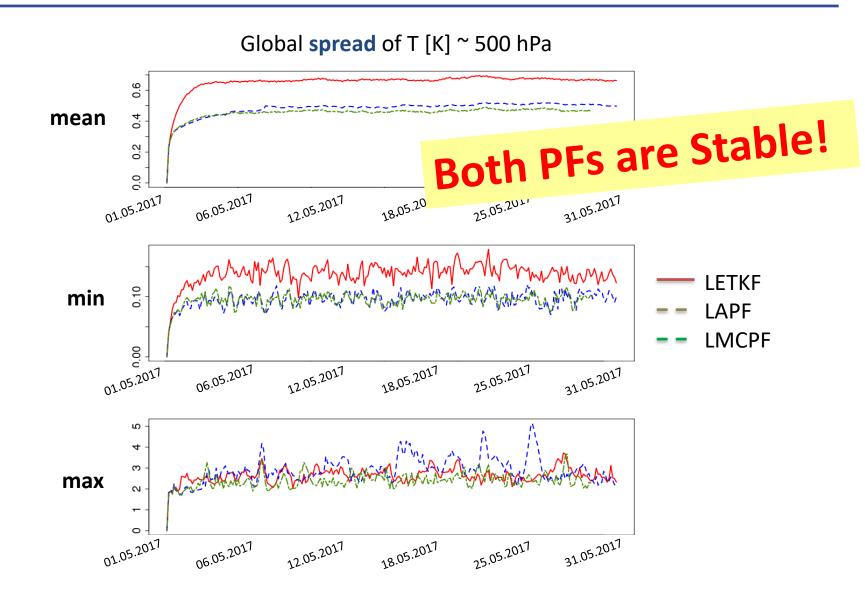




### LAPF Spread vs LMCPF & LETKF

**Deutscher Wetterdienst** Wetter und Klima aus einer Hand





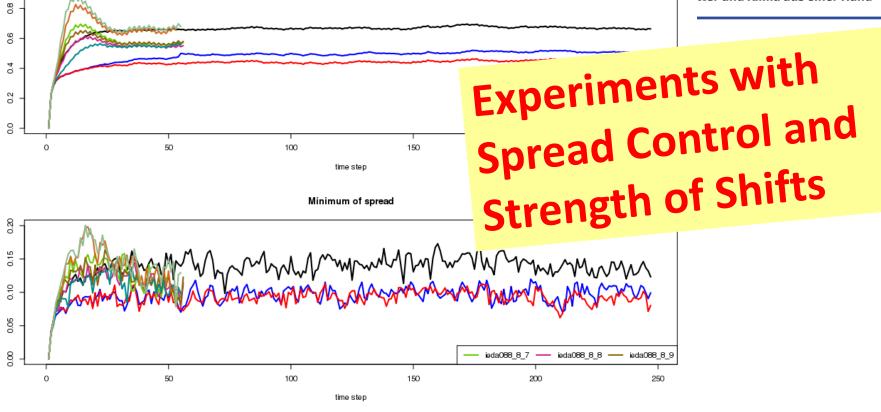
#### Statistics for spread at level 64 for variable T

Mean of spread

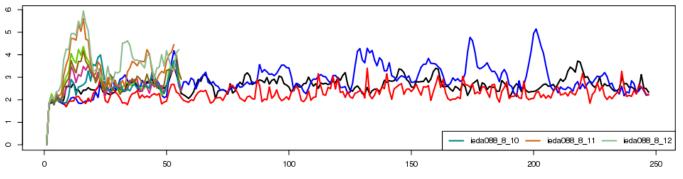
**utscher Wetterdienst** 



tter und Klima aus einer Hand



Maximum of spread



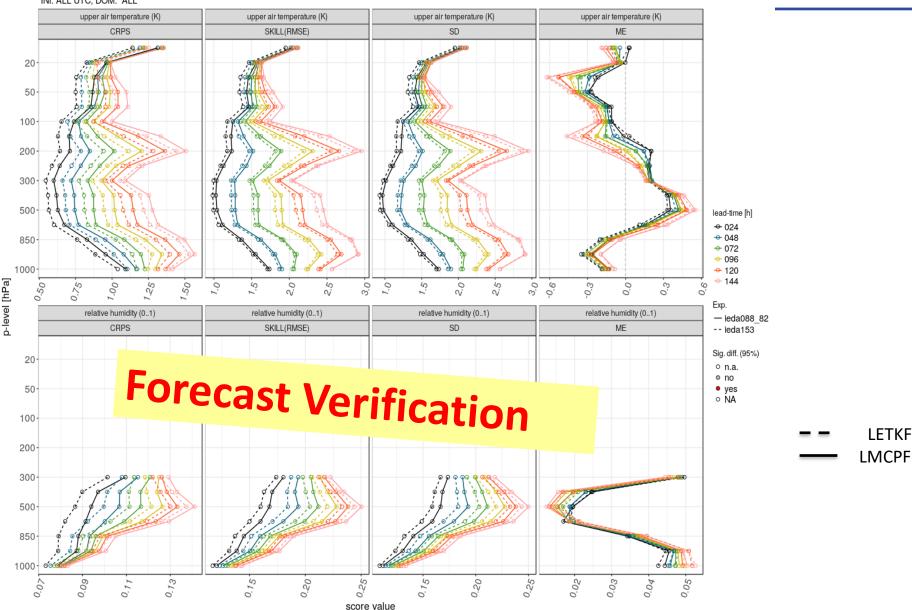
time step

### LMCPF Scores vs LETKF

**Deutscher Wetterdienst** 6 a aus einer Hand

DWD

LETKF



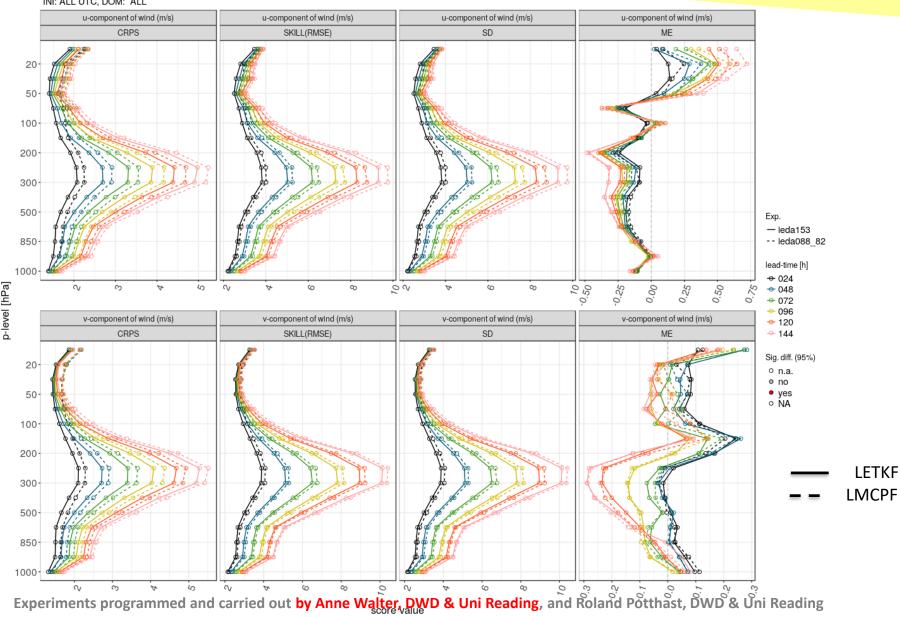
Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni Reading

2016/05/02 - 2016/05/24 INI: ALL UTC, DOM: ALL

### **LMCPF Scores vs LETKF**

## **Forecast Verification**

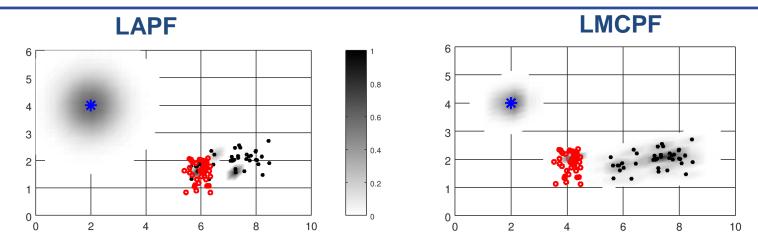
2016/05/02 - 2016/05/24 INI: ALL UTC, DOM: ALL



### Summary LAPF and LMCPF

Deutscher Wetterdienst Wetter und Klima aus einer Hand





- LAPF and LMCPF are implemented in an operational NWP system: Globally + mesoscale, convective scale
- Both Particle Filters are able to provide reasonable atmospheric analysis in a large-scale (high-dimensional) environment and are running stably over a period of one month
- The LMCPF outperforms the LAPF but not yet the LETKF, but both Particle Filters are not far behind the operational LETKF

Both Particle Filters are showing promising results; further tuning and development is in progress.

### Many Thanks!



### **Inverse Modeling**

An introduction to the theory and methods of inverse problems and data assimilation

Gen Nakamura Roland Potthast

