Utilizing Gauss-Legendre Quadrature for Computation of Radiative Fluxes in Atmospheric Models

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Environnement et Changement Climatique Canada
• GCMs RT models (Li and Barker 2018):
  - sort stochastic CWP... low-order GLQ... SW and LW operate on common atmos.
  - reduce noise for NET fluxes... boundary fluxes OK; HRs not so much
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- GCMs RT models (Li and Barker 2018):
  - sort stochastic CWP... low-order GLQ... SW and LW operate on common atmos.
  - reduce noise for NET fluxes... boundary fluxes OK; HRs not so much

- State-of-the-art LESs, CSRMs, NWP models run routinely using:
  - domain sizes...... 50 - 1,000 km
  - grid-spacings..... 0.1 - 1.0 km... 10⁵ - 10⁷ columns
  - N layers............ 64 - 256
  - 1D RT in ICA....... yes
  - RT timesteps...... every 15 - 30 dynamics timesteps
  - RT of total CPU... 15 - 35%

- an attempt at a catastrophic reduction of CPU time consumed by RT algorithms

(per. comm. S. Krueger, J. Manners, P. Vaillancourt, 2018)
Dealing with RT’s CPU demand

1. full ICA... but only every $N$ dynamics timesteps (probably the most common?)
2. full ICA... as in (1) but intermediate steps using CRE bands only (Manners et al.)
3. ICA... employing stochastic spectral sampling (Pincus et al.)

Goal:

*Full-resolution* (time, space, spectra) $Q_{rad}$ using much less CPU time than the above methods and resulting in simulated (cloud) properties that differ insignificantly from those obtained with the full-ICA.

• Towards the modelling “chasm” as described by Lawrence et al. (2018)
• NB. still adhering to the 1D-ICA paradigm...
Something to bear in mind...

Is moving to 3D RT considered to be intractable or unwarranted?

**SW surface irradiance**

1D

3D

$153.6 \text{ km}$

$15.62 \text{ W m}^{-2}$

$\mu_0 = 0.5$
Something to bear in mind...

Is moving to 3D RT considered to be intractable or unwarranted?

If yes, then the 1D RT errors to be shown have to be considered almost negligible!

\[ \mu_0 = 0.5 \]

153.6 km

SW surface irradiance

1D

3D

15.62 W m\(^{-2}\)

0.09 W m\(^{-2}\)
1. Partitioning a domain’s columns

Partition into sub-domains such that radiative flux profiles are “distinctive”

cf. K-means

** $\text{max}(Q_{rad})$ near cloudtops exposed-to-space

$\Delta x = 0.25 \text{ km}$
2. Sort columns within partitions

Partition into sub-domains such that radiative flux profiles are “distinctive”

cf. K-means

** max($Q_{\text{rad}}$) near cloudtops exposed-to-space

\[ \Delta x = 0.25 \text{ km} \]

[Image of cloud map with color legend:
- 4.6% 0: cloudless
- 34.7% 1: cloudtops > 10 km
- 13.0% 2: 5 km < cloudtops < 10 km (liq + ice)
- 0.2% 3: 5 km < cloudtops < 10 km (ice-only)
- 14.9% 4: 5 km < cloudtops < 10 km (liq-only)
- 32.6% 5: cloudtops < 5 km

\[
\langle F \rangle = \frac{1}{N} \sum_{n=1}^{N} F(n) \quad \text{full ICA}
\]

\[
= \frac{\sum_{m=1}^{M} N_s(m) \left[ \frac{1}{N_s(m)} \sum_{n=1}^{N_s(m)} F_m(n) \right]}{\sum_{m=1}^{M} N_s(m)} = \frac{\sum_{m=1}^{M} N_s(m) \langle F_m \rangle}{\sum_{m=1}^{M} N_s(m)}
\]

partitioned into $M$ categories

for each partition, sort and index according to CWP

\[
s_{m,n} = \frac{n - 1}{N_s(m) - 1}; \quad n = 1, \ldots, N_s(m)
\]

\[
\langle F_m \rangle = \int_0^1 F(s_m) ds_m
\]
3. Apply GLQ to sorted partitions

Partition into sub-domains such that radiative flux profiles are “distinctive”

cf. K-means

** max($Q_{rad}$) near cloudtops exposed-to-space

$\Delta x = 0.25$ km

$\langle F \rangle = \frac{1}{N} \sum_{n=1}^{N} F(n)$ ← full ICA

$= \frac{1}{N} \sum_{m=1}^{M} N_s(m) \left[ \frac{1}{N_s(m)} \sum_{n=1}^{N_s(m)} F_m(n) \right] = \frac{\sum_{m=1}^{M} N_s(m) \langle F_m \rangle}{\sum_{m=1}^{M} N_s(m)}$

partitioned into $M$ categories

for each partition, sort and index according to CWP

$M \cdot n_G \ll N$ 1D RT apps. →

$\langle F_m \rangle = \int_0^1 F(s_m) ds_m$
2. Sort columns within partitions

\[
\frac{1}{N} \sum_{n=1}^{N} F_n = \frac{1}{N} \sum_{n=1}^{N} F_{m(n)} \equiv \int_0^1 F(s) \, ds \approx \sum_{n=1}^{n_G} w_n F_{\tilde{s}_n}
\]

full ICA (benchmark) \hspace{2cm} \text{sorted ICA} \hspace{2cm} \text{integral form} \hspace{2cm} n_G\text{-point GLQ McICA}

\[
m(1) \leftarrow \min \{W_n\} \quad s = 0 \leftarrow \min \{W_n\} \\
m(N) \leftarrow \max \{W_n\} \quad s = 1 \leftarrow \max \{W_n\}
\]

\[
s_i = \frac{i - 1}{N - 1}
\]
4. Associate and distribute $F(s_{m,n})$

Partition into sub-domains such that radiative flux profiles are “distinctive”

cf. K-means

** $\max(Q_{rad})$ near cloudtops exposed-to-space

$\Delta x = 0.25 \text{ km}$

$\mathcal{M} \cdot n_{G} \ll \mathcal{N}$ 1D RT apps.

$\langle F \rangle = \frac{1}{N} \sum_{n=1}^{N} F(n)$ ← full ICA

$$
= \frac{1}{\mathcal{N}_{s}(m)} \sum_{m=1}^{\mathcal{M}} \mathcal{N}_{s}(m) \left[ \sum_{n=1}^{\mathcal{N}_{s}(m)} F_{m}(n) \right] = \frac{\mathcal{M}}{\sum_{m=1}^{\mathcal{M}} \mathcal{N}_{s}(m)} \sum_{m=1}^{\mathcal{M}} \mathcal{N}_{s}(m) \langle F_{m} \rangle
$$

for each partition, sort and index according to CWP

$\mathcal{M} \cdot n_{G} \ll \mathcal{N}$ 1D RT apps.

$w_{m,n}$

$$
\begin{cases}
  n = 1 & \frac{1}{2} \mathcal{N}_{s}(m) \\
  n = 2 & \frac{s_{1} + s_{2}}{2} \mathcal{N}_{s}(m) + 1, \quad \frac{s_{2} + s_{3}}{2} \mathcal{N}_{s}(m) \\
  \vdots & \vdots \\
  n = n_{G}(m) & \frac{s_{nG(m)-1} + s_{nG(m)}}{2} \mathcal{N}_{s}(m), \mathcal{N}_{s}(m)
\end{cases}
$$
A stringent test: Deep tropical convection

M. Khairoutdinov (2005)

- 0.1 km horizontal grid-spacing
- 1536 x 1536 = 2,359,296 columns
- 76 layers from 0 to 20 km; 15 layers from 20 to 100 km
- uniform ocean surface
- $\mu_0 = 0.5$
- total cloud fraction = 0.58... 1,368,082 cloudy columns

- partition according to CTES... $\max(Q_{rad})$

nadir reflectance

1D RT

3D RT

153.6 km

cloudbase altitude (km)
- 20 ranges of CTES altitude: (0, 1.5] km... (11.43, 15.6] km
- most cover 0.025 to 0.035 of the domain
- clear-sky is the 21st range... 990,904 columns

- range 12: cloudtops \( \in [8.127, 8.427] \) km
- fraction = 0.024... 55,678 columns

- sort CWP (quicksort) with \((i,j)\) going along passively

A stringent test: Deep tropical convection
\[ n_G = 12 \text{ of } 20 \]
\[ \text{cloudtops} \in [8.127, 8.427] \text{ km} \]
\[ \text{cloudy columns in this partition} = 55,678 \]
\[ n_G = 12 \text{ of } 20 \]

cloudbases \( \in [8.127, 8.427] \) km

cloudy columns in this partition = \( 55,678 \)

\[
\int_0^1 f(x) \, dx \approx \sum_{n=1}^{n_G} w_n f(x_n)
\]

- \( f(x) \) is approximated by a poly. deg. \( \leq (2n_G - 1) \) on \([0, 1]\)

- apply full BB models to each GLQ column...

\[
\frac{1536 \times 1536}{20 \times 20 (+10)} = 5,754x \text{ fewer}
\]
A stringent test: Deep tropical convection

Net SW + LW at SFC (GLQ - ICA)
A stringent test: Deep tropical convection

- domain avg fluxes (W m\(^{-2}\))

<table>
<thead>
<tr>
<th></th>
<th>cloudy-sky</th>
<th>all-sky</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ICA</td>
<td>GLQ</td>
</tr>
<tr>
<td>(\uparrow) SW TOA</td>
<td>316.53</td>
<td>316.37</td>
</tr>
<tr>
<td>(\downarrow) SW SFC</td>
<td>190.82</td>
<td>190.96</td>
</tr>
<tr>
<td>(\uparrow) LW TOA</td>
<td>229.57</td>
<td>229.78</td>
</tr>
<tr>
<td>(\downarrow) LW SFC</td>
<td>-33.10</td>
<td>-33.29</td>
</tr>
<tr>
<td>(\downarrow) NET TOA</td>
<td>133.90</td>
<td>133.85</td>
</tr>
<tr>
<td>(\downarrow) NET SFC</td>
<td>157.72</td>
<td>157.64</td>
</tr>
</tbody>
</table>

\[
\frac{1536 \times 1536}{20 \times 20 (+ 10)} = 5,754 \times \text{fewer}
\]

- application of full BB models to each GLQ column... but only that column...
Cloudless columns

\[ \frac{1}{N} \sum_{n=1}^{N} F_n = \frac{1}{N} \sum_{n=1}^{N} F_{m(n)} \equiv \int_0^1 F(s) \, ds \approx \sum_{n=1}^{n_G} w_n F_{\tilde{s}_n} \]

- full ICA (benchmark)
- sorted ICA
- integral form
- \( n_G \)-point GLQ McICA

\[
m(1) \leftarrow \min \{W_n\} \quad s = 0 \leftarrow \min \{W_n\}
\]
\[
m(N) \leftarrow \max \{W_n\} \quad s = 1 \leftarrow \max \{W_n\}
\]

\[
W_n = \text{water vapour path (g m}^{-2})
\]
\[
n_G = 10
\]

\[ \begin{cases} 
W_n = \text{water vapour path (g m}^{-2}) \\
n_G = 10
\end{cases} \quad \rightarrow \quad \text{only 10 columns to represent 990,904!} \]
Boundary fluxes v. heating rate profiles

\[ n_G = 12 \text{ of } 20 \]


\[ \text{cloudbtops } \in [8.127, 8.427] \text{ km} \]

Net SW + LW at SFC

4137 columns

\[ k=1, \ldots, \mathcal{K} \]

sorted CWP

CWP at GLQ pts

flux at GLQ pts

10-polynomial

CWP (g m\(^{-3}\))

normalized sorted column number

\[ \mathcal{F}_N = \sum_{k=1}^{\mathcal{K}} c_k F_{x_{GLQ}(N),k} \]

regular

1-col McICA

column number

net SW + LW at SFC

flux (W m\(^{-2}\))
Boundary fluxes v. heating rate profiles

\[ n_G = 12 \text{ of } 20 \]

Cloudtops \( [8.127, 8.427] \text{ km} \)

\[ k=1, \ldots, K \]

\[ c_{k} \rightarrow \text{McICA} \]

\[ 1/2 \text{ to } 1/3 \text{ less noise} \]
MBE and RMSE or quantiles?

20 cloudtop altitude partitions

\[ n_G = 20 \]
Structure function analysis

\[ S_q(L) = \langle |r(x) - r(x + L)|^q \rangle; \quad q \geq 0; \quad \Delta x \leq L \leq N\Delta x; \quad r_1 \leq r \leq r_2 \]

\[ S_2(L) \sim L^{\zeta(2)} \quad \rightarrow \quad P(k) \sim k^{-\beta} \quad \rightarrow \quad \zeta(2) = \beta - 1 \]

- \( r \) can be either a single field or the difference between two renderings of a field
- Analyses performed for surface fluxes and HRs
- Focus on \( q = 2 \)
- \( L \) from 2 km to 64 km
- \( \zeta(q) \) estimated by LLSR
Structure function analysis

surface solar irradiance

1D GLQ

1D ICA

GLQ - ICA

1D ICA

3D

1D - 3D

$S_2(L)$
Structure function analysis

\[ R^2 = 0.6 \text{ to } 0.9 \]

**GLQ:**
- ICA \( \approx \) GLQ
- dif : closer to “noise” than fields

**1D v. 3D:**
- similar to 1D GLQ (degraded res. + homogenized WV)
- dif close to fields themselves

\[ S_2(L) \sim L^{\zeta(2)} \]

\( \zeta(2) \) at surface

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>LW</th>
<th>NET</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICA</td>
<td>0.64 (0.84)</td>
<td>0.52 (0.84)</td>
<td>0.64 (0.84)</td>
</tr>
<tr>
<td>GLQ</td>
<td>0.63 (0.84)</td>
<td>0.49 (0.84)</td>
<td>0.64 (0.84)</td>
</tr>
<tr>
<td>ICA - GLQ</td>
<td>0.01 (0.76)</td>
<td>0.31 (0.84)</td>
<td>0.04 (0.85)</td>
</tr>
</tbody>
</table>

\[ 1D: \quad 0.47 (0.60) \]
\[ 3D: \quad 0.53 (0.60) \]
\[ 1D - 3D: \quad 0.51 (0.59) \]
Variable SW irradiance across domains

500 x 500 km @ latitude = 48°N

near Sunset (22-Dec)
\[ <\mu_0> = 0.077 \]

near noon (22-Jun)
\[ <\mu_0> = 0.911 \]

apply RT using domain-average \( \cos(SZA) \) and scale when re-positioning

\[
Q_{rad}(i, j) = \left[ \frac{\mu_0(i, j)}{\langle \mu_0 \rangle} \right] Q_{rad}(n_{GLQ})
\]
Information overload

If the entire domain cannot be handled at once, sub-domains can be defined and the process applied in parallel.

- 2,359,296 columns
- reduce RT by 2,000x
- ~1,180 RT executions

1. entire domain, 20 partitions @ $n_G = 59$ (59 partitions @ $n_G = 20$)

2. 25 sub-domains, 10 partitions, $n_G = 5$
Variable surface conditions

- surface type: water v. land
  - use surface type as a partition and allocate $n_c$ proportional to areas

- variable surface temperature
  - again, partition according to ranges (cf. cloudtop altitudes)

- surface elevation
  - more complicated given terrain-following vertical coordinates
  - partition according to altitude ranges followed by interpolation???

- a “partitioning algorithm”... somewhat tantamount to McICA’s generator
Summary

To date

• currently RT accounts for 15% - 35% of a hi-res cloud (NWP) model’s CPU time
  - RT is always 1D-ICA and usually applied with (relatively) long timesteps
  - move to 3D RT... warrant??... If not, then:

• proposal: partition, sort, GLQ, and redistribute

• > 3,000x fewer calculations than full ICA... full-resolution $Q_{rad}$ at every dynamic timestep
  - sorting and indirect accessing overheads... 1,000 - 2,000x should be possible

Ongoing activities

• verification: more diagnostic tests with various cloud and surface conditions

• validation: SAM (v6.11)... testing for a range of cloud and meteorological conditions
  - especially ones in which cloud-radiation interactions are demonstrably important
- Thank You -