Physically-based stochastic parameterisation

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Types of model error / uncertainty

Classification

Model uncertainty

Unknown physics

Known physics
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Representation

- Structural
- Parametric
- Statistical description
- Monte Carlo

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- **Representation**
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- **Implementation**
  - Multi-model ensemble
  - Multi-parameter ensemble
  - Error covariance matrix
  - Stochastic parameterization

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Monte Carlo
Stochastic physics example: Turbulent fluctuations

Convective boundary layer scaling
• All lengths proportional to depth of layer $H$
• Second order moments, including transport (covariances), proportional to buoyancy flux $\langle w'\theta' \rangle$
• e.g. (Mellor and Yamada 1982)
  $$\langle \theta'^2 \rangle = \Lambda_2 \operatorname{tke}^{-1/2} \langle w'\theta' \rangle \partial_z \Theta$$

Theory implies stochastic variability over a certain range of spatial scales
• Perturbations correlated over distance $H$
• Variability small if grid length $dx$ large compared to $H$
• Magnitude increases as $dx$ approaches $H$ (then decreases as eddies start to be resolved)
Physically-based Stochastic Perturbations (PSP)

Implementation in COSMO model (2.8 km grid length)
• Add random increments to model variables
• Amplitude scaled using turbulence theory
• Rescaled to account for averaging over effective horizontal resolution
• Perturbations are coherent in height and over 10 min in time

\[
\left( \frac{\partial \Phi}{\partial t} \right)_{\text{stoch}} = \frac{\partial \Phi}{\partial t} + \alpha_{sh} \cdot \eta_{sh} \cdot \langle \Phi^2 \rangle^{1/2} 
\]

\[
\frac{\partial \Phi}{\partial t} : \text{tendency of } \Phi \text{ of all physical parameterizations} \\
\Phi : \text{resolved variable (T, w, q)} \\
\alpha_{sh} : \text{scaling factor} \\
\eta_{sh} : \text{Gaussian random perturbation} \\
\langle \Phi^2 \rangle : \text{variances from turbulence parameterization}
\]

\[
\alpha_{sh} = \alpha_{sh,\Phi} \cdot \frac{\ell_\infty}{5 \cdot dx} \cdot \frac{1}{dt}
\]

\[
dt : \text{temporal resolution of model} \\
\ell_\infty : \text{asymptotic mixing length} \\
dx : \text{horizontal resolution of model grid} \\
\alpha_{sh,\Phi} : \text{scaling factor}
\]

(Kober and Craig 2016)
Example of a PSP-SH field

Smoothed Gaussian random field

$\langle \theta' \rangle$ diagnosed from turbulence parameterization

$\theta$ increment

(Kober and Craig 2016)
Impact of PSP-SH

2 m Temperature
• Spread ~ 0.5 – 1 K
• RMSE increase up to 0.3 K when convection is active
• Increments with constant amplitude (yellow) cause large errors early and late in the day

Domain-integrated precipitation
• Strength of diurnal cycle much improved in comparison to radar
• Increments with constant amplitude trigger convection at places and times where it should not occur

(Kober and Craig 2016)
Two questions

1. What about other sources of small-scale variability?
   
   For example:
   
   - orographic forcing
   - cold pools
   - surface-forced mesoscale circulations
   - etc.

2. Wouldn't SPPT achieve the same effect?
Criteria for a stochastic parameterisation

1. Is the scheme stable and well-behaved in the full model? (e.g. resolution dependence)

2. Is the variability contributed by the scheme significant? (compared to initial condition uncertainty, etc.)

3. Is the forecast skill superior to that obtained with a deterministic scheme? (on some score!)
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4. Are there nontrivial interactions with the resolved flow? (Could the same skill be obtained by postprocessing output of model with deterministic scheme?)

5. Could the same skill be achieved with an inexpensive *ad hoc* scheme?
A physically based stochastic convection scheme

**Deterministic:** bulk plume represents mean of convective ensemble

**Stochastic:** plumes with different mass flux drawn randomly from equilibrium PDF

(Plant and Craig 2008)
The Plant-Craig stochastic convection scheme

1. Closure assumption scales a pdf of cloud radii

2. Draw clouds randomly from this pdf

![Diagram showing cloud size over time with probability distribution](image-url)
Reflectivity examples \textit{(COSMO 7 km)}

strongly forced:
25 June 2008
13 UTC

weakly forced:
1 July 2009
12 UTC

(Kober et al. 2015)
Summary of evaluation results

1. Resolution-dependence in aquaplanet simulations
   *(Keane et al. 2014)*
   - Realistic precipitation variability? Not unrealistic
   - Variability scales correctly? Yes

2. Spread in a regional ensemble prediction system
   *(Groenemeijer et al. 2012)*
   - Spread comparable to other sources? When synoptic forcing weak

3. Skill in a regional ensemble prediction system
   *(Kober et al. 2015)*
   - Forecast skill improved? For some scores and weather regimes

4. Upscale error growth at different resolutions
   *(Selz and Craig 2015)*
   - Realistic impact on large scale dynamics? Yes
   - Impact scales correctly with resolution? Yes
Set-up of error growth experiment

- ECMWF forecast
- COSMO 2.8km no conv. scheme
- 19.07.2007, 0UT
- Temperature-Perturbation, Sigma = 0.01K
- +15h +21h +27h +33h
- Control run
- 4.250km 7.000km

- Diagnostics will be based on **differences** to Ctl and **averaged** over all perturbation experiments
- Weather maps will show the first perturbation experiment

(Selz and Craig 2014)
Multi-scale error growth

1. Initial growth in regions of precipitation, rapid saturation
2. Spreading of perturbations in space to radius of deformation over inertial time $f^{-1}$
3. Exponential growth of synoptic scale perturbation
4. Further growth to planetary scales(?)

Color: Difference total energy
Contour: large-scale 500hPa geopotential perturbation

Quantitative results on poster of Tobias Selz
Upscale perturbation growth

- Difference Total Energy on medium (dashed) and large (solid) scales after 60 hours perturbation growth
- No parameterization (black) – growth damped at low resolution
- Default Tiedtke scheme (green) – too little growth
- Plant-Craig stochastic (red) – realistic growth

(Selz and Craig 2015)
Geostrophic adjustment after convection

1. Perturb convective mass flux $M$
2. Changes upper-level divergence
3. Changes geostrophically balanced wind

Scalings from theory

1. Transients propagate with gravity wave speed
   \[ c = \frac{N}{m} \approx 30 \text{ ms}^{-1} \]
2. Half-width of balanced response is Rossby radius
   \[ R_d \sim \frac{N}{f_0 m} \approx 300 \text{ km} \]
3. Adjustment time
   \[ T \sim \frac{c}{R_d} = \frac{f_0^{-1}}{f_0} \approx 6 \text{ hr} \]
4. Balanced vortex strength
   \[ v_g \sim \frac{Q_o f_0 m}{N^2} \]

- $f_0$ = Coriolis parameter
- $N$ = Brunt-Väisälä frequency
- $m$ = vertical wavenumber
- $Q_o$ = buoyancy source strength

A temperature perturbation of 1 K over a 100 km region will produce a balanced wind perturbation of about $v_g \approx 1 \text{ ms}^{-1}$ over 600 km after 6 hours

(Bierdel et al. in prep.)
Law of large numbers

- $M$ total mass flux over region
- $M$ divided into $N$ clouds of mass flux $m$
- Overbar is ensemble average
  \[ \sigma_M = \overline{M} / \overline{N}^{1/2} = \overline{M^{1/2}} \overline{m^{1/2}} \]
- Perturbations accumulate like random walk

Scaling of IFS temperature tendencies

- T159 forecasts coarse-grained (250 km, 12 hr) \( \rightarrow \overline{M} \)
- T1279 forecasts coarse-grained \( \rightarrow \sigma_M \) as function of $\overline{M}$

(Shutts and Callado Pallarès 2015)
Concluding remarks

- Types of model error
  - Errors associated with limited model complexity can be parameterized (stochastically)
  - But these are not the only errors
    Empirical estimates of error may not distinguish among types
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- Examples of stochastic parameterizations useful at current resolutions
  - Turbulent fluctuations in convection-permitting models
  - Cumulus convection in global models
    Significant impact, but not enough to account for all model error
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• Impact of small-scale perturbations
  – Multiscale process (not cascade)
  – Variability does not average out on synoptic scale
    Which aspects of model error influence forecast error?