A finite-volume module for the IFS

Christian Kühnlein, Piotr Smolarkiewicz, Sylvie Malardel

Operational configuration of IFS at ECMWF

Current operational configuration of the Integrated Forecasting System (IFS):

- hydrostatic primitive equations (nonhydrostatic option available; see Benard et al. 2014)
- hybrid $\eta - p$ vertical coordinate (Simmons and Burridge, 1982)
- spherical harmonics discretisation in horizontal (Wedi et al., 2013)
- finite-element discretisation in vertical (Untch and Hortal, 2004)
- semi-implicit semi-Lagrangian (SISL) integration scheme (Temperton et al. 2001, Diamantakis 2014)
- cubic-octahedral ("$T_{co}$") grid (Wedi, 2014, Malardel et al. 2016, Smolarkiewicz et al. 2016)
- HRES: $T_{co}1279$ (O1280) with $\Delta_h \sim 9$ km and 137 vertical levels
- ENS (1+50 perturbed members): $T_{co}639$ (O640) with $\Delta_h \sim 16$ km and 91 vertical levels

→ in the near future, spectral approach (as in IFS) is assumed to remain highly competitive in terms of time-to-solution
→ however, uncertainties concerning scalability/efficiency of spectral discretisation with respect to future HPC architectures
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Supplement IFS with a finite-volume module (FVM) that introduces:

→ finite-volume (→ compact stencil, conservative)
→ all-scale compressible Euler equations
→ flexible meshes
→ steep orography capability
FVM formulation – key features

- compressible Euler equations in geospherical coordinates
- generalised curvilinear coordinates (Prusa & Smolarkiewicz JCP 2003; Wedi & Smolarkiewicz JCP 2003; Kühnlein et al. JCP 2012)
- fully unstructured edge-based finite-volume discretisation in horizontal (Szmelter & Smolarkiewicz, JCP 2010)
- structured flux-form finite-difference discretisation in vertical (Smolarkiewicz et al. JCP 2016)
- all prognostic variables are co-located
- two-time-level semi-implicit integration scheme with 3d implicit acoustic, buoyant and rotational modes (Smolarkiewicz, Kühnlein, Wedi JCP 2014)
- preconditioned generalised conjugate residual iterative solver for elliptic problems arising in semi-implicit integration schemes
- Eulerian advection with non-oscillatory forward-in-time MPDATA scheme (Smolarkiewicz and Szmelter JCP 2005; Kühnlein and Smolarkiewicz, prep. to JCP)

\[
\int_{\Omega} \nabla \cdot \mathbf{A} = \int_{\partial \Omega} \mathbf{A} \cdot \mathbf{n} = \frac{1}{V_i} \sum_{j=1}^{l(i)} A_j^\perp S_j
\]

dual volume: \(V_i\), face area: \(S_j\)
Octahedral reduced Gaussian grid

- suitable for spherical harmonics transforms applied in spectral IFS
  - Gaussian latitudes ⇒ Legendre transforms
  - uniform spacing of points along latitudes ⇒ Fourier transforms
- Notation is ‘OX’, for octahedral grid with X latitudes between pole and equator
- quasi-uniform resolution over the sphere
- operational at ECMWF with HRES and ENS since March 2016
- Malardel et al. ECMWF Newsletter 2016, Smolarkiewicz et al. JCP 2016
Octahedral reduced Gaussian grid

- FVM develops median-dual mesh around nodes of octahedral grid
- operational spectral IFS and FVM can operate on same horizontal grid
- FVM formulation is not restricted to this grid
- Parallel data structures and mesh generator provided by Atlas (Deconinck et al., in prep. for Comput. Phys. Commun.)
FVM compressible Euler equations for moist-precipitating dynamics

\[
\frac{\partial \rho G \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{v} \rho G \mathbf{u}) = \rho G \left( -\Theta_d \tilde{\mathbf{G}} \nabla \varphi' - \frac{g}{\theta_a} \left( \theta' + \theta_a (\varepsilon q'_v - q_v - q_p) \right) - f \times (\mathbf{u} - \gamma_c \mathbf{u}_a) + \mathbf{M} \right)
\]

\[
\frac{\partial \rho G \theta'}{\partial t} + \nabla \cdot (\mathbf{v} \rho G \theta') = \rho G \left( -\tilde{\mathbf{G}}^T \mathbf{u} \cdot \nabla \theta_a - \frac{L \theta}{c_p T} (C_d + E_p) + \mathcal{H} \right)
\]

\[\varphi' = c_p \theta_0 \left[ \left( \frac{R_d}{p_0} \rho \theta (1 + q_v / \varepsilon) \right)^{R_d/c_v} - \pi_a \right]\]

\[
\frac{\partial \rho G q_v}{\partial t} + \nabla \cdot (\mathbf{v} \rho G q_v) = \rho G \left( -C_d - E_p + D_{q_v} \right)
\]

\[
\frac{\partial \rho G q_c}{\partial t} + \nabla \cdot (\mathbf{v} \rho G q_c) = \rho G \left( C_d - A_p - C_p + D_{q_c} \right)
\]

\[
\frac{\partial \rho G q_p}{\partial t} + \nabla \cdot (\mathbf{v} \rho G q_p) = \rho G \left( A_p + C_p + E_p + D_{q_p} \right) - \nabla \cdot (\mathbf{v} \rho G q_p)
\]

with:

\[
\Theta_d := \frac{\theta (1 + q_v / \varepsilon)}{\theta_0 (1 + q_t)} \equiv \frac{\theta_d}{\theta_0} \quad \gamma_c \equiv \frac{\theta}{\theta_a} \quad \varepsilon := \frac{R_d}{R_v} \quad \varepsilon = 1 / \varepsilon - 1 \quad \mathbf{v} = \tilde{\mathbf{G}}^T \mathbf{u}
\]

→ "a" subscript denotes ambient state which satisfies subset of full equations, "0" subscript refers to constant reference, all primed variables are deviations with respect to the ambient state \((\psi' = \psi - \psi_a \quad \psi = u, v, w, \theta, ..)\)
Two-time-level semi-implicit solution of compressible Euler equations (Smolarkiewicz et. al JCP 2014)

Mass continuity:

$$\rho_{i}^{n+1} = \mathcal{A}_{i}(\rho^{n}, (\mathbf{v}G)^{n+1/2}, G^{n}, G^{n+1}) \Rightarrow (\mathbf{v} \cdot G\rho)^{n+1/2}$$

Conservation laws for primitive variables $\Psi$ ($\Psi = u, v, w, \theta', ...$):

$$\Psi_{i}^{n+1} = \mathcal{A}_{i}(\tilde{\Psi}^{n}, (\mathbf{v} \cdot G\rho)^{n+1/2}, (G\rho)^{n}, (G\rho)^{n+1}) + 0.5 \delta t R_{i}^{\Psi} |_{n+1}^{n}$$

with $\tilde{\Psi}^{n} \equiv \Psi^{n} + 0.5 \delta t R_{i}^{\Psi} |^{n}$

$\mathcal{A}(\cdot)$-operator is FV-MPDATA for homogeneous equation (Smolarkiewicz & Szmeleter, 2005):

$$\frac{\partial G\psi}{\partial t} + \nabla \cdot (\mathbf{V} \psi) = 0 \quad \rightarrow \quad \psi_{i}^{n+1} = \psi_{i}^{n} - \frac{\delta t}{G_{i} V_{i}} \sum_{j=1}^{l(i)} F_{j}^{\perp} (\psi_{j}^{n}, \psi_{j}^{n}, V_{j}^{\perp}) S_{j}$$

with normal upwind flux: $F_{j}^{\perp} (\psi_{i}, \psi_{j}, V_{j}^{\perp}) = [V_{j}^{\perp}]^{+} \psi_{i} + [V_{j}^{\perp}]^{-} \psi_{j}$ where $V^{\perp} := \mathbf{V} \cdot \mathbf{n}$

- First step is classical upwind scheme with advection of $\psi$ by physical flow $\mathbf{V}$
- Error-compensative step with pseudo-velocity $\tilde{V}^{\perp} := -\psi^{-1} \times \text{Error}(\delta r, \delta t)$ from modified equation analysis, e.g.

$$\text{Error} = -\frac{1}{2} |V_{j}^{\perp}| \left( \frac{\partial \psi}{\partial r} \right)_{s_{j}}^{*} (r_{j} - r_{i}) + \frac{1}{2} \delta t \frac{V_{j}^{\perp}}{G_{j}} \{ \mathbf{V} \cdot \nabla \psi \}^{*}_{s_{j}} + \frac{1}{2} \delta t \frac{V_{j}^{\perp}}{G_{j}} \{ \psi (\nabla \cdot \mathbf{V}) \}^{*}_{s_{j}} + \text{HOT}$$
Finite-volume MPDATA for compressible atmospheric dynamics

Two-time-level semi-implicit solution of compressible Euler equations
(Smolarkiewicz et. al JCP 2014)

Mass continuity:

\[ \rho_i^{n+1} = \mathcal{A}(\rho^n, (v \mathcal{G})^{n+1/2}, \mathcal{G}^n, G^{n+1}) \Rightarrow (v \mathcal{G} \rho)^{n+1/2} \]

Conservation laws for primitive variables \( \Psi = (u, v, w, \theta', \ldots) \):

\[ \Psi_i^{n+1} = \mathcal{A}(\widetilde{\Psi}^n, (v \mathcal{G} \rho)^{n+1/2}, (\mathcal{G} \rho)^n, (\mathcal{G} \rho)^{n+1}) + 0.5 \delta t \mathcal{R} \Psi |^{n+1} \]

with \( \widetilde{\Psi}^n \equiv \Psi^n + 0.5 \delta t \mathcal{R} \Psi |^n \)

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with normal upwind flux: \( F_j^\perp (\psi_i, \psi_j, V_j^\perp) = [V_j^\perp]^+ \psi_i + [V_j^\perp]^+ \psi_j \) \( \text{where } V^\perp \equiv \mathbf{V} \cdot \mathbf{n} \)

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Finite-volume MPDATA for compressible atmospheric dynamics

Two-time-level semi-implicit solution of compressible Euler equations (Smolarkiewicz et. al JCP 2014)

**Mass continuity:**

\[
\rho^{n+1}_i = A_i(\rho^n, (vG)^{n+1/2}, G^n, G^{n+1}) \Rightarrow (v^\perp G \rho)^{n+1/2}
\]

**Conservation laws for primitive variables \( \Psi \) \((\Psi = u, v, w, \theta', ...)\):**

\[
\Psi^{n+1}_i = A_i(\bar{\Psi}^n, (v^\perp G \rho)^{n+1/2}, (G \rho)^n, (G \rho)^{n+1}) + 0.5 \delta t R^\Psi |^{n+1}_i
\]

with \( \bar{\Psi}^n \equiv \Psi^n + 0.5 \delta t R^\Psi |^n \)

FV-MPDATA solely using face-normal vector components (Kühnlein and Smolarkiewicz, in prep. for JCP):

**Error**

\[
Error = -\frac{1}{2} |V_j^\perp| \left( \frac{\partial \psi}{\partial r} \right)^*_{s_j} (r_j - r_i) + \frac{1}{2} \delta t \frac{V_j^\perp}{G_j} \{ \nabla \cdot (V \psi) \}^*_{s_j} + \text{HOT}
\]

**Gauss-divergence theorem:**

\[
\int_{\Omega} \nabla \cdot A = \int_{\partial \Omega} A \cdot n = \frac{1}{V_i} \sum_{j=1}^{l(i)} A_j^\perp S_j
\]
Finite-volume MPDATA for compressible atmospheric dynamics

Orographically-forced internal gravity waves in sheared ambient flow on small planet (Keller 1994, Wedi & Smolarkiewicz 2009) with octahedral grid O90:

Vertical velocity (m/s) along equator after 2 h

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<tr>
<th>Simulation</th>
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<tbody>
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⇒ Proposed FV-MPDATA reproduces reference results of established formulation and enables effective semi-implicit integration of compressible Euler equations on arbitrary meshes.
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Vertical velocity along centreline of trapped wave at 3.5 km height with different model configurations (see Table)

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Vertical discretisation and incorporation of orography in FVM

Generalised terrain-following vertical coordinate

Held-Suarez benchmark N640 (16 km) with realistic IFS orography at day 90
(relative vorticity at $z=2$ km)

stratified flow past steep orography with maximum slope $\sim 70^\circ$ on small planet
(vertical velocity in m/s, lon-height section at lat=0, lon-lat section at $z=2$ km)

⇒ robust and efficient handling of steep orography
Tropical cyclone with FVM at day 10 ($\Delta_h = 0.25^\circ$):

Supercell evolution (0.5, 1, 1.5, 2h) with FVM at 500 m grid spacing on small planet:

Vertical velocity (m/s) at 5 km

Rainwater (g/kg) at 5 km

→ FVM with simplified cloud microphysics and PBL parametrisations
Finite-volume versus spectral solutions in the IFS

Dry baroclinic instability, FVM (O640) versus the spectral IFS (Tco639):

**Finite-volume**

**Spectral**

*Day 10*
Finite-volume versus spectral solutions in the IFS

Dry baroclinic instability, FVM (O640) versus the spectral IFS (Tco639):

Finite-volume

Surface pressure

Spectral

Surface temperature

day 15
Dry baroclinic instability, FVM (O640) versus the spectral IFS (T_{co}639):

Meridional wind (m/s) in zonal-height section at 50° N

Finite-volume

day 15

Spectral
Dry baroclinic instability, FVM (O160) versus the spectral IFS (T_co159):

[Diagram showing comparison between finite-volume and spectral solutions on day 10]
Finite-volume versus spectral solutions in the IFS

Dry baroclinic instability, FVM (O160) versus the spectral IFS (Tco159):

Finite-volume

Spectral

Surface pressure

Surface temperature

day 15
Finite-volume versus spectral solutions in the IFS

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day 15

Spectral
Finite-volume versus spectral solutions in the IFS

Held-Suarez climate benchmark N640 (16 km) with realistic
IFS orography at day 90 (rel. vorticity at 2 km)

Outlook:

- Efficiency at high resolution and extreme scaling (→ ESCAPE)
- Comprehensive comparison of FVM with spectral IFS
- Towards more realism (Initial conditions, parametrisations,...)
Further reading

The modelling infrastructure of the Integrated Forecasting System: Recent advances and future challenges


Research Department

November 2015

Special topic paper presented at the 44th session of ECMWF's Scientific Advisory Committee, Reading, UK.

European Centre for Medium-Range Weather Forecasts
Europäisches Zentrum für mittel- und langfristige Wettervorhersage
Centre européen pour les prévisions météorologiques à moyen terme

Christian Kühnlein, Piotr Smolarkiewicz, Sylvie Malardel
Orographically-forced internal gravity waves in sheared ambient flow on small planet (Keller 1994, Wedi & Smolarkiewicz 2009) with octahedral grid O90:
Baroclinic instability (Jablonowski and Williamson, 2006) with FVM (from top: anelastic, pseudo-incompressible, compressible)

$\rightarrow$ Generic nonhydrostatic formulation with consistent options:

* fully compressible Euler equations (default)
* pseudo-incompressible (Durran, JAS 1989)
* anelastic (Lipps and Hemler, JAS 1982)

$$\frac{\partial G\varphi}{\partial t} + \nabla \cdot (vG\varphi) = 0$$

$$\frac{\partial G\varphi u}{\partial t} + \nabla \cdot (vG\varphi u) = -G\varphi \left( \Theta G\nabla \varphi' + g \tau_B \frac{\theta'}{\theta_b} + f \times (u - \tau_C u_a) + M \right)$$

$$\frac{\partial G\varphi \theta'}{\partial t} + \nabla \cdot (vG\varphi \theta') = -G\varphi \left( \tilde{G}^T u \cdot \nabla \theta_a \right)$$

with optional coefficients:

$\varphi := [\rho(x, t), \rho_b \frac{\theta_b(z)}{\theta_0}, \rho_b(z)]$, $\varphi' := [c_p \theta_0 \pi', c_p \theta_0 \pi', c_p \theta_b \pi']$

$\Theta := \left[ \frac{\theta}{\theta_0}, \frac{\theta}{\theta_0}, 1 \right]$, $\tau_B := \left[ \frac{\theta_b(z)}{\theta_a(x)}, \frac{\theta_b(z)}{\theta_a(x)}, 1 \right]$, $\tau_C := \left[ \frac{\theta}{\theta_a(x)}, \frac{\theta}{\theta_a(x)}, 1 \right]$