From 1D to 4D: Towards a gravity-wave parameterization for NWP and climate models beyond the wave-dissipation paradigm

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Motivation: Gravity-wave (GW) parameterizations

GW parameterizations based on **WKB theory** (e.g. Grimshaw 1975)

- **Simplifications** for efficiency:
  - Single column
  - Steady state (turbulence needed)
- **Limitations**:
  - Without these simplifications non-interaction theorem does not need turbulence for wave/mean-flow interaction (Bühler & McIntyre 1998, 2003, 2005)

- More **general approach** yet to be fully implemented
- Some issues:
  - Dependence on stratification
  - Vortical/geostrophic mode
  - Numerical implementation
  - Direct wave-mean-flow interaction/impact by wave breaking
  - Mesoscale/submesoscale interaction
Classic WKB (Grimshaw 1975, ...) for illustration 1D:

Locally monochromatic fields of the form \( b'(x, t) = \Re B(z, t)e^{i \phi(x, t)} \)

Local wavenumber and frequency:

\[
\begin{align*}
 k(z, t) &= ke_x + me_z = \nabla \phi, \\
 \omega(z, t) &= -\frac{\partial \phi}{\partial t}
\end{align*}
\]

wave-action density \( A(z, t) \) so that (e.g.)

\[
E_{GW}(z, t) = A(z, t) \dot{\omega}(m)
\]

Along rays, defined by \( \frac{dz}{dt} = c_g \)

\[
\begin{align*}
 \frac{dm}{dt} &= -k \frac{\partial U}{\partial z}, \\
 \frac{dA}{dt} &= -A \frac{\partial c_g}{\partial z}
\end{align*}
\]

Mean flow:

\[
\frac{\partial U}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \left( \bar{\rho} u'w' \right) = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \left( c_g k A \right)
\]
Ray tracing with caustics: Numerics for fully coupled WKB

GW packet refracted by a jet

(c) (d) 

$\times 10^{-3}$ $\times 10^{-3}$

z (km) time km

LES WKB

Rieper et al (2013)
Ray tracing with caustics: 1D for illustration

Locally monochromatic fields

wave-action density $A(z, t)$ so that (e.g.)

$$E_{GW}(z, t) = A(z, t) \hat{\omega}(m)$$

Along rays, defined by $dz/dt = c_g$

$$\frac{dm}{dt} = -k \frac{\partial U}{\partial z}, \quad \frac{dA}{dt} = -A \frac{\partial c_g}{\partial z}$$

Mean flow:

$$\frac{\partial U}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} \bar{u}' \bar{w}') = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (c_g k A)$$
Ray tracing with caustics: 1D for illustration

Locally monochromatic fields

wave-action density $A(z, t)$ so that (e.g.)

$$E_{GW}(z, t) = A(z, t) \tilde{\omega}(m)$$

Along rays, defined by $dz/dt = c_g$

$$\frac{dm}{dt} = -k \frac{\partial U}{\partial z}, \quad \frac{dA}{dt} = -A \frac{\partial c_g}{\partial z}$$

Mean flow:

$$\frac{\partial U}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} u' w') = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (c_g k A)$$

Crossing rays (caustics): uniqueness problem for $A$ and $m$!
Ray tracing with caustics: examples for caustic situations

**Nonuniqueness** of wave number and wave-action density arises easily:

e.g. reflection at a jet
Ray tracing with caustics: examples for caustic situations

**Nonuniqueness** of wave number and wave-action density arises easily:

e.g. overtaking rays
Nonuniqueness of wave number and wave-action density arises easily:

e.g. by wave-induced mean flow
Ray tracing with caustics:
1D for illustration

**linear limit:** wave field can be decomposed into fields with singlevalued wavenumbers

\[
\hat{B} = \Re \left\{ b_1(\zeta, \tau) e^{i \left[ kx + \phi_1(\zeta, \tau)/\epsilon \right]} + b_2(\zeta, \tau) e^{i \left[ kx + \phi_2(\zeta, \tau)/\epsilon \right]} \right\}
\]

\[
\frac{\partial \phi_1}{\partial \zeta} = m_1 \quad \frac{\partial \phi_2}{\partial \zeta} = m_2
\]

\[
\frac{D_{g\alpha} A_\alpha}{D\tau} = \frac{\partial A_\alpha}{\partial \tau} + c_{g\alpha} \frac{\partial A_\alpha}{\partial \zeta} = -\frac{\partial c_{g\alpha}}{\partial \zeta} A_\alpha + D_\alpha \quad (\alpha = 1, 2)
\]

case dependent surgery: **very complex**
**Ray tracing with caustics:**

**spectral approach**

**linear limit:** wave field can be **decomposed** into fields with singlevalued wavenumbers


**wave-action density**

\[ \mathcal{N}(m, z, t) = \int d\alpha A_\alpha(z, t) \delta[m - m_\alpha(z, t)] \]

satisfies conservation equation

\[
\frac{\partial \mathcal{N}}{\partial t} + \frac{\partial}{\partial z}(c_g \mathcal{N}) + \frac{\partial}{\partial m}(\dot{m} \mathcal{N}) = 0 \\
\dot{m} = -k \frac{\partial U}{\partial z}
\]

**Mean flow:**

\[
\frac{\partial U}{\partial t} = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} u'w') = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\int dm c_g k \mathcal{N})
\]

**Generalization to 3D straightforward**
Ray tracing with caustics: efficient numerics (Muraschko et al 2015)

Phase-space velocity is non-divergent

\[
\frac{\partial c_g}{\partial z} + \frac{\partial \dot{m}}{\partial m} = \frac{\partial}{\partial z} \frac{\partial \Omega}{\partial m} + \frac{\partial}{\partial m} \left( - \frac{\partial \Omega}{\partial z} \right) = 0
\]

Hence
- flow is volume preserving
- rays cannot cross
- Wave-action density conserved on rays

\[
\frac{D \mathcal{N}}{Dt} = \frac{\partial \mathcal{N}}{\partial t} + c_g \frac{\partial \mathcal{N}}{\partial z} + \dot{m} \frac{\partial \mathcal{N}}{\partial m} = 0
\]

- region of nonzero \( \mathcal{N} \) approximated by rectangular ray volumes
- ray volumes move with central ray
- ray volumes change height \( (\Delta z) \) and width \( (\Delta m) \) in area-preserving manner
Ray tracing with caustics: efficient numerics (Muraschko et al 2015)

hydrostatic wave packet (Boussinesq)

Rays are no wavepackets

No turbulence taken into account!
Ray tracing with caustics: no numerical instabilities (Bölöni et al. 2016, submitted)

GW packet refracted by a jet

Altitude (km)

Time (N x t)

LES

WKB ray tracer

WKB finite volume
– transient GWs can interact with the mean flow without the onset of turbulence (eg Dosser & Sutherland 2011)
– GW parameterizations (steady-state approximation) only rely on wave breaking

comparative role of wave transience (direct interaction) vs wave breaking?
diagram wave-mean-flow interaction vs wave breaking (Bölöni et al 2016)

horizontally infinite GW packets in interaction with mean flow
- **1D**: \( U(z, t), A(z, t), m(z, t) \)
- direct GW-mean-flow interaction always active
- WKB: \( E_{\text{mean}} + E_{\text{wave}} = \text{const} \).

**tools:**
- wave resolving LES (reference data)
- fully coupled WKB
- turbulence onset
  - once static instability threshold can be surpassed
    \[
    \int dm \ m^2 \frac{d|B|^2}{dm} = \int dm \ N \ f(m) > \alpha N^2
    \]
  - parameter \( \alpha \in [1,2] \) accounting for phase cancellations between spectral components
  - (scale selective) **eddy viscosity/diffusivity** reduces wave amplitude to inst. threshold
direct wave-mean-flow interaction vs wave breaking ( Bölöni et al 2016)

static instability hydrostatic wave packet

LES (wave-resolving)

WKB

Time (N x t)
direct wave-mean-flow interaction vs wave breaking ( Bölöni et al 2016)

static instability hydrostatic wave packet

LES (wave-resolving)

WKB

WKB with saturation (turbulence param.)
direct wave-mean-flow interaction vs wave breaking ( Bölöni et al 2016)

static instability non-hydrostatic wave packet

LES (wave-resolving)  
WKB with saturation (turbulence param.)
direct wave-mean-flow interaction vs wave breaking (Bölöni et al. 2016)

static instability non-hydrostatic wave packet

LES (wave-resolving)
WKB with saturation (turbulence param.)
steady-state (GW parameterization)
role of lateral propagation
linear large-scale dynamics in interaction with GWs

From **GCM data** (HAMMONIA, Schmidt et al 2006):

- Seasonally dependent reference climatology \( \overline{u}(\lambda, \phi, z), \overline{T}(\lambda, \phi, z) \)
- Diurnal heating cycle \( \Re \sum_n Q_n(\lambda, \phi, z) e^{in\Omega t} \)

**Linear model** (Achatz et al 2008, based on KMCM, Becker and Schmitz 2003)

\[
\begin{align*}
\mathbf{u} &= \overline{\mathbf{u}} + \mathbf{u}'(\lambda, \phi, z, t) \\
T &= \overline{T} + T'(\lambda, \phi, z, t)
\end{align*}
\]

\[
\left( \frac{\partial}{\partial t} + \overline{\mathbf{u}} \cdot \nabla_h \right) \mathbf{u}' + \cdots = -\frac{1}{\bar{\rho}} \nabla \cdot (\bar{\rho} \, \mathbf{v}_{GW} \mathbf{u}_{GW})
\]

\[
\left( \frac{\partial}{\partial t} + \overline{\mathbf{u}} \cdot \nabla_h \right) T' + \mathbf{v}' \cdot \nabla \overline{T} + \cdots = \Re \sum_n Q_n(\lambda, \phi, z) e^{in\Omega t} - \nabla_h \cdot (\mathbf{u}_{GW} T_{GW})
\]

GW fluxes from **4D WKB model with rays propagating on** \( (\overline{\mathbf{u}} + \mathbf{u}', \overline{T} + T') \)

**First implementation of a fully coupled transient ray tracer into a global model**
linear large-scale dynamics in interaction with GWs (Ribstein et al 2015, Ribstein & Achatz 2016)

3D effects (beyond single column)

- **Horizontal GW propagation**
  \[
  \frac{dx_h}{dt} = c_{gh}, \quad \frac{dz}{dt} = c_{gz}
  \]

- **Horizontal gradients in reference climatology and tides**
  \[
  \frac{dk_h}{dt} = -k \nabla_h (\bar{u} + u') - l \nabla_h (\bar{v} + v'), \quad \frac{dm}{dt} = -k \frac{d}{dz} (\bar{u} + u') - l \frac{d}{dz} (\bar{v} + v')
  \]

- **Horizontal GW flux convergence**
  \[
  \left( \frac{\partial}{\partial t} + \bar{u} \cdot \nabla_h \right) u' + \cdots = -\frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} \ u_{GW} u_{GW}) - \frac{1}{\bar{\rho}} \nabla_h \cdot (\bar{\rho} \ u_{GW} u_{GW})
  \]
  \[
  \left( \frac{\partial}{\partial t} + \bar{u} \cdot \nabla_h \right) T' + v' \cdot \nabla\bar{T} + \cdots = \Re \sum_n Q_n(\lambda, \phi, z) e^{i n \Omega t} - \nabla_h \cdot (\bar{u}_{GW} \ T_{GW})
  \]
Tidal model in interaction with GWs (Ribstein et al 2015, Ribstein & Achatz 2016)

3D effects (beyond single column)

``Full'' experiment

``Single-column'' experiment

zonal-mean daily-mean GW forcing (December)
Summary

- **numerical implementation** fully interactive WKB
  - no instabilities due to caustics
  - very efficient
- **transient-GW dynamics**
  - direct GW-mean-flow interaction dominates over GW breaking
  - GW parameterizations not reliable
- **lateral-propagation effects matter in middle atmosphere**


Investigation **multi-scale dynamics of GWs** in 6 projects

**prognostic WKB GW parameterization** to be developed for NWP and climate model

To be addressed:

- Sources
- Propagation
- Dissipation

Combined effort:

- Theory,
- Modelling,
- Measurements,
- Laboratory experiments