Prospect for radar and lidar cloud assimilation

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ABSTRACT

Space-borne active instruments, providing a three-dimensional characterization of clouds, promise a new dimension of information to be used in numerical weather prediction (NWP) systems. Observations from CloudSat and CALIPSO (Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observations) are already available and new missions, such as EarthCARE (Earth Clouds, Aerosol and Radiation Explorer) should appear in the near future. The challenge is to assimilate such new data sources into a NWP system to achieve better knowledge about the atmospheric state, and possibly to improve the weather forecasts. Research activities are ongoing at the European Centre for Medium-Range Weather Forecasts (ECMWF) to exploit these data for monitoring and assimilation.

The methodology explored for assimilation studies is described. Information about the observation operators, observation error definition and handling of observations (such as quality control and bias correction) is also provided. The results from assimilation experiments using a technique combining one-dimensional variational (1D-Var) assimilation with four-dimensional variational (4D-Var) data assimilation indicate that 1D-Var analyses get closer to assimilated and also independent observations. The performed 1D+4D-Var assimilation experiments suggest a positive impact of the new observations on the subsequent forecast.

1 Introduction

During the last decade the representation of precipitation and clouds in the global numerical weather prediction (NWP) models has greatly improved and it reaches a reasonable degree of realism. This opens new possibilities for assimilation of data related to clouds from active and passive instruments. Research activities are on-going at ECMWF to exploit observations providing three-dimensional information on clouds from space-borne active instruments on board of CloudSat and CALIPSO for monitoring and assimilation. The work paves the way for the use of similar observations from the planned EarthCARE (Earth Clouds, Aerosols and Radiation Explorer) mission in the ECMWF system.

In order to study the impact of the cloud radar and lidar observations on analyses and subsequent forecasts, a technique combining one-dimensional variational (1D-Var) assimilation with fourdimensional variational (4D-Var) data assimilation has been selected. In this two-step approach, 1D-Var assimilation can already on its own provide very useful information about potential benefits and problems related to assimilation of new observational types. This was, for instance, demonstrated in ECMWF studies towards the assimilation of cloud and rain-affected microwave/infrared radiances (Marécal and Mahfouf, 2000; Moreau *et al.*, 2004; Bauer *et al.*, 2006a). First assimilation experiments with cloud radar data were also performed applying the 1D-Var approach for Atmospheric Radiation Measurement (ARM) observations (Janisková *et al.*, 2002; Benedetti and Janisková, 2004; Janisková, 2004). Furthermore, use of a 2D-Var technique (the second dimension being time) has been explored by Lopez *et al.* (2006) for the ARM cloud radar data combined with the ground-based precipitation measurements and GPS total column water-vapour retrievals. The two-step 1D+4D-Var approach has been developed and used for assimilation of rain affected observations at ECMWF (Bauer *et al.*, 2006a,b; Lopez and Bauer, 2007) in the past. This technique has also been used in the study of Janisková *et al.* (2012) which demonstrated the positive impact that the 4D-Var assimilation of temperature and humidity pseudo-observations derived from CloudSat has on the analysis and forecast of temperature, humidity and winds. More recently, assimilation experiments have been extended when not only observations of Cloudsat cloud radar reflectivity, but also CALIPSO lidar backscatter, either separately or in combination, are used in the 1D+4D-Var system (Janisková, 2014).

2 1D-Var assimilation

2.1 Methodology

The principle of 1D-Var is similar to that of 4D-Var, but the control vector \mathbf{x} represents a single column only and the time dimension is not included. The goal of 1D-Var is to search for the optimal model state that simultaneously minimizes the distance to the observations \mathbf{y}^{o} and to a background model state \mathbf{x}^{b} (i.e. a short-range forecast valid at the time of assimilation). The cost function minimized during the assimilation processes is defined as:

$$\mathscr{J}(\mathbf{x}) = \frac{1}{2} \left(\mathbf{x} - \mathbf{x}^{\mathrm{b}} \right)^{\mathrm{T}} \mathbf{B}^{-1} \left(\mathbf{x} - \mathbf{x}^{\mathrm{b}} \right) + \frac{1}{2} (H(\mathbf{x}) - \mathbf{y}^{0})^{\mathrm{T}} \mathbf{R}^{-1} (H(\mathbf{x}) - \mathbf{y}^{0})$$

where **B** is the covariance matrix of the background error. Its role is to pass to the variational analysis the appropriate information about statistical structure of the forecast errors. **R** represents the observation and representativeness error covariance matrix. H is the observation operator providing the equivalent of the data from the model variable **x**.

(a) Observation operator

In the case of radar and lidar assimilation, the observation operator H employs physical parametrization schemes for moist processes, i.e. convection scheme and cloud scheme simulating large-scale condensation and precipitation processes (Lopez and Moreau, 2005; Tompkins and Janisková, 2004; Janisková and Lopez, 2013). Further parametrizations of radar reflectivity and lidar backscatter due to clouds (Di Michele *et al.*, 2012, 2014a,b) are required to convert model fields to reflectivity and backscatter, respectively. Input cloud and precipitation fields to radar/lidar model are computed by the moist physics parametrizations, the main input variables of which are temperature and humidity fields.

(b) Observations

In our study, measurements of cloud radar reflectivity, converted to $mm^6 m^{-3}$, from the CloudSat 94 GHz radar and/or lidar backscatter (km⁻¹ sr⁻¹) due to clouds at 532 nm from CALIPSO are assimilated by the 1D-Var system.

(c) Observation errors

The impact of any type of observation in data assimilation is partly determined by the errors that are assigned to them. These errors take into account the instrument errors, together with forward modelling and representativity errors (due to the narrow field of view). For CloudSat, the instrument random error is used (Di Michele *et al.*, 2014a). For CALIPSO, instrument errors are evaluated from level-1 data according to Liu *et al.* (2006). Forward modelling error is based on evaluation of uncertainty in the microphysical assumption by defining error through the differences between the perturbed state and the reference configuration (Di Michele *et al.*, 2014a,b). This is done for the different ranges of temperature. The representativity error used in our experiments is flow dependent and it is estimated based on the structure function maximum according to Stiller (2010). This error is defined for the different altitudes and geographical regions.

M. Janisková: Prospect for radar and lidar cloud assimilation

(d) Bias correction and quality control

For a proper handling of observations in the context of an assimilation system, an appropriate quality control strategy and a scheme for bias correction (Di Michele *et al.*, 2014a,b) are required. Based on the statistics of first-guess (FG) departures (differences between the model first guess and observations), the quality control excludes situations when discrepancies between observations and model equivalents are large. The bias correction scheme uses temperature and altitude as predictors. It is defined for different seasons and geographical regions to account for the variability linked to the different weather regimes and phase of hydrometeors. By applying correction, a more Gaussian distribution of the FG departures is obtained.

(e) Background values

The background values have been taken from the 12-hour forecast of the ECMWF model with T799 spectral truncation (corresponding to approximately 25 km) and 91 vertical levels. The forecast results have been stored every half an hour in order to use observations within 1D-Var in similar way as in the operational 4D-Var system where all observations are split into half-hour time slots. The profiles of temperature and specific humidity (control variables of 1D-Var), along with tendencies, surface pressure and other surface quantities are used as inputs to the observation operator.

(f) Background error statistics

The covariance matrix of the background error \mathbf{B} is taken from the ECMWF 4D-Var system. No cross correlations between the background errors of specific humidity and temperature are considered.

2.2 Results

Using the 1D-Var system, different experiments have been performed using observations of cloud radar reflectivity from CloudSat (**RADAR**) and cloud lidar backscatter from CALIPSO (**LIDAR**), either separately or in combination (**COMBI**). Observations have been averaged in the model grid-box and the full error definition, the quality control and the bias corrections (mentioned above) have been applied. The performance of 1D-Var assimilation has been verified against independent observations (i.e. observations which were not assimilated), such as cloud optical depth from MODIS (at the standard reference wavelength of 0.55 μ m) or radar reflectivity and lidar backscatter when not assimilated. The results are presented either (i) for the single satellite track between 23:50 UTC on 23 January 2007 and 00:26 UTC on 24 January 2007 over the whole Pacific Ocean from 62°N to 62°S (covering a variety of meteorological situations, e.g. tropical convection and an extratropical cyclone in the north) or (ii) for multiple tracks covering the 12-hour period from 21:00 UTC on 23 January 2007 to 09:00 UTC on 24 January 2007 (corresponding to the length of the 4D-Var assimilation window used at ECMWF).

Figure 1 shows a comparison of the first guess and 1D-Var retrieved radar reflectivity versus CloudSat 94 GHz cloud radar observations over the Pacific Ocean. The 1D-Var analysis is closer to the observations for most of the profiles. However, one can notice that convective clouds between 8°N and 8°S are less modified and remain closer to the first guess values. The latter is due to the larger representativity errors of the observations in areas of convection.

Results from 1D-Var assimilation of CALIPSO cloud lidar observations (Figure 2) for the same period indicate contrary to the radar case that the analysis fit to observations is only marginally better than that for the background. This is partly related to the small observation field of view, which leads to large values of representativity error. These larger errors than reduce the weight given to these observations in analysis. 1D-Var analysis get closer to cloud lidar backscatter observations when combination of cloud radar reflectivity and lidar backscatter is used (Figure 2d).



Figure 1: Cross-section of radar reflectivity (in dBZ) on 24 January 2007 over the Pacific Ocean: (a) CloudSat observations from 94 GHz radar, (b) model first guess (FG) and (c) 1D-Var analysis.

Analysis increments of temperature (T) and specific humidity (q) (Figure 3) have been also evaluated since they can provide information about an impact of the assimilated observations on the control of the 1D-Var system. This evaluation revealed that both of them are modified by the assimilation of cloud radar reflectivity and/or lidar backscatter. Therefore the pseudo-observations of both T and qprofiles from 1D-Var retrievals need to be included into the 4D-Var system. Comparing to radar, the lidar increments occur at higher altitudes and are therefore complementary. At altitudes where both radar and lidar observations are available, the increments are consistent.

Verification of the 1D-Var assimilation performance indicated that the 1D-Var analyses get closer not only to assimilated, but also to independent observations. However, the impact of lidar backscatter from clouds is smaller than that of cloud radar reflectivity. This is demonstrated in Figure 4 displaying the root mean square (rms) error differences between the first-guess and analysis (AN) departures for both CloudSat radar reflectivity and CALIPSO lidar backscatter. A comparison of the background and analysis departures for MODIS cloud optical depth is also shown in Figure 4c.

3 1D+4D-Var assimilation

In the past years, a 1D+4D-Var approach has been developed and used for assimilation of rain affected observations at ECMWF (Bauer *et al.*, 2006b; Lopez and Bauer, 2007). This technique has also been applied in the study of Janisková *et al.* (2012), in which temperature and humidity pseudo-observations derived from 1D-Var assimilation of CloudSat data were included in 4D-Var. As extension of the latter study, 1D+4D-Var assimilation experiments have been performed with both CloudSat cloud radar reflectivity and CALIPSO cloud lidar backscatter observations, either separately or in combination (Janisková, 2014), in order to study the impact of the new observations on the 4D-Var analysis and subsequent forecast.



Figure 2: Cross-section of lidar cloud backscatter (in $km^{-1} sr^{-1}$ using logarithmic scale) for the same situation as in Figure 1: (a) CALIPSO observations, (b) the model first guess, (c) 1D-Var analysis using cloud lidar backscatter alone and (d) in combination with cloud radar reflectivity.

3.1 Methodology

The simple diagram (Figure 5) provides the schematic description of 1D+4D-Var assimilation. In the first step, a 1D-Var technique is used to assimilate CloudSat reflectivity and/or CALIPSO cloud backscatter. The 1D-Var assimilation is performed with the aim of adjusting the model temperature and specific humidity profiles. The second step consists in the assimilation of specific humidity (q) and temperature (T) profiles retrieved from 1D-Var into the 4D-Var system.

The observation errors for T and q pseudo-observations correspond to the 1D-Var retrieval errors which depend on the background error assumed for the 1D-Var control variables, the observation errors (for reflectivity and backscatter) and the accuracy of the observation operator used by the 1D-Var system. As originally shown by Rodgers (2000) and later used in 1D+4D-Var approaches described by Bauer *et al.* (2006b); Benedetti *et al.* (2006); Lopez and Bauer (2007) or Janisková *et al.* (2012), the errors on the retrieved variables can be derived directly from the 1D-Var analysis error covariance matrix, **A**, as:

$$\mathbf{A} = [\mathbf{B} + \mathbf{K}^{\mathrm{T}}(\mathbf{x}) \, \mathbf{R}^{-1} \, \mathbf{K}(\mathbf{x})]^{-1}$$
⁽²⁾



Figure 3: Analysis increments for (a, b) specific humidity in $g kg^{-1}$ and (c, d) temperature in K from 1D-Var assimilation of: (a, c) CloudSat reflectivity and (b, d) CALIPSO lidar backscatter. Situation over the Pacific Ocean on 24 January 2007.

where $K = \left[\frac{\partial H(\mathbf{x})}{\partial \mathbf{x}}\right]$ stands for the Jacobian matrix formed by the partial derivatives of the 1D-Var observation operator H with respect to the control variable \mathbf{x} . This error computation could be influenced by the problem of possible non-linearity of the observation operator. However, an evaluation of the 1D-Var convergence performance indicated that the linearity of the observation operator is ensured for the majority of cases. Experiments have also been performed with the errors twice as large as computed in order to have errors closer to those for radiosonde T and q measurements.

3.2 Results

Several 1D+4D-Var experiments have been run over one assimilation cycle of 12-hours (the current length of 4D-Var assimilation window at ECMWF) assimilating T and q pseudo-observations retrieved from 1D-Var. These pseudo-observations have been added to the full system of regularly assimilated observations by the ECMWF system. From obtained 4D-Var analyses, 10 day forecasts have been run.



Figure 4: Difference of (a) CloudSat radar reflectivity rms errors (in dBZ) and (b) CALIPSO lidar backscatter rms errors (in 1000 km⁻¹ sr⁻¹) in terms of the differences between the first guess (FG) departures and analysis (AN) departures when assimilating cloud radar reflectivity (black solid line) and lidar backscatter either separately (red solid line), or in combination (green dotted line). Numbers on the right side of (a) and (b) indicate the average number of observations included in the statistics. (c) Bias (in black) and standard deviation (in red) of the FG (dashed bar) and AN (solid filled bar) departures from MODIS cloud optical depth. Results are displayed for the 12-hour period from 23 January 2007 21:00 UTC to 24 January 2007 09:00 UTC.



Figure 5: Schematic description of 1D+4D-Var system for assimilation of cloud related observations.

Verification of the assimilation runs has been carried out against other assimilated observation types in 4D-Var. The results indicated (not shown here) that mainly when verified against conventional observations (such as TEMP radiosonde, PILOT or AIREP observations) there is some reduction in bias of the analysis departures, but standard deviations are systematically larger in the experimental runs compared to the reference one. This indicates that the new cloud related observations bring more variability into the system which may be related to the fact that clouds often represent small scale features. Small, but systematic improvements coming from the lidar observations when combined with the radar have also been noticed. For all other types of observations assimilated in 4D-Var, no significant changes were found. Achieving a significant improvement between the experimental runs and the reference run over a domain well covered by a large amount of other measurements is always a big challenge. Therefore any improvement is encouraging since it indicates a potential benefit from assimilating cloud information.

The impact on the subsequent forecasts started from the 1D+4D-Var analyses obtained by assimilating pseudo-observations retrieved from 1D-Var with the cloud radar and lidar observations over the whole 12-hour assimilation window has also been assessed. An evaluation has been done for temperature, specific humidity and wind by considering differences in root-mean-square forecast errors between experiments and the reference run (computed with respect to the operational analysis). Zonal means of these rms error differences are shown in Figure 6 a,c,e for 24-hour forecast. Generally, errors are reduced in the experimental runs compared to the reference run. Even though the positive impact of the new assimilated observations on the subsequent forecast decreases in time, it is still noticeable up to 48-hour forecast (Figure 6 b,d,f). Small, but systematic improvements coming from the lidar observations when combined with the radar are also observed.

4 Conclusions and perspectives

The performed studies have provided indications on the potential which assimilation of cloud observations from space-borne radar and lidar instruments could offer. The feasibility to assimilate such observations has triggered desirability to use them. However, there are certain requirements/constraints to be fulfilled in order to succeed. One of them is availability of accurate enough observation operators. When model is not accurate (realistic enough), the model equivalents would be too far from observations to be assimilated. Another requirement is a linearity of observation operator to be used in the variational assimilation which is based on the strong assumption that the analysis is performed in quasi-linear framework. Without treatment of most serious threshold processes, the tangent-linear and adjoint model can turn to be useless as demonstrated for instance in Janisková and Lopez (2013). In addition, for a proper handling of observations in the context of an assimilation system, an appropriate quality control strategy and bias correction scheme are required. An observation error definition accounting for spatial representativeness of observations with the rather narrow horizontal field-of-view is also important. An example in Figure 7 shows that in the case of 1D-Var analysis with lidar backscatter, the analysis only gets closer to the independent CloudSat radar reflectivity observations when the above mentioned treatment of lidar observations is applied.

Though the first results from cloud radar and lidar assimilation are encouraging, to achieve the full benefit from these observations in an operational context will still require a substantial amount of work. More experiments and statistical evaluation of the model equivalents to the observations need to be done for different situations to refine data quality control and error definition. The used 1D+4D-Var assimilation approach requires to define errors for pseudo-observations retrieved from 1D-Var. Their computation from the 1D-Var analysis covariance matrix is quite expensive for profiling observations and only affordable for non-operational application. Therefore for any future operational implementation, the use of a direct 4D-Var assimilation of cloud related observations should be considered.

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Figure 6: Differences of (a, b) specific humidity (in $g kg^{-1}$), (c, d) temperature (in K) and (e, f) wind (in $m s^{-1}$) rms errors for the differences between forecasts starting from analysis created by 4D-Var assimilation of T, q pseudo-observations and the operational analysis, and between the forecast starting from the reference analysis and the operational analysis. (a,c,e) zonal mean of rms errors for 24-hour forecasts from experiments with cloud radar reflectivity alone (reduction, resp. increase of rms errors for the experimental run is shown with blue, resp. red shadings). (b,d,f) rms errors presented as global values for 12-, 24- and 48-hour forecasts from experiments with cloud radar reflectivity alone (ashed line) or in combination with lidar backscatter (solid line).



Figure 7: Difference of (a) CALIPSO lidar backscatter rms errors (in 1000 km⁻¹ sr⁻¹) and (b) CloudSat radar reflectivity rms errors (in mm⁶ m⁻³) for differences between the first guess (FG) departures and analysis (AN) departures when assimilating observations of cloud lidar backscatter. Results are shown for the experiments with rough quality control, no bias correction and simple observation error definition (**nreg**), with applied bias correction and quality control (**qcbc**) on top of **nreg** and with complex observation error definition (**qcbcer**) applied to **qcbc**. Colour numbers on the left side of (a) and (b) indicate number of observations considered for statistics by the different experiments for the situation over the Pacific Ocean on 24 January 2007.

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M. JANISKOVÁ: PROSPECT FOR RADAR AND LIDAR CLOUD ASSIMILATION

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