

**Workshop on parameter estimation and inverse modelling
for atmospheric composition**
22 to 24 October 2013

Estimating emission rates of reacting constituents by variational inversion

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2. specific problems
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Data assimilation and Inverse Modelling: What makes the difference?

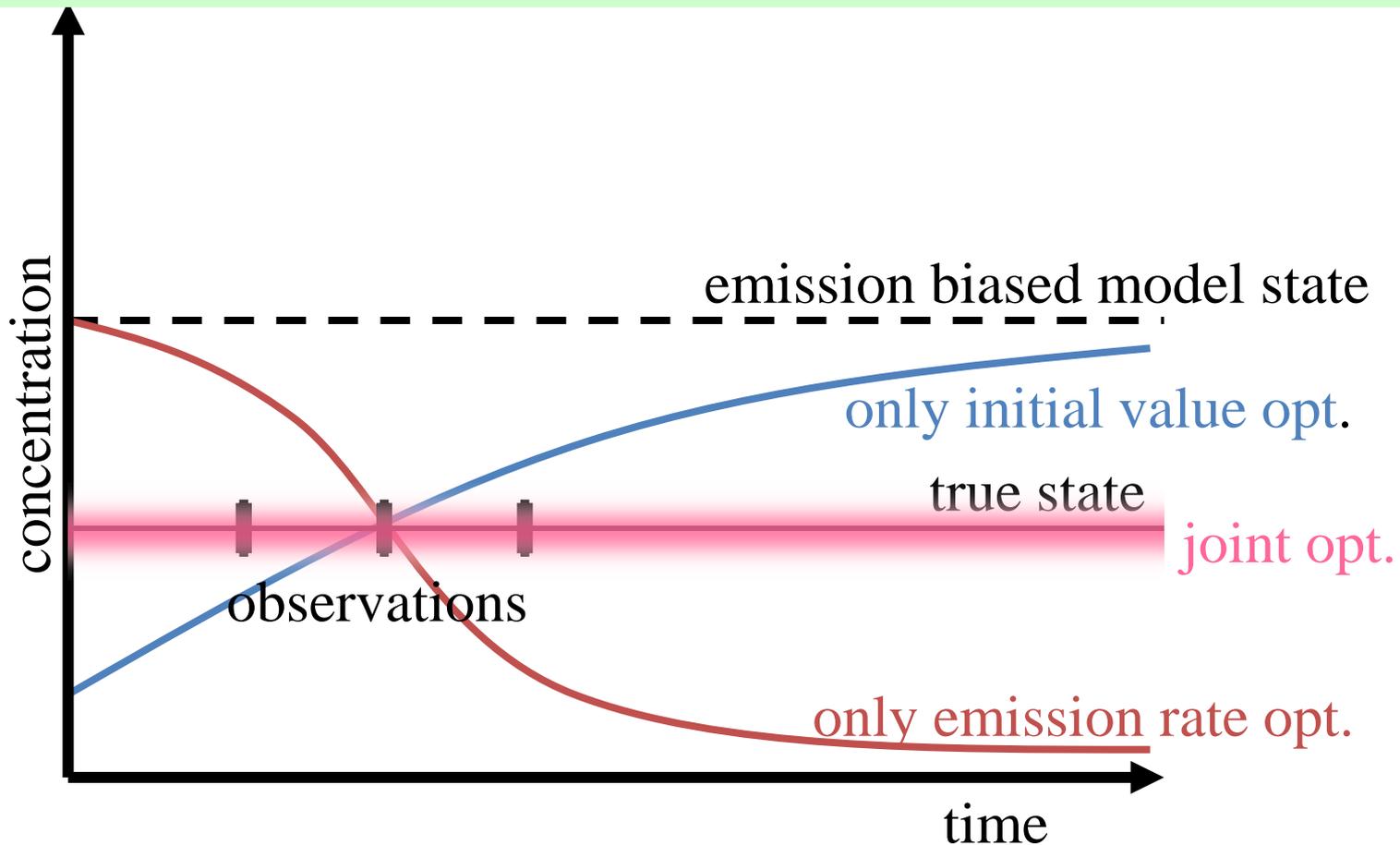
- **Data assimilation:** estimate
 - transient parameters:
 - prognostic parameters, state variables of prognostic systems
 - **for reactive chemistry: initial values**
 - **Inverse Modelling:** estimate
 - intransient system control parameters,
 - forcing parameters ,
 - diagnostic parameters
 - **for reactive chemistry: emission rates, (deposition velocities)**
- time and spatial scales carefully to be considered,
if combined**

Problem formulation (increasingly ill-posed)

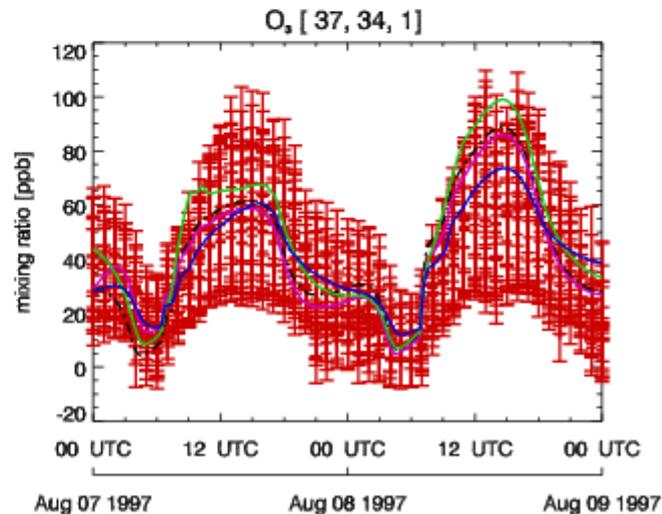
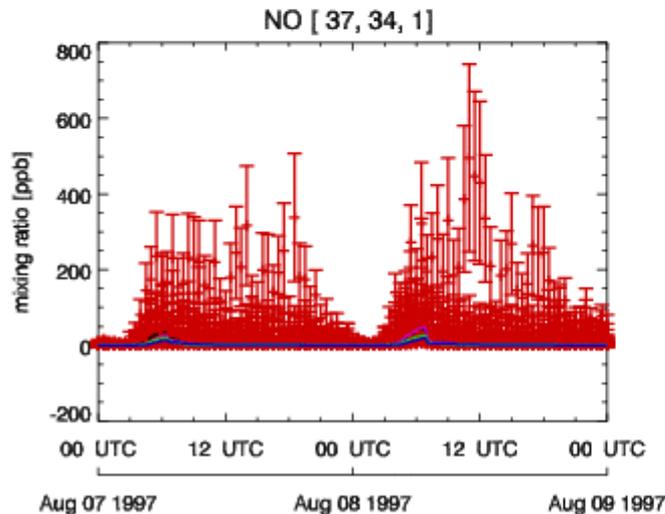
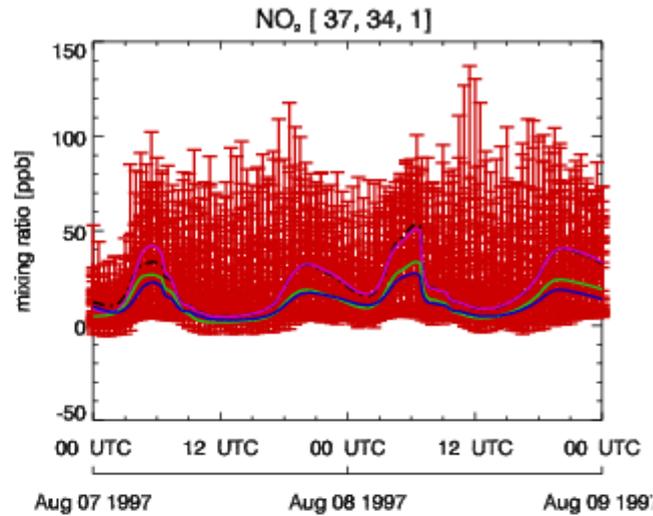
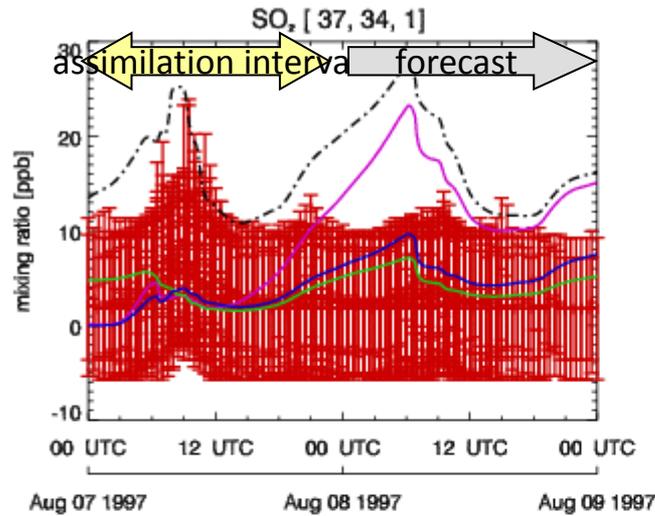
- which parameter sets are to be optimized at all, and are the observations available?
- poor representativity of observations
- often only integral measurements available
(NB: PM_x , $NO_{x/y}$, column soundings from satellites)
- reactive chemistry, and its observability
(with poorly observed emitted compounds)

SPECIFIC PROBLEMS

In the troposphere, for **emission rates**, the product (*paucity of knowledge* * *importance*) is high



The representativity problem (1):
 non-Gaussian distribution example:
 Rhine-Main area box: (Frankfurt-Mainz)
 9.-10. August 1997



+ observations
 no optimisation

initial value opt.

emis. rate opt.

joint emis +
 ini val opt.

Incremental Formulation

- Analysis State:
$$\mathbf{x}^a = \mathbf{x}^b + \delta \mathbf{x}^a$$
$$\mathbf{u}^a = \mathbf{u}^b + \delta \mathbf{u}^a$$

- New „State“ Variables:
$$\mathbf{v} = \mathbf{B}^{-1/2} \delta \mathbf{x}$$
$$\mathbf{w} = \mathbf{K}^{-1/2} \delta \mathbf{u}$$

- Cost Function:

$$J(\mathbf{v}, \mathbf{w}) = \frac{1}{2} \mathbf{v}^T \mathbf{v} + \frac{1}{2} \mathbf{w}^T \mathbf{w} + \frac{1}{2} [\mathbf{H} \delta \mathbf{x}_i - \mathbf{d}_i]^T \mathbf{R}^{-1} [\mathbf{H} \delta \mathbf{x}_i - \mathbf{d}_i]$$

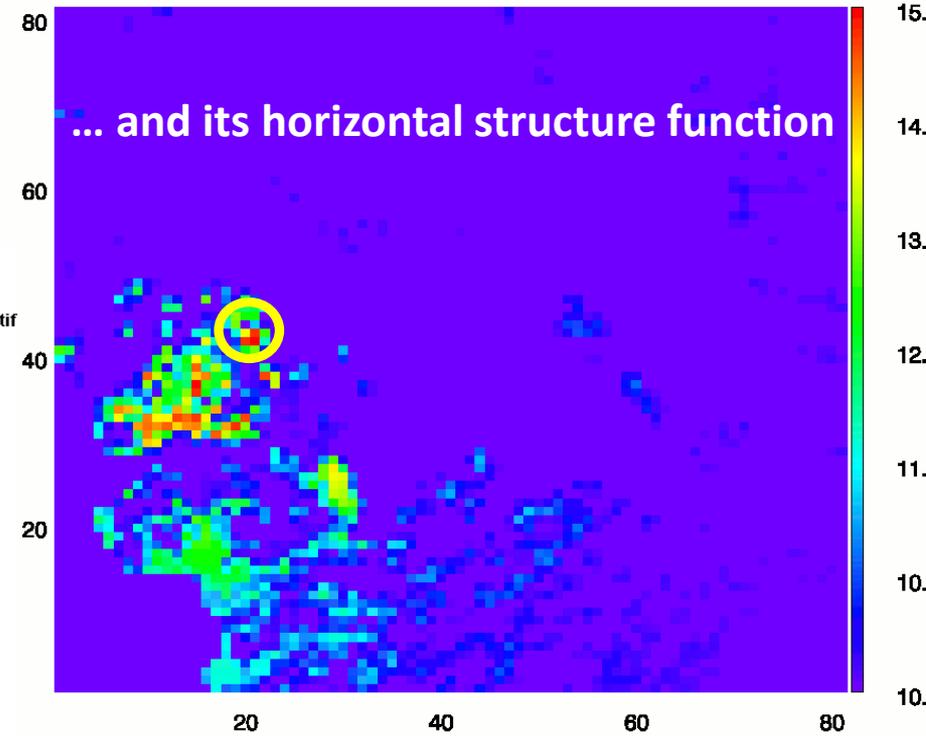
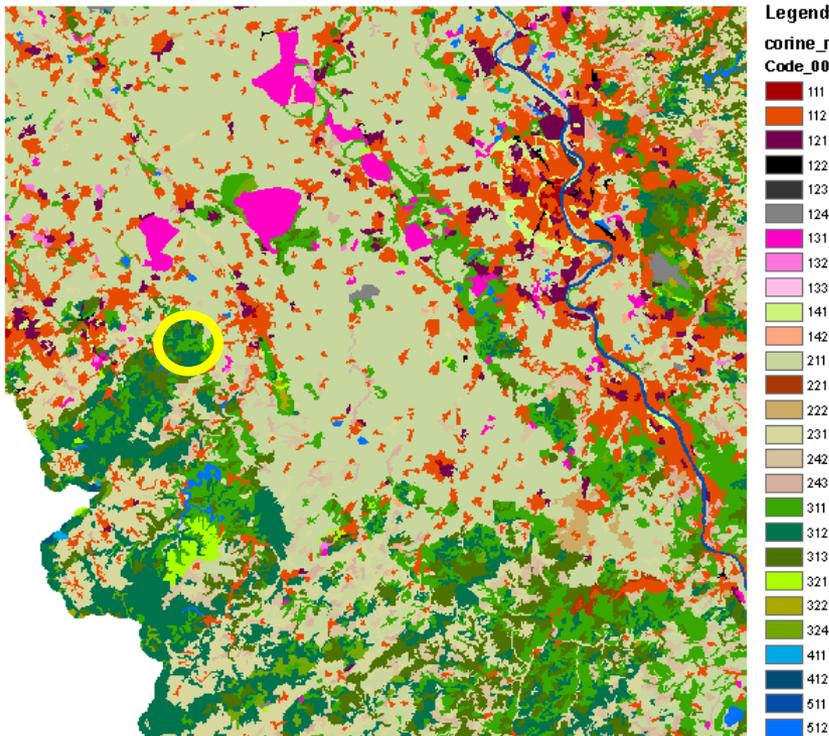
- Gradient:

$$\nabla_{\mathbf{v}} J = \nabla_{\mathbf{v}} J_{IV} + \nabla_{\mathbf{v}} J_O = \mathbf{v} + \mathbf{B}^{T/2} \nabla_{\delta \mathbf{v}} J_O$$
$$\nabla_{\mathbf{w}} J = \nabla_{\mathbf{w}} J_{EF} + \nabla_{\mathbf{w}} J_O = \mathbf{w} + \mathbf{K}^{T/2} \nabla_{\delta \mathbf{w}} J_O.$$

The representativity problem (2):

Tessellated terrain and associated covariances

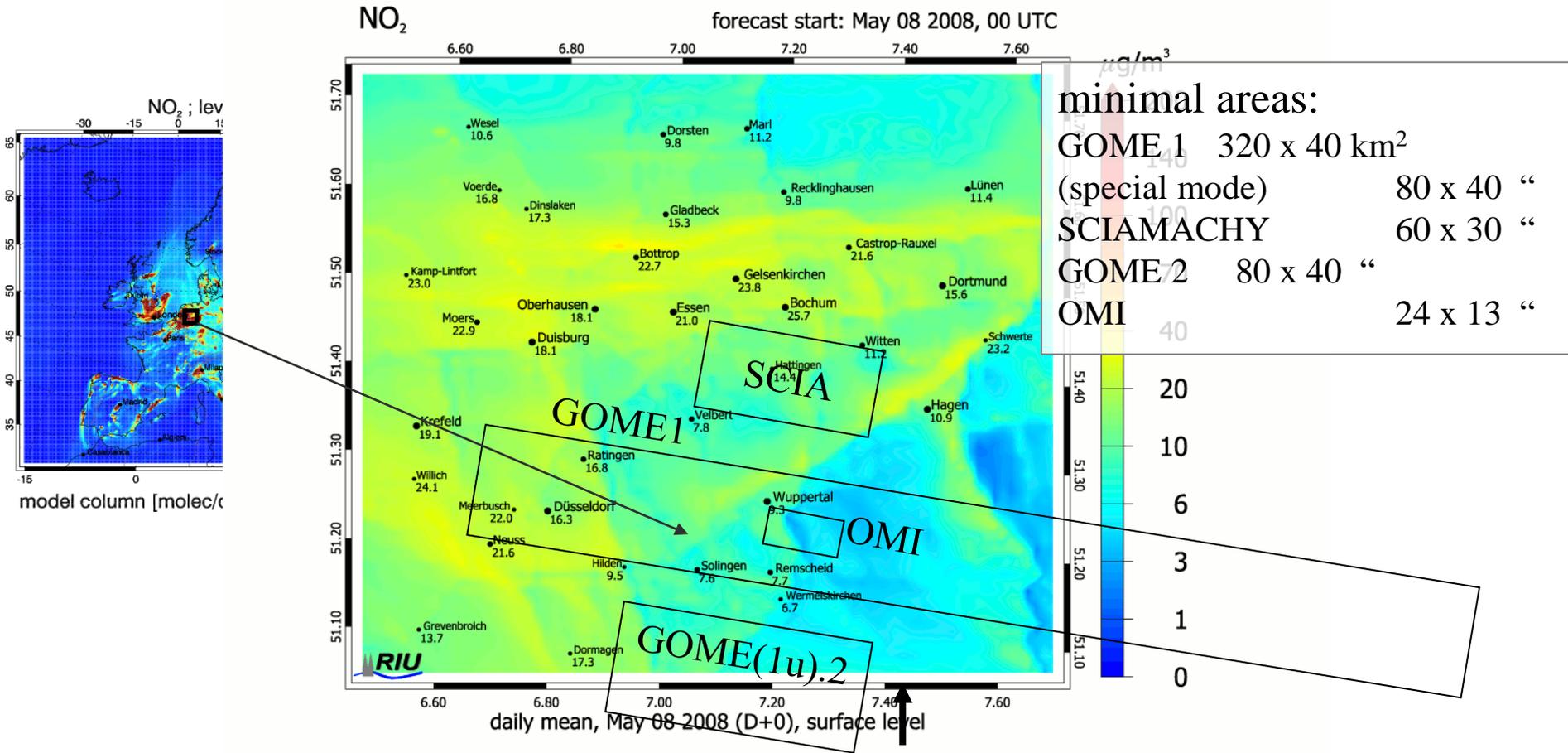
High resolution CORINE land use of the Cologne Aix-la-Chapelle area with a forest harboured station 



For high resolution, correlation functions need to bridge gaps

Integral information - Satellite data:

ESA UV-VIS satellite footprints Ruhr area comparison

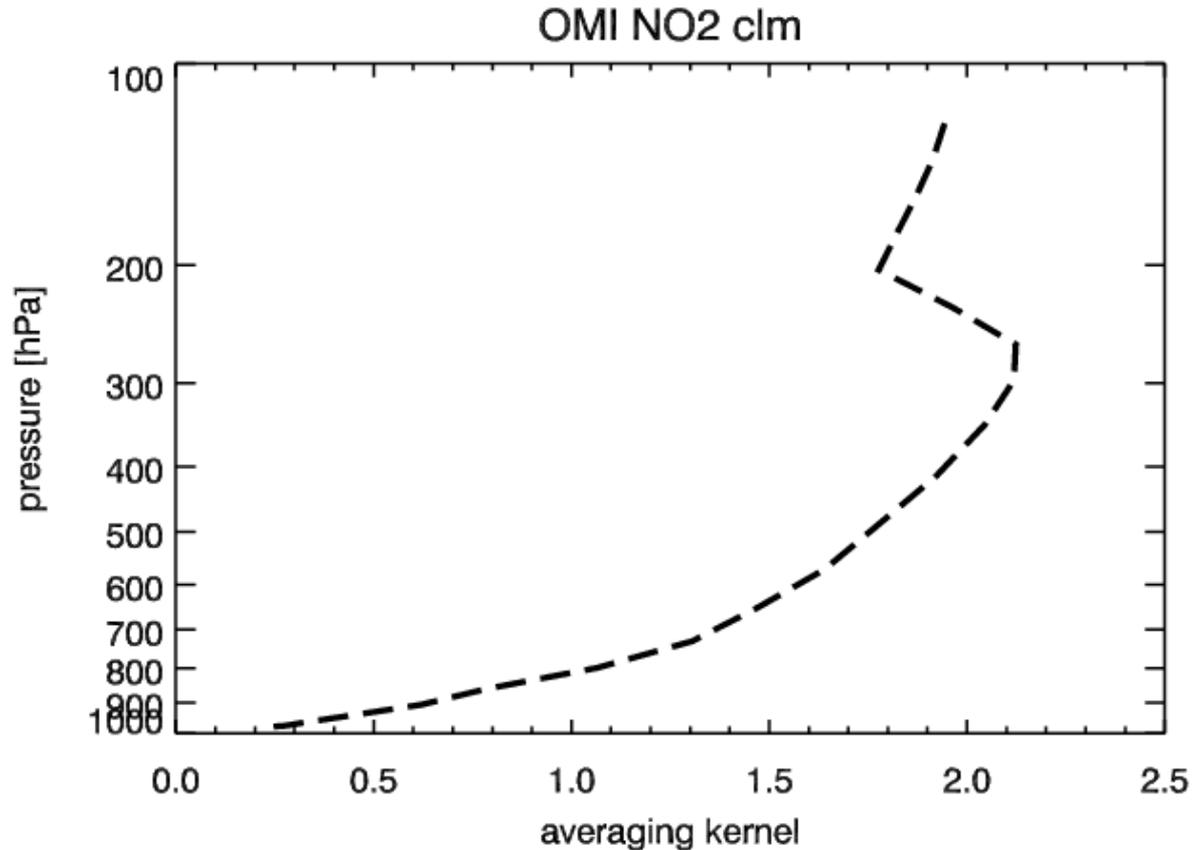


Ruhr area 90 km² x 80 km²
(~12 000 000 inhabitants)

Integral information –

Do NO₂ columns provide useful information?

*Average OMI averaging kernel profile
over model domain for July 9th 2006*



model domain mean averaging kernel.

Integral information – Exploitation of NO₂ column averaging kernel information

- AK profile mostly dependent on optical properties of the atmosphere (cloud cover), rather than NO₂
- typical maximal sensitivity **above** the boundary layer
- **does not allow** a clear distinction between PBL or lower free troposphere pollution burden

Integral information – Observation operator \mathbf{H}

Formally an integral equation to be solved for vertical NO_2 molecule density function x (σ vertical coordinate)

$$y = \int_1^0 w(\sigma)x(\sigma)d\sigma$$
$$y = \sum_{k=1}^K h_k x_k$$

At the minimum $\mathbf{x} =: \mathbf{x}_a$

$$d\mathbf{x}_a := \mathbf{x}_a - \mathbf{x}_b = (\mathbf{B}_0^{-1} + \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{R}^{-1} \{ \mathbf{y}^0 - H[\mathbf{x}_b] \}$$
$$= \mathbf{B} \mathbf{H}^T (\mathbf{R} + \mathbf{H} \mathbf{B} \mathbf{H}^T)^{-1} \{ \mathbf{y}^0 - H[\mathbf{x}_b] \}$$

For scalar column retrieval:

$$d\mathbf{x}_a^c = \underbrace{\mathbf{B} \mathbf{h}^T}_{(r+b)} (r+b)^{-1} \{ y^0 - h[\mathbf{x}_b] \}$$

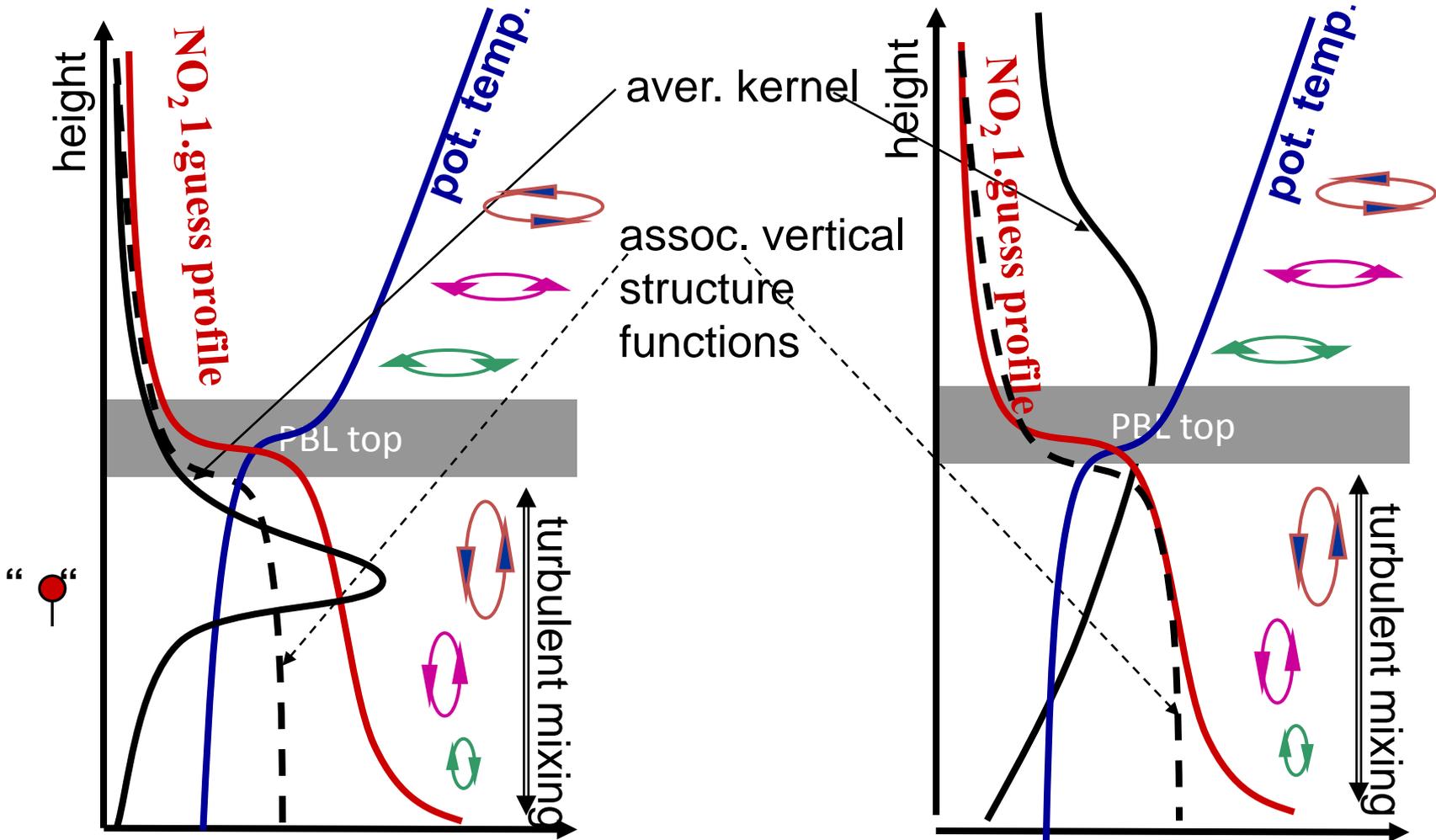
adjoint representer

→ vertical structure function in \mathbf{B} essential!

Vertical structure function:

Extending the information from observation location by vertical exchange of pollutants and information

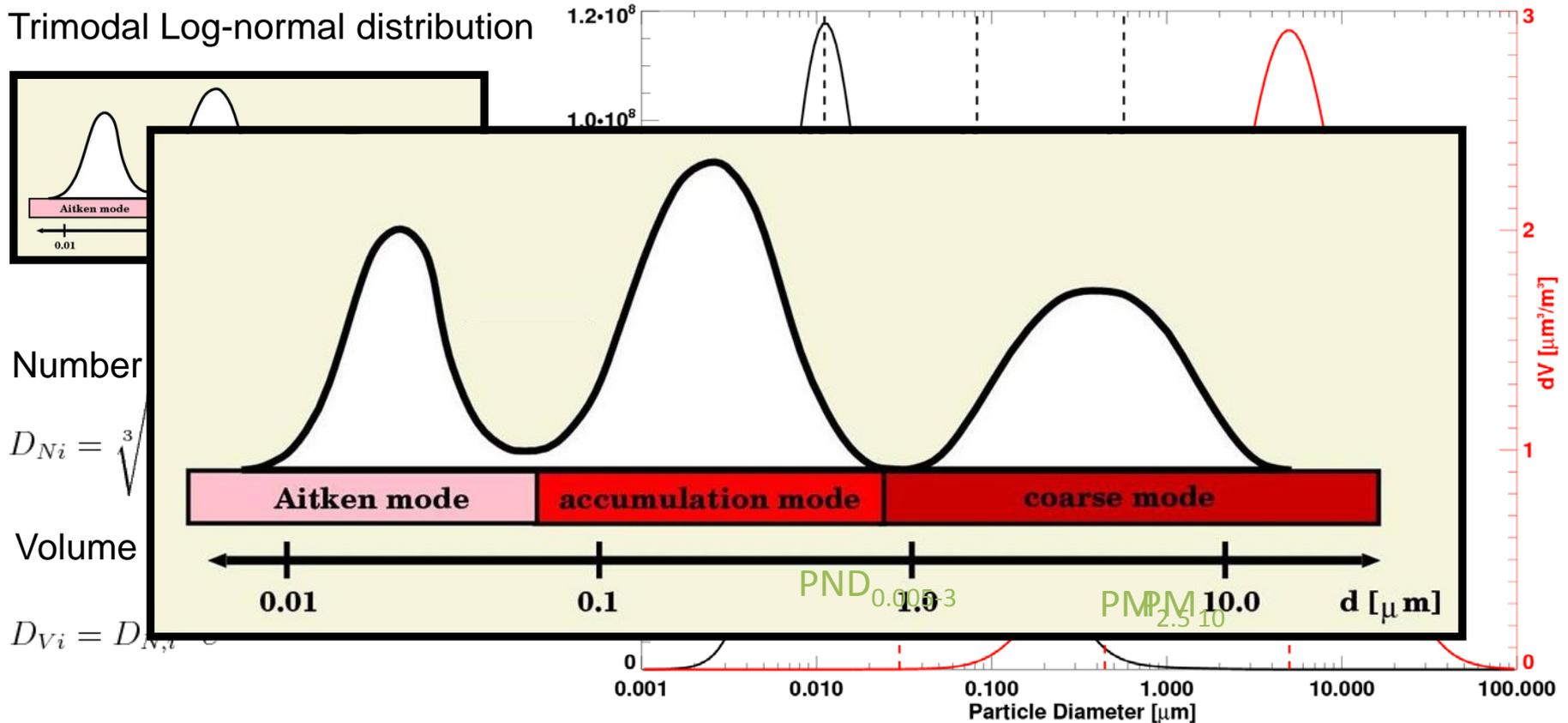
ideal *case* *real*



Integral information – Modal representation of particles

Particulate Matter (PM_x) and Particle Number Density ($PND_{0.005-3.0}$)

Trimodal Log-normal distribution



Mass/Number
Concentrations

Integrator \rightarrow
 \leftarrow Adjoint Integrator

PM_x / PND

Integral information -

Example:

Aerosol Chemistry in
MADE

Modal Aerosol Dynamics for
EURAD/Europe
(Ackerman et al., 1998, Schell
2000)

$$dM_i^k/dt = \text{nuk}_i^k + \text{coag}_{ij}^k + \text{coag}_{ji}^k + \text{cond}_i^k + \text{emi}_i^k$$

M_i^k : = k^{th} Moment of i^{th} Mode

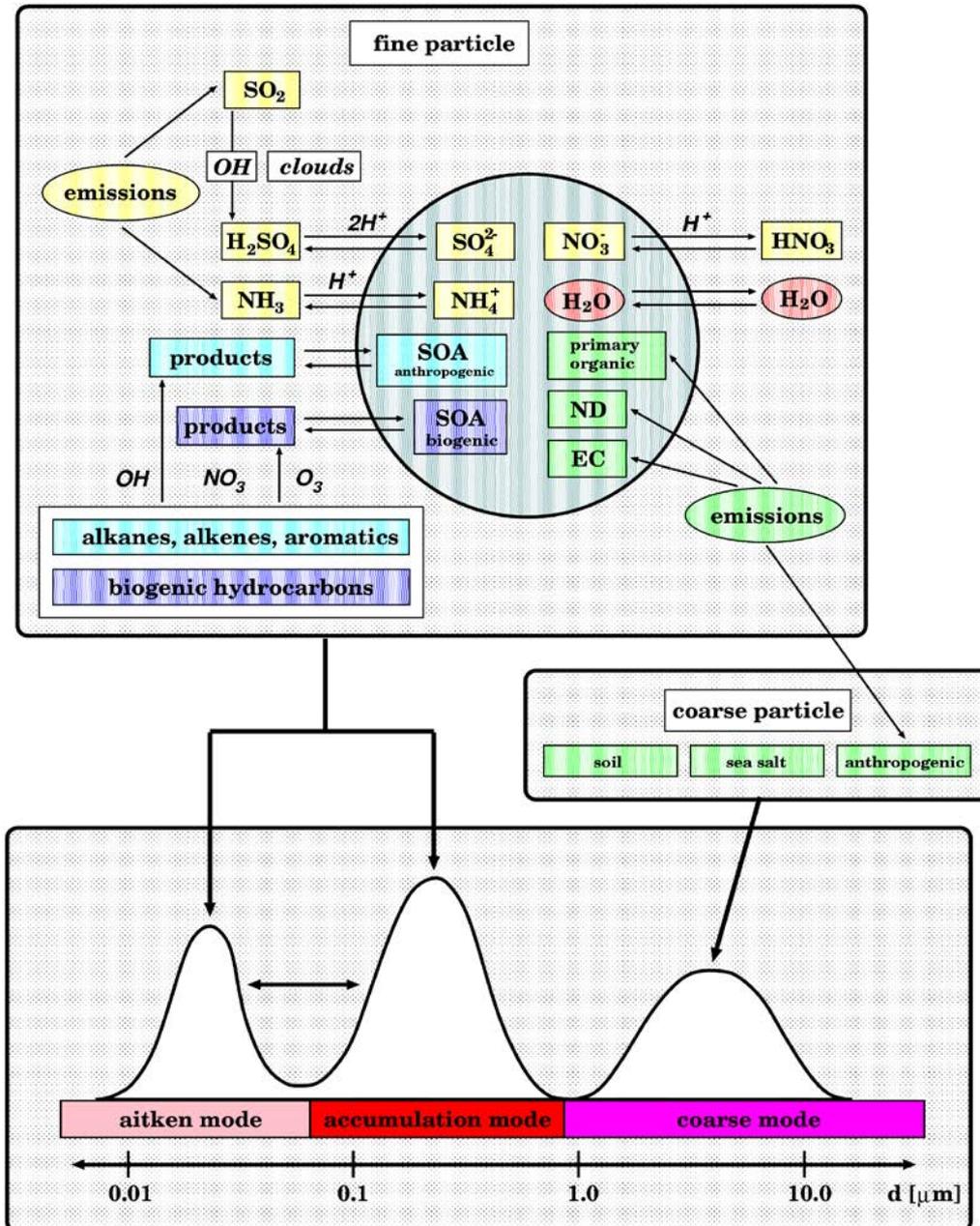
assimilation of aerosol

By satellite retrievals: e.g.

Bridge from optical to chemical
properties

MERIS MODIS

AATSR+SCIAMACHY
...



SYNAER retrieval algorithm

Species Mapping

EURAD-IM [$\mu\text{g}/\text{m}^3$]		SYNAER - AOT
SO ₄ , NH ₃ , NO ₃ , H ₂ O, SOA		WASO (WATER SOLuble)
Unidentified PM		INSO (water INSOLuble)
Elemental Carbon		SOOT
Sea Salt		SEAS
Mineral Dust		DUST

radiative transfer model



adjoint radiative transfer model

Further significant problems, not considered here

- Meteorological driver model with boundary layer, convection, and precipitation
- chemistry model errors
- deposition velocities
-

ISSUES ON OBSERVABILITY

Observability ad a simple forcing model

Consider two one-way coupled linear models:

$A(t_i, t_0)$ atmospheric model,
 $E(t_i, t_0)$ emission model, and
 C the coupling between both.

$$\begin{pmatrix} H(t_i), & 0 \end{pmatrix} \begin{pmatrix} x(t_i) \\ e(t_i) \end{pmatrix} = \begin{pmatrix} H(t_i, t_0), & 0 \end{pmatrix} \underbrace{\begin{pmatrix} A(t_i, t_0), & C \\ 0, & E(t_i, t_0) \end{pmatrix}}_{:=M(t_i, t_0)} \begin{pmatrix} x(t_0) \\ e(t_0) \end{pmatrix}$$

$M(t_i, t_0)$ model integration from time t_0 to t_i

$H(t_i)$ observation operator at time t_i

$x(t)$ atmospheric model state,

$e(t)$ emission parameter.

Observability

$H(t_i)$ observation operator at time t_i

$$O_v := \begin{bmatrix} H(t_0) \\ H(t_1)M(t_1, t_0) \\ H(t_2)M(t_2, t_0) \\ \vdots \\ H(t_v)M(t_v, t_0) \end{bmatrix}$$

Consider rank of matrix, Singular Vectors and -Values of

$$O_v^T O_v$$

to be above some noise level.

How can we identify the degree of observability?

Given CTM (here RACM and EURAD-IM) acting as tan.-lin. model operator \mathcal{L} :

$$\delta \mathbf{c}(t_F) = \mathcal{L}_{t_I, t_F} \delta \mathbf{c}(t_I), \quad \mathcal{L}_{t_I, t_F} = \left. \frac{\partial \mathcal{M}_{t_I, t_F}}{\partial \mathbf{c}} \right|_{\mathbf{c}(t_I)}$$

1. Berliner et al., (1998) Statistical design:
 “Minimize” the analysis error covariance matrix \mathbf{A} (say, via trace):

$$\min_{\mathbf{H}} \mathbf{A} = \mathbf{B} - \underbrace{\mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1} \mathbf{H} \mathbf{B}}_{\text{to be maximized by } \mathbf{H}}$$

For this find maximal eigenvectors as observation operators \mathbf{H} , which configure observations.

$$\mathcal{L}_{t_I, t_F} \mathbf{B} \mathcal{L}_{t_I, t_F}^T \mathbf{H}^T = \lambda \mathbf{H}^T$$

2. Palmer (1995) Singular vector analysis:
 Observe maximal SV configuration:

$$\max_{\delta \mathbf{c}(t_I)} \frac{\|\delta \mathbf{c}(t_F)\|_{\mathbf{B}}^2}{\|\delta \mathbf{c}(t_I)\|_{\mathbf{B}}^2} = \max_{\delta \mathbf{c}(t_I)} \frac{\delta \mathbf{c}(t_I)^T \mathcal{L}_{t_I, t_F}^T \mathbf{B} \mathcal{L}_{t_I, t_F} \delta \mathbf{c}(t_I)}{\delta \mathbf{c}(t_I)^T \mathbf{B} \delta \mathbf{c}(t_I)},$$

E_{mpirical} **K**_{inetic} **M**_{odel} **A**_{pproach} **scheme** **A prototype non-linearity example:**

- Nitrogen oxides and numerous hydrocarbons act highly nonlinearly as precursors of ozone.
- Chemical conditions are either controlled by NO_x or VOC deficit, delineating the “chemical regime”.
- Both 4D-var and Kalman filter should start with the proper chemical regime.

EKMA diagram
(Empirical Kinetic Model Approach)

NO_x

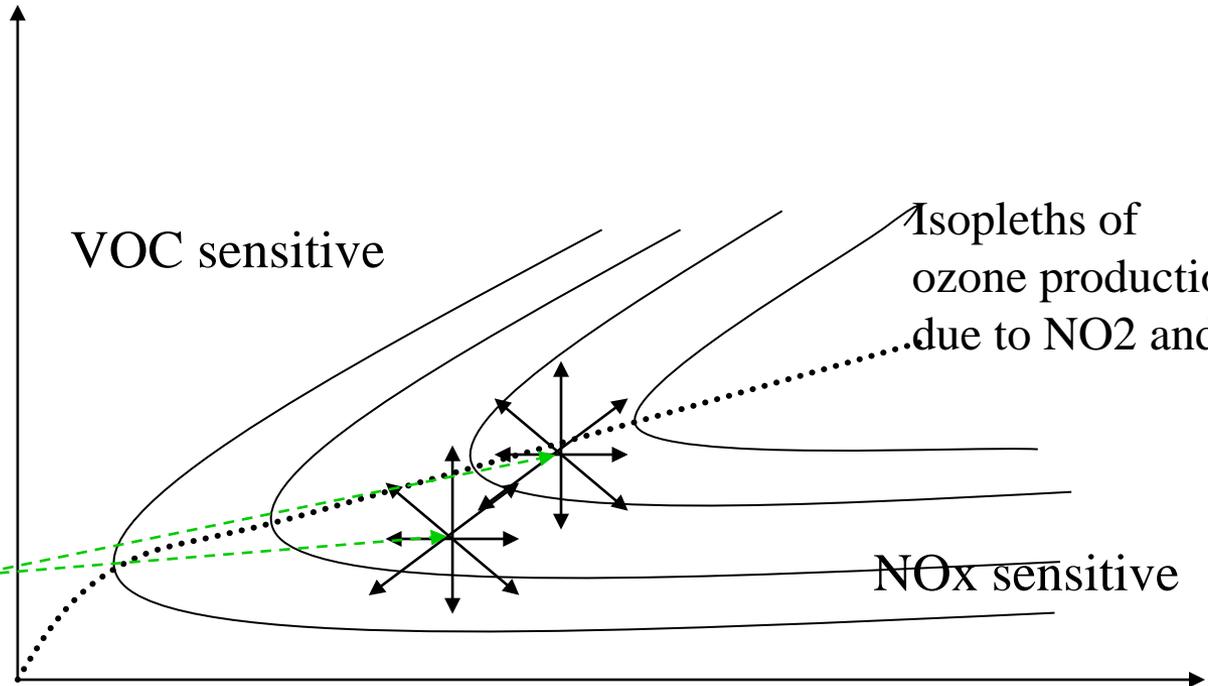
VOC sensitive

Isopleths of ozone production, due to NO₂ and VOC

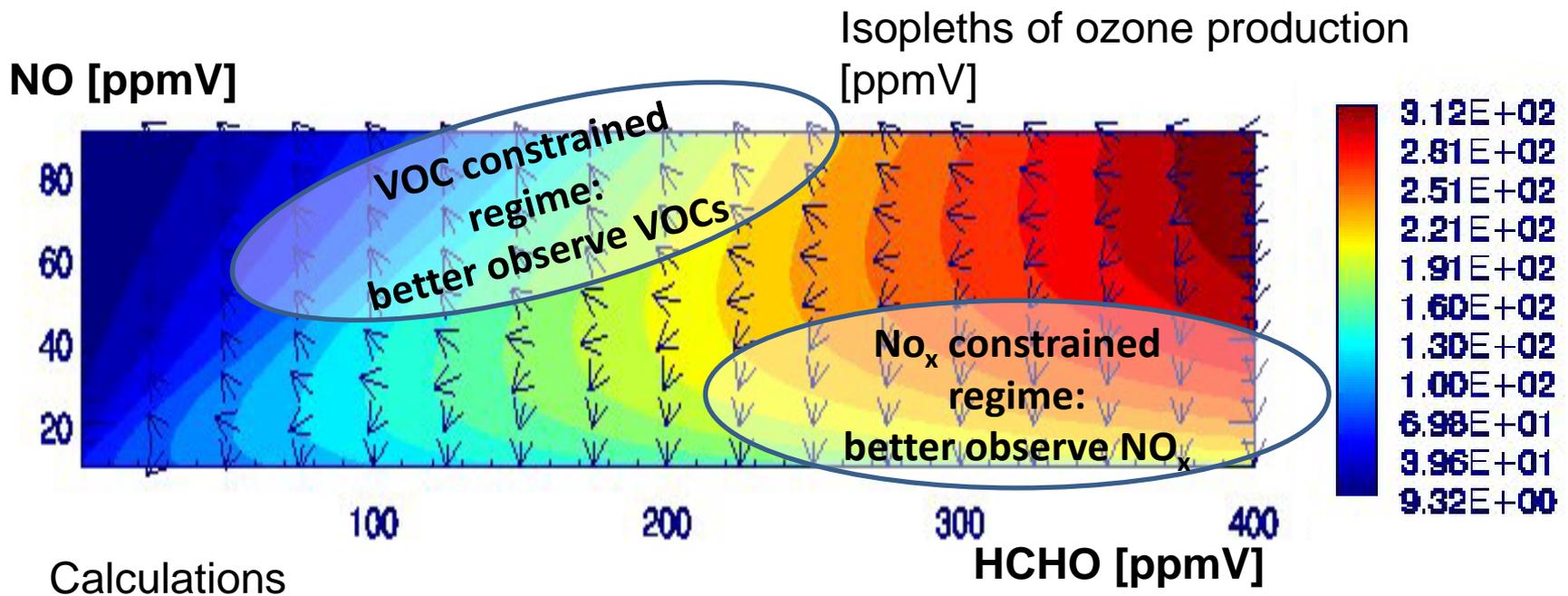
NO_x sensitive

VOC_s

approximate conditions
20.7. and 21.7.

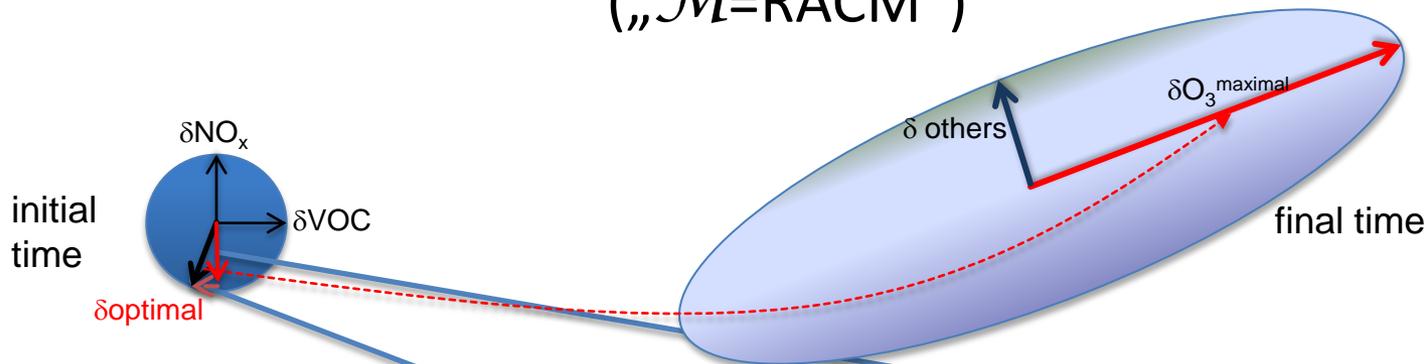


Is NO_x the key to ozone production? And consequently, its observation the key to better forecast?



- ✓ within a fixed time span
- ✓ initial concentrations of NO / HCHO were varied
- ✓ change of final concentration is given by colour
- ✓ gradients (SVs) of maximyl ozone production given by arrows

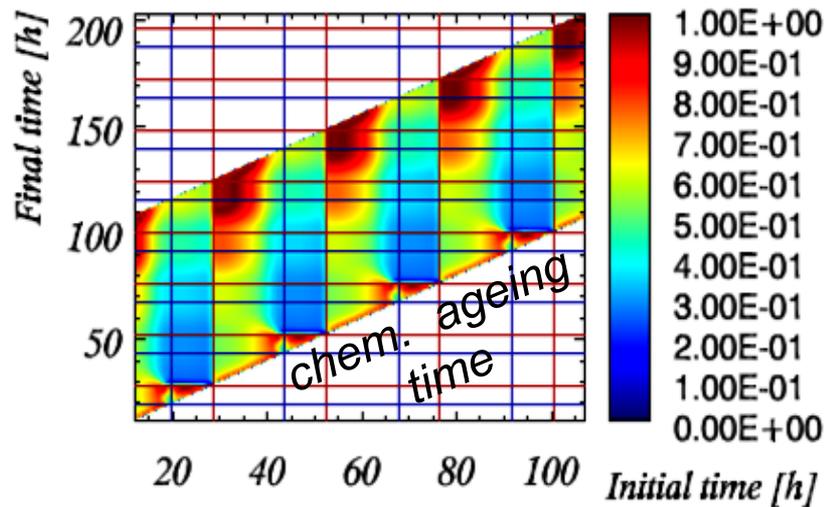
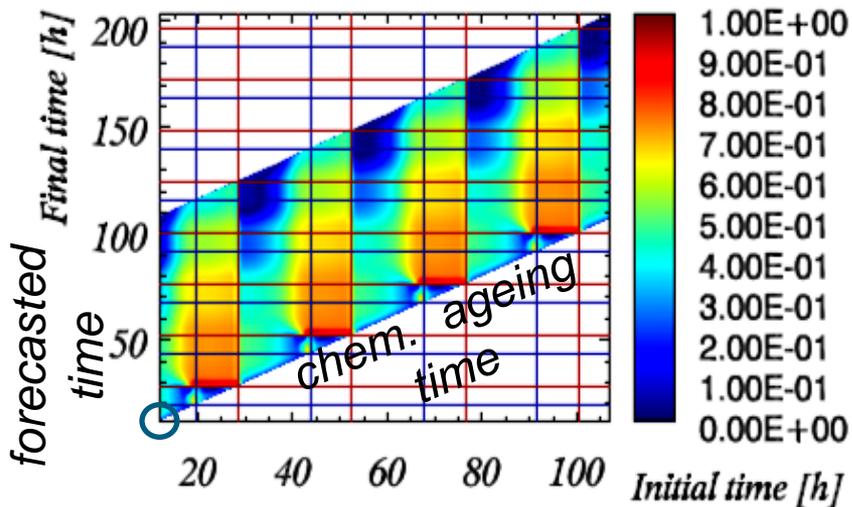
Basic 0-D Regional Atm. Chemistry Mechanism („ \mathcal{M} =RACM“)



- Optimal perturbations (Singular Vectors) for scenario MARINE**

1st Grouped Singular Vectors (δVOC)

1st Grouped Singular Vectors (δNO_x)



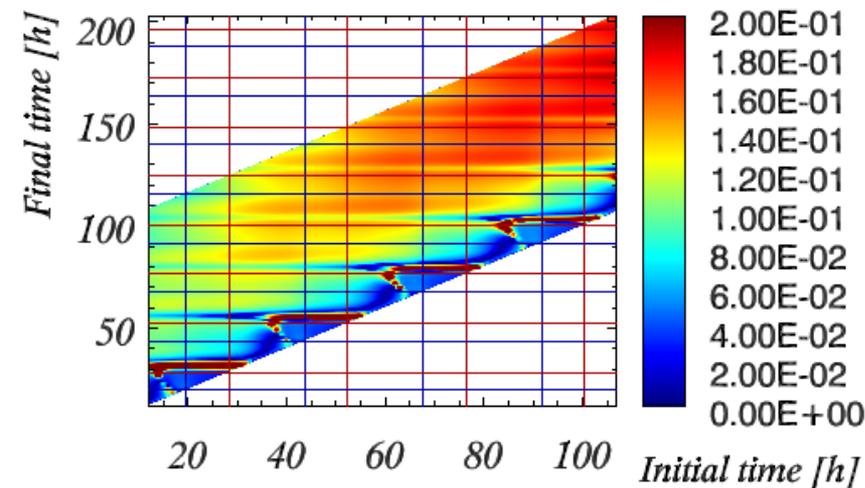
— sunrise — sunset

not | very important to observe

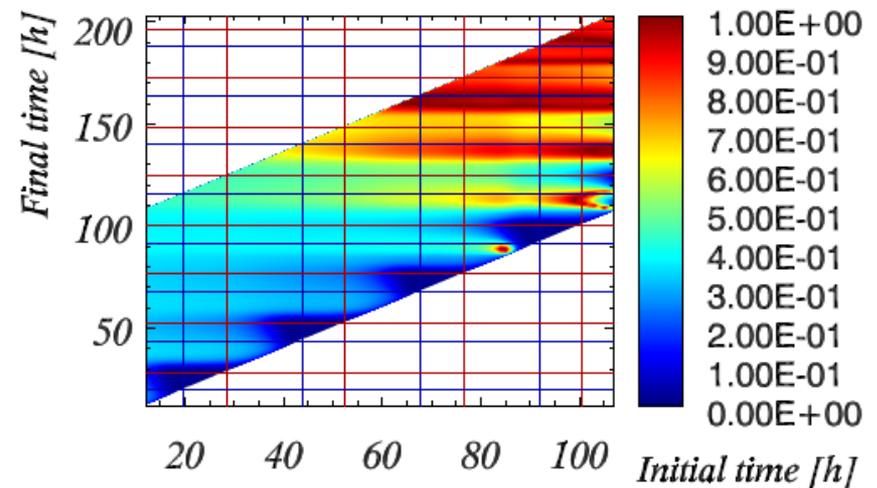
When is the chemical regime sensitive?

Leading **emission** singular vectors for VOCs (vs Nox)

1st Grouped Relative Singular Vectors (VOC)



1st Grouped Relative Singular Vectors (VOC)



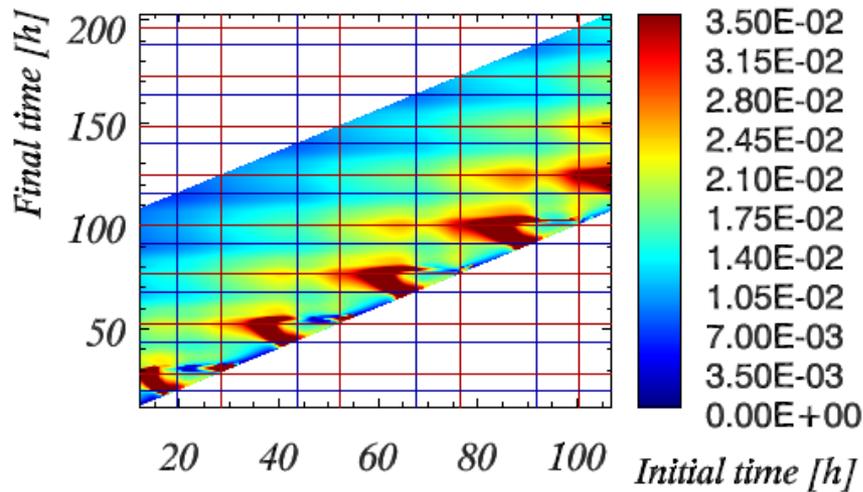
TSVD of the optimal grouped relative singular vectors with respect to emission uncertainties for scenarios **(a)** PLUME and **(b)** URBAN.

Both panels depict VOC-sections of the grouped relative singular vectors.

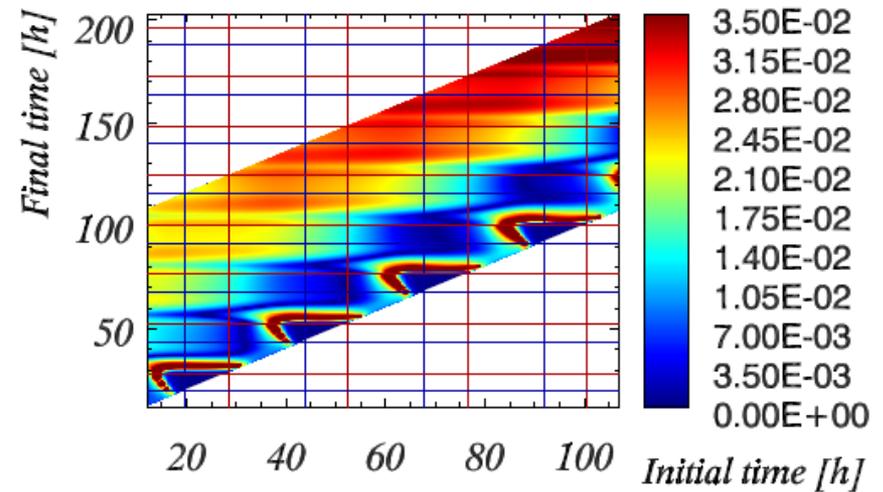
When is the chemical regime sensitive?

Leading **emission** singular vectors
for olefines (left) and lower alkanes (vs Nox)

1st Projected Singular Vectors (OLI)



1st Projected Singular Vectors (HC3)

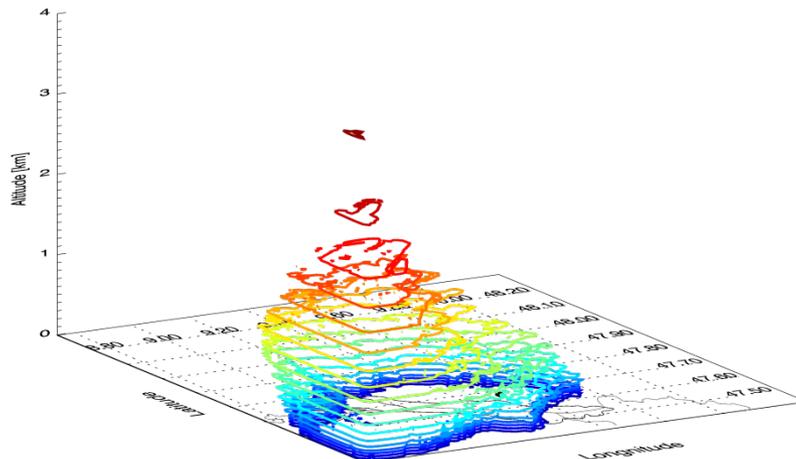
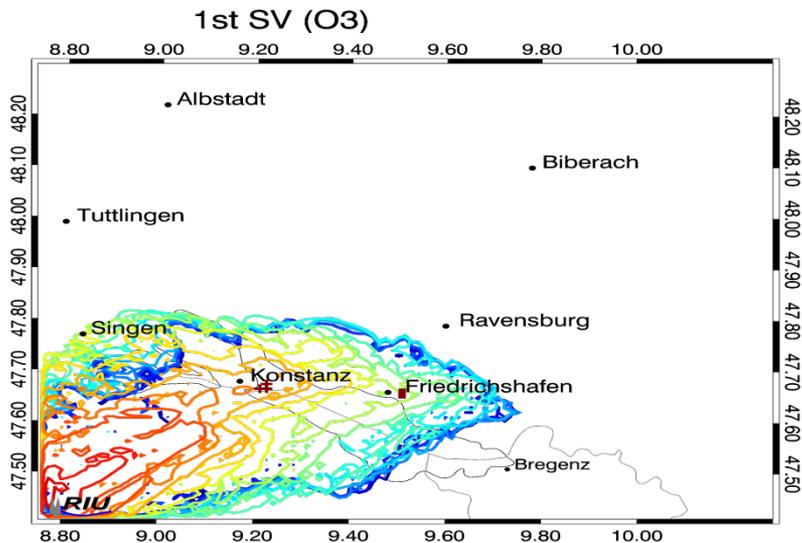
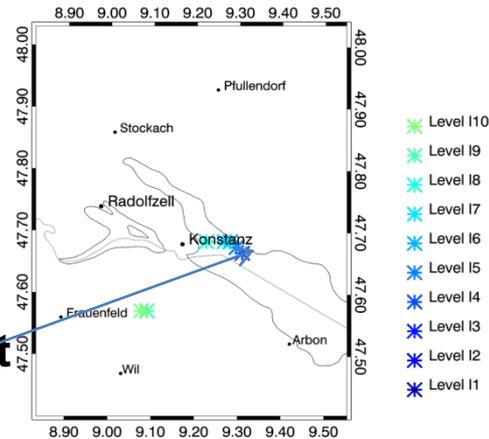


3D-targeted observations for flight missions

Given the need for a forecast at some final time t_F of a mission flight, what and where should be observed at the beginning t_I ?

Airship cruising area
Lake Constance

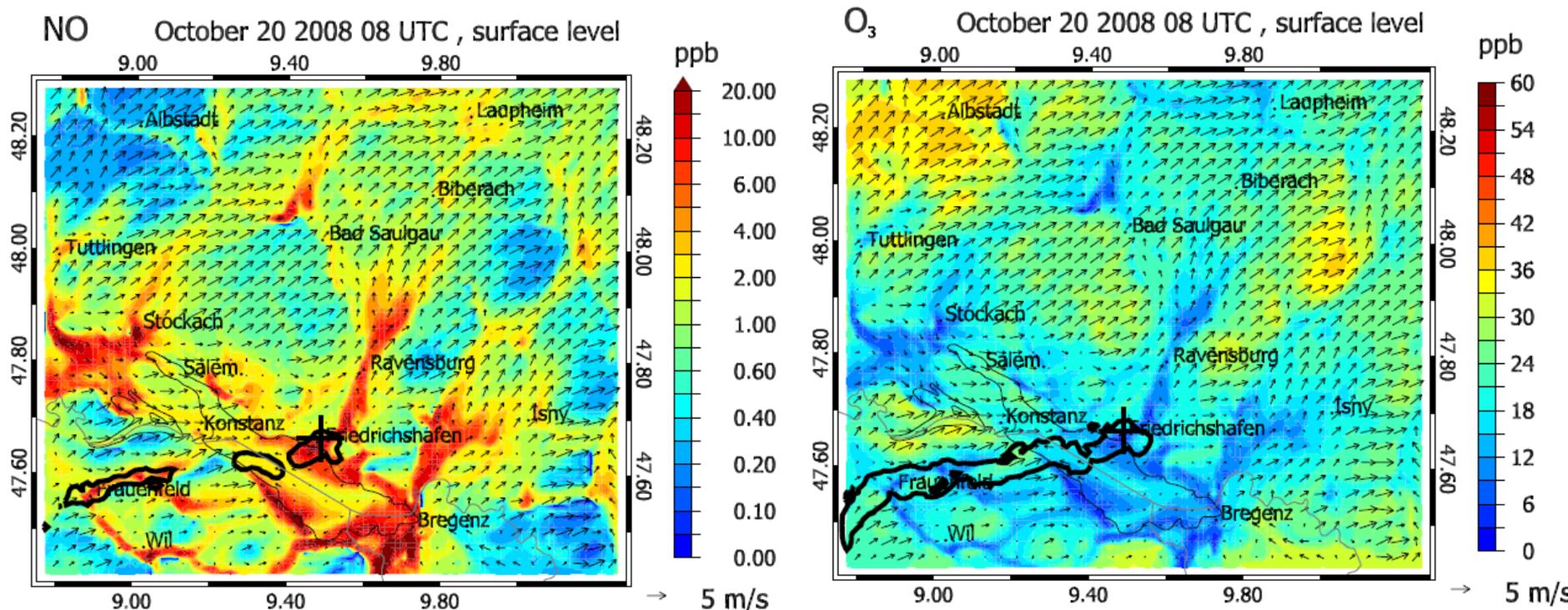
Optimal O_3 forecast
sought for
Friedrichshafen airport



- + flight (12h)
- flight (14h)
- Level 118
- Level 117
- Level 116
- Level 115
- Level 114
- Level 113
- Level 112
- Level 111
- Level 110
- Level 109
- Level 108
- Level 107
- Level 106
- Level 105
- Level 104
- Level 103
- Level 102
- Level 101

October 18 2008 12 UTC (H+02)

Example observation targeting: SV optimal placement of observation sites



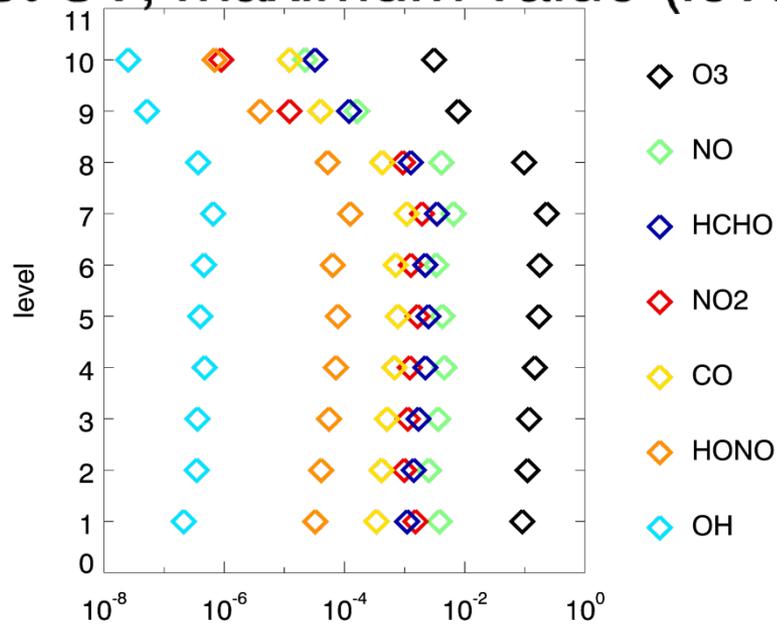
Initial concentrations and optimal horizontal placement of NO (left) and O₃ (right) at surface level . Isopleths of the optimal horizontal placement are indicated with black lines.

What is important to observe for O₃ prediction?

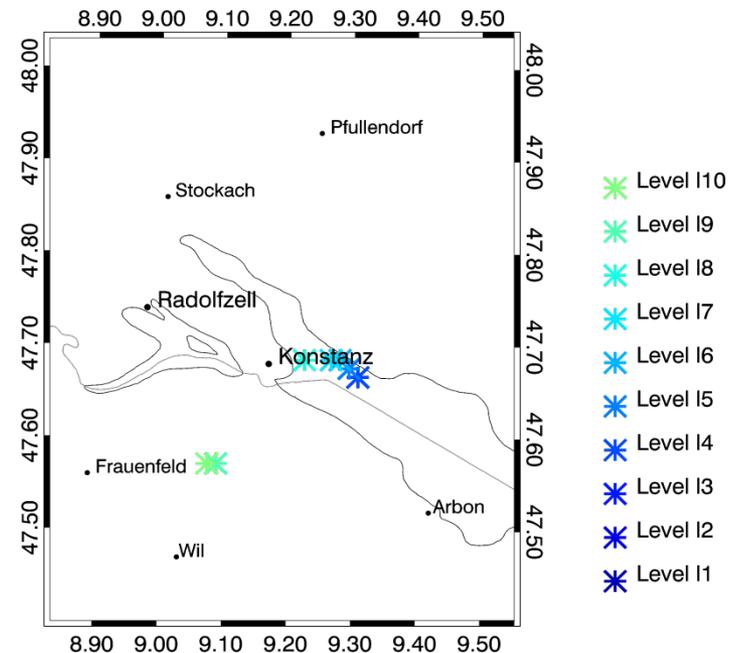
$$\delta O_3 > \delta HCHO > \delta CO > \delta HONO > \delta OH$$

with NO and NO₂ typically varying between $\delta O_3 > \delta NO_x > \delta HONO$
dependent on the specific chemical scenario.

1st SV, maximum value (level)



October 18 2008 12 UTC (H+02)



Orders of magnitude of different compounds (left) and position of maximal values (right) of maximal singular vector entries per model level.

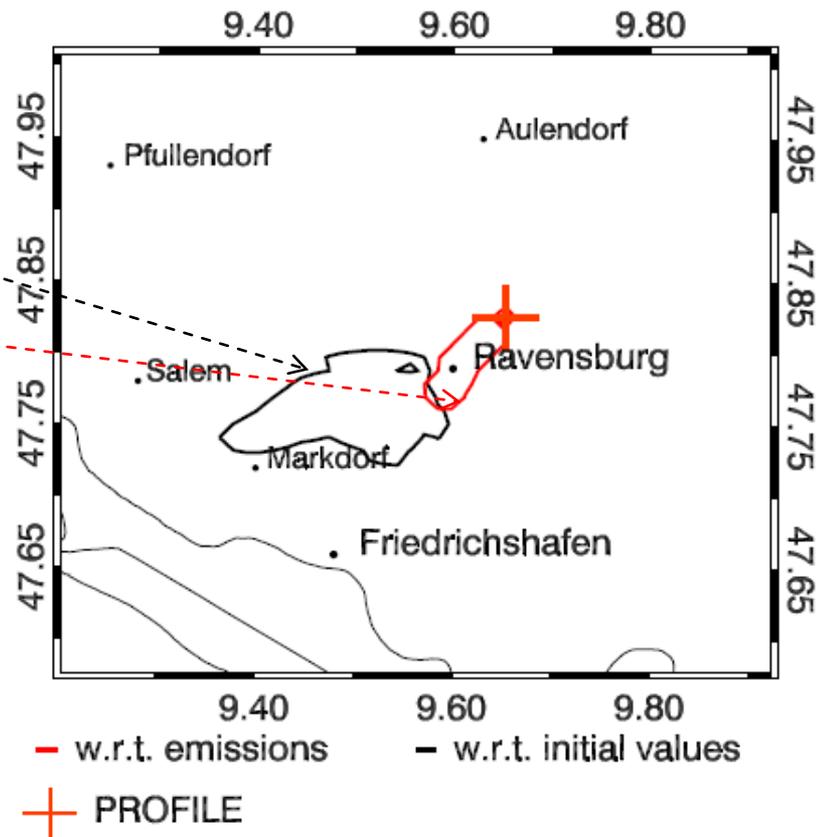
Where is it important to observe for O₃ prediction?

Example of maximal sensitivity contours of SV for

- HCHO initial values (black) and
- emission rates (red) for flight

19.Oct. 2008, $t_i=14:00$, $t_F=15:20$ at
+ for final location.

Emission sensitivity is concentrated in the Ravensburg urban area.



RESULTS

Implementation of emission rate optimisation

In the troposphere, for **emission rates**, the product (*paucity of knowledge * importance*) is high

Emission Rate Optimization

minimize cost function

$$J(\mathbf{x}(t_0), \mathbf{e}) = \frac{1}{2}(\mathbf{x}^b(t_0) - \mathbf{x}(t_0))^T \mathbf{B}_0^{-1}(\mathbf{x}^b(t_0) - \mathbf{x}(t_0)) + \frac{1}{2} \int_{t_0}^{t_N} (\mathbf{e}_b(t) - \mathbf{e}(t))^T \mathbf{K}^{-1}(\mathbf{e}_b(t) - \mathbf{e}(t)) dt + \frac{1}{2} \int_{t_0}^{t_N} (\mathbf{y}^0(t) - H[\mathbf{x}(t)])^T \mathbf{R}^{-1}(\mathbf{y}^0(t) - H[\mathbf{x}(t)]) dt$$

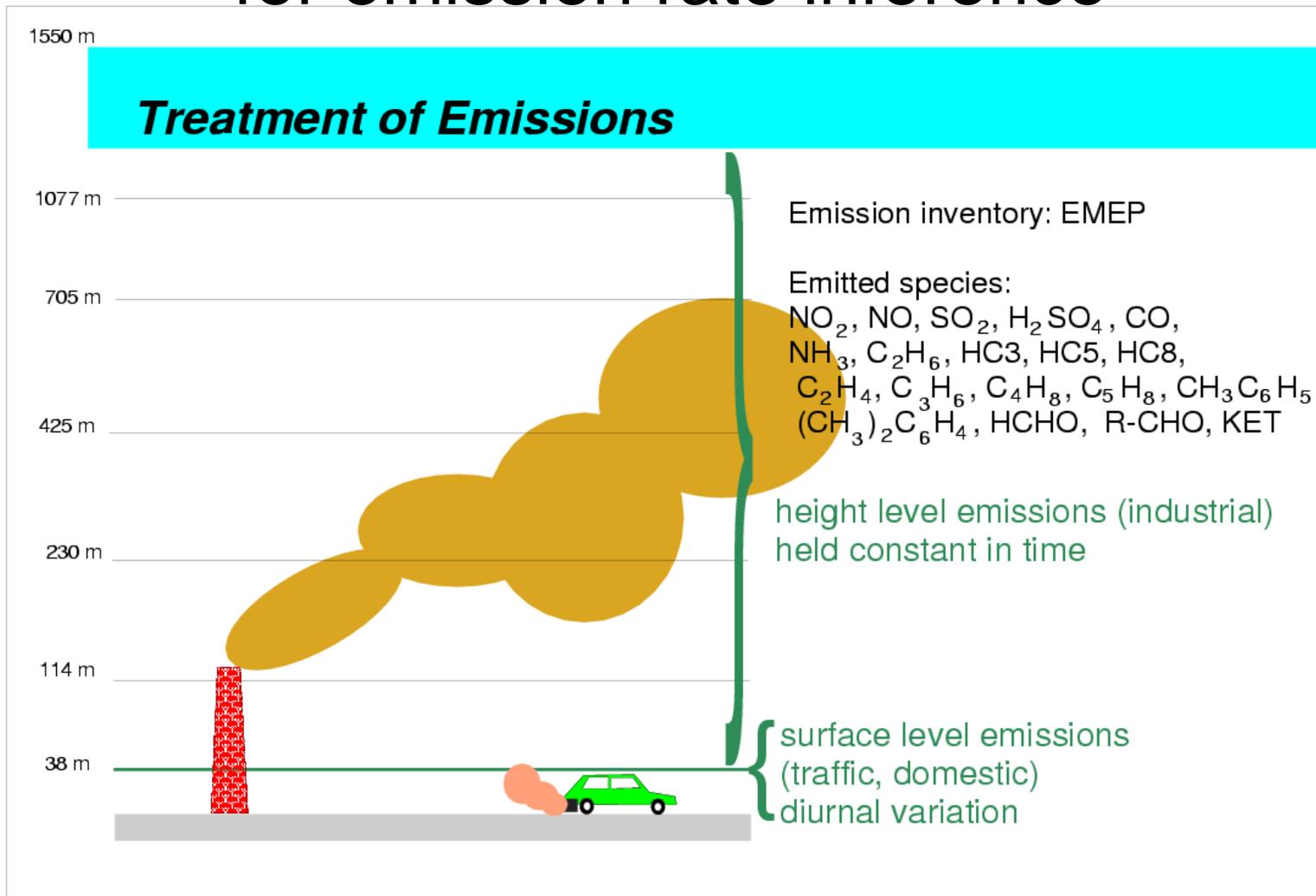
deviations from background initial state

deviations from a priori emission rates

model deviations from observations

- $\mathbf{x}^b(t_0)$ background state at $t = 0$
- $\mathbf{x}(t)$ model state at time t
- $\mathbf{e}_b(t_0)$ background emission rate at $t = 0$
- $\mathbf{e}(t)$ emission rate field at time t
- \mathbf{K} emission rate error covariance matrix
- $H[]$ forward interpolator
- $\mathbf{y}^0(t)$ observation at time t
- \mathbf{B}_0 background error covariance matrix

Treatment of the inverse problem for emission rate inference

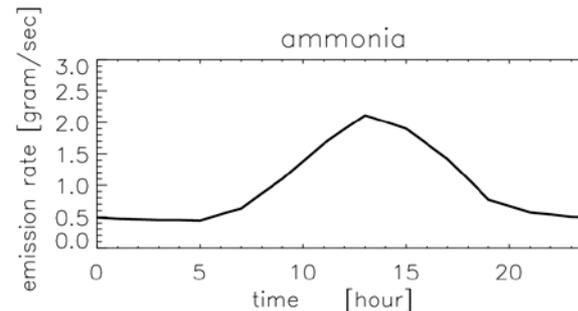
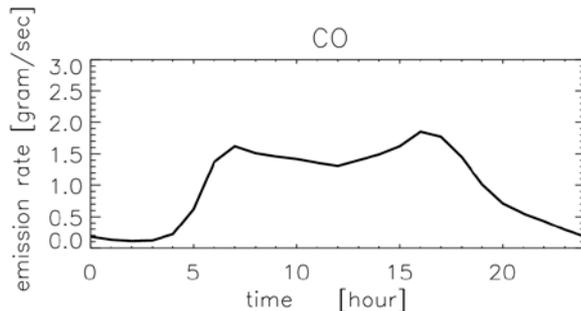
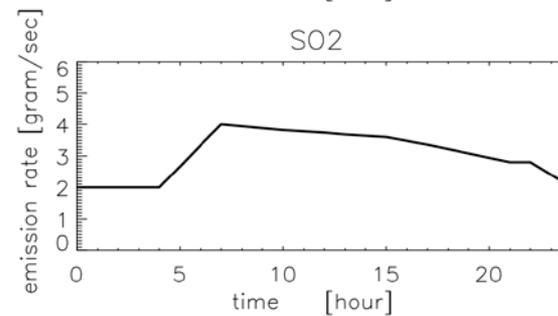
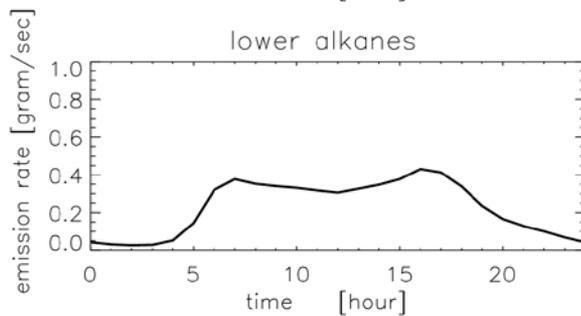
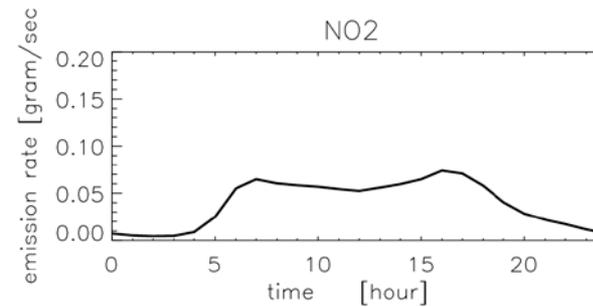
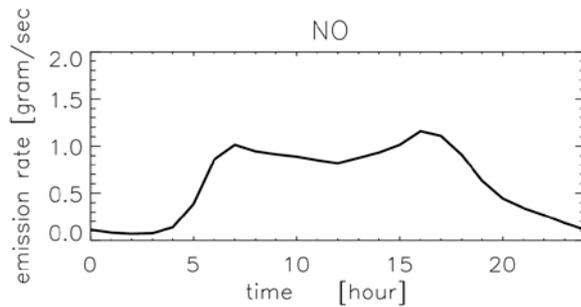


How can the degree of freedom of emission variation be reduced? a “strong constraint regularisation”

Normalised diurnal cycle of anthropogenic surface emissions $f(t)$

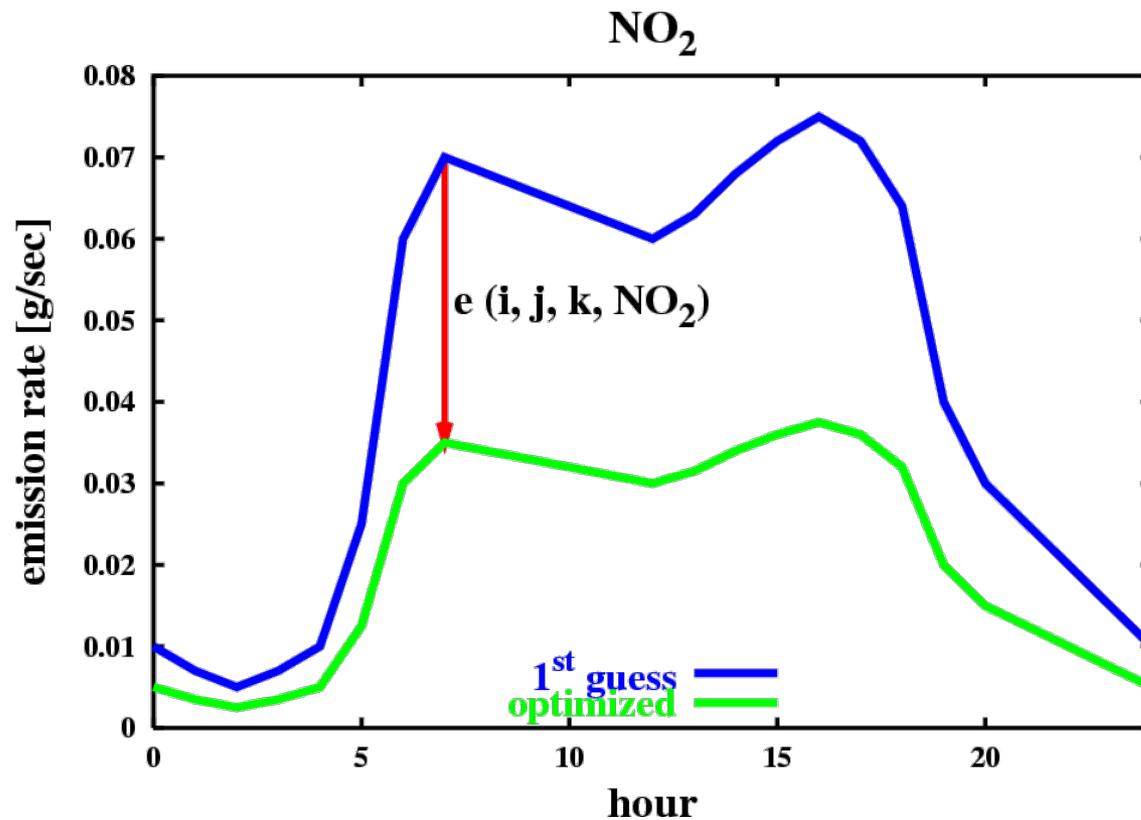
$$\text{emission}(t) = f(t; \text{location}, \text{species}, \text{day}) * v(\text{location}, \text{species})$$

day in {working day, Saturday, Sunday} v optimization parameter



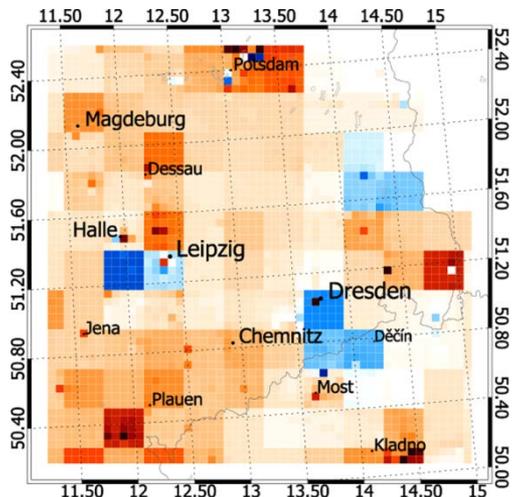
Optimisation of emission rates

amplitude optimisation for each emitted species
and grid cell

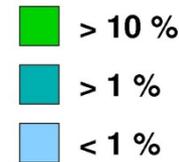


Novel setup of joint emission rate-initial value optimisation after upgrade of regional emission inventory

- Preparation:
 - adaptation to extended emitted species, („adjoint emission sources“)
 - more biogenic emission



	SO2	SO4	NO2	NO	ALD	HCHO	NH3	HC3	HC5	HC8	ETH	CO	OL2	OLT	OLI	TOL	XYL	KET	ISO
SO2	100	40	12	12	0.4	0.4	0.4	0.4	0.4	0.4	0.4	9	0.4	0.4	0.4	0.4	0.4	0.4	0
SO4	100	12	12	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	9	0.4	0.4	0.4	0.4	0.4	0.4	0
NO2	100	26	8	8	7	8	8	8	8	8	8	12	8	8	8	7	7	8	0.4
NO	100	8	8	7	8	8	8	8	8	8	8	12	8	8	8	7	7	8	0.4
ALD	100	26	5	6	6	6	6	6	6	6	5	10	6	6	5	5	26	0.3	0.3
HCHO	100	5	6	6	6	6	6	6	6	6	5	10	6	6	5	5	26	0.3	0.3
NH3	100	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0.3
HC3	100	23	23	23	5	18	15	15	6	6	6	6	6	6	6	6	6	6	0.3
HC5	100	23	23	5	18	15	15	6	6	6	6	6	6	6	6	6	6	6	0.8
HC8	100	23	5	18	15	15	6	6	6	6	6	6	6	6	6	6	6	6	0.8
ETH	100	5	18	15	15	6	6	6	6	6	6	6	6	6	6	6	6	6	0.8
CO	100	6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0.3
OL2	100	23	23	6	6	10	1												
OLT	100	27	5	5	6	15													
OLI	100	5	5	6	15														
TOL	100	24	5	0.3															
XYL	100	5	0.3																
KET	100	0.3																	
ISO	100																		



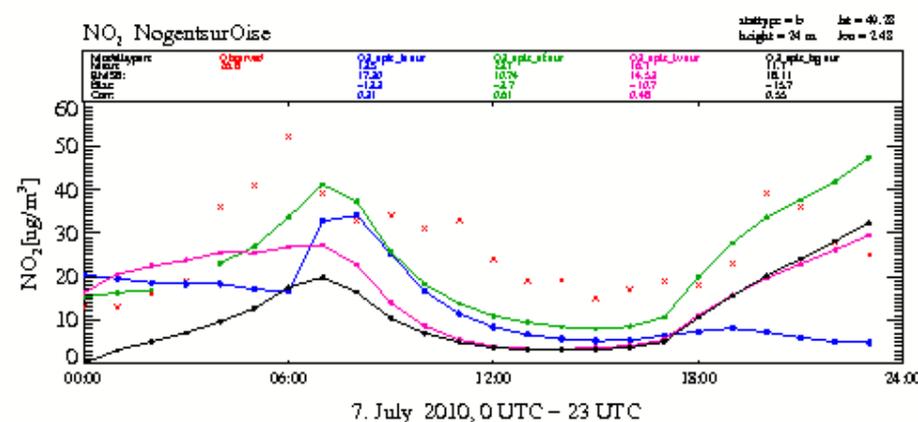
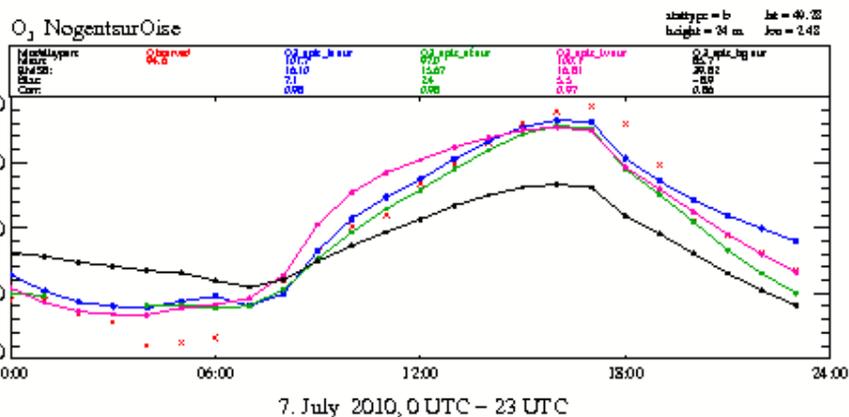
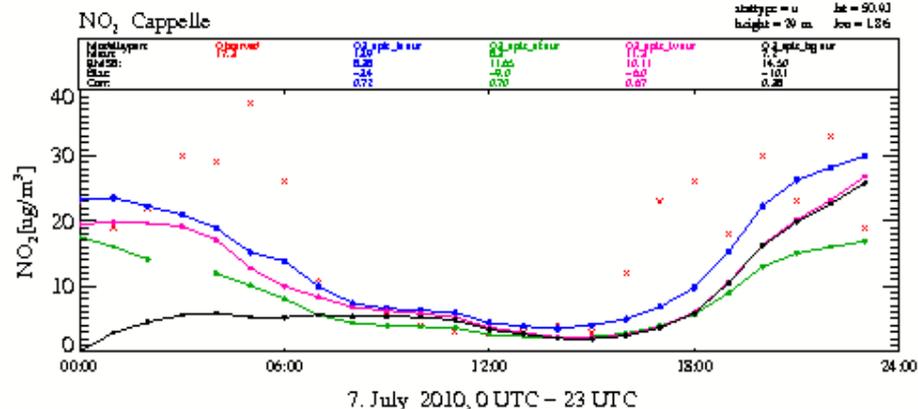
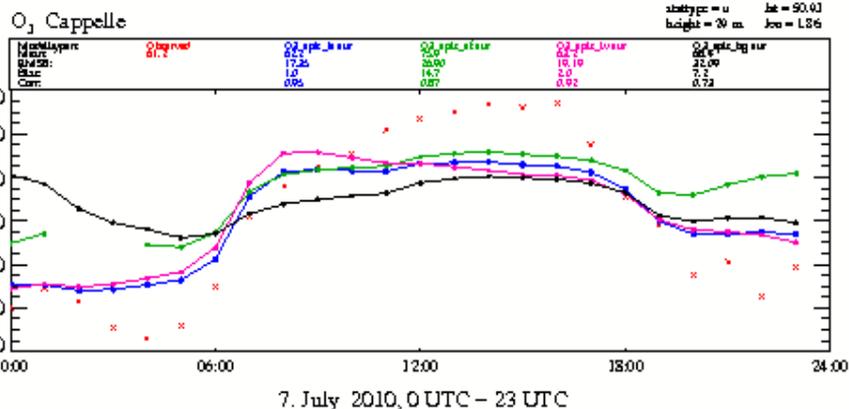
4D-var optimisation EURAD-IM

24 h window

- background (no assim)
- initial values only
- emission rates only
- joint emi-ini. values

A potential constraint: consider O_x -concept [Guicherit 1988]:

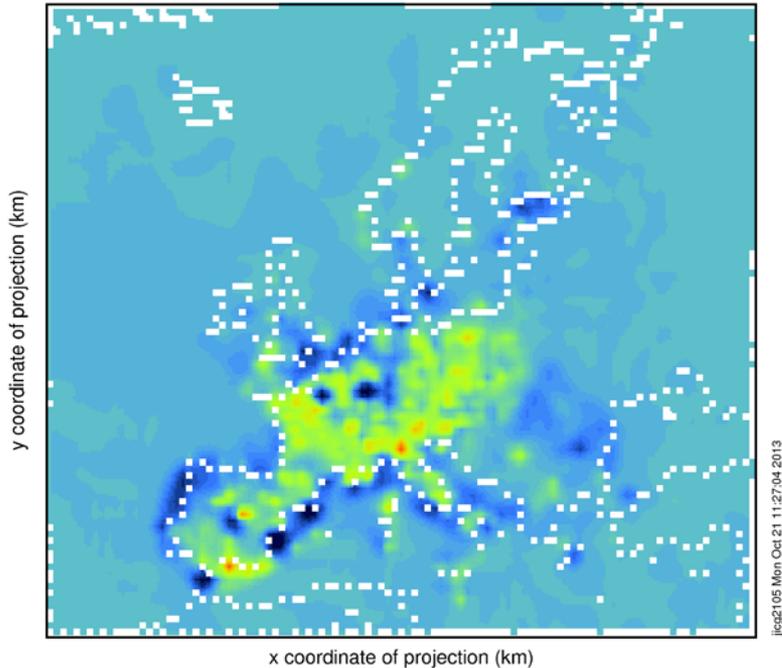
$$O_x = NO_2 + O_{zon} = const$$



NO₂ emission correction factors summer 2010

7. July

Emission Factor of NO2 (1)

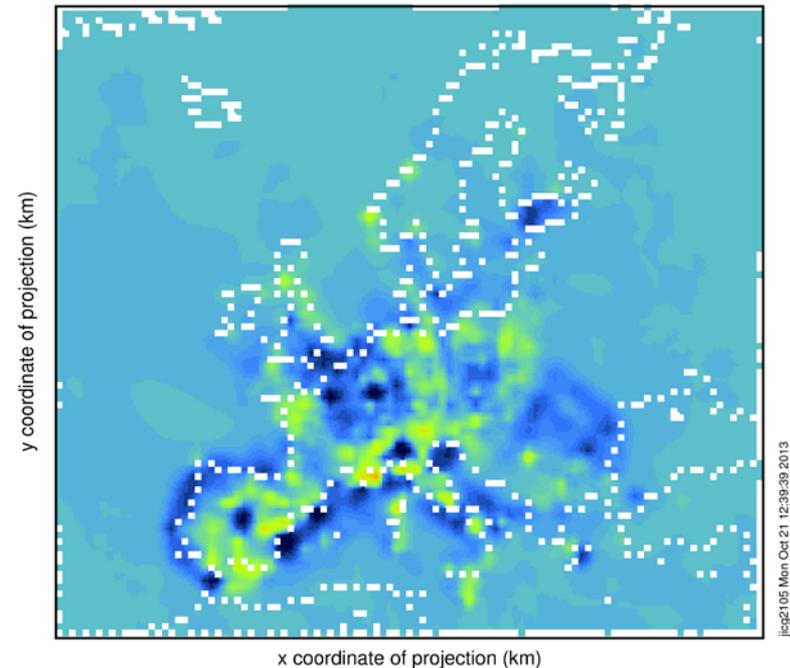


Output from EURAD-IM

Range of Emission Factor of NO2: 0 to 20 1
Range of x coordinate of projection: 22.5 to 4792.5 km
Range of y coordinate of projection: 22.5 to 4342.5 km
Current time: 19 hours since 2010-07-07 00:00:00
Current sigma at layer midpoints: 0.9975 sigma_level
File gradout_O3_epis_ivef_4dvar_eur_188.nc

16. July

Emission Factor of NO2 (1)



Output from EURAD-IM

Range of Emission Factor of NO2: 0 to 20 1
Range of x coordinate of projection: 22.5 to 4792.5 km
Range of y coordinate of projection: 22.5 to 4342.5 km
Current time: 5 hours since 2010-07-16 00:00:00
Current sigma at layer midpoints: 0.9975 sigma_level
File gradout_O3_epis_ivef_4dvar_eur_197.nc



analyses of NO_2 concentrations summer 2010

7. July

16. July

nitrogen dioxide ($1\text{e-}9$)

nitrogen dioxide ($1\text{e-}9$)

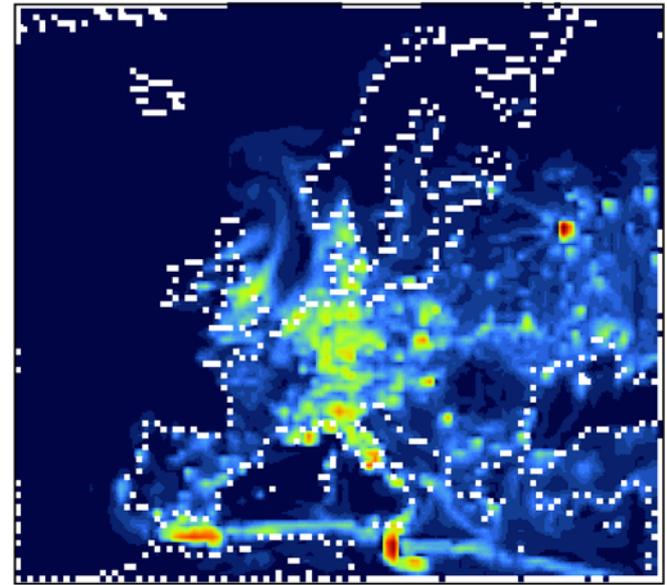
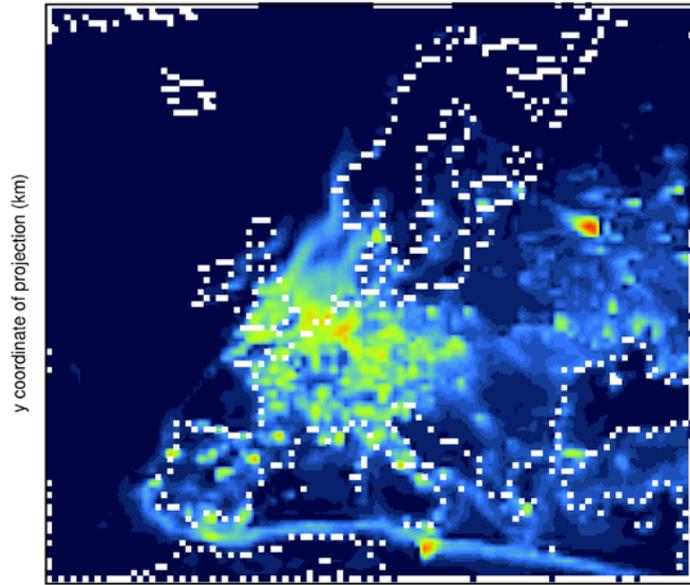


fig2105 Mon Oct 21 11:10:25 2013

fig2105 Mon Oct 21 10:46:09 2013

x coordinate of projection (km)

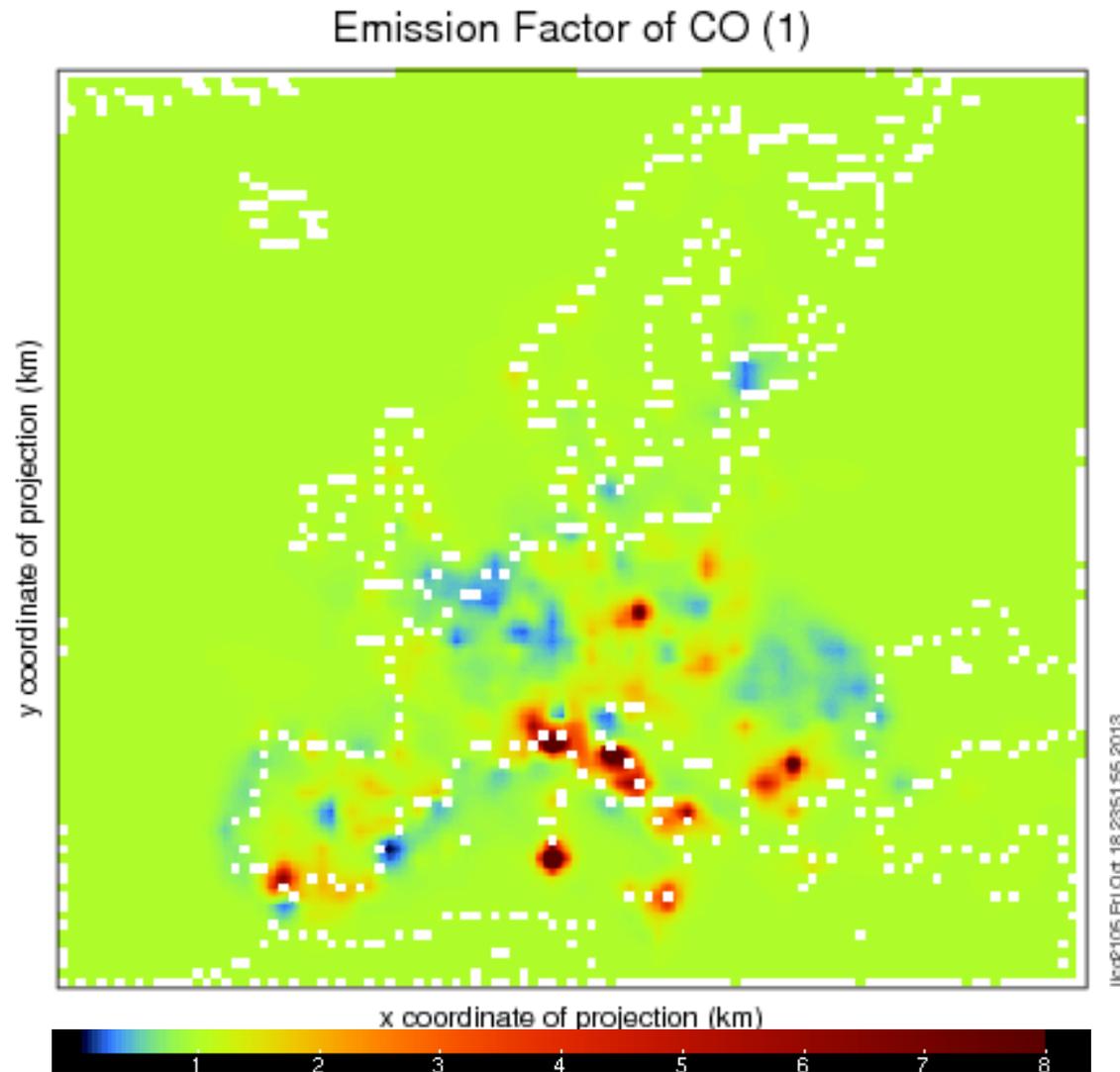
x coordinate of projection (km)



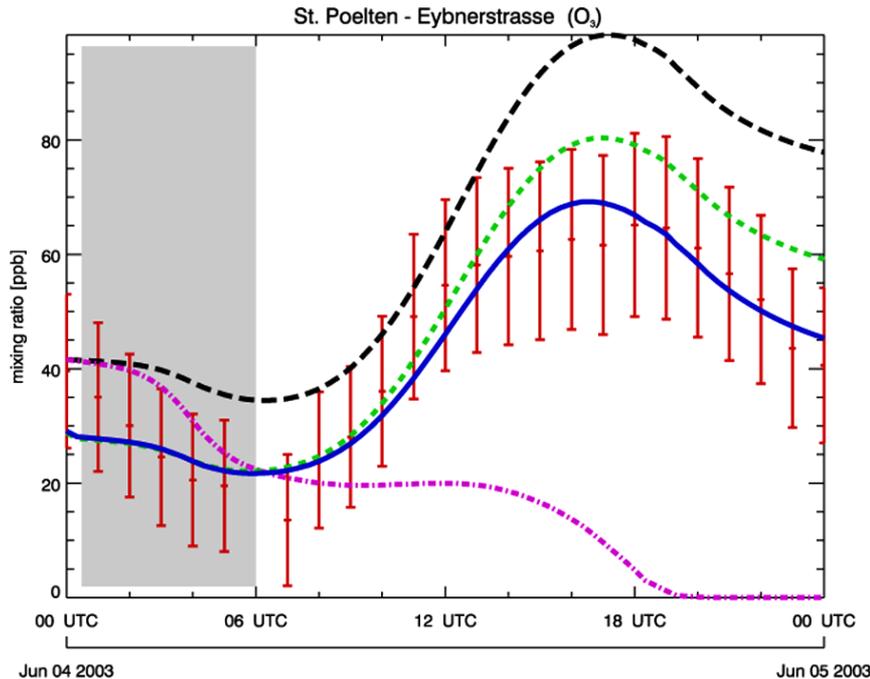
Output from EURAD-IM
 Range of nitrogen dioxide: 0 to 100 $1\text{e-}9$
 Range of x coordinate of projection: 22.5 to 4792.5 km
 Range of y coordinate of projection: 22.5 to 4342.5 km
 Current time: 19 hours since 2010-07-07 00:00:00
 Current sigma at layer midpoints: 0.9975 sigma_level
 File ctmout_O3_epis_ivef_frw_eur_188.nc

Range of nitrogen dioxide: 0 to 100 $1\text{e-}9$
 Range of x coordinate of projection: 22.5 to 4792.5 km
 Range of y coordinate of projection: 22.5 to 4342.5 km
 Current time: 19 hours since 2010-07-16 00:00:00
 Current sigma at layer midpoints: 0.9975 sigma_level
 File ctmout_O3_epis_ivef_frw_eur_197.nc

CO emission correction factors

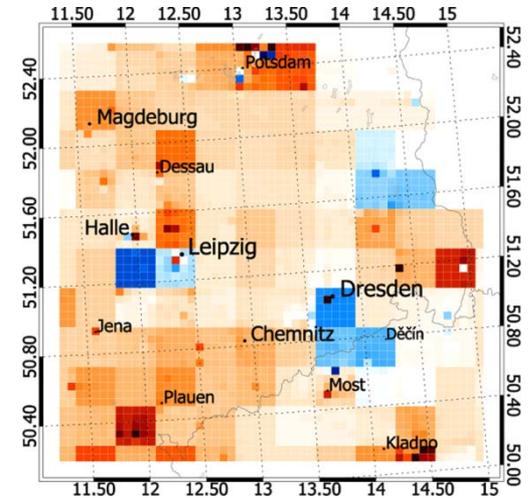


Analyses, Example (i): Analysis of emissions by 4D-var (VERTIKO)

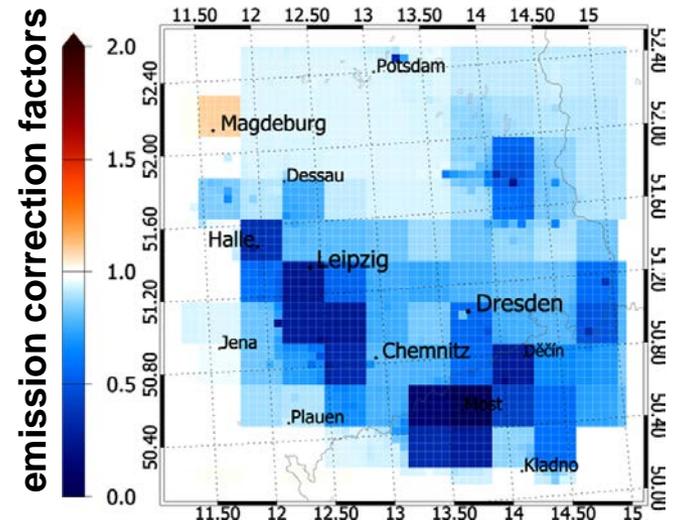


Observed and analysed ozone evolution at St. Poelten
Vertical bars: ozone observations with error estimates.
- - - - Control run without data assimilation.
..... initial value optimisation.
- . - . emission factor optimisation.
_____ joint initial value and emission factor optimisation (Strunk et al., 2011)

NO2



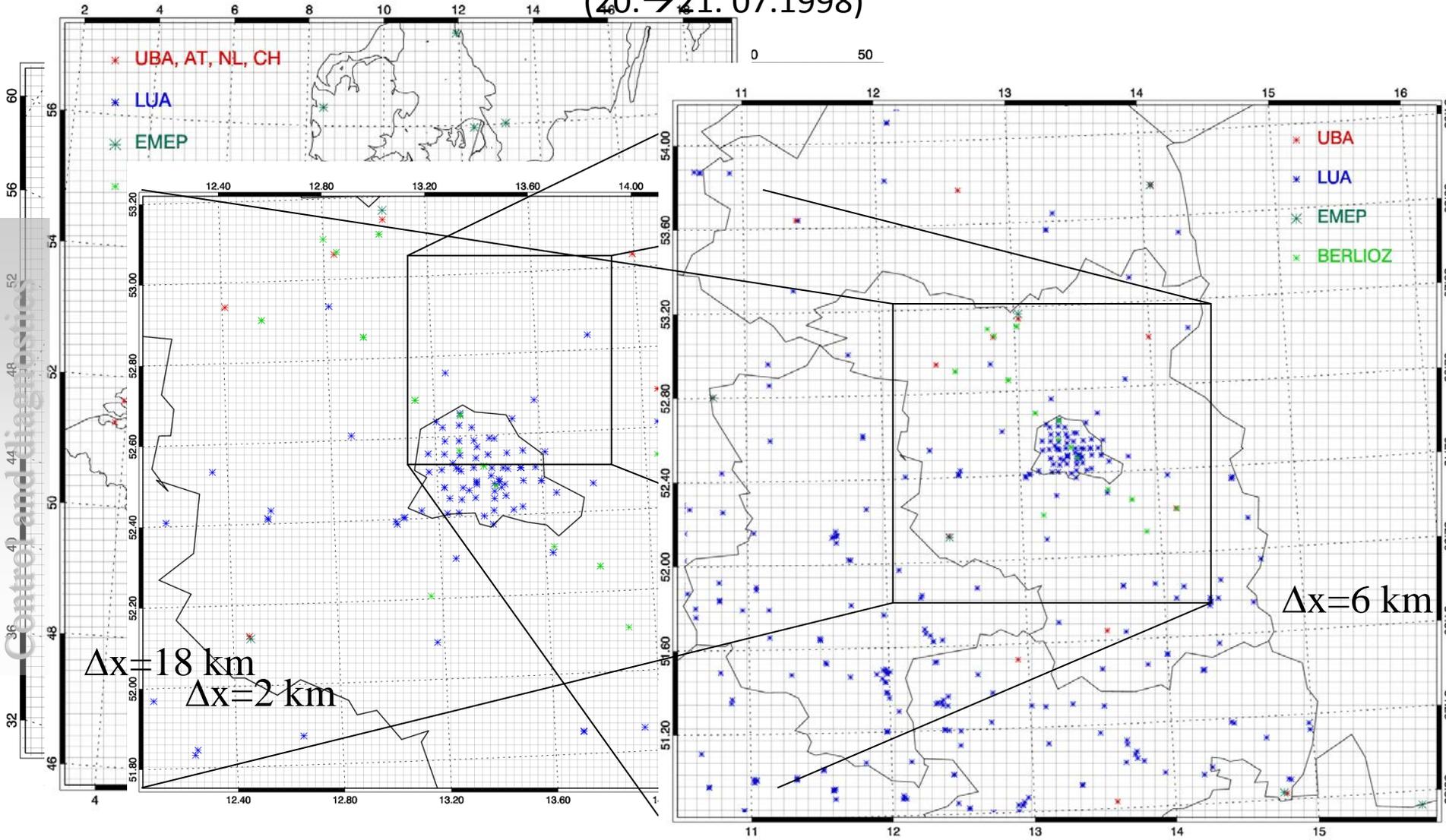
SO2



Which is the requested resolution?

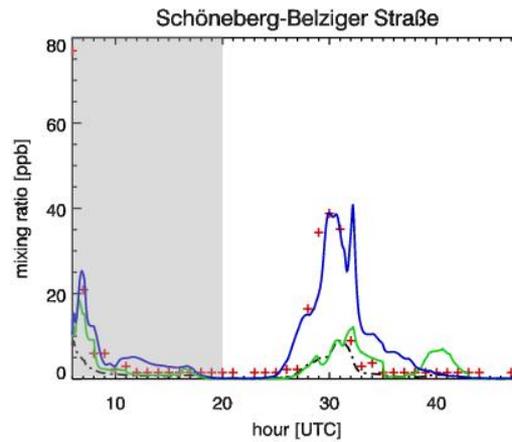
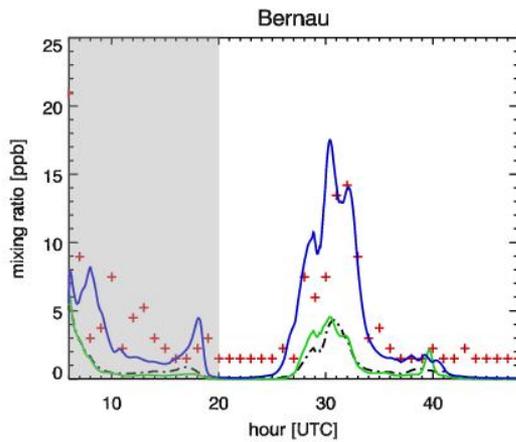
BERLIOZ grid designs and observational sites

(20. → 21. 07.1998)



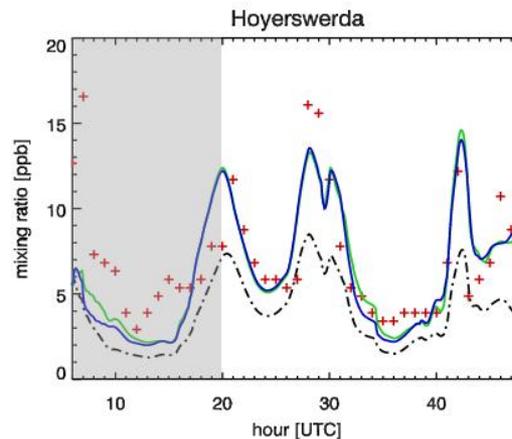
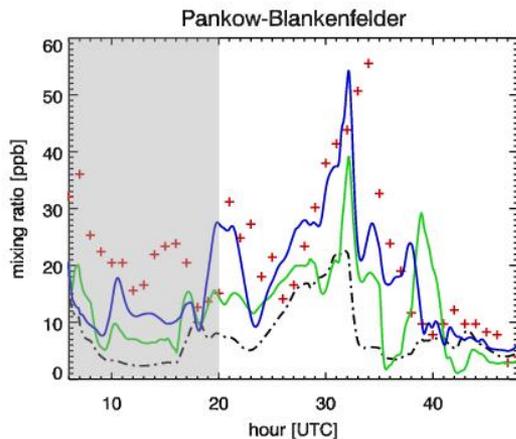
Some BERLIOZ examples of NOx assimilation (20. → 21. 07.1998)

NO



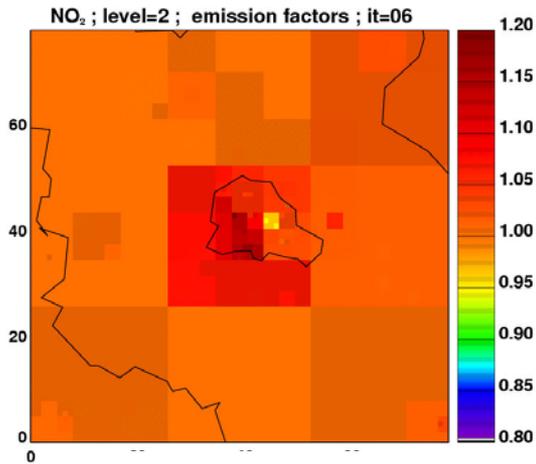
Time series for selected NOx stations on nest 2.
+ observations,
- - - no assimilation,
- N1 assimilation (18 km),
- N2 assimilation (6 km),
- grey shading: assimilated observations, others forecasted.

NO₂

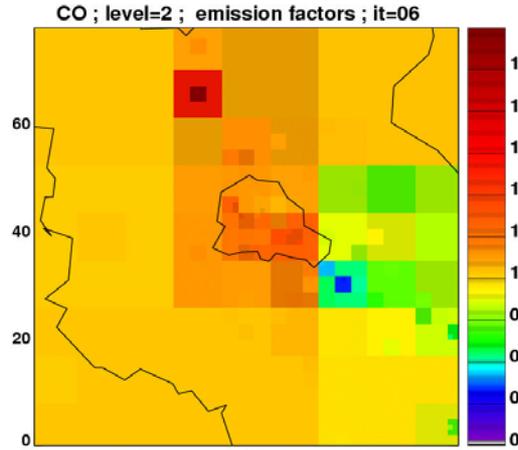


Emission source estimates by inverse modelling

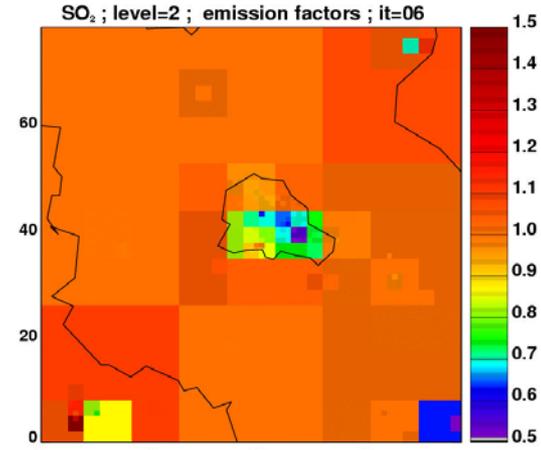
Optimised emission factors for Nest 3



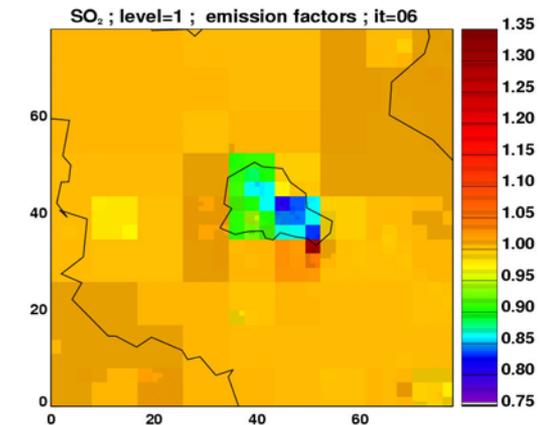
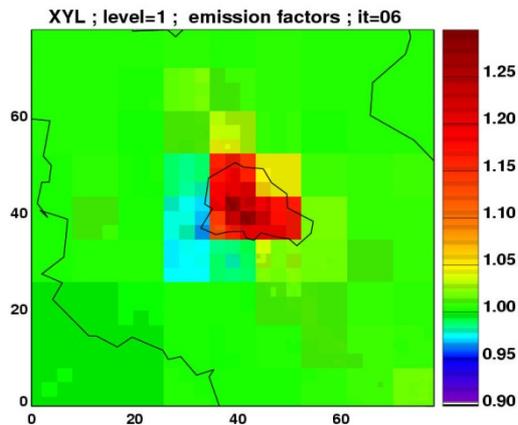
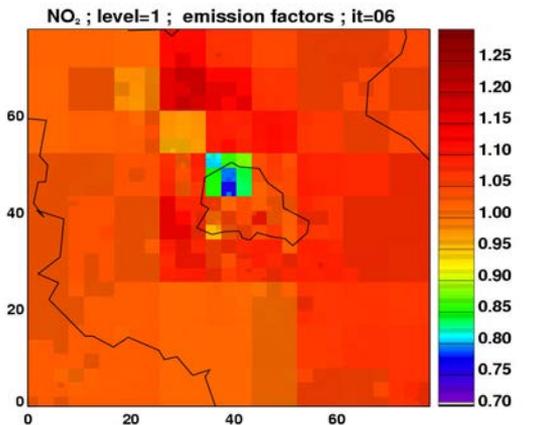
NO₂,



(xylene (bottom), CO (top))



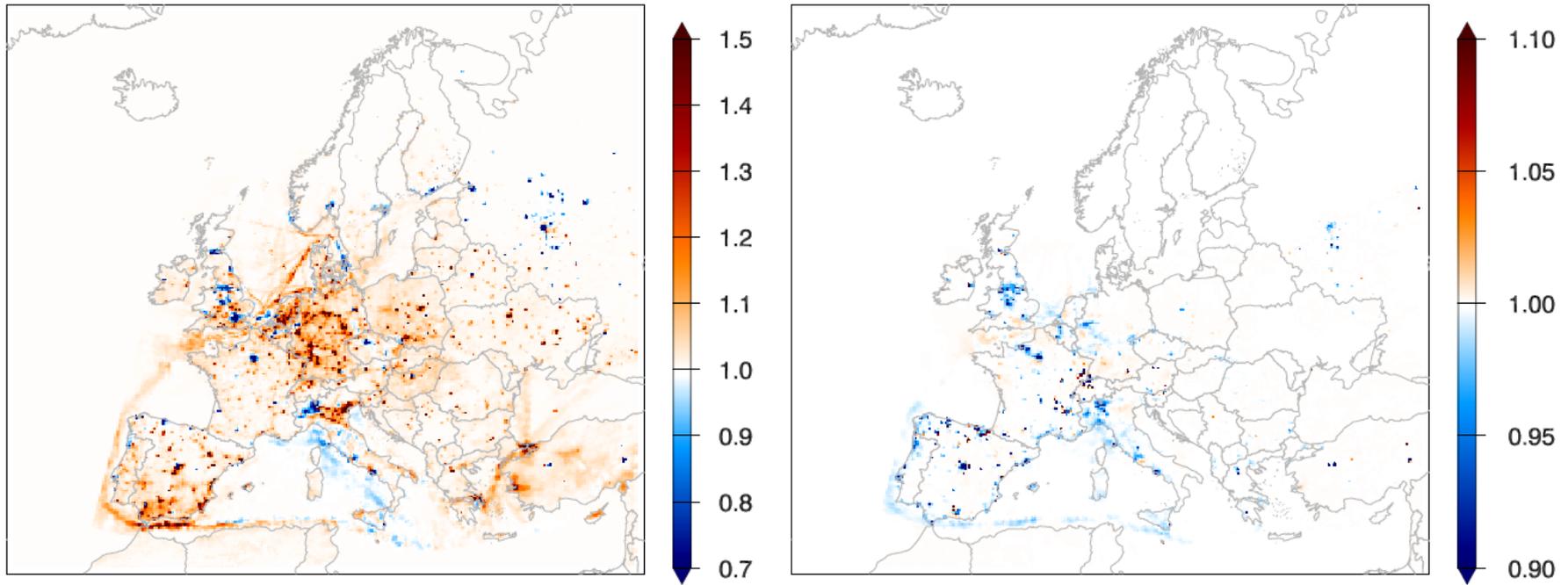
SO₂.



height layer ~32-~70m

surface

NO2 Emission factors TNO emission inventory

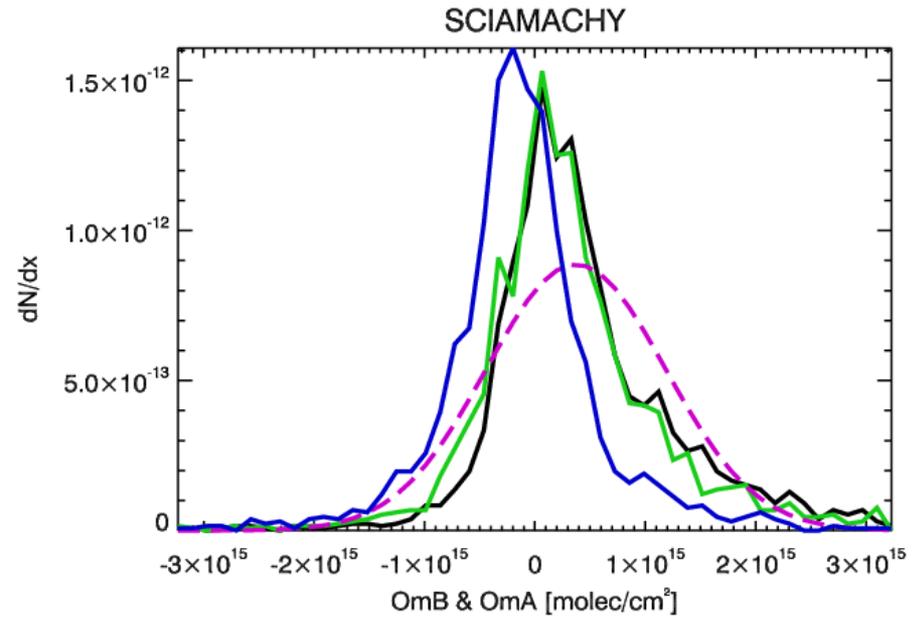
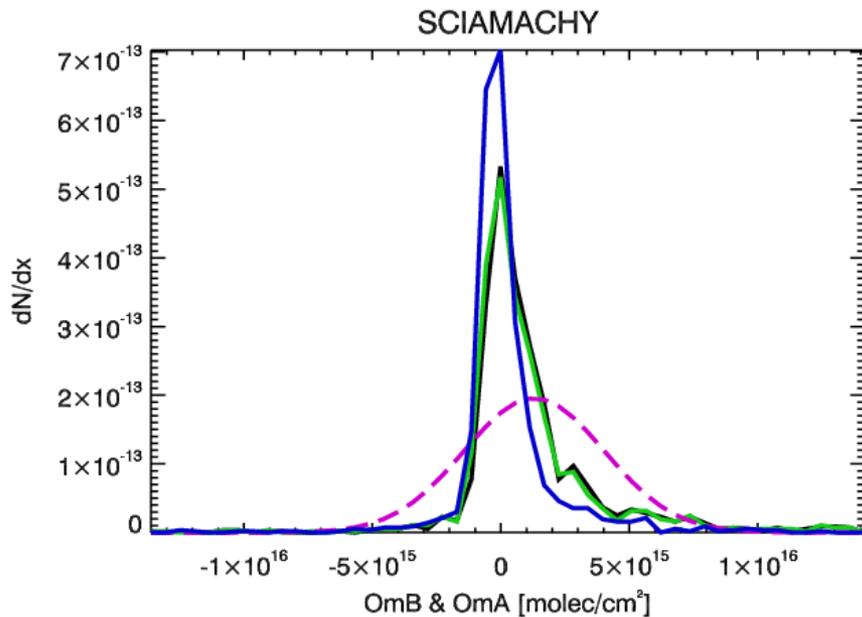


Comparison of nitrogen dioxide emission factor results for the summer case study (left panel) and the winter episode (right panel). Please note the different ranges on the color scale.

SCIAMACHY

$O_{\text{SCIA}} mX_{\text{OMIT}}$ probability density functions

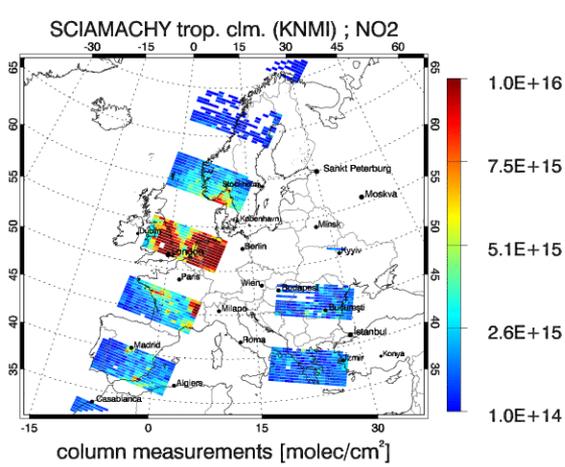
for July 6th (left), and July 8th, (right).



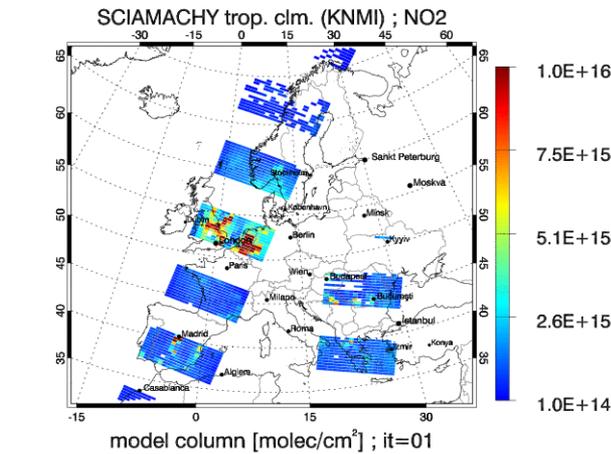
Control run (OmC) (no data assimilation at all,) black bold line,
forecasted values (OmF) green bold line,
analyses (OmA) blue bold line.

For comparison: Gaussian fit to OmF pdf by
mean and standard deviation given by broken purple line.

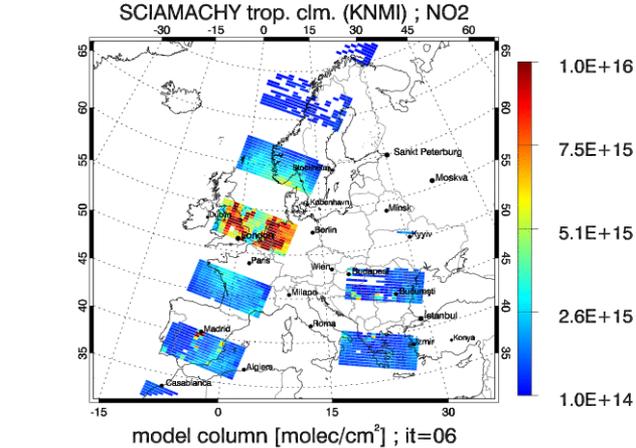
Comparison of NO₂ tropospheric columns in molecules/cm² for July 6th, 2006, 09-12 UTC.



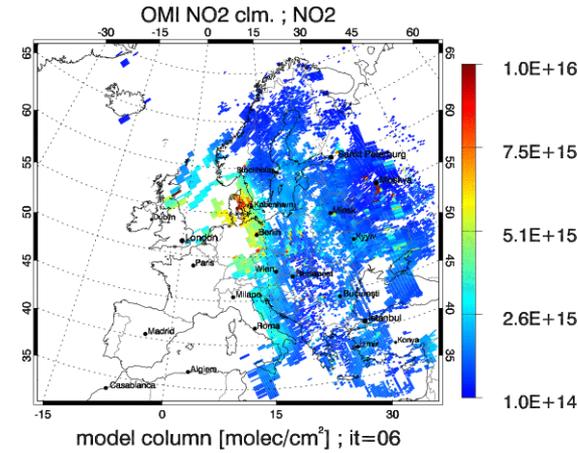
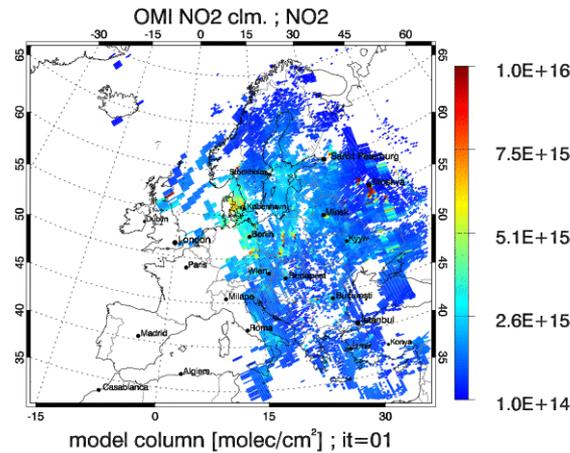
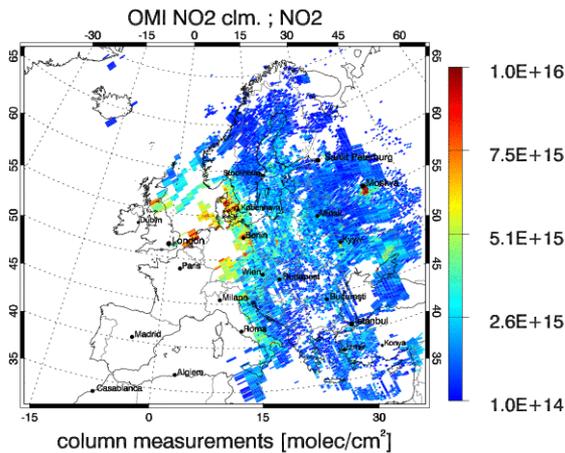
assimilated values (\mathbf{y});



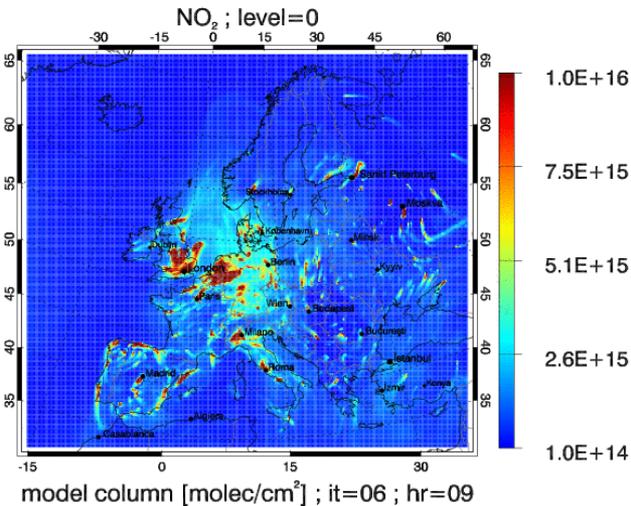
:EURAD forecasted (\mathbf{Hx}_b);



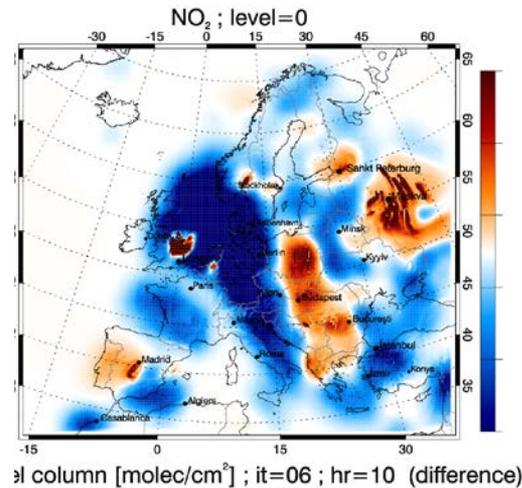
column analyses (\mathbf{Hx}_a).



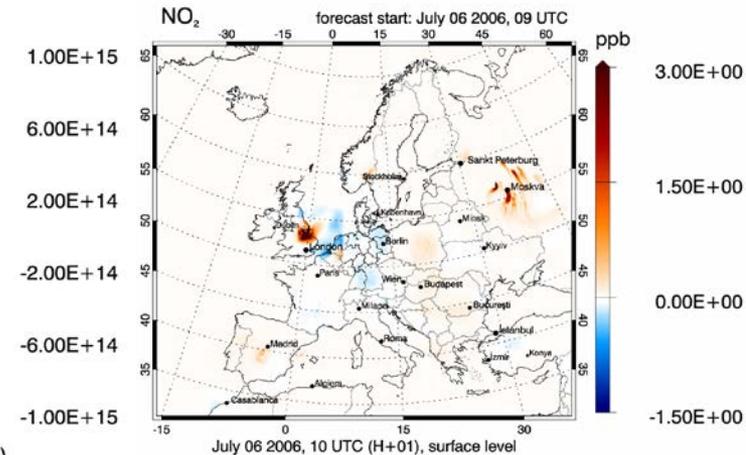
Data assimilation result in terms of tropospheric columns for July 6th, 2006. NO₂ model columns based on OMI and SCIAMACHY assimilation within interval, 09-12 UTC.



Analysed column densities

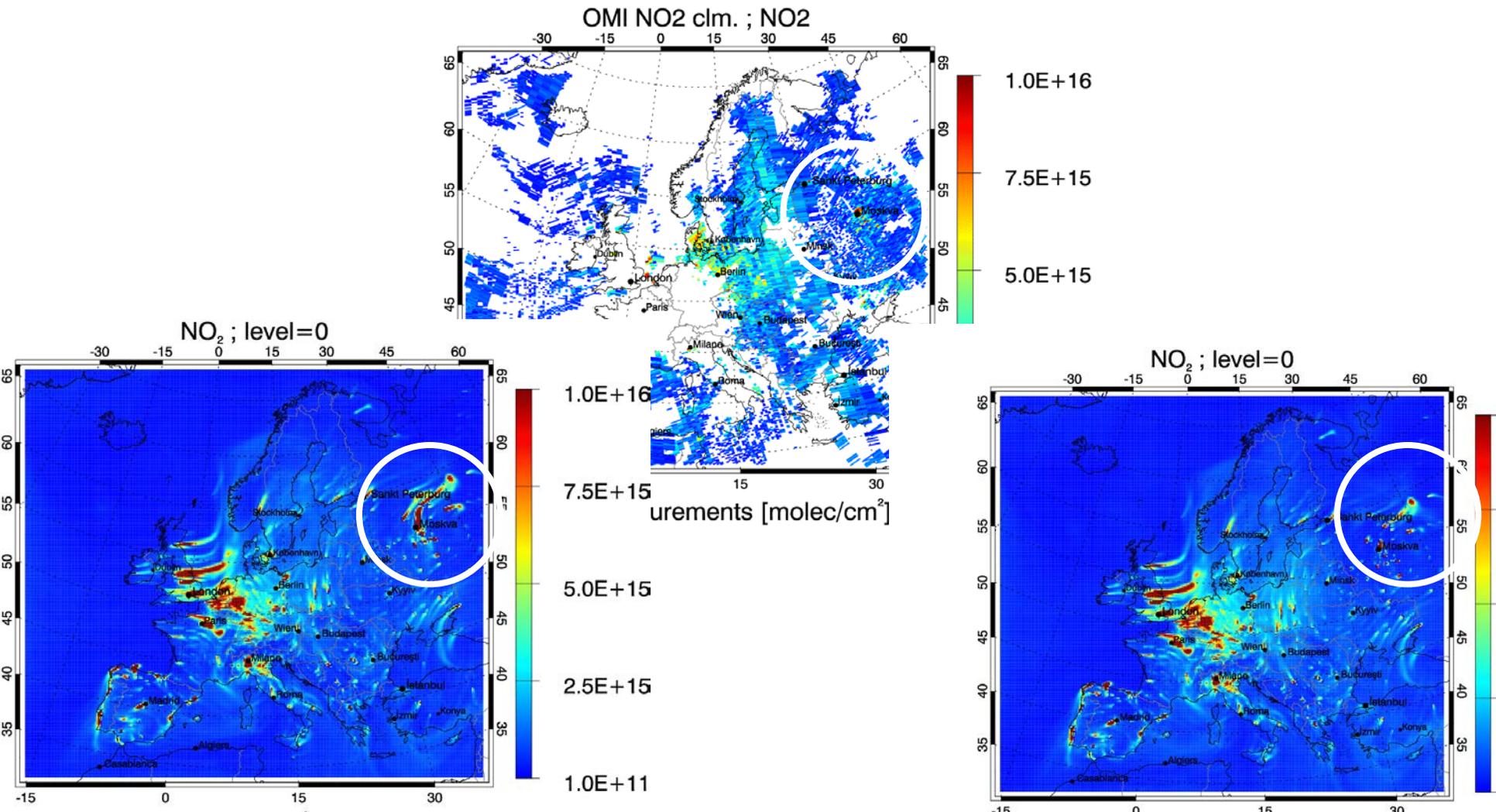


difference field from background

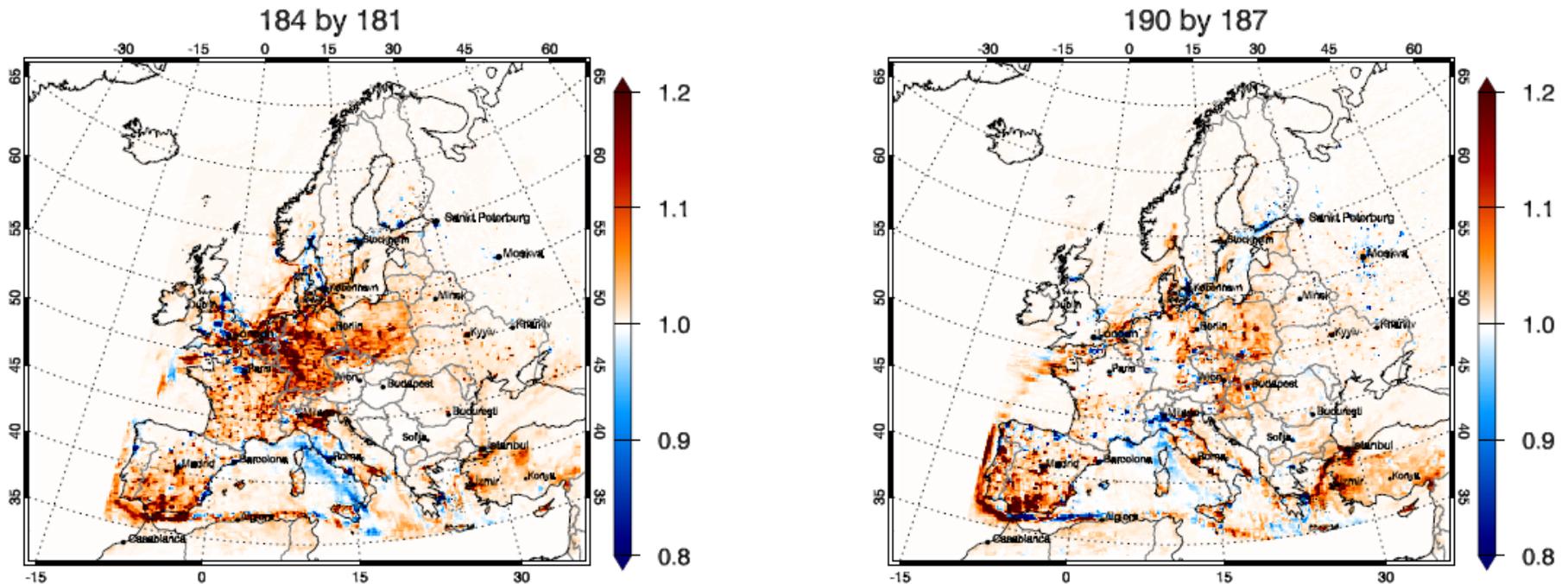


induced surface concentration changes by NO₂ ppb

Data assimilation result in terms of tropospheric columns for **July 7th, 2006**. NO₂ model columns based on OMI and SCIAMACHY assimilation within the assimilation interval, 09-12 UTC.



Emission rate optimisation factors for NO₂ after assimilation of OMI retrieved NO₂ tropospheric columns



2 x 4 days assimilation sequence. Left panel shows results after assimilation procedures from July 1.-4. 2006, right panel for July 7.-11., 2006. OMI data from KNMI

Summary

- Emission rate estimation cannot be performed isolated. Rather, other controlling parameters (initial values) must be included.
- A smoother type spatio-temporal data assimilation algorithm is required (integration „backward in time and process“: 4D-var)
- As ill-conditioned problem, regularisation is needed.
- Future: chemical scenario dependent covariances need to be designed
- A rigorous (control) theory based balance between initial value and emission rate weights is needed.
- Emission rates need be optimized SNAP-wise.

Some references

Elbern, H. A. Strunk, L. Nieradzik, Inverse modelling and combined state-source estimation for chemical weather, In: "Data Assimilation: Making sense of observations", Springer, eds. Lahoz, Khattatov, Menard, pp 491-513, doi 10.1007/978-3-540-74703-1_19, 2010.

Elbern, H., A. Strunk, H. Schmidt, and O. Talagrand, Emission rate and chemical state estimation by 4-dimensional variational inversion, ACP, 3749-3769, 2007.

Elbern, H., H. Schmidt, O. Talagrand, A. Ebel, 4D variational data assimilation with an adjoint air quality model for emission analysis, Environ. Mod. Softw., 15, 539-548, 2000.

Goris, N., and H. Elbern Singular vector decomposition for sensitivity analyses of tropospheric chemical scenarios Atmos. Chem. Phys., 13, 5063-5087, 2013.