The COSMO model: towards cloud-resolving NWP

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Outline

• The new fast waves solver of the COSMO model - consequences from the vertically stretched grid
• An new analytic solution to test LAM and global dynamical cores
• Influence of the water loading in strong convective situations
• Staggered vs. Unstaggered grids
  … and what does this mean for discontinuous Galerkin methods
The operational Model Chain of DWD: GME, COSMO-EU and -DE

GME
- hydrostatic parameterised convection
  \( \Delta x \approx 20 \text{ km} \)
  1482250 * 60 GP
  \( \Delta t = 66.7 \text{ sec.}, T = 7 \text{ days} \)
- will be replaced by ICON in 2014

COSMO-EU
- non-hydrostatic parameterised convection
  \( \Delta x = 7 \text{ km} \)
  665 * 657 * 40 GP
  \( \Delta t = 66 \text{ sec.}, T = 78 \text{ h} \)
- will be replaced by ICON in ~2015

COSMO-DE & -EPS
- non-hydrostatic convection-permitting
  \( \Delta x = 2.8 \text{ km} \)
  421 * 461 * 50 GP
  \( \Delta t = 25 \text{ sec.}, T = 27 \text{ h} \)
Revision of the current dynamical core

- redesign of the fast waves solver

Time integration scheme of COSMO dynamical core:

*Wicker, Skamarock (2002) MWR:* split-explicit 3-stage Runge-Kutta
→ stable integration of 5th order upwind advection (large $\Delta T$);
these tendencies are added in each fast waves step (small $\Delta t$)
'Fast waves' processes (p'T'-dynamics):

\[
\begin{align*}
\frac{\partial u}{\partial t} & = -\frac{1}{\rho} \frac{1}{r \cos \phi} \left( \frac{\partial p'}{\partial \lambda} + \frac{\partial \zeta}{\partial \lambda} \frac{\partial p'}{\partial \zeta} \right) + \frac{1}{\rho} \frac{1}{r \cos \phi} \left( \frac{\partial \alpha_{\text{div}}^h \rho D}{\partial \lambda} + \frac{\partial \zeta}{\partial \lambda} \frac{\partial \alpha_{\text{div}}^h \rho D}{\partial \zeta} \right) + f_u \\
\frac{\partial v}{\partial t} & = -\frac{1}{\rho} \frac{1}{r} \left( \frac{\partial p'}{\partial \phi} + \frac{\partial \zeta}{\partial \phi} \frac{\partial p'}{\partial \zeta} \right) + \frac{1}{\rho} \frac{1}{r} \left( \frac{\partial \alpha_{\text{div}}^h \rho D}{\partial \phi} + \frac{\partial \zeta}{\partial \phi} \frac{\partial \alpha_{\text{div}}^h \rho D}{\partial \zeta} \right) + f_v \\
\frac{\partial w}{\partial t} & = -\frac{1}{\rho} \left( \frac{\partial \zeta}{\partial z} \frac{\partial p'}{\partial \zeta} \right) + g \left( \frac{p_0 T'}{p T_0} - \frac{p'}{p} + \frac{p_0}{p} \frac{T}{T_0} q_x \right) + \frac{1}{\rho} \frac{1}{\partial z} \left( \frac{\partial \zeta}{\partial \zeta} \frac{\partial \alpha_{\text{div}}^v \rho D}{\partial \zeta} \right) + f_w \\
\frac{\partial p'}{\partial t} & = -\frac{c_p}{c_v} \rho D + g \rho_0 w + f_p \\
\frac{\partial T'}{\partial t} & = -\frac{R}{c_v} T D - \frac{\partial T_0}{\partial z} w + f_T
\end{align*}
\]

\[D = \text{div } \mathbf{v}\]

\[f_u, f_v, \ldots\] denote advection, Coriolis force and all physical parameterizations

Spatial discretization: centered differences (2nd order)
Main changes towards the old fast waves solver:

1. improvement of the vertical discretization: use of weighted averaging operators for all vertical operations
2. divergence in strong conservation form
3. optional: complete 3D (=isotropic) divergence damping

Additionally some 'technical' improvements; hopefully a certain increase in code readability

Overall goal: improve numerical stability of COSMO

→ new version fast_waves_sc.f90 contained in official COSMO 4.24

M. Baldauf (2013) COSMO Technical report No. 21 (www.cosmo-model.org)
Discretization in stretched grids

Example: calculate 1st derivative $\frac{\partial y}{\partial z}$ by an (at most) 3-point formula ($\leftrightarrow$ tridiagonal solver)

Approach 1: by weightings in 'original space'

\[
\frac{dy}{dz} \bigg|_{z_k} = \frac{z_{k+1} - z_k}{z_{k+1} - z_{k-1}} \cdot \frac{y_k - y_{k-1}}{z_k - z_{k-1}} + \frac{z_k - z_{k-1}}{z_{k+1} - z_{k-1}} \cdot \frac{y_{k+1} - y_k}{z_{k+1} - z_k}
\]

(e.g. Ikeda, Durbin (2004) JCP)

Approach 2: use of a coordinate transformation $z_k = f(\zeta_k)$, $\zeta_k = k \Delta \zeta$

$\frac{\partial y}{\partial z} = \frac{\partial \zeta}{\partial z} \frac{\partial y}{\partial \zeta}$

centered diff.

straightforward in unstaggered A-grid, less clear in staggered C-grid
Improvement of the vertical discretization

COSMO: Half levels (w-positions) are defined by a stretching function \( z_k = f(\zeta_k) \); Main levels (\( p^*, T^* \)-pos.) lie in the middle of two half levels

Arithmetic average from half levels to main level:
\[
\overline{\psi}_i^\zeta \equiv A_\zeta \psi|_{i,j,k} := \frac{1}{2}(\psi_{i,j,k-\frac{1}{2}} + \psi_{i,j,k+\frac{1}{2}})
\]

Weighted average from main levels to half level
\[
\overline{\psi}_i^{N,\zeta} \equiv A_\zeta^N \psi|_{i,j,k-\frac{1}{2}} := g_{k-\frac{1}{2}} \psi_{i,j,k} + (1 - g_{k-\frac{1}{2}}) \psi_{i,j,k-1}
\]

Derivatives always by centered differences (appropriate average used before)
\[
\delta_{\zeta} \psi|_{i,j,k} := \frac{\psi_{i,j,k+\frac{1}{2}} - \psi_{i,j,k-\frac{1}{2}}}{\Delta\zeta}
\]

G. Zängl could show the advantages of weighted averages in the explicit parts of the fast waves solver.
New: application to all vertical operations (also the implicit ones)
How to inspect truncation errors in stretched grids?

... by Taylor expansion

Equidistant grids: let $\Delta x, \ldots \to 0$ (easy)

Non-equidistant grids: infinitely many possibilities to refine the grid!

**Variant A.** Define grid by a (fixed) stretching function

$$z_k = f(\zeta_k), \quad \zeta_k = k \Delta \zeta,$$

then let $\Delta \zeta \to 0$

important: grid becomes locally increasingly linear (locally nearly non-stretched)

**Variant B.** Constant stretching ratio $s$ between neighbouring grid cells:

$$\ldots, \quad z_{k+\frac{3}{2}} = \ldots, \quad z_{k+\frac{1}{2}} = z_{k+\frac{3}{2}} + \frac{1}{s} \Delta z, \quad z_{k-\frac{1}{2}} = z_{k+\frac{1}{2}} + \Delta z, \quad z_{k-\frac{3}{2}} = z_{k-\frac{1}{2}} + s \Delta z, \quad \ldots$$

then let $\Delta z \to 0$
Buoyancy ($\sim g \ T'/T_0$) – grid stretching variant A

buoyancy term with **weighted** average of $T'$
($T_0$ exact):

\[
\frac{1}{T_0} A^{N}_\zeta T' = \frac{T'}{T_0} + d\zeta^2 \frac{1}{T_0} \left[ \frac{1}{8} \left( \frac{\partial z}{\partial \zeta} \right)^2 \frac{\partial^2 T'}{\partial z^2} \right] + O(d\zeta^4)
\]

buoyancy term with **arithmetic** average of $T'$
($T_0$ exact):

\[
\frac{1}{T_0} A^{T}_\zeta T' = \frac{T'}{T_0} + d\zeta^2 \frac{1}{T_0} \left[ \frac{1}{8} \left( \frac{\partial z}{\partial \zeta} \right)^2 \frac{\partial^2 T'}{\partial z^2} + \frac{1}{4} \frac{\partial^2 z}{\partial \zeta^2} \frac{\partial T'}{\partial z} \right] + O(d\zeta^4)
\]
Buoyancy ($\sim g \frac{T'}{T_0}$) - grid stretching variant B

buoyancy term with **weighted** average of $T'$
(T$_0$ exact):

$$\frac{1}{T_0} A^N \zeta T' = \frac{T'}{T_0} + dz^2 \frac{1}{2} \frac{s}{(s+1)^2} \frac{1}{T_0} \frac{\partial^2 T'}{\partial z^2} + O(dz^3)$$

buoyancy term with **arithmetic** average of $T'$
(T$_0$ exact):

$$\frac{1}{T_0} A^N \zeta T' = \frac{T'}{T_0} + dz \frac{1}{2} \frac{s-1}{s+1} \frac{1}{T_0} \frac{\partial T'}{\partial z} + O(dz^2)$$

buoyancy term with weighted average for $T'$ and $T_0$:

$$\frac{1}{A^N T_0} A^N \zeta T' = \frac{T'}{T_0} + dz^2 \frac{1}{2} \frac{s}{(s+1)^2} \left[ \frac{1}{T_0} \frac{\partial^2 T'}{\partial z^2} - \frac{1}{T_0^2} \frac{\partial^2 T_0}{\partial z^2} T' \right] + O(dz^3)$$
Truncation error in stretched grids

Divergence – grid stretching variant A

Divergence with **weighted average** of \( u \) (and \( v \)) to the half levels:

\[
\text{div} \mathbf{v} = \frac{1}{g} \left[ \frac{\partial g u(x, \zeta)}{\partial x} + \frac{\partial}{\partial \zeta} \left( \frac{\partial z}{\partial x} u - w \right) \right] \\
\]

\[
\begin{align*}
\text{div} \mathbf{v} &= \frac{\partial u(x, z)}{\partial x} + \frac{\partial w(x, z)}{\partial z} + d\zeta^2 \left( -\frac{1}{4} \frac{\partial^2 z}{\partial x \partial \zeta} \frac{\partial z}{\partial \zeta} \frac{\partial^2 u}{\partial x \partial z^2} + \frac{1}{4} \frac{\partial^2 z}{\partial \zeta^2} \frac{\partial z}{\partial \zeta} \frac{\partial^2 u}{\partial x \partial z^2} \right) \\
&\quad - \frac{1}{6} \left( \frac{\partial z}{\partial \zeta} \right)^2 \frac{\partial z}{\partial x} \frac{\partial^3 u}{\partial z^3} + \frac{1}{24} \left( \frac{\partial z}{\partial \zeta} \right)^2 \frac{\partial^3 w}{\partial z^3} \\
&\quad + dx^2 () + \ldots
\end{align*}
\]

Divergence with **only arithmetic average** of \( u \) (and \( v \)) to the half levels:

\[
\begin{align*}
\text{div} \mathbf{v} &= \frac{\partial u(x, z)}{\partial x} + \frac{\partial w(x, z)}{\partial z} + d\zeta^2 \left( -\frac{1}{4} \frac{\partial^2 z}{\partial x \partial \zeta} \frac{\partial z}{\partial \zeta} \frac{\partial^2 u}{\partial x \partial z^2} + \frac{1}{2} \frac{\partial^2 z}{\partial \zeta^2} \frac{\partial z}{\partial \zeta} \frac{\partial^2 u}{\partial x \partial z^2} \right) \\
&\quad - \frac{1}{6} \left( \frac{\partial z}{\partial \zeta} \right)^2 \frac{\partial z}{\partial x} \frac{\partial^3 u}{\partial z^3} + \frac{1}{24} \left( \frac{\partial z}{\partial \zeta} \right)^2 \frac{\partial^3 w}{\partial z^3} \\
&\quad - \frac{1}{4} \frac{\partial z}{\partial z} \frac{\partial^2 z}{\partial x \partial \zeta} \frac{\partial^2 u}{\partial x \partial z} - \frac{1}{4} \frac{\partial z}{\partial \zeta^3} \frac{\partial^2 u}{\partial \zeta \partial x} \\
&\quad + dx^2 () + \ldots
\end{align*}
\]
**Truncation error in stretched grids**

**Divergence – grid stretching variant B**

Divergence with **weighted average** of \( u \) (and \( v \)) to the half levels:

\[
\text{div } \mathbf{v} = \frac{\partial u}{\partial x} \bigg|_z + \frac{\partial w}{\partial z} \bigg|_x + \frac{1}{8} \frac{\partial^2 u}{\partial x \partial z^2} \left( \frac{1}{s} - s \right) + O(dz^2, dx^2)
\]

Divergence with **only arithmetic average** of \( u \) (and \( v \)) to the half levels:

\[
\text{div } \mathbf{v} = \frac{\partial u}{\partial x} \bigg|_z + \frac{\partial h \partial u}{\partial x \partial z} \left( \frac{1}{2} - \frac{1}{4} \left( s + \frac{1}{s} \right) \right) + \frac{\partial w}{\partial z} \bigg|_x + \\
+ \frac{1}{8} \frac{\partial^2 u}{\partial x \partial z^2} \left( \left( \frac{1}{s} - s \right) + \frac{1}{2} \left( \frac{1}{s^2} - s^2 \right) \right) + O(dz^2, dx^2)
\]

*not a consistent discretization if \( s \neq 1 \)!*
Summary

- New fast waves solver since COSMO 4.24
- in operational use at DWD since 16 Jan. 2013
- the higher numerical stability (in particular in steeper terrain) stems at least partly from a better and more consistent discretization in a vertically stretched grid
- Remind: in a stretched and staggered grid not every information is contained in the metric coefficients. The relations between main and half levels influence the discretization
- Proper derivation (use the exact positions of half and main levels!) of truncation errors helps in the decision in which way weightings should be used.

M. Baldauf (2013) COSMO Technical report No. 21 (www.cosmo-model.org)
How can we check the correctness of the previous considerations?

An analytic solution for linear gravity waves in a channel as a test case for solvers of the non-hydrostatic, compressible Euler equations
Motivation

For the development of dynamical cores (or numerical methods in general) idealized test cases are an important evaluation tool.

• Idealized standard test cases with (at least approximated) analytic solutions:
  • stationary flow over mountains
  • Balanced solutions on the sphere: Staniforth, White (2011) ASL
  • non-stationary, linear expansion of gravity waves in a channel
    Skamarock, Klemp (1994) MWR for Boussinesq-approx. atmosphere
  • most of the other idealized tests only possess 'known solutions' gained from other numerical models.

There exist even fewer analytic solutions which use exactly the same equations as the numerical model used, i.e. in the sense that the numerical model converges to this solution. One exception is given here:
  linear expansion of gravity/sound waves in a channel
Non-hydrostatic, **compressible**, 2D Euler equations in a flat channel (shallow atmosphere) on an f-plane

For analytic solution only one further approximation is needed: linearisation (\(=\) **controlled** approximation) around an **isothermal**, steady, **hydrostatic** atmosphere at rest \((f \neq 0\) possible) or with a constant basic flow \(U_0\) (and \(f=0\))
Bretherton-, Fourier- and Laplace-Transformation →

Analytic solution for the Fourier transformed vertical velocity $w$

$$\dot{w}_b(k_x, k_z, t) = -\frac{1}{\beta^2 - \alpha^2} \left[ -\alpha \sin \alpha t + \beta \sin \beta t + \left( f^2 + c_s^2 k_x^2 \right) \left( \frac{1}{\alpha} \sin \alpha t - \frac{1}{\beta} \sin \beta t \right) \right] \frac{g \hat{\rho}_b(k_x, k_z, t = 0)}{\rho_s}$$

analogous expressions for $u_b(k_x, k_z, t)$, ...

The frequencies $\alpha, \beta$ are the gravity wave and acoustic branch, respectively, of the dispersion relation for compressible waves in a channel with height $H$;

$$k_z = \left( \frac{\pi}{H} \right) \cdot m$$

Baldauf, Brdar (2013) QJRMS
Linear, unsteady gravity wave initialization similar to Skamarock, Klemp (1994) MWR
Initialization similar to Skamarock, Klemp (1994)

\[ T'(x, z, t = 0) = \Delta T \cdot e^{\frac{1}{2} \delta z} \cdot e^{-\frac{(x-x_0)^2}{a^2}} \cdot \sin \pi \frac{z}{H} \]

\[ p'(x, z, t = 0) = 0 \]

**Small scale test**
with a basic flow \( U_0 = 20 \text{ m/s} \)
\( f = 0 \)

Black lines: analytic solution
*(Baldauf, Brdar (2013) QJRMS)*

Shaded: COSMO
Convergence test with vertically stretched grid

initial condition for $T'$ and grids for the first 3 resolutions
Convergence test with vertically stretched grid for old and new fast waves solver

The improvement is best for coarse resolutions, because here the highest relative stretching for neighbouring grid boxes occurs.
The analogous linearized solution on the sphere …
Non-hydrostatic, compressible, shallow atmosphere, adiabatic, 3D Euler equations on a sphere with a rigid lid

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p - g e_z - 2\Omega \times \mathbf{v}
\]
\[
\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = -\rho \nabla \cdot \mathbf{v}
\]
\[
\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p = c_s^2 \left( \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho \right)
\]
\[
c_s = \sqrt{\frac{c_p}{c_v} \frac{\rho}{p}}
\]

Boundary conditions:
\[
w(r=r_s) = 0
\]
\[
w(r=r_s+H) = 0
\]

most global models using the compressible equations should be able to exactly use these equations in the dynamical core for testing.

For an analytic solution only one further approximation is needed: **linearisation** (= controlled approximation) around an **isothermal, steady, hydrostatic atmosphere**
Analytic solution for the vertical velocity \( w \) (Fourier component with \( k_z \), spherical harmonic with \( l,m \))

\[
\hat{w}_{lm}(k_z, t) = -\frac{1}{\beta^2 - \alpha^2} \left[ -\alpha \sin \alpha t + \beta \sin \beta t + \left( f^2 + c_s^2 \frac{l(l+1)}{r_s^2} \right) \left( \frac{1}{\alpha} \sin \alpha t - \frac{1}{\beta} \sin \beta t \right) \right] \frac{g \hat{\rho}_{lm}(k_z, t=0)}{\rho_s}
\]

analogous expressions for \( \hat{u}_{lm}(k_z, t) \), ...

The frequencies \( \alpha, \beta \) are the gravity wave and acoustic branch, respectively, of the dispersion relation for compressible waves in a spherical channel of height \( H \);
\( k_z = (\pi / H) \cdot n \)

Baldauf, Reinert, Zängl (2013) acc. by QJRMS
Test scenarios

(A) Only gravity wave and sound wave expansion

(B) Additional Coriolis force (‘global f-plane approx. on a sphere‘)

\[ 2\Omega (\lambda, \phi) = f \cdot e_r (\lambda, \phi), \quad f=\text{const.} \quad (\text{and } v_0 = 0) \]

→ test proper discretization of inertia-gravity modes, e.g. in a C-grid discretization.

For problems with C-grid discretizations on non-quadrilateral grids see

Nickowicz, Gavrilov, Tasic (2002) MWR,
Thuburn, Ringler, Skamarock, Klemp (2009) JCP,
Gassmann (2011) JCP

(C) Additional advection by a solid body rotation velocity field \( v_0 = Q \times r \)

→ test the coupling of fast (buoyancy, sound) and slow (advection, Coriolis) processes

Problem: solid body rotation field generates centrifugal forces!

Solution: \( Q = -\Omega \rightarrow \) similar to (A) in the absolute system

(analogous to Läuter et al. (2005) JCP)
**ICON** (joint development of DWD/MPI-M) simulation

in $z=5$ km ($f=0$)

**Small earth simulations**

*Wedi, Smolarkiewicz (2009) QJRMS*

- $r_s = r_{\text{earth}} / 50 \sim 127$ km
  simulations with $\Delta \varphi \sim 1^\circ ... 0.0625^\circ$
  $\rightarrow \Delta x \sim 2.2 \text{ km} \ldots 0.14 \text{ km}$
  $\rightarrow$ non-hydrostatic regime

- for runs *with* Coriolis force:
  $f = f_{\text{earth}} \cdot 10 \sim 10^{-3}$ $1/s$
  $\rightarrow$ dimensionless numbers

\[ Ro = 0.2 \cdot Ro_{\text{earth}} \]
\[ f / N = 10 \cdot f_{\text{earth}} / N \sim 0.05 \]
Time evolution of $T'$

$f=0$

test scenario (A)

$f\neq 0$

test scenario (B)

Black lines: analytic solution

Colours: ICON simulation
Time evolution of $T'$

$f=0$

test scenario (A)

$f \neq 0$

test scenario (B)

Black lines: analytic solution

Colours: ICON simulation
Time evolution of $T'$

$f=0$

test scenario (A)

$f\neq 0$

test scenario (B)

Black lines: analytic solution

Colours: ICON simulation
Time evolution of $T'$

$f=0$

test scenario (A)

$f \neq 0$

test scenario (B)

Black lines: analytic solution
Colours: ICON simulation
Time evolution of $T'$

\[ f=0 \]

Test scenario (A)

\[ f \neq 0 \]

Test scenario (B)

Black lines: analytic solution

Colours: ICON simulation
Time evolution of $T'$

\[ f=0 \]

test scenario (A)

\[ f \neq 0 \]

test scenario (B)

Black lines: analytic solution
Colours: ICON simulation
Time evolution of $T'$

$f=0$

Test scenario (A)

$f \neq 0$

Test scenario (B)

Black lines: analytic solution

Colours: ICON simulation
Time evolution of $T'$

$f=0$

test scenario (A)

$f\neq 0$

test scenario (B)

Black lines: analytic solution

Colours: ICON simulation
Time evolution of $T'$

$f=0$

test scenario (A)

$f\neq 0$

test scenario (B)

Black lines: analytic solution
Colours: ICON simulation
Convergence rate of the ICON model

- The ICON simulation with/without Coriolis force produces almost similar $L_2$, $L_\infty$ errors
- Spatial-temporal convergence order of ICON is $\sim 1$
Test scenarios

(A) Only gravity wave and sound wave expansion

(B) ...

(C) Additional advection by a solid body rotation velocity field \( v_0 = Q \times r \)
   \( \rightarrow \) test the coupling of fast (buoyancy, sound) and slow (advection) processes

Problem: solid body rotation field generates centrifugal forces!
Solution: \( Q = -\Omega \) \( \rightarrow \) similar to (A) in the absolute system
(analogous to Läuter et al. (2005) JCP)

Euler equations in spherical coordinates

\[
\begin{align*}
\frac{\partial u}{\partial t} + v \cdot \nabla u - \frac{\tan \phi}{r} uv + \frac{1}{r} uw &= - \frac{1}{\rho r \cos \phi} \frac{\partial p}{\partial \lambda} + 2\Omega (v \sin \phi - w \cos \phi) \\
\frac{\partial v}{\partial t} + v \cdot \nabla v + \frac{\tan \phi}{r} u^2 + \frac{1}{r} vw &= - \frac{1}{\rho r} \frac{\partial p}{\partial \phi} - 2\Omega u \sin \phi - \Omega^2 r \cos \phi \sin \phi \\
\frac{\partial w}{\partial t} + v \cdot \nabla w - \frac{u^2 + v^2}{r} &= - \frac{1}{\rho} \frac{\partial p}{\partial r} - g + 2\Omega u \cos \phi + \Omega^2 r \cos^2 \phi
\end{align*}
\]

Most deep terms are needed now for the analytic solution!
... but not all are contained in ICON
Test scenario (C) with the ICON model

Nevertheless:
The missing deep terms in the horizontal equations are not visible until 0.0625°: ICON converges ~ 1st order
Summary

• Analytic solution of the linearized, compressible, non-hydrostatic Euler equations on the sphere (for global models) and on a plane (for LAM's) have been derived → a reliable solution for a well known test exists and can be used not only for qualitative comparisons but even as a reference solution for convergence tests

• This solution/test exercises several important processes/terms and the time integration scheme of the numerical model

• On the sphere the test setup is quite similar to one of the DCMIP 2012 test cases

• 'standard' approximations used: shallow atmosphere, 'global f-plane approx.' can be easily realised in every atmospheric model

• only one further approximation: linearisation (=controlled approx.)

• For fine enough resolutions ICON has a spatial-temporal convergence rate of about 1, no drawbacks visible

• Such tests can be used to evaluate improved discretizations. Example: vertical discretizations in the new fast waves solver in COSMO

References:
Baldauf, Brdar (2013) QJRMS (DOI:10.1002/qj.2105) partly financed by MetStröm
Baldauf, Reinert, Zängl (2013) accepted by QJRMS
Influence of the water loading in strong convective simulations

Motivation: a bad forecast quality of COSMO-DE at 20 June 2013
'new'

Front coming in at evening;
convergence line during afternoon with heavy precipitation
'nnew' Radar

M. Baldauf (DWD) 44
Bug fix in the buoyancy term of COSMO:

\[ \frac{\partial w}{\partial t} + \mathbf{v} \cdot \nabla w = -\frac{1}{\rho} \frac{\partial p'}{\partial z} + g \left( \frac{p_0 T'}{p T_0} - \frac{p'}{p} + \frac{p_0 T}{p T_0} q_x \right) + \ldots \]

Moisture correction in the ideal gas law (water loading):

\[ q_x := \left( \frac{R_v}{R_d} - 1 \right) q_v - q_c - q_r - \ldots \]

RK-scheme with new fast waves solver:

until COSMO 4.27: moisture variables \( q_v, q_c, \ldots \) in \( q_x \) at time level \( n_{new} \)

bug fix: moisture variables \( q_v, q_c, \ldots \) in \( q_x \) at time level \( n_{now} \)

reason: during the RK scheme, \( n_{new} \) means 'old' for the moisture variables!
Front coming in at evening;
convergence line during afternoon with heavy precipitation

M. Baldauf (DWD)
Radar 'nnow' = old

RADAR COMPOSITE
Valid: 20 Jun. 2013 14 - 15 UTC
Forecast time: 20 June 2013 15:00 UTC
Total precipitation [mm/h] (shaded) Geoplot at 700 hPa [gpm] (dist. last 1gpm)

'Slave' = old

RADAR COMPOSITE
Valid: 20 Jun. 2013 14 - 15 UTC
Forecast time: 20 June 2013 15:00 UTC
Total precipitation [mm/h] (shaded) Geoplot at 700 hPa [gpm] (dist. last 1gpm)
COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

'nnew' = old

Radar

RADAR COMPOSITE
valid: 20 JUN 2013 16 - 17 UTC

'M. Baldauf (DWD) 49

'nnow'}
COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

'nnew' = old

Radar

RADAR COMPOSITE
valid: 20 JUN 2013 17 - 18 UTC
Total precipitation [mm/h] (shaded)
Geoplot at 700 hPa [gpmh] (cost, last 1hpm

1h PRECIPITATION

Start time: 20.06.2013 12:00 UTC
Forecast time: 20.06.2013 18:00 UTC
Total precipitation [mm/h] (shaded) Geoplot at 700 hPa [gpmh] (cost, last 1hpm

M. Baldauf (DWD)
COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

\( \text{\textquoteleft nnew\textquotepright} = \text{old} \)

Radar

\( \text{RADAR COMPOSITE} \)

Valid: 20 JUN 2013 18 - 19 UTC

Total precipitation [mm/h] (shaded) Geoplot, at 700 hPa [gpm] (dist. last 1gpm)

Start time: 20.06.2013 12:00 UTC
Forecast time: 20.06.2013 16:00 UTC

\( \text{\textquoteleft nnow\textquotepright} \)
COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

'new' = old

Radar

RADAR COMPOSITE
Valid: 20 JUN 2013 19 - 20 UTC
1h PRECIPITATION

Start time: 20.06.2013 12:00 UTC
Forecast time: 20.06.2013 26:00 UTC
Total precipitation [mm/1h] (shaded)
Geopr: at 700 hPa [gpmn] (dist. last 1gpmn)

M. Baldauf (DWD)
COSMO-DE, 20 June 2013, 12 UTC run
1h precipitation sum

'nnew' = old

Radar

RADAR COMPOSITE
valid: 20 JUN 2013 20 - 21 UTC
1h PRECIPITATION

'nnow'

Total precipitation [mm/1h] (shaded) Geopot. at 700 hPa [gpm] (cont. last 1gpm)

Start time: 20.06.2013 12:00 UTC
Forecast time: 20.06.2013 21:00 UTC
Total precipitation [mm/1h] (shaded) Geopot. at 700 hPa [gpm] (cont. last 1gpm)

Start time: 20.06.2013 12:00 UTC
Forecast time: 20.06.2013 21:00 UTC
Total precipitation [mm/1h] (shaded) Geopot. at 700 hPa [gpm] (cont. last 1gpm)
vertical velocity $w$ [m/s] in $z \sim 5$ km

'new' = old

w.1 t=3:00, lev=26

'now'

w.1 t=3:00, lev=26
Idealised convection test at the resolution limit of the model

- T-perturbation $\Delta T=+2K$ in only one grid box (in $z \sim 500m$)
- Stratification analogous to Weismann, Klemp (1982) MWR
- Atmosphere at rest
- No turbulence, only cloud physics
- Non-stretched grid
Idealised convection test at the resolution limit of the model

\[ w_{\text{max}} \ [\text{m/s}] \]

\[ \text{cloud water } q_c \ [0.01 \text{ g/kg}] \]

graupel-scheme used \((q_v, q_c, q_i, q_r, q_s, q_g)\)
Idealised convection test at the resolution limit of the model

**rain** $q_r$ [0.01 g/kg]

**precipitation rate** [mm/day]

graupel-scheme used ($q_v$, $q_c$, $q_i$, $q_r$, $q_s$, $q_g$)
Summary

- Convection-permitting models are quite sensitive to (among others) the treatment of the buoyancy term (not a new insight, of course)
- The water loading contribution to the buoyancy is relatively large and even the 'small' error of using moisture variables one time level too late has a strong influence in the evolution of convection
- Experience: the largest improvements in weather forecasting stem from bug removals …
Staggered vs unstaggered grids

... and what does this mean for discontinuous Galerkin methods?
Dynamical core (Euler solver) developments in COSMO

- Current Runge-Kutta dynamical core
  - further maintenance (DWD) (~0.5 FTE)
  - higher order discretizations (Univ. Cottbus) (~1 FTE)

COSMO priority project 'Conservative dynamical core (2008-2012):
- EULAG as a candidate for the future COSMO dyn. Core

  Ziemiański et al. (2011) Acta Geophysica
  Rosa et al. (2011) Acta Geophysica
  Kurowski et al. (2011) Acta Geophysica

→ follow up PP ‚COSMO-EULAG operationalization (2012-2015) (IMGW, Poland) (~3 FTE)
- fully implicit FV solver ‚CONSOL‘ (CIRA, Italy) (~0.5 FTE)

Jameson (1991) AIAA

Project in the framework of the German research foundation
- Dynamical core based on Discontinuous Galerkin methods
  (DWD, Univ. Freiburg) (~1.08 FTE)

the last three dynamical core developments use an unstaggered (!) grid

1 FTE (full time equivalent) = 1 person/year
Linear 1D wave equation as a prototype for hyperbolic equations

\[
\begin{align*}
\frac{\partial u}{\partial t} &= -g \delta_x h \\
\frac{\partial h}{\partial t} &= -H_0 \delta_x u \\
\end{align*}
\]

\[ c = \sqrt{gH_0} \]

wave ansatz: \( \phi(x, t) = \tilde{\phi} e^{i(kx - \omega t)} \)

continuous: \( \delta_x \phi = \frac{\partial \phi}{\partial x} \rightarrow ik_x \tilde{\phi} \)

unstaggered \( \delta_x \phi_j = \frac{\phi_{j+1} - \phi_{j-1}}{2\Delta x} \rightarrow i \frac{\sin k_x \Delta x}{\Delta x} \tilde{\phi} \)

staggered: \( \delta_x \phi_j = \frac{\phi_{j+\frac{1}{2}} - \phi_{j-\frac{1}{2}}}{\Delta x} \rightarrow i \frac{2\sin \frac{k_x \Delta x}{2}}{\Delta x} \tilde{\phi} \)

e.g. D. R. Durran: Numerical methods … 'modified’ wavenumber
Dispersion relation of the 1D wave equation

Frequency $\omega$

- $\omega$ = 0 for 2$\Delta x$ waves

Phase-, group-velocity

- Negative group velocity

Staggered grid:

$\Delta t_{\text{stagg}} = \frac{1}{2} \Delta t_{\text{unstagg}}$

Unstaggered grid:

$\omega = 0$ for 2$\Delta x$ waves
1D wave expansion with a Discontinuous Galerkin (DG) discretization

\[
\begin{align*}
\frac{\partial u}{\partial t} &= -\frac{\partial g}{\partial x} h \\
\frac{\partial h}{\partial t} &= -\frac{\partial H_0}{\partial x} u
\end{align*}
\]

\[c = \sqrt{gH_0}\]

Literature:

Hu, Hussaini, Rasetarinera (1999) JCP: 1D advection-, 2D wave-equation

Hu, Atkins (2002) JCP: non-uniform grids \(k = k(\omega)\)


\(\rightarrow\) talk by F. Giraldo
Discontinuous Galerkin (DG) methods in a nutshell

\[ \frac{\partial q^{(k)}}{\partial t} + \nabla \cdot f^{(k)}(q) = S^{(k)}(q), \quad k = 1, \ldots, K \]

\[ \int_{\Omega_j} dx \; v(x) \cdot \ldots \rightarrow \text{weak formulation (increases solution space)} \]

\[ q^{(k)}(x, t) = \sum_{l=0}^{p} q^{(k)}_{j,l}(t) \; p_l(x - x_j) \]

Finite-volume ingredient

\[ f(q) \rightarrow f^{num}(q^+, q^-) = \frac{1}{2} \left( f(q^+) + f(q^-) - \alpha (q^+ - q^-) \right) \]

Lax-Friedrichs flux

\[ \rightarrow \text{ODE-system for } q^{(k)}_{j,l} \]

\[ \rightarrow \text{talk by F. Giraldo} \]

From Nair et al. (2011) in ‘Numerical techniques …

Cockburn et al. (1989) JCP

e.g. Legendre-Polynomials

M. Baldauf (DWD)
DG with p=0
(=classical FV-method)

dispersion relation is the same as for the 2nd order cent. diff. scheme on an *unstaggered* grid + 2nd order (hyper-)diffusion
DG with $p=1$ → 2 physically relevant (!) modes (not spurious/parasitic mode)
DG with $p=2$

- The lowest mode has completely wrong behaviour near $k \Delta x = \pm \pi$.
- $\alpha > 0.15 \, c$ necessary!

### Graphs

- **$\alpha = c$**
- **$\alpha = 0.11 \, c$**
- **$\alpha = 0$**

### Key Points

- Frequency gap
DG with $p=0,1,2,3$ ($\alpha=c$ used)

| $p$ | $\max |\omega| \Delta x/c$ |
|-----|---------------------------|
| 0   | 1                         |
| 1   | 3.9                       |
| 2   | 7.51                      |
| 3   | 11.83                     |
| 4   | 16.86                     |
| 5   | 22.58                     |
| 6   | 28.96                     |
| 10  | 60.75                     |
| 15  | 113.68                    |

$\rightarrow \max |\omega| \Delta x/c \approx 1 + 2.6 \ p + 0.33 \ p^2$

increases slightly stronger than linear with $p$.

Choose not too large $p$!
Conclusions from 1D wave expansion with DG method:

• Wave expansion with DG methods behaves as on an unstaggered grid, but with strong damping of short waves.
• There is no spurious (or 'parasitic') mode in wave expansion: the dispersion relation is continuous and smooth until wavelength $2 \frac{dx}{(p+1)}$ if the numerical diffusive flux is not too small (but this is automatically fulfilled if $\alpha = \text{max EV of } f'(q)$).
• Maximum of frequency increases slightly stronger than linear with $p$ → Choose not too large polynomial degree $p$. 

Thank you very much for your attention