

Modelling convective boundary layers in the terra-incognita

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ABSTRACT

With increasing supercomputer power, numerical weather prediction forecasts at grid lengths in the range of 100 m to 1 km are now possible. Within this range, the convective boundary layer is partially resolved, but not a full Large-eddy simulation (LES). Wyngaard (2004) called this regime the "terra-incognita". Previous studies have examined the role of the sub-grid model in the terra-incognita. Here we will use an LES model to investigate the sensitivity of the convective boundary layer to horizontal grid length and advection scheme. The boundary layer top entrainment shows significant sensitivity to the advection scheme in the terra-incognita, indicating that it must be considered alongside the sub-grid model.

1 Introduction

During the diurnal cycle, the typical length scale (L) of the boundary layer eddies varies between of order 1 km by day to much smaller values at night. Until recently, numerical weather prediction (NWP) models used a horizontal grid length much larger than the size of the boundary-layer eddies. The scale separation meant that column-based parametrizations were formally justified. However, with current supercomputer power, and in order to provide more skilful regional forecasts, operational weather centres now run limited area models at horizontal grid lengths as small as a few kilometres. For example, the 1.5 km grid length configuration of the Met Office Unified Model (MetUM) provides high-resolution forecasts of severe convective precipitation and fog for domains covering the UK. The ratio of the horizontal grid length to the boundary layer eddy size is now of order one; such a regime was named the terra-incognita by Wyngaard (2004). For several years, the terra-incognita has been an issue for NWP of convective systems (Craig and Dörnbrack, 2008). The terra-incognita is now a pressing practical issue for NWP of the atmospheric boundary layer. Whilst many weather centres run models at these resolutions, there is currently little theoretical and numerical modeling basis for how to represent the boundary layer.

Figure 1 illustrates the terra-incognita in more detail. One limit is the mesoscale in which the grid length (D) is such that $D \gg L$ and there is a good scale separation between the grid and the boundary layer eddies. Here, a column-based parametrization of the boundary layer is formally justified. An example of a column-based parametrization is a vertical diffusion operator. In the Large-eddy simulation (LES) limit, $D \ll L$ and the main boundary-layer eddies are well resolved. The terra-incognita is the intermediate regime characterised by the absence of scale separation, where $D \sim L$. Here the assumptions of the column based boundary-layer parametrization break down. Simulations in this regime are dependent on the model formulation close to the grid scale D , and so the truncation errors from the numerical methods (e.g. the advection scheme), can significantly affect the representation of boundary-layer eddies. Despite such sensitivity, the terra-incognita is a regime of immediate and long term practical importance for NWP. Although NWP models are reaching the terra-incognita for the boundary layer, they are unlikely to reach the resolution of the LES regime for several decades at least, and therefore it is essential that numerical modellers address it. Sullivan and Patton (2011) showed that grid lengths smaller than 20 m

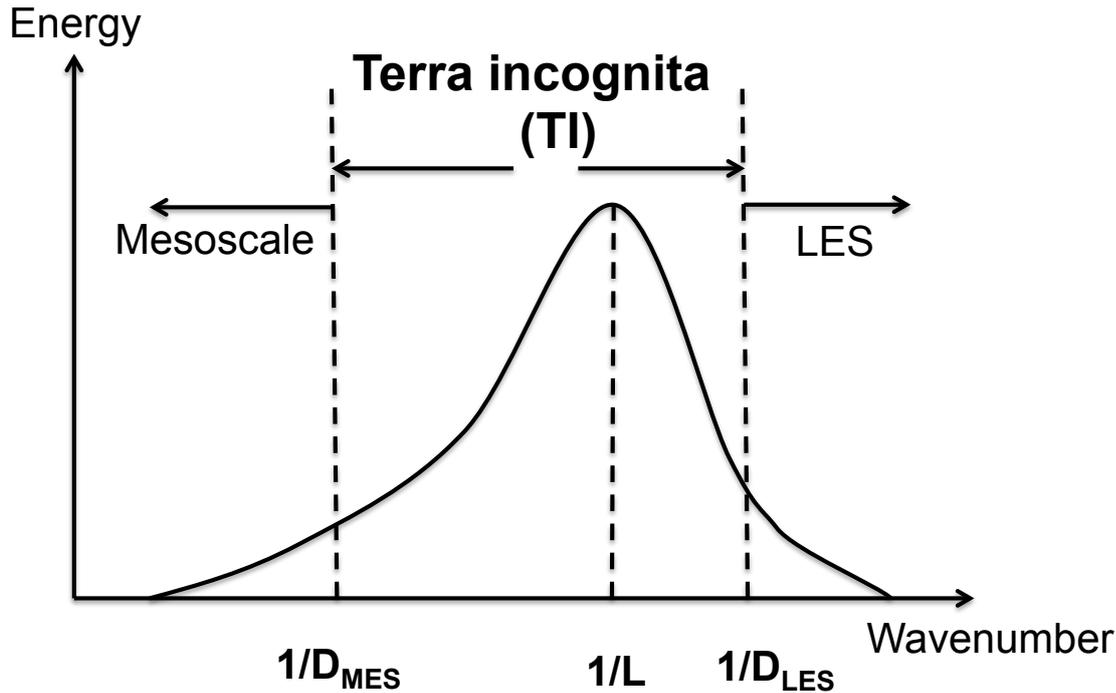


Figure 1: Schematic illustrating the terra-incognita. Plotted is the energy spectrum against total horizontal wavenumber. L is the length scale of the boundary layer turbulence, and D_{MES} and D_{LES} are the grid-lengths associated with a Mesoscale and LES simulation respectively. Figure based on Wyngaard 2004.

were required for resolving the convective boundary layer (CBL), around 100 times smaller than current high resolution NWP.

1.1 Advection schemes and sub-grid models

Wyngaard (2004) derived modifications to the column-based parametrizations in the terra-incognita using horizontal arrays of surface layer observations. He replaced the scalar diffusion in a column-based scheme with a tensor that accounted for backscatter from smaller to larger scales. In an LES model, Honnert et al. (2011) evaluated the sub-grid model term as the grid length changes from the LES regime into the terra-incognita. Whilst sub-grid schemes are often explicitly formulated as a diffusion operator, advection schemes can have implicit diffusion due to truncation errors (errors due to finite resolution, Chow and Moin (2003)). Since the truncation errors are at the same scale as the boundary layer eddies in the terra-incognita, the implicit diffusion from the advection scheme could be as significant as the sub-grid model. However, the role of the advection scheme in the boundary layer terra-incognita has so far been overlooked. In addition to dissipative effects, the truncation can also give dispersion errors.

This paper is an initial study of the role of the advection scheme in the terra-incognita. We base our approach on experiences at the LES limit, where several authors have considered the compensating diffusion between advection and sub-grid schemes. Brown et al. (2000) show how changing the diffusion in the advection scheme (by switching between centred-difference and monotone schemes) can compensate for that in the sub-grid model. In fact, some have discarded the sub-grid model entirely and use the dissipation from just the advection scheme, the so-called implicit LES, ILES (Grinstein et al., 2007). Beare et al. (2007) configured both the MetUM and the Met Office Large-eddy Model (MetLEM) with the same Smagorinsky sub-grid model for a convective boundary layer LES. They demonstrated that the

semi-lagrangian scheme in the MetUM has more implicit diffusion than the centre-difference schemes in the MetLEM, and this had a significant impact on modelling the CBL. Whilst these studies are for the LES limit (Figure 1), it is likely that the issues of dissipation from the advection and sub-grid schemes will be even more significant in the terra-incognita because the truncation errors and sub-grid schemes will act at the scale of the most energetic eddies.

1.2 Entrainment

We will also focus on entraining CBLs in the diurnal cycle. These boundary layers have turbulent length scales of order 1 km, close to the grid length of high resolution NWP models. In contrast, the night-time stable boundary layer has a terra-incognita below around 10 m resolution (Beare and MacVean, 2004). Sullivan and Patton (2011) showed that the negative entrainment flux at the top of the CBL is very sensitive to resolution within the terra-incognita. This flux is an important controlling factor for the boundary layer evolution. Beare (2008), for example, showed that the timing of the onset of the early morning boundary layer is sensitive to the entrainment at the boundary-layer top. We will therefore use the entrainment flux as a key diagnostic for comparing techniques in the terra-incognita.

2 Convective boundary layer LES

The MetLEM is configured as a free convective boundary layer, with a weak geostrophic wind and significant surface sensible heat flux. The initial potential temperature is a mixed layer up to an inversion height (z_{i0}), and then an overlying stratification above:

$$\begin{aligned} \theta &= \theta_0 \quad z < z_{i0} \\ &= \theta_0 + S(z - z_{i0}); \quad z > z_{i0} \\ \theta_0 &= 293K; \quad z_{i0} = 1000m; \quad S = 0.003Km^{-1} \end{aligned} \tag{1}$$

where S is the overlying vertical temperature gradient respectively. The Coriolis parameter (f_0) is $0.0001 s^{-1}$. The geostrophic wind in the x direction is $2 ms^{-1}$ and zero in the y direction. A random perturbation of amplitude 0.1 K is applied below 250 m to initiate turbulence. Roughness lengths of 0.1 m for momentum and 0.01 m for heat are used, typical of a rural land surface. The surface sensible heat flux is constant at $150 Wm^{-2}$. A domain of 10 km x 10 km x 5 km is used; the horizontal domain size allows room for several convective eddies.

The simulations are run for 3 hours so that initial transients have decayed away, leaving a steadily entraining CBL. Runs were performed at horizontal resolutions of 1600, 800, 400, 200, 100, 50, and 25 m respectively. The vertical grid length is fixed at 100 m for horizontal grid lengths of 100 m and above. In order to provide a fine-scale reference, the 50 m and 25 m horizontal grid lengths are run with 50 m and 25 m vertical grid lengths respectively. The Smagorinsky sub-grid model was used, with the Smagorinsky constant set to 0.17, following Lilly (1967). A centred-difference advection scheme (Piacsek and Williams, 1970, P-W) is used on momentum throughout. For the tests of sensitivity to grid length, a Total Variation Diminishing scheme (TVD, Leonard et al., 1993) is used on potential temperature. When looking at the sensitivity to advection scheme, the P-W scheme will also be used on potential temperature.

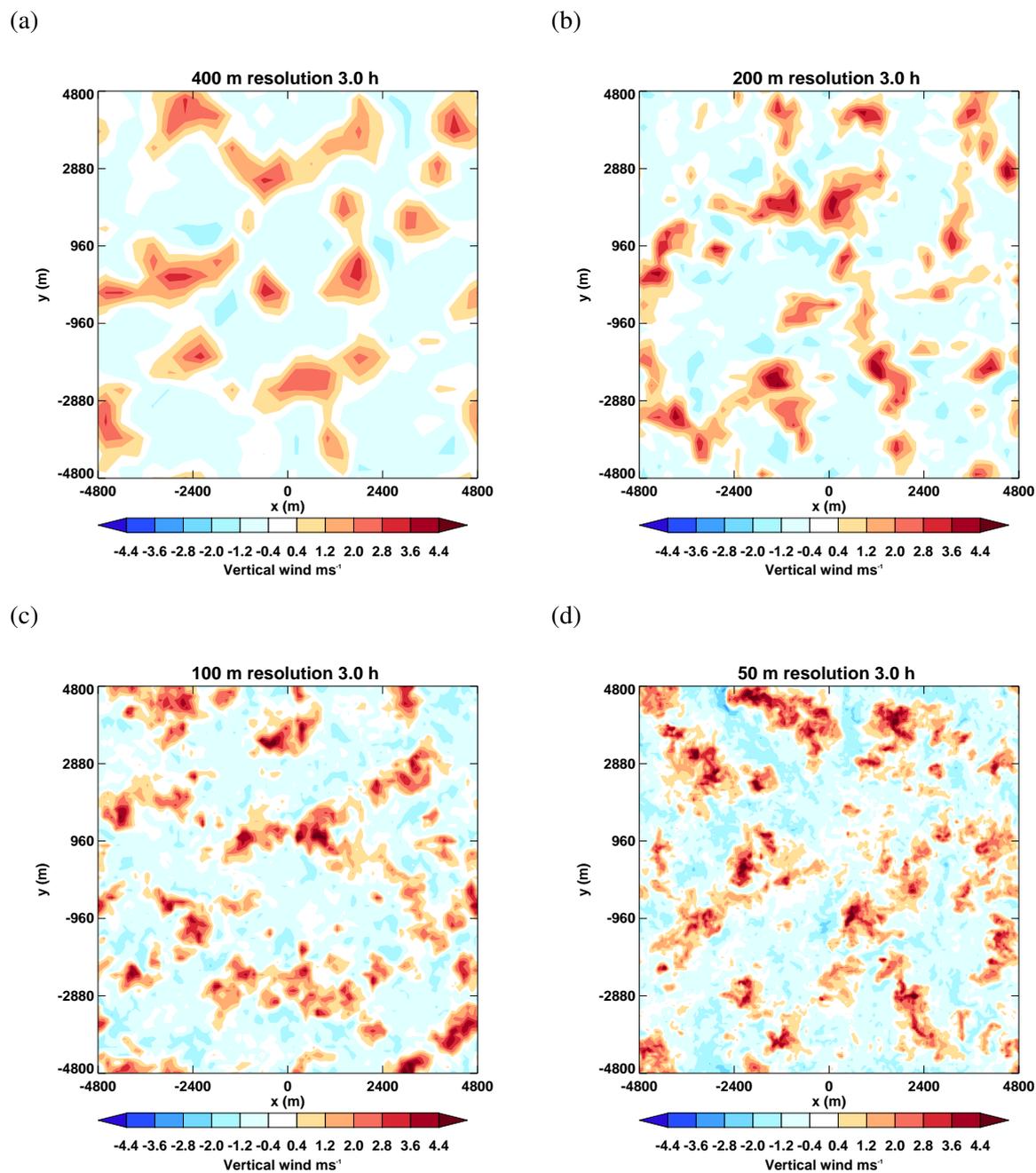


Figure 2: Horizontal cross-sections of vertical velocity at height 1 km at time 3 hours for horizontal grid lengths of: (a) 400 m, (b) 200 m, (c) 100 m and (d) 50 m. Contour interval 0.8 ms^{-1} .

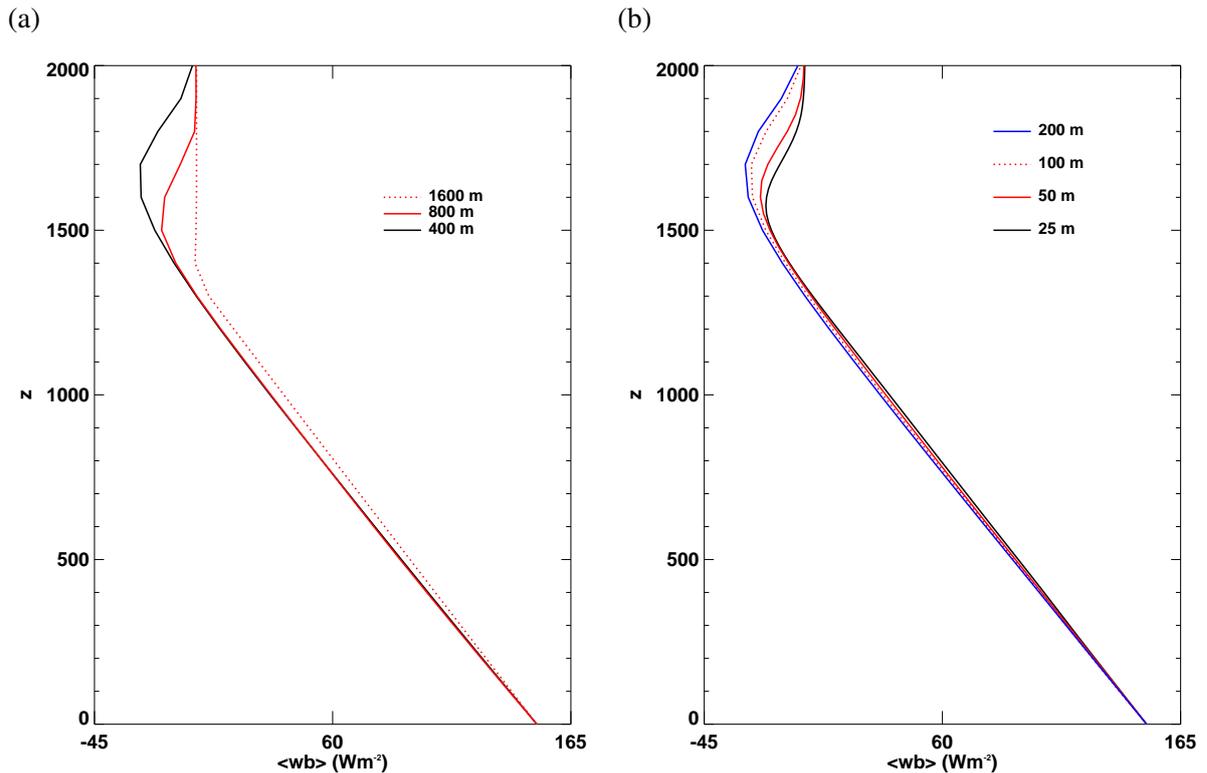


Figure 3: Total (resolved plus sub-grid) buoyancy flux for horizontal grid lengths of (a) 1600 m to 400 m, (b) 200 m to 50 m. The height (z) is in metres.

3 Initial results

3.1 Vertical velocity cross sections

Figure 2 shows the vertical velocity at height 1 km (about 0.6 of the boundary layer depth) for horizontal grid lengths from 400 m down to 50 m. At 400 m horizontal grid length, the simulations are in the terra-incognita; here, the horizontal grid length is 0.25 of the boundary layer depth. At 50 m grid length the simulations are close to an LES. The updrafts at 400 m resolution are both broader horizontally and smaller in amplitude compared with the LES limit at 50 m. As the grid length is decreased from 400 m to 50 m, the updrafts narrow and increase in magnitude; moreover the edges of the updrafts develop increasing turbulent fine structure. Whilst the 50 m grid length simulation is visibly a turbulent simulation, the smoothness of the fields at 400 m grid length mean that the LES is compromised. However, given there is still motion in the simulation, it is not sufficient to represent this regime with a single-column simulation.

3.2 Sensitivity of entrainment to resolution

Figure 3 shows vertical profiles of the area-time averaged buoyancy flux from grid lengths of 1600 m down to 50 m. The buoyancy flux shown is the sum of resolved and sub-grid contributions. Whilst the flux in the bottom half of the boundary layer is relatively unchanged, the entrainment flux at the boundary layer top is very sensitive to horizontal grid length. The entrainment flux is also not a monotonic function of grid length. At 1600 m horizontal grid length (Fig 3a), where little turbulence is resolved, the Smagorinsky sub-grid model is providing all the buoyancy flux. The Smagorinsky model is local in

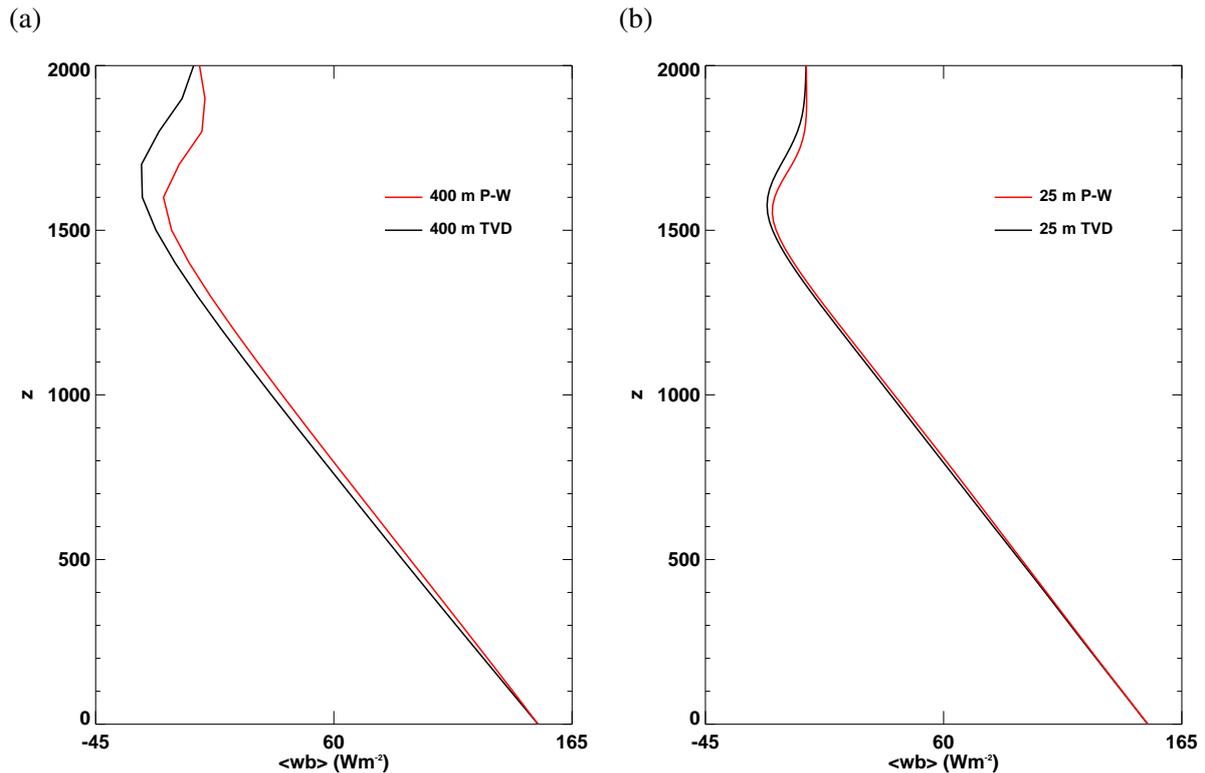


Figure 4: Total buoyancy flux for either P-W (red) or TVD (black) advection for horizontal grid lengths of (a) 400 m, (b) 25 m. The height (z) is in metres.

formulation and is incapable of providing an entrainment flux. As the grid length is decreased, some turbulence is resolved and the entrainment flux begins to increase (Fig 3a). However, as the horizontal grid length is decreased from 200 m (Fig 3a), the entrainment flux decreases again to an almost-converged value at 25 m horizontal grid length. Convergence of the entrainment was accelerated by also decreasing the vertical grid length to be the same as the horizontal grid length for the 50 m and 25 m simulations.

3.3 Sensitivity of entrainment to advection scheme

Figure 4 shows the effect of changing from the Total Variation Diminishing (TVD) scheme to the centre-difference P-W scheme operating on potential temperature. Whilst the TVD scheme maintains monotonicity of scalars, it is also more dissipative than the P-W scheme. At 400 m horizontal grid length (Fig 4a) the entrainment flux is much more sensitive to the advection scheme than at 25 m horizontal grid length (Fig 4b). The P-W scheme at 400 m grid length (Fig 4a) is closer to the high resolution simulations (Fig 4b). Since it is less dissipative than the TVD, the P-W can give smaller scale variability than the TVD scheme. However, there is also evidence of a spurious positive flux from the P-W simulation at height 1800 m. One key reason for the increased sensitivity to advection scheme at 400 m is that the truncation errors scale with the grid length and these are much closer to the length scales of the energy-containing eddies.

4 Conclusions

In this initial study we used LES to understand the behaviour of the dry convective boundary layer in the terra-incognita. The simulated entrainment was not only sensitive to horizontal grid length, but also to advection scheme. The entrainment was not a monotonic function of horizontal grid length, reaching maximum amplitude within the terra-incognita, but decreasing as the LES limit was approached. Such functionality presents challenges for parametrization. We chose to focus on the advection scheme, as it often receives less attention in boundary-layer meteorology relative to the sub-grid model (e.g. [Honnert et al., 2011](#)). Such sensitivity to model components is a very uncomfortable regime for the LES modeller, where the ideal is insensitivity to model components ([Mason, 1994](#)). However, given that NWP models will increasingly be in the terra-incognita, it is one of practical importance for forecasting.

We have only scratched the surface of modelling the boundary layer terra-incognita. For example, there is the practical issue of combining the column-based parametrizations typically used for the boundary layer in NWP with the 3D Smagorinsky model. It would also be beneficial to look at adapting sophisticated sub-grid formulations used in LES for the terra-incognita such as the stochastic ([Mason and Thomson, 1992](#)) or dynamic ([Meneveau and Katz, 2000](#)) models.

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