TOWARD A UNIFIED MICROPHYSICS SCHEME FOR NWP MODELS IN CANADA

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Outline

1. Overview of current Canadian NWP models and plan to 2017

2. Multi-moment microphysics for high resolution NWP

3. Development of unified microphysics
   a) Sub-grid scale cloud and precipitation fractions
   b) Consolidation of hydrometeor categories

4. Future research

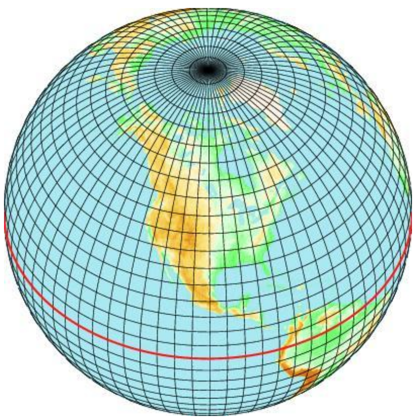
1. Overview of current Canadian NWP models and plan to 2017

Environment Canada's NWP Model

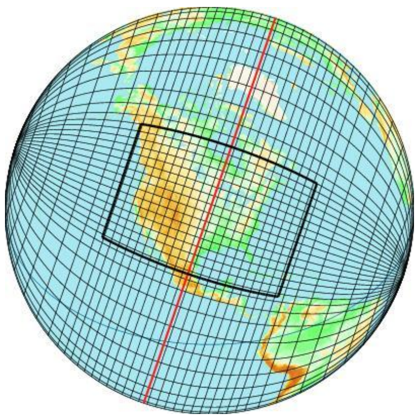
GEM (<u>G</u>lobal <u>E</u>nvironmental <u>M</u>ultiscale)

<u>Various grid configurations</u>:

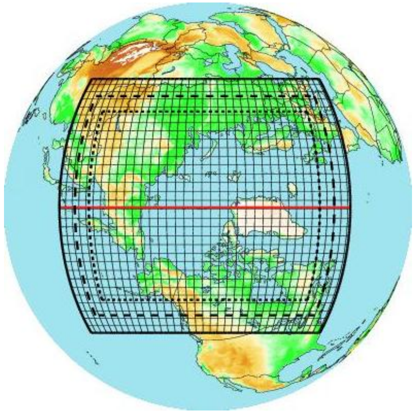
Global Uniform



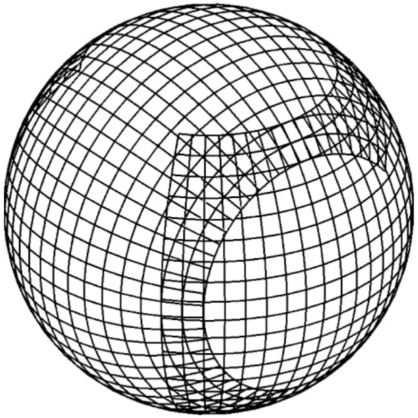
Global Variable



Limited Area (LAM)



Yin-Yang



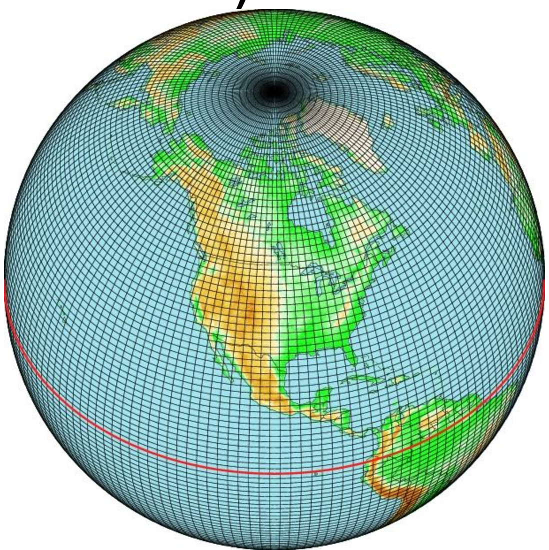
- **non-hydrostatic**
- **fully compressible**
- **semi-implicit**
- **semi-Lagrangian**
- **one-way self-nesting**
- **staggered vertical grid (Charney-Phillips)**

Côté et al. (1998) *Mon. Wea. Rev.*

Current NWP Suite – Global Scale

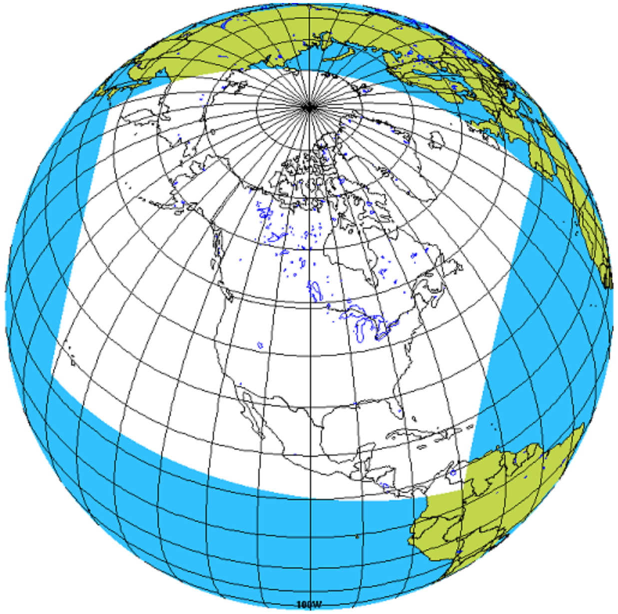
- Deterministic (GDPS)
  - Initial condition: 4D-Var
  - Grid spacing ~33 km at mid-latitudes
  - Lead time: 10 days

- Probabilistic (GEPS, 20 members+1 control)
  - Initial condition: EnKF
  - Grid spacing ~67 km at equator
  - Lead time: 16 days



*Courtesy of Martin Charron

Current NWP Suite – Regional Scale

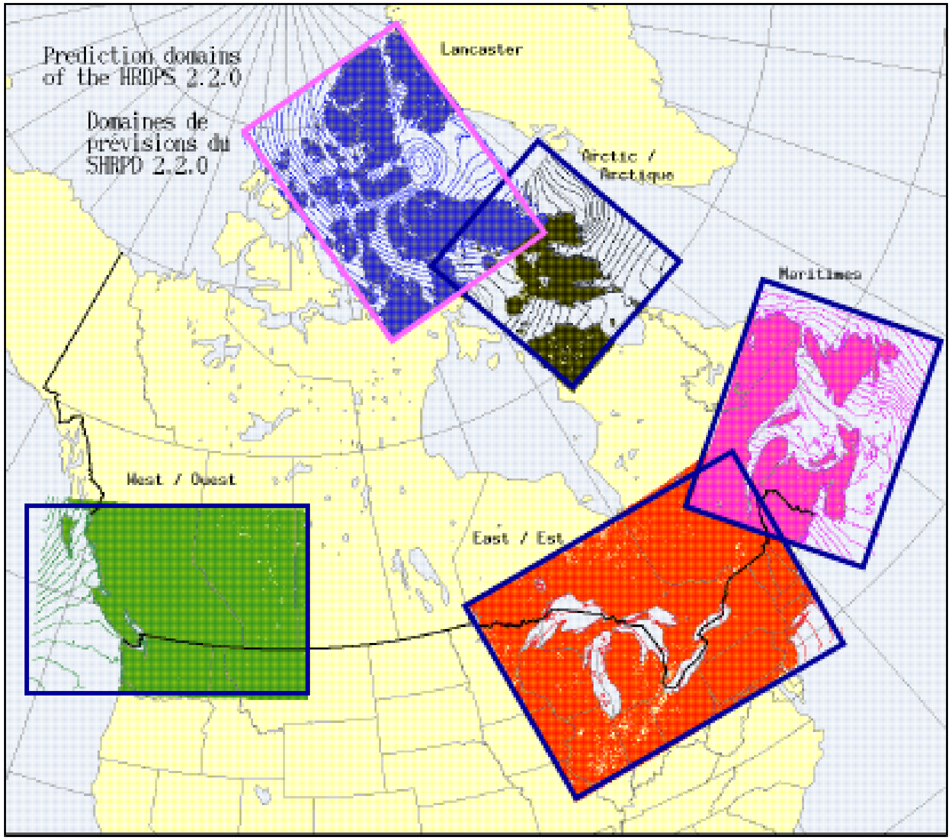


- Deterministic (RDPS)
  - Initial condition: 3D-Var (soon 4D-Var)
  - Grid spacing: 15 km (soon 10 km)
  - Lead time: 2 days
- Probabilistic (REPS, 20 members+1 control)
  - Initial condition: Interpolated from global EnKF
  - Grid spacing: 33 km
  - Lead time: 3 days

*Courtesy of Martin Charron

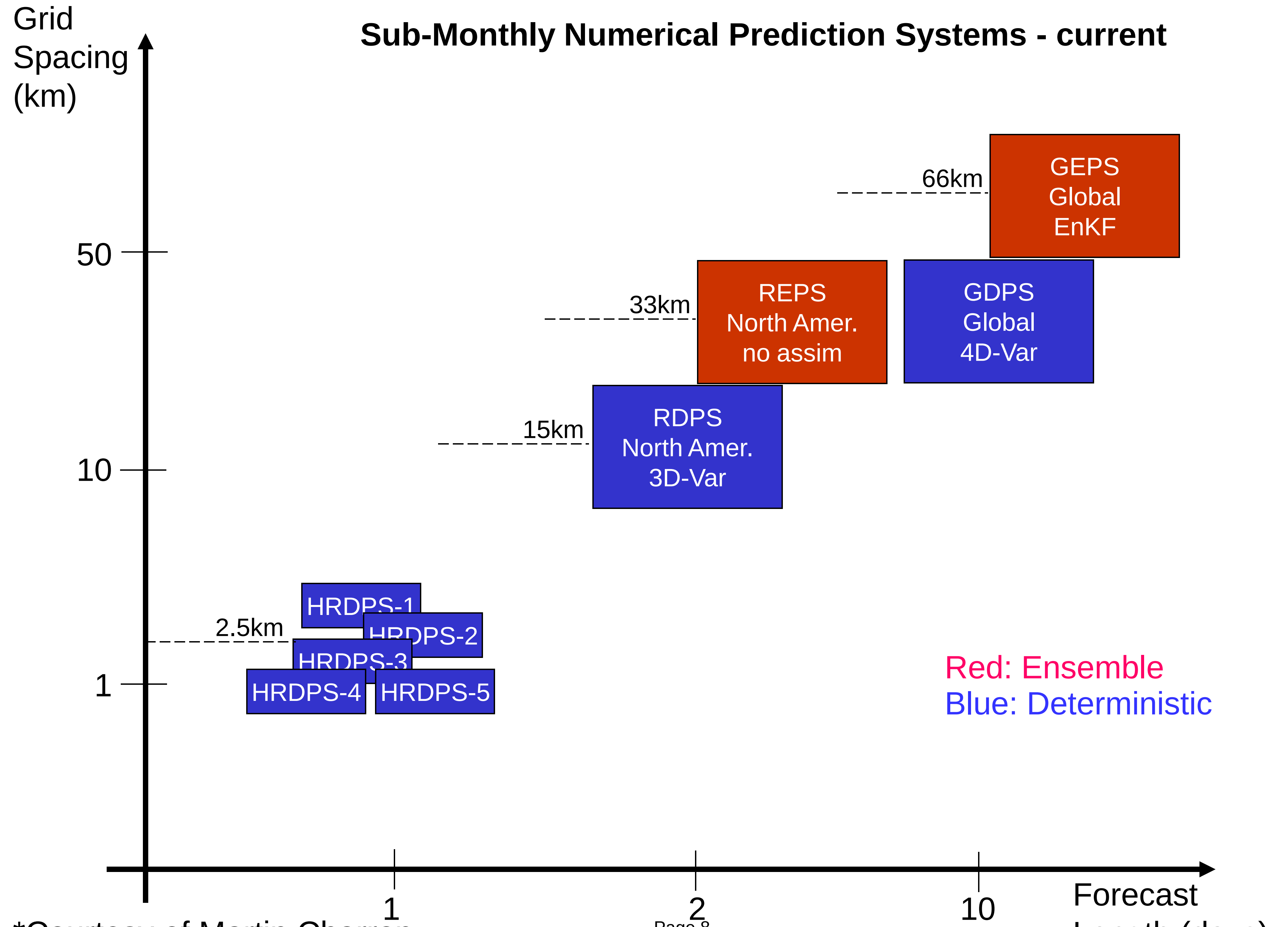
Current NWP Suite – Local Scale

- Deterministic (HRDPS)
  - Initial condition: Interpolation from RDPS
  - Grid spacing: 2.5 km
  - 5 windows (national grid in 2013)
  - Lead time: 1 day



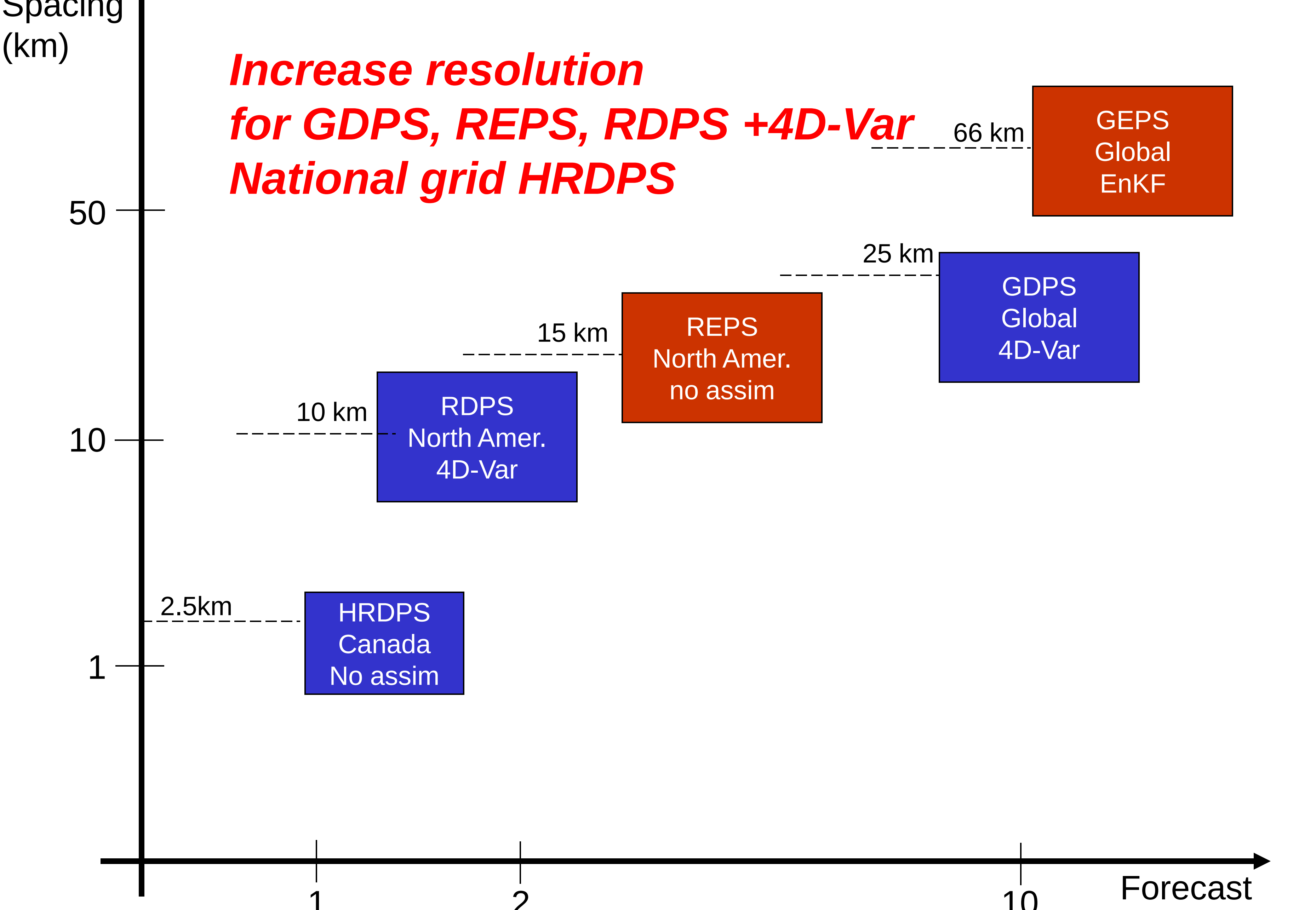
- Probabilistic (HREPS)
  - None

*Courtesy of Martin Charron



*Courtesy of Martin Charron

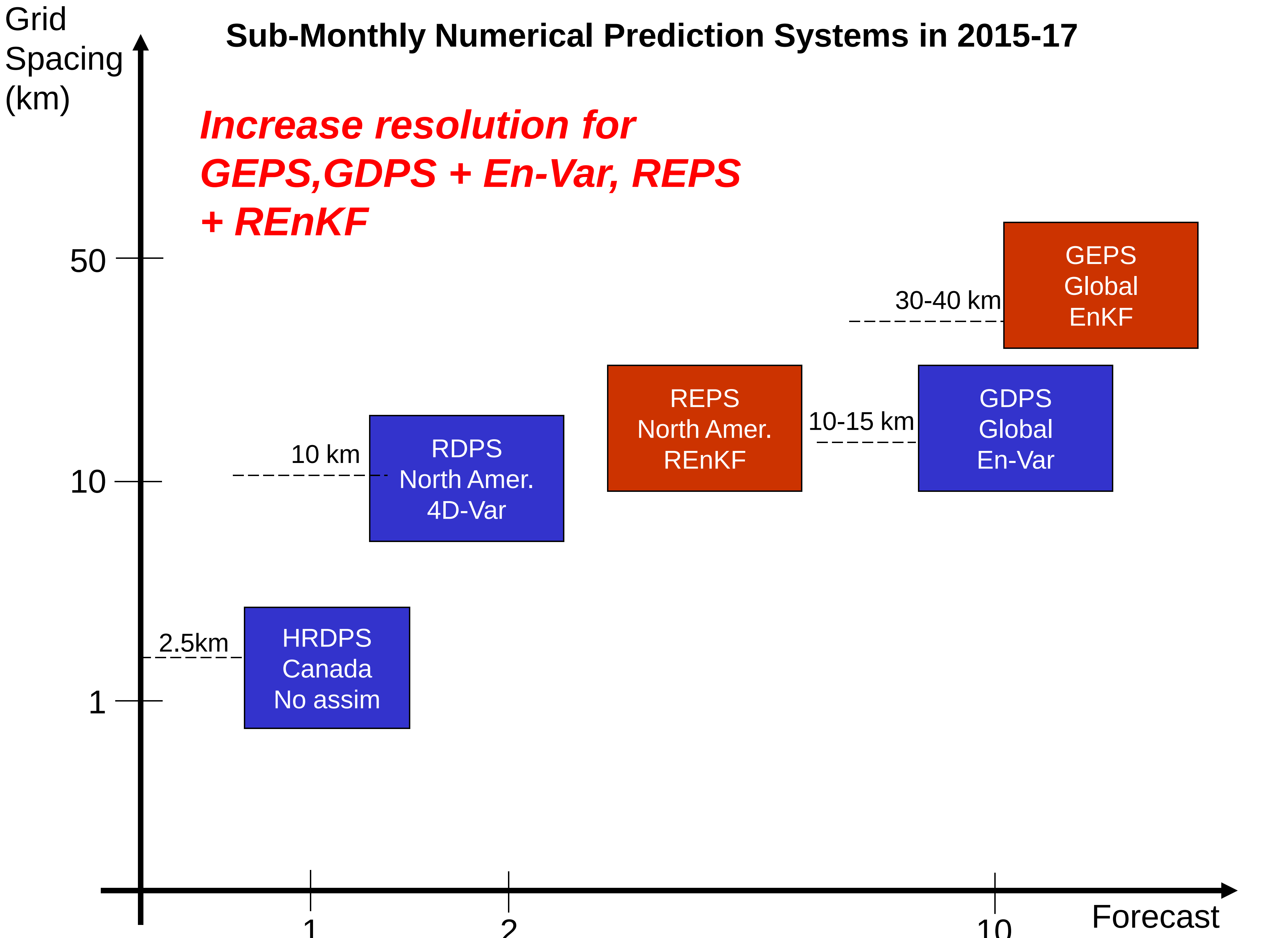
Sub-Monthly Numerical Prediction Systems by the End of 2012-2015



*Courtesy of Martin Charron

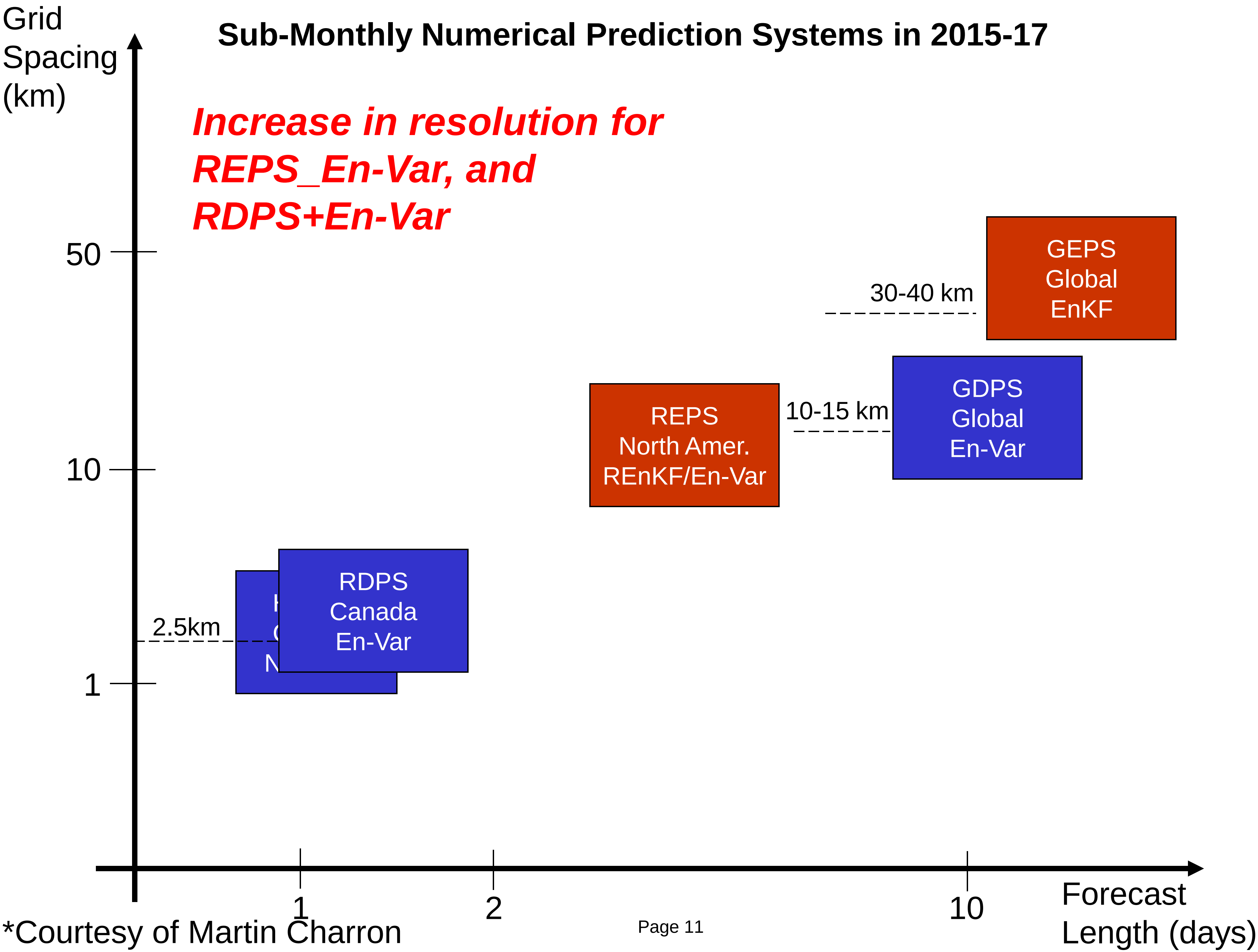
Sub-Monthly Numerical Prediction Systems in 2015-17

Increase resolution for GEPS,GDPS + En-Var, REPS + REnKF



*Courtesy of Martin Charron

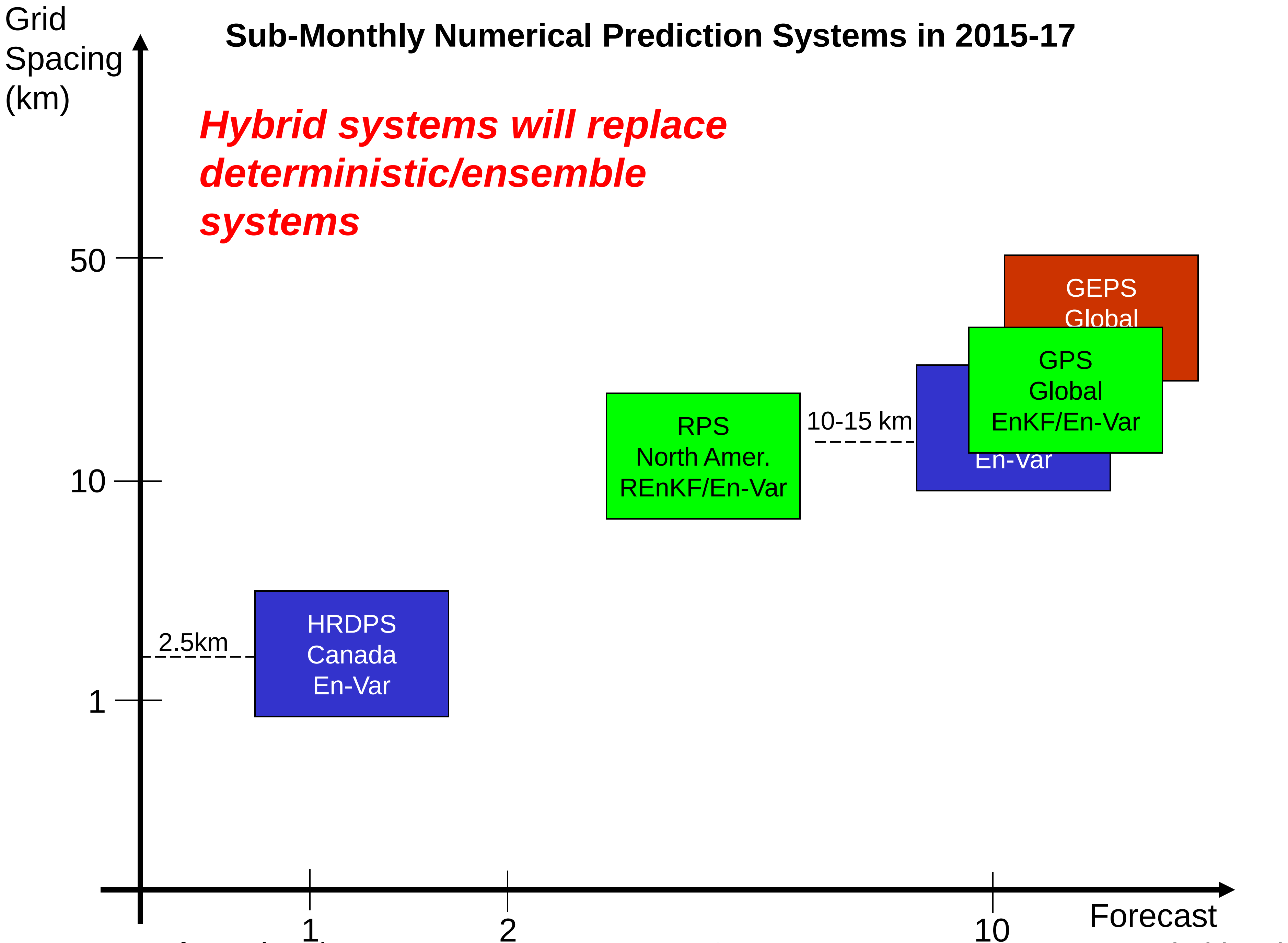
Sub-Monthly Numerical Prediction Systems in 2015-17



*Courtesy of Martin Charron

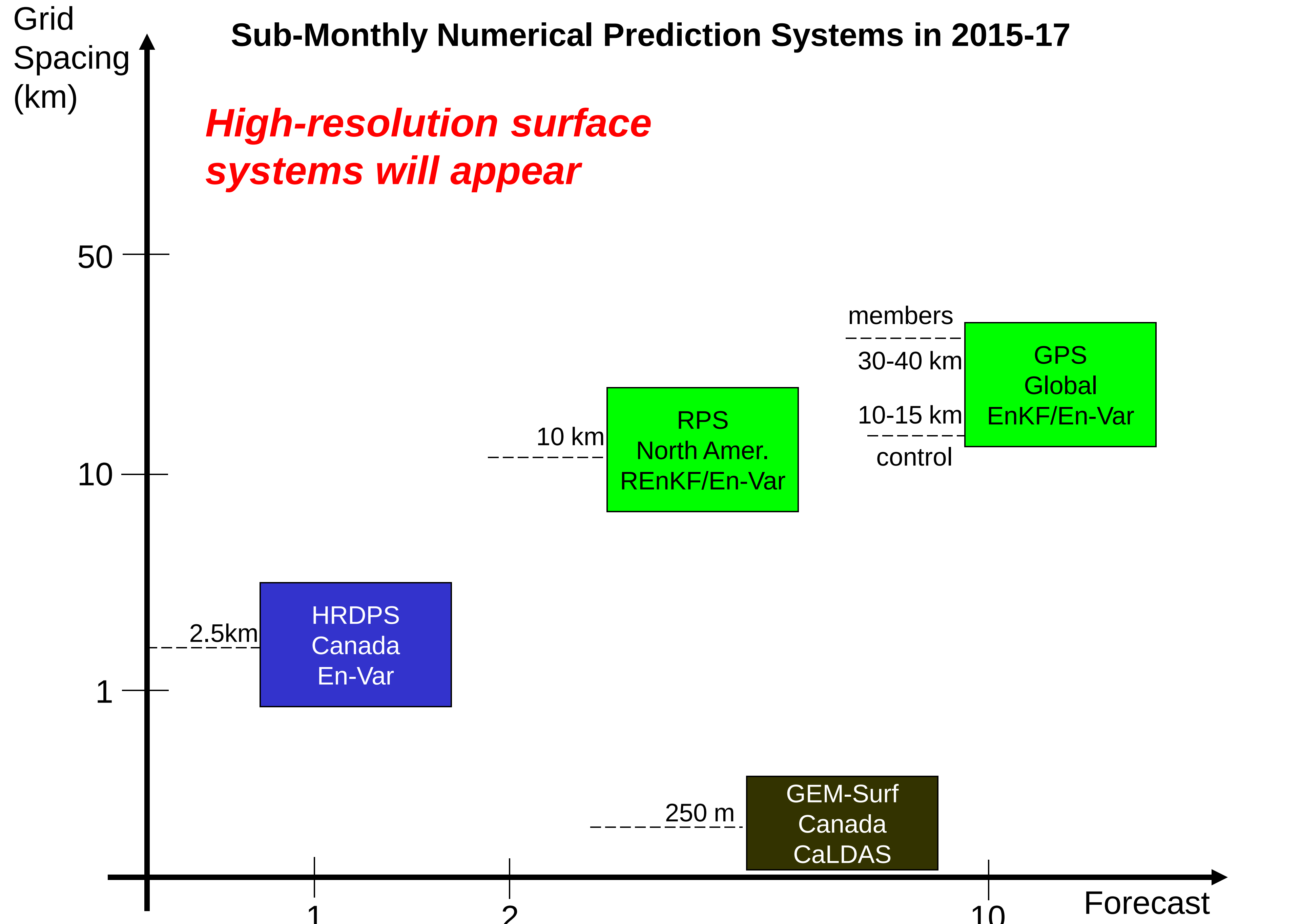
Sub-Monthly Numerical Prediction Systems in 2015-17

Hybrid systems will replace deterministic/ensemble systems



*Courtesy of Martin Charron

Sub-Monthly Numerical Prediction Systems in 2015-17



*Courtesy of Martin Charron

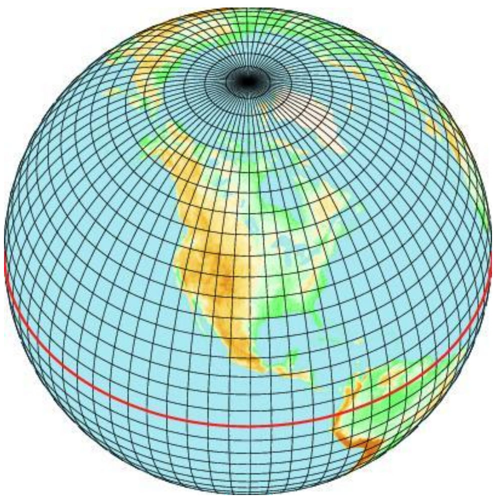
2. Multi-moment microphysics for high resolution NWP

Current Microphysics in Environment Canada's forecast model

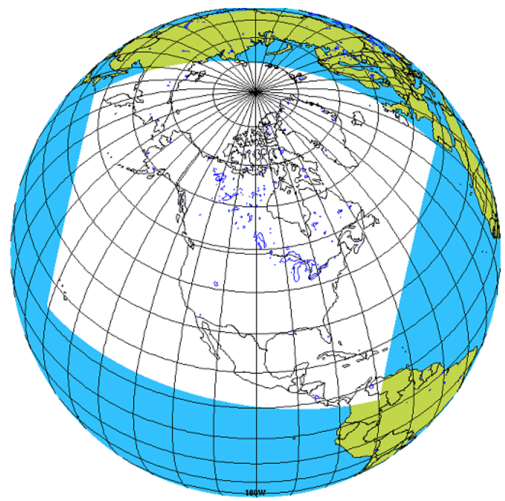
GEM (**G**lobal **E**nvironmental **M**ultiscale)

Grid configurations:

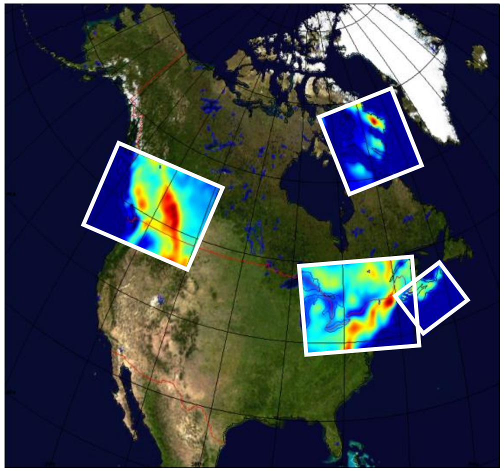
Global



Regional



Local

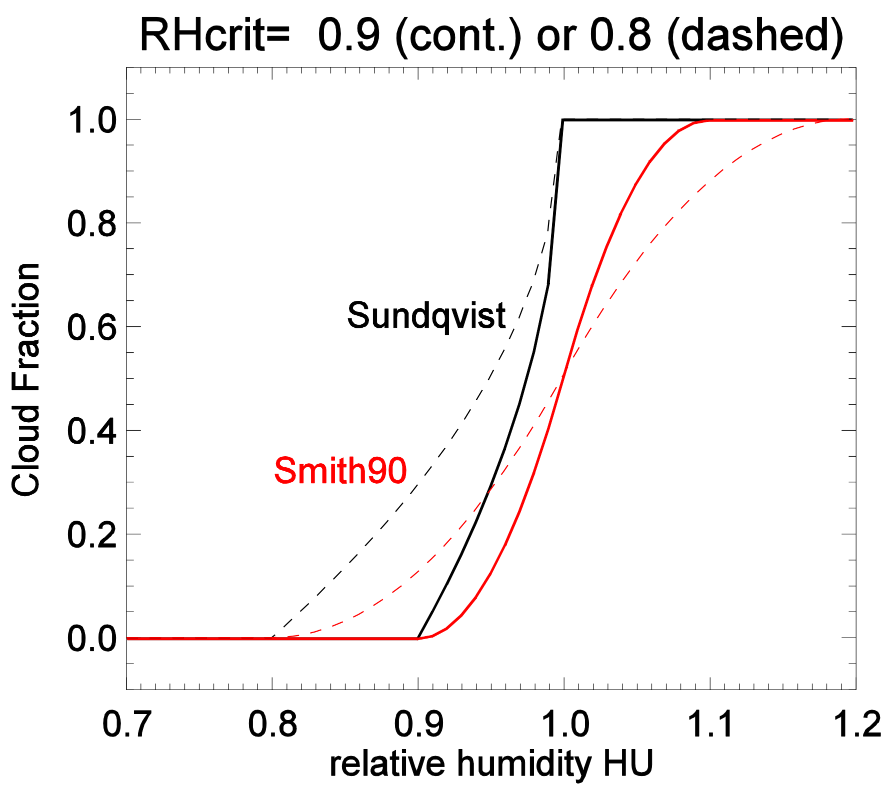


Sundqvist Cloud Scheme

Double moment Milbrandt-Yau Scheme

The Sundqvist cloud scheme

- Cloud-cover fraction idiagnosed (function of RH)
- Condensation occurs when RH > 80% near surface
- Total condensate (cloud water/ice) predicted
- Precipitation falls instantly to the ground

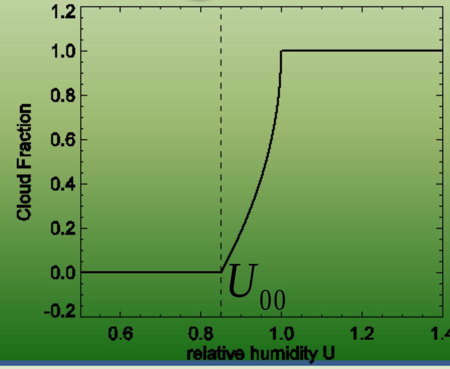


Sundqvist: Subgrid Cloud Fraction

$$a = 1 - \sqrt{\frac{1-U}{1-U_{00}}}$$

cloud fraction

=



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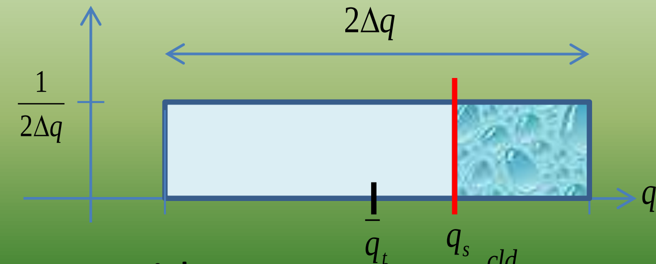
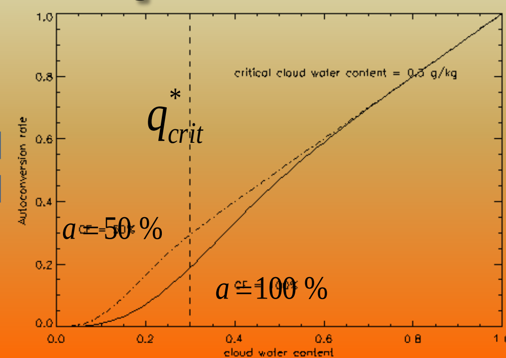


with assumption $q_v^{cld} = q_s$

Sundqvist: Precipitation Generation

$$G_p = c_0^* q_c \left[1 - \exp\left(- \frac{q_c}{a.q_{crit}^*} \right)^2 \right]$$

=



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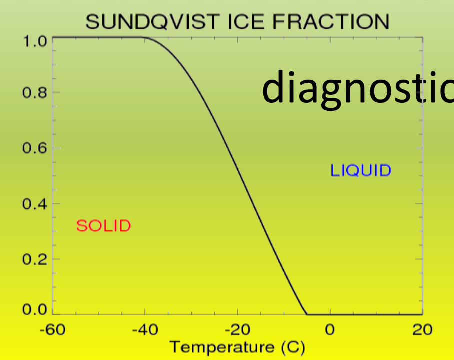
$$P = \frac{1}{g} \int_{top}^{above} G_p dp$$

$$P_{ice} = \frac{1}{g} \int_{top}^{above} f_{ice} G_p dp$$

Sundqvist: Ice Fraction Assumption

$$q_{tot} = q_v + q_c$$

=



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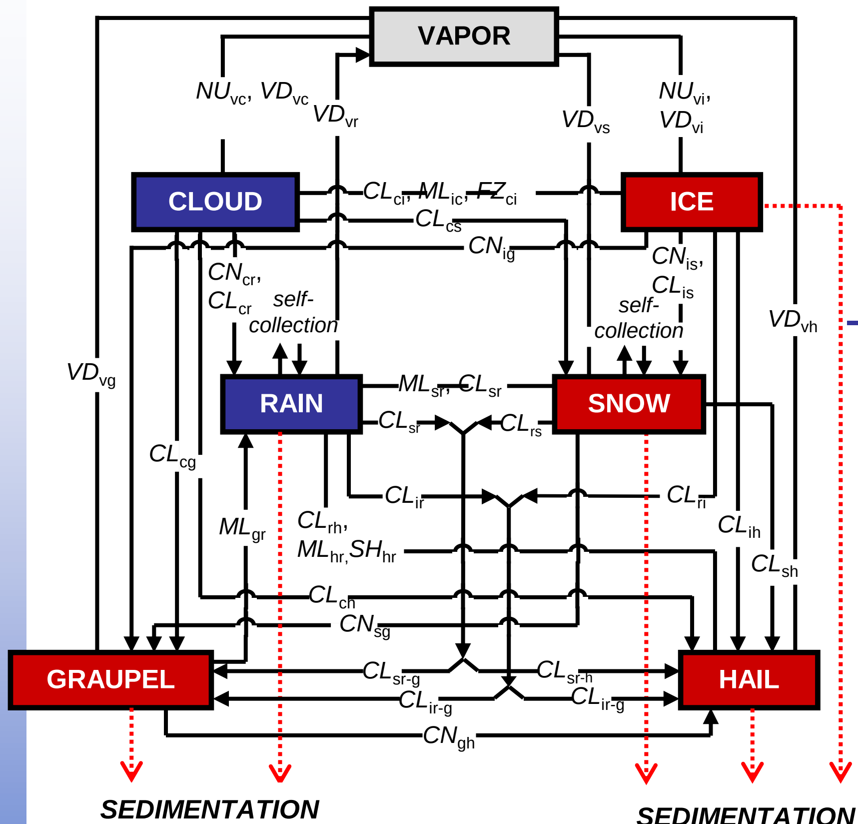
for clouds (Boudala et al., 2004)

$$f_{ice}(T) = 1 - q_c^{\alpha} e^{\beta T}$$

The Milbrandt-Yau microphysics scheme



Six hydrometeor categories:

2 liquid: ***cloud***, ***rain***

4 frozen: ***ice***, ***snow***, ***graupel***, ***hail***

Multi-moment scheme

Milbrandt and Yau (JAS 2005 a,b)
Milbrandt and Yau (JAS, 2006 a,b)
Gultepe and Milbrandt (Pure App. Geoph.,2007)
Milbrandt et al. (MWR, 2008)
Milbrandt et al. (MWR, 2010)
Dawson et al. (MWR, 2010)
Morrison and Milbrandt (MWR, 2011)
Milbrandt and Morrison (JAS, 2012, accepted)

Scheme implemented in

GEM-Local (Canada)

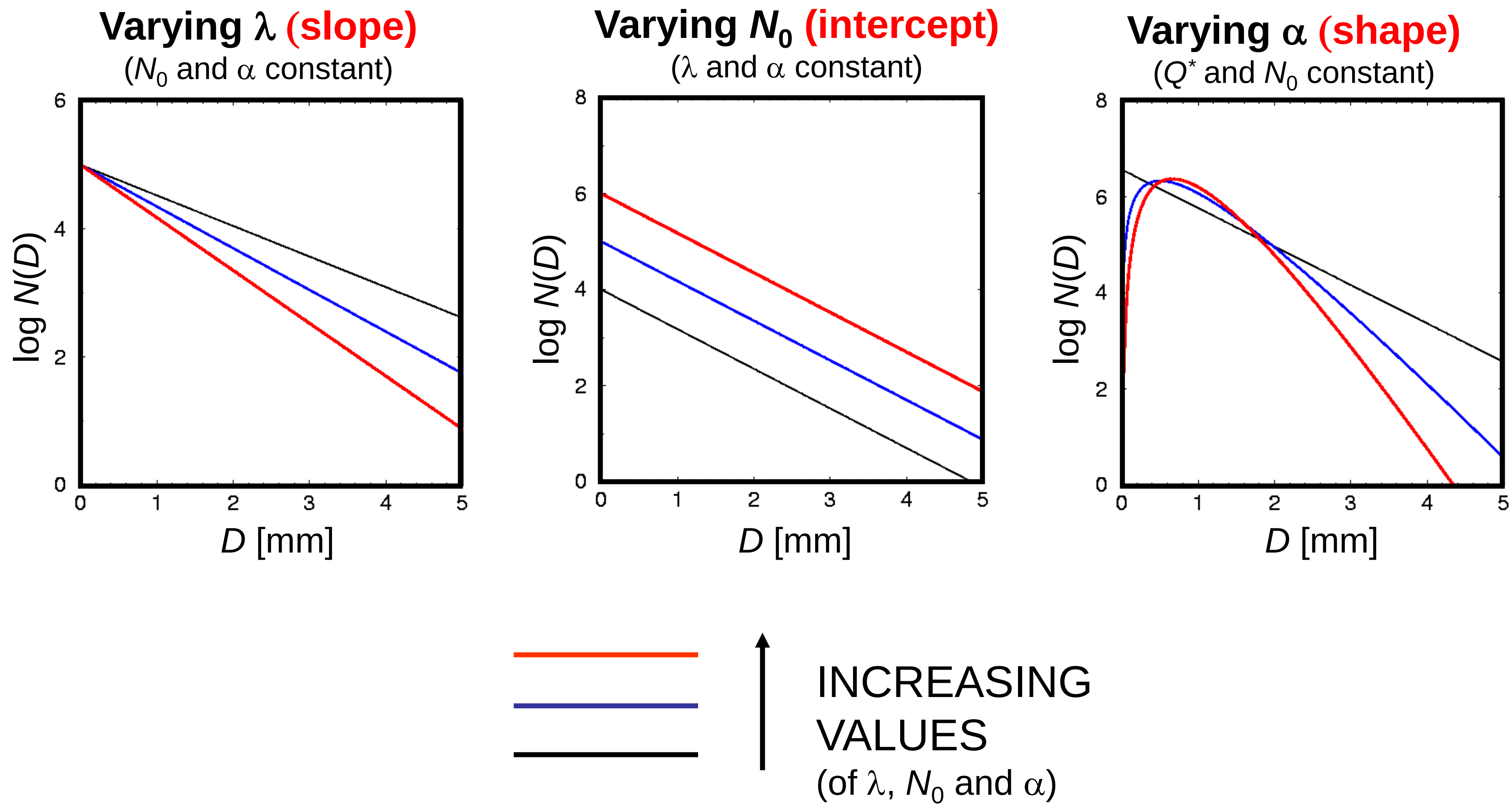
ARPS (U Oklahoma, US)

WRF 3.2 (US)

COAMPS (US)

Gamma Distribution Function:

$$N(D) = N_0 D^{\alpha} e^{-\lambda D}$$



* $Q = \rho q$ (mass content)

BULK METHOD

Predict evolution of specific moment(s)

e.g. q_x, N_{Tx}, ...

↓

Implies prediction of evolution of parameters

i.e. N_{0x}, λ_x, ...

For every predicted moment, there is one prognostic parameter.

The remaining parameters are prescribed or diagnosed.

e.g.

One-moment scheme:

q_x is predicted;
$\rightarrow \lambda_x$ is prognosed
(N_{0x} and α_x are specified)

Two-moment scheme:

q_x and N_{Tx} are predicted;
$\rightarrow \lambda_x$ and N_{0x} are prognosed;
(α_x is specified)

Three-moment scheme:

q_x, N_{Tx} and Z_x are predicted;
$\rightarrow \lambda_x$, N_{0x} and α_x is prognosed

***Size Distribution Function*:**

$$N_x(D) = N_{0x} D^{\alpha_x} e^{-\lambda_x D}$$

***p*th moment:**

$$M_x(p) \equiv \int_0^\infty D^p N_x(D) dD = N_{0x} \frac{\Gamma(1+\alpha_x+p)}{\lambda_x^{p+1+\alpha_x}}$$

CLOSURE OF SYSTEM

Solve for shape parameter α from

$$\frac{c^2 N_T Z}{(\rho q)^2} = G(\alpha) = \frac{(\alpha+6)(\alpha+5)(\alpha+4)}{(\alpha+3)(\alpha+2)(\alpha+1)},$$

$$where\ m(D) = cD^3, and\ \rho = air\ density$$

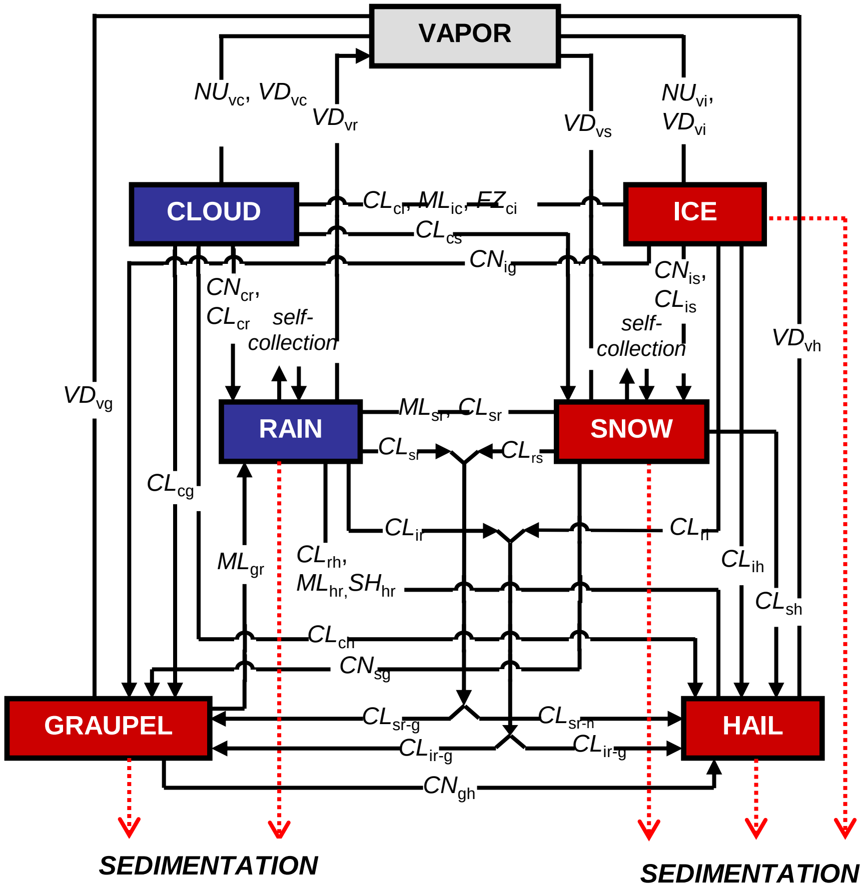
Solve for slope parameter λ from

$$\lambda = \left(\frac{cN_T \Gamma(\alpha+4)}{\rho q \Gamma(\alpha+1)} \right)^{\frac{1}{3}}$$

Solve for intercept parameter N_0 from

$$N_0 = \frac{N_T \lambda^{\alpha+1}}{\Gamma(\alpha+1)}$$

Milbrandt-Yau* 2-Moment Microphysics Scheme (MY2) in operational GEM - Local



Six hydrometeor categories:

2 liquid: ***cloud***, ***rain***

4 frozen: ***ice***, ***snow***, ***graupel***, ***hail***

For each category *x = c, r, i, s, g, h*:

$$N_x(D) = N_{0x} D^{\alpha_x} e^{-\lambda_x D}$$

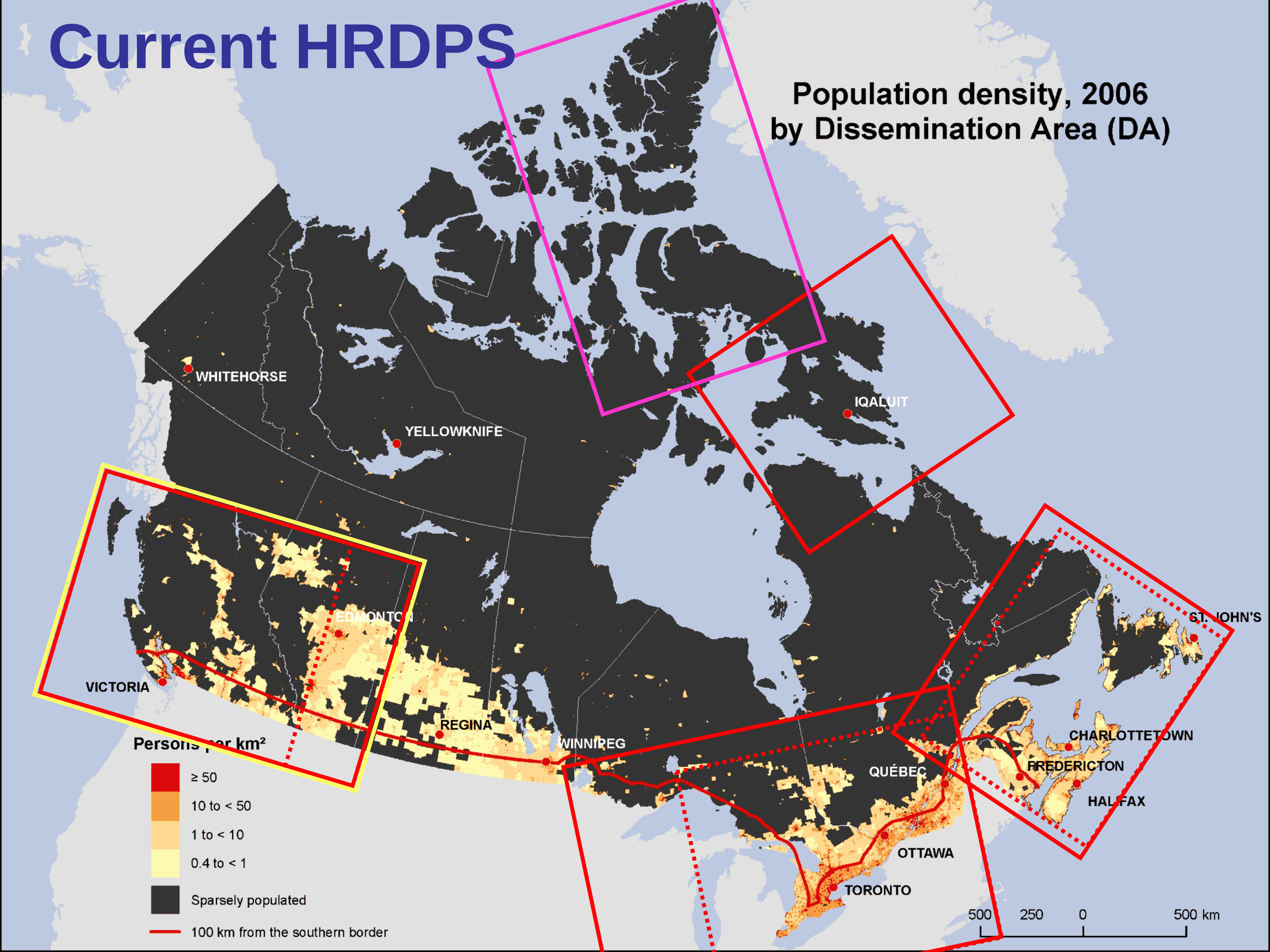
$$\alpha_x = 3 \; for \; c, \; \alpha_x = 0 \;\; for \;\; the \;\; rest$$

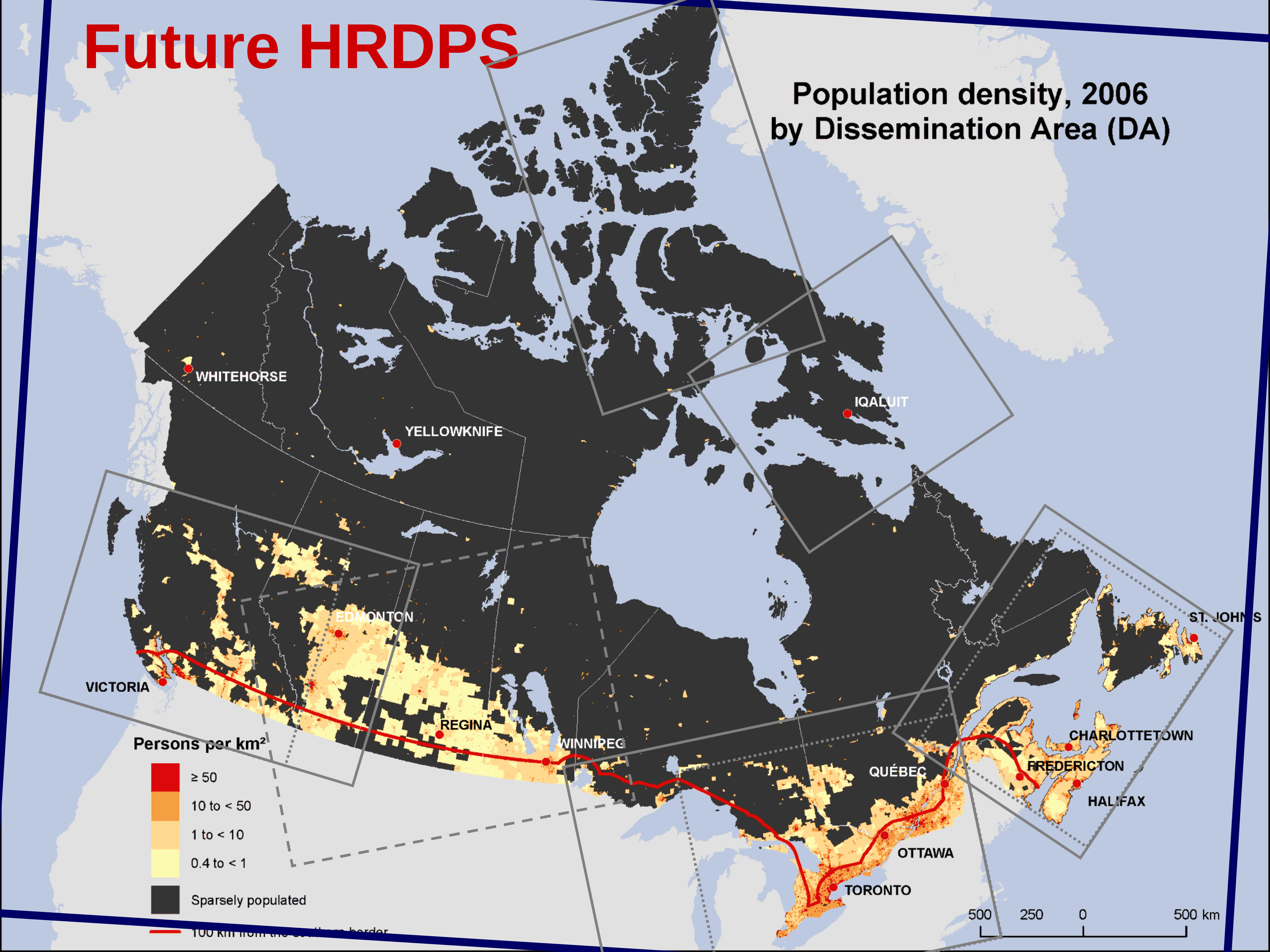
Prognostic variables

q_x, N_x (12)

* Milbrandt and Yau (2005a,b)

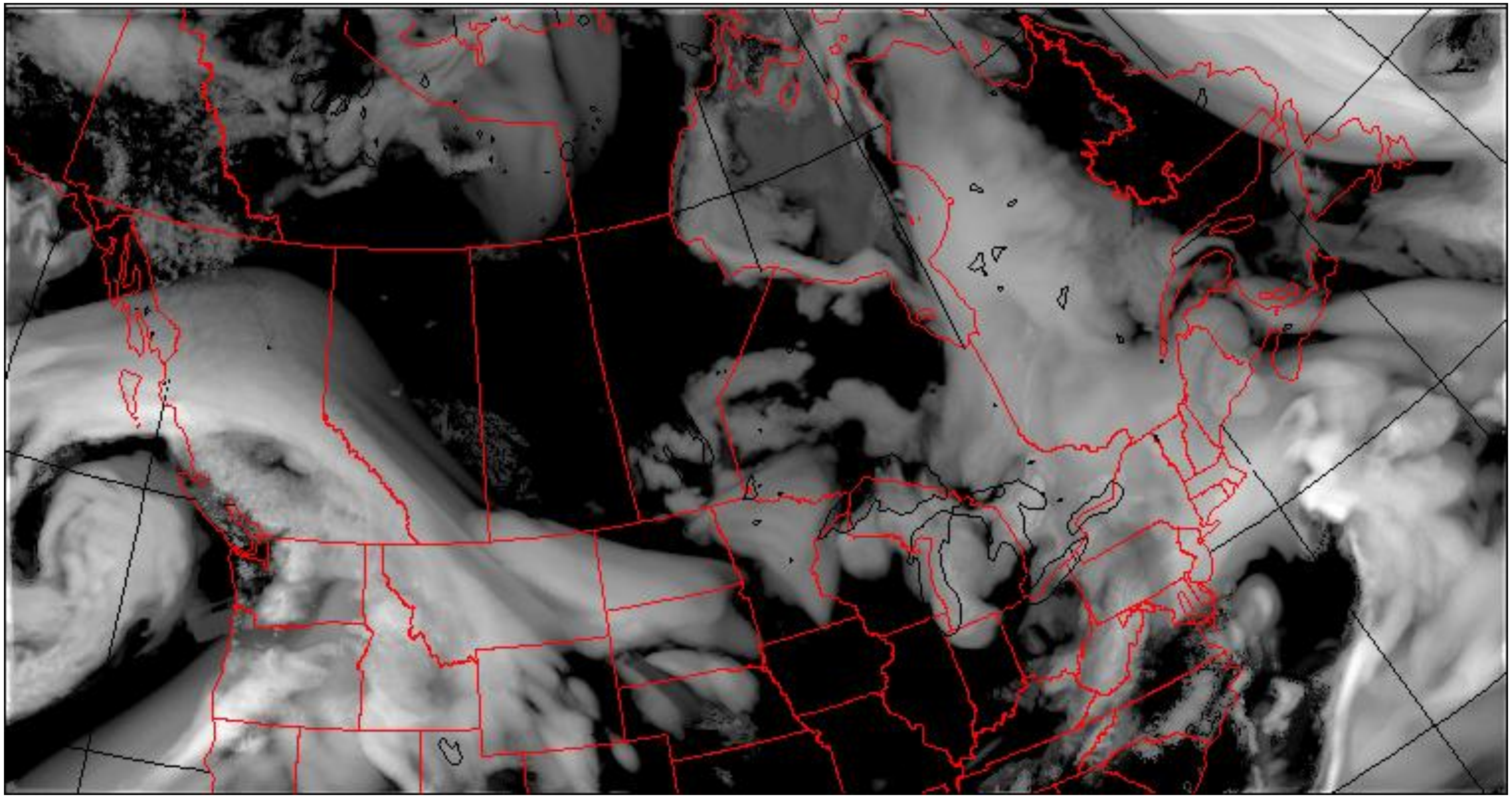
Current HRDPS




Future HRDPS
Population density, 2006
by Dissemination Area (DA)
WHITEHORSE
YELLOWKNIFE
IQALUIT
EDMONTON
VICTORIA
REGINA
WINNIPEG
QUÉBEC
OTTAWA
TORONTO
CHARLOTTETOWN
FREDERICTON
HALIFAX
Persons per km²
≥ 50
10 to < 50
1 to < 10
0.4 to < 1
Sparsely populated
500
250
0
500 km

MY2 MICROPHYSICS IN High-Resolution Simulations

(Courtesy of Martin Charron)



Physical-dynamical processes generating realistic cloud cover.

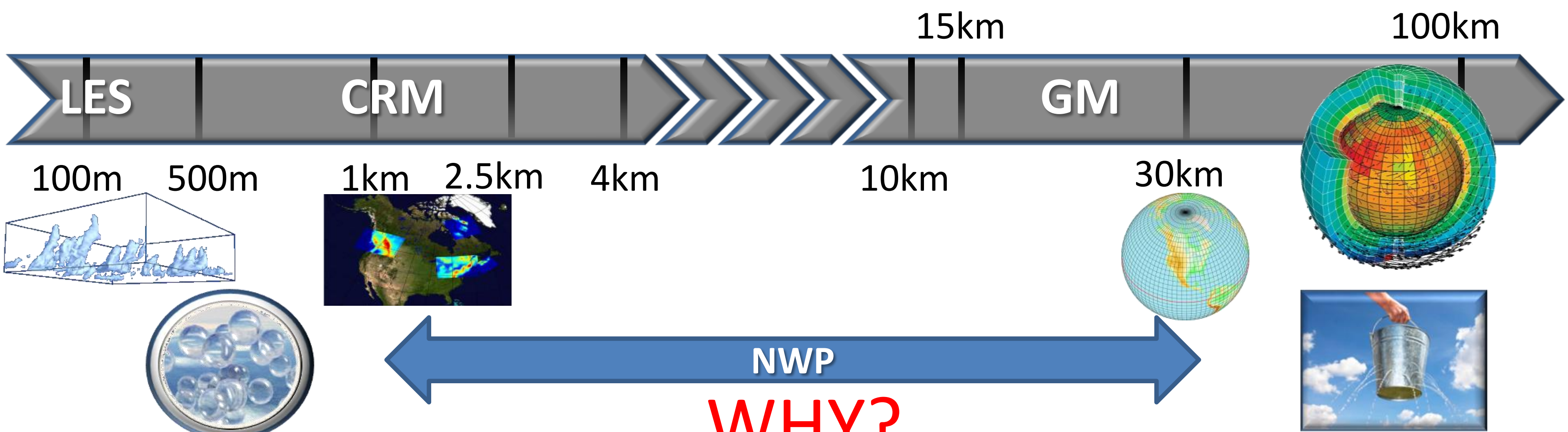
In a year or two: Routine high-resolution (2.5 km grid spacing) model forecasts over the entire Canadian territory.

3. Development of unified microphysics for coarser resolution to about 40 km

   a) Sub-grid scale cloud and precipitation fractions for MY2

OBJECTIVE

Develop a unified double moment, multiclass microphysical scheme that can be used on the largest possible scale range in NWP models



WHY?

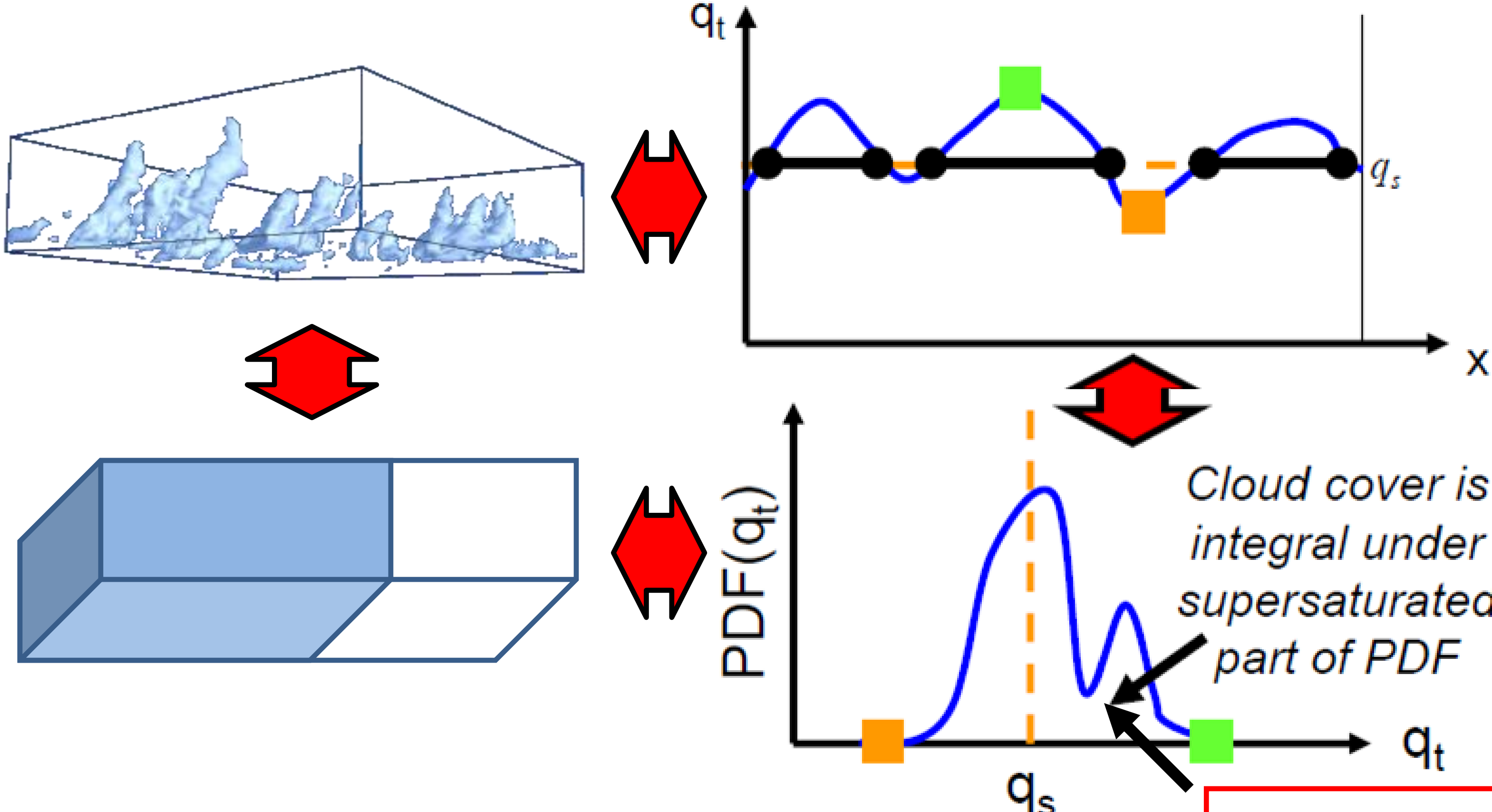
- Consistency amongst models: deterministic and probabilistic prediction systems
- Decrease spin-up and more consistent background field for data assimilation
- Better microphysics for coarser resolution models

HOW ?

multi-moment, multi-class microphysical schemes has been developed for small scales

•<u>Extend to larger spatial scales: implement subgrid cloud/precip fractions</u>

Subgrid Cloud Fraction



$$a = \int_{q_s}^{\infty} PDF dq_t$$

DEFINITIONS :

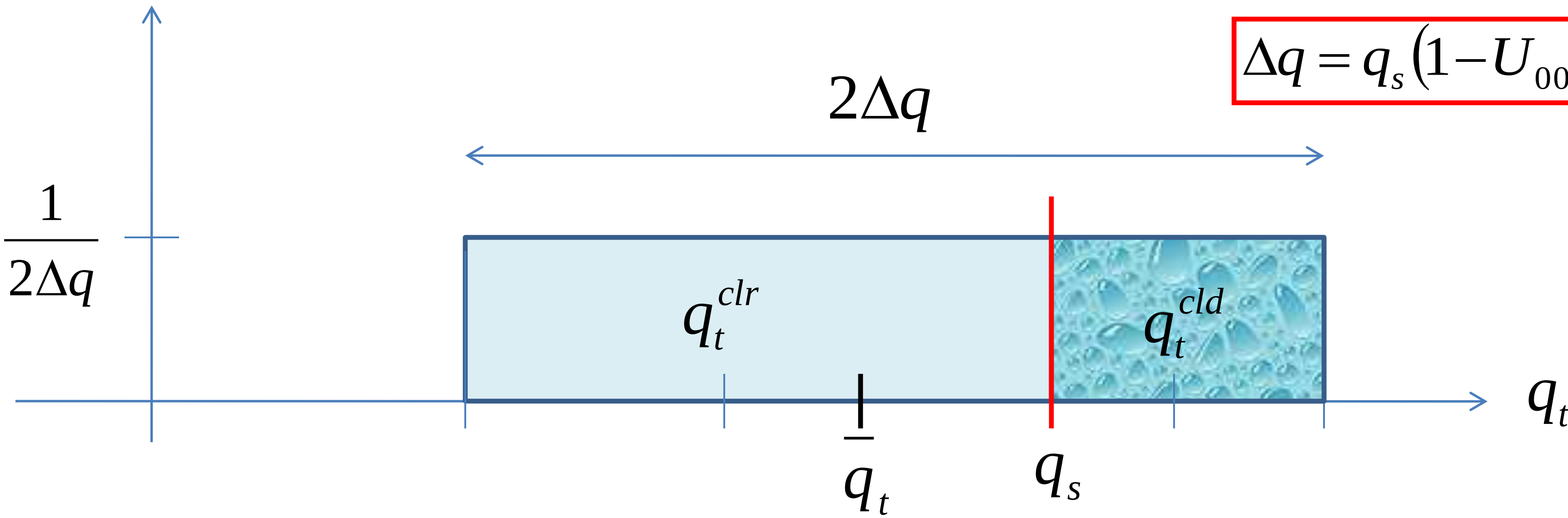
1. The cloud fraction is the fraction of the grid that is **supersaturated**
2. The cloud fraction eventually contains the **cloudy species** : cloud droplets, small ice particles, and snow particles (hydrometeors experiencing water vapour deposition)

Subgrid Cloud Fraction

PDF of total water content

PDF width

$$\Delta q = q_s\left(1 - U_{00}\right)$$



water vapor
in-cloud part

$$q_v^{cld} = q_{tot}^{cld} - q_c^{cld} = \frac{\overline{q}_t + \Delta q + q_s}{2} - \frac{\overline{q}_c}{a}$$

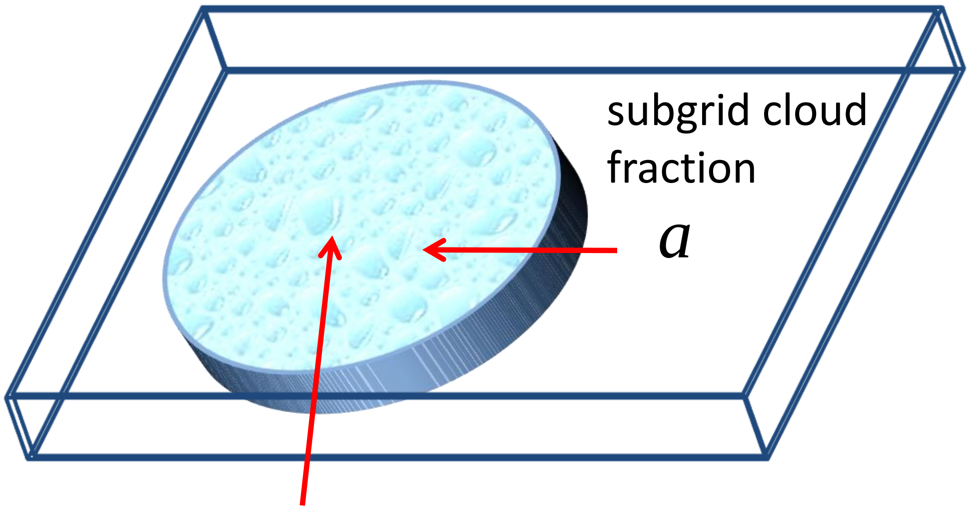
cloud
fraction

$$a = \frac{\overline{q}_t + \Delta q - q_s}{2\Delta q}$$

water vapor
clear-sky part

$$q_v^{clr} = q_t^{clr} = \frac{\overline{q}_t - \Delta q + q_s}{2}$$

Subgrid Cloud Fraction



In-cloud microphysical processes

CLOUDY CLASSES: **Pristine Ice**, **Snow**, **Cloud droplets** — q_x, n_x

$$\left.\frac{\partial q_x}{\partial t}\right|_{proc.} = F(n_x, q_x)$$

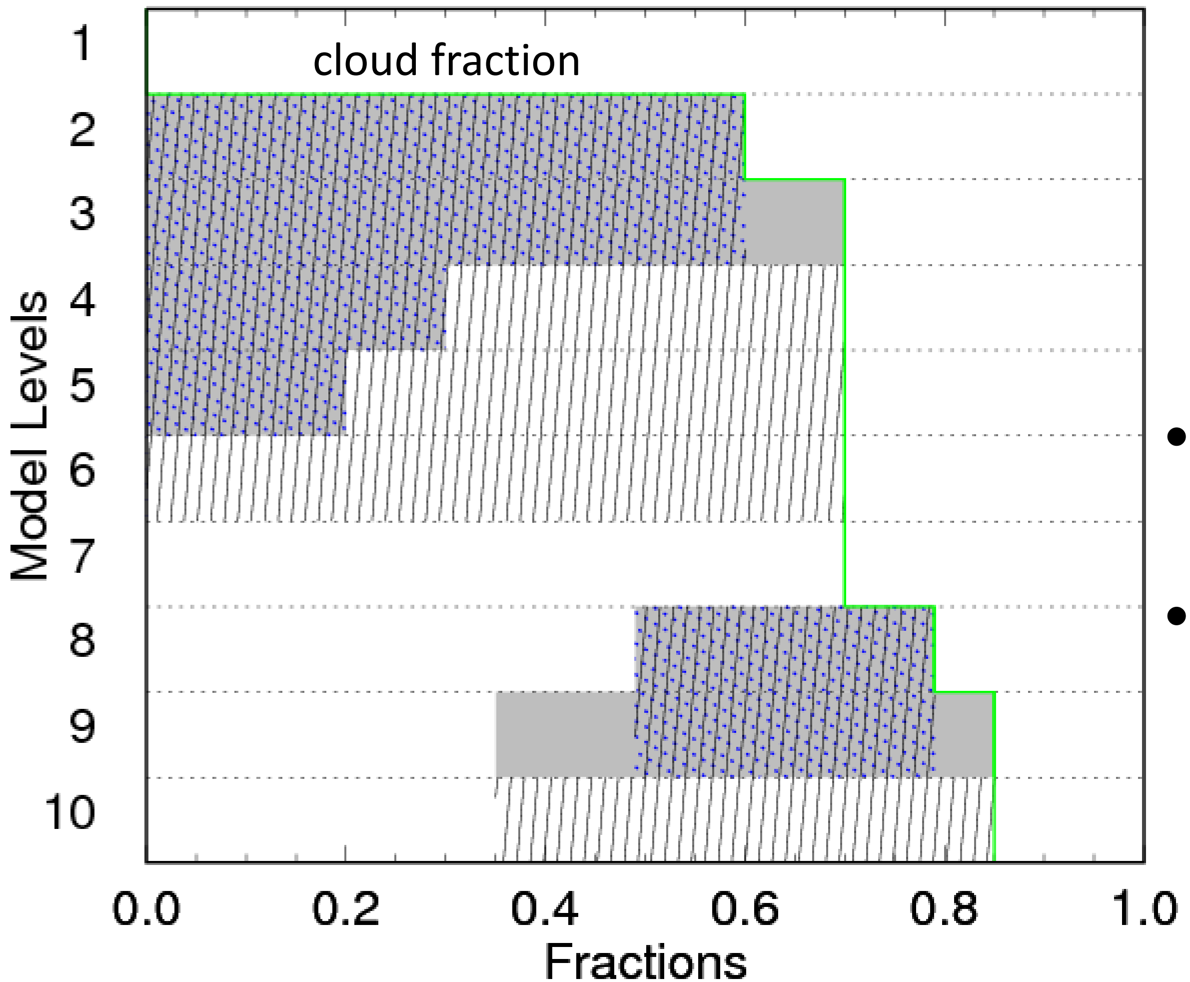
$$\left.\frac{\partial \overline{q}_x}{\partial t}\right|_{proc.} = F\left(\frac{\overline{n}_x}{a}, \frac{\overline{q}_x}{a}\right).a$$

$$\overline{q}_x = a\, q_x$$

$$\overline{n}_x = a\, n_x$$

Subgrid Precipitation Fraction

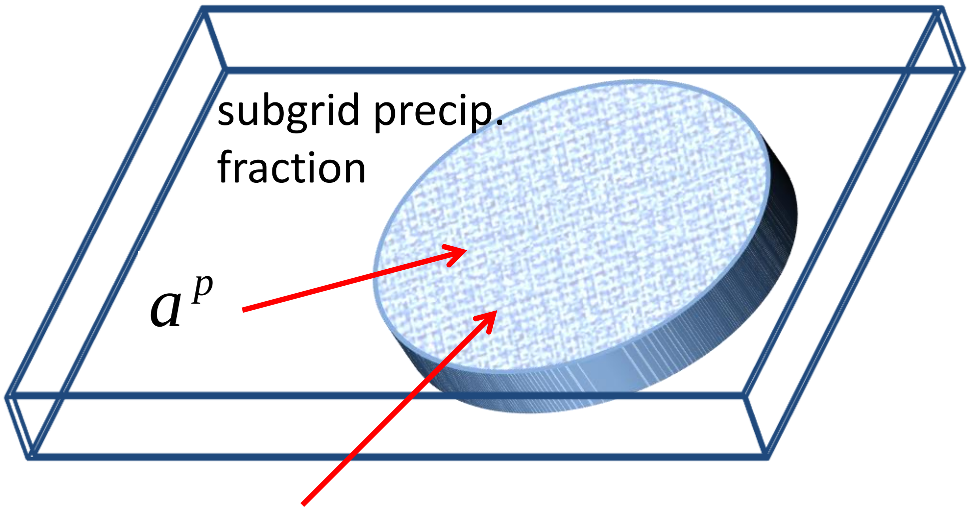
Details in Chosson et al. (2012, JAS, submitted)



$$a_k^p = a_{k\ clr}^p + a_{k\ cld}^p$$

- Maximum-random overlap
- Precip. comes from overlying clouds

Subgrid Precipitation Fraction



Precipitation microphysical processes

PRECIP. CLASSES: **Rain** **Graupel** **Hail** q_x n_x

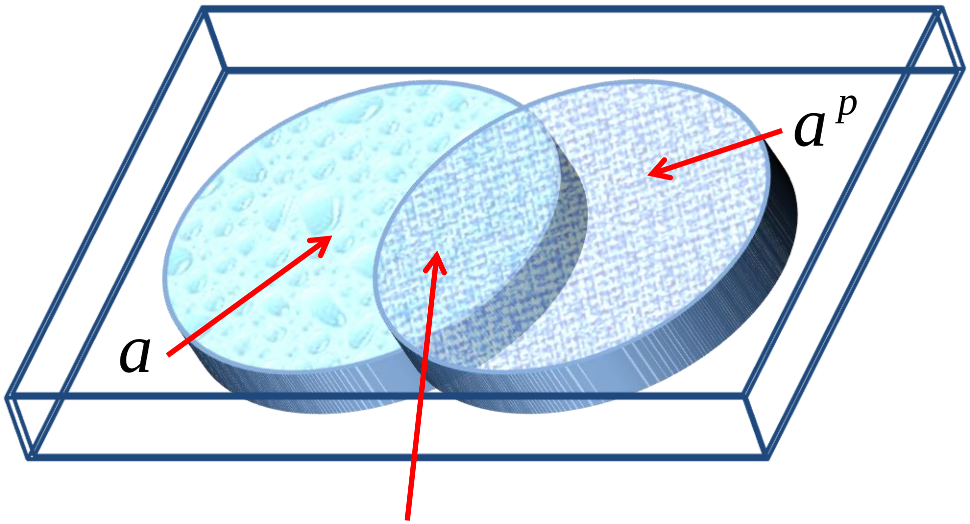
$$\left.\frac{\partial q_x}{\partial t}\right|_{proc.} = F(n_x, q_x)$$

$$\left.\frac{\partial \overline{q}_x}{\partial t}\right|_{proc.} = F\left(\frac{\overline{n}_x}{a^p}, \frac{\overline{q}_x}{a^p}\right) . a^p$$

$$\overline{q}_x = a^p \; q_x$$

$$\overline{n}_x = a^p \; n_x$$

Overlap assumptions



OVERLAP OF PRECIPITATION AND CLOUD FRACTIONS

a^{p}_{cld}

cloud-precip. interaction microphysical processes

$$\left.\frac{\partial q_c}{\partial t}\right|_{proc} = F(q_c, q_r)$$

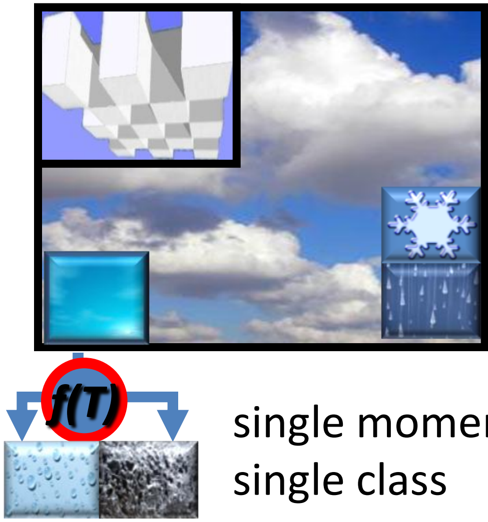
$$\left.\frac{\partial \overline{q}_c}{\partial t}\right|_{proc} = F\left(\frac{\overline{q}_c}{a}, \frac{\overline{q}_r}{a^p}\right). a^{p}_{cld}$$

Intercomparison using GEM-REGIONAL

Sundqvist et al. (1989)

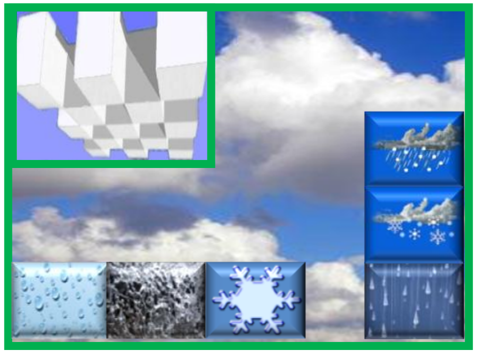
Milbrandt and Yau (2005) original

Milbrandt and Yau (2005) + SCF



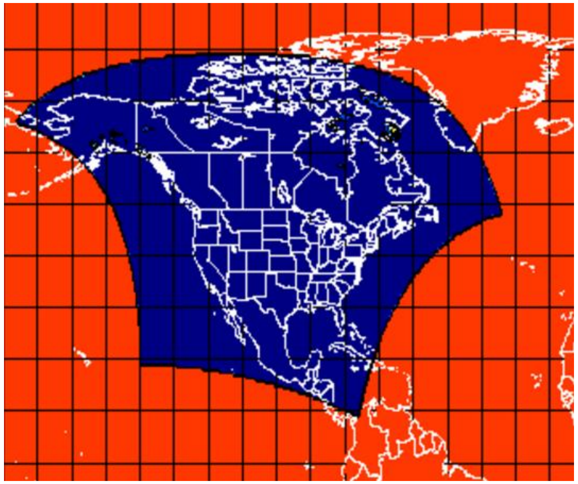
single moment
single class

double moment
six hydrometeor classes



+ Subgrid Cloud/Precip. Fraction scheme

Operational regional GEM
Δx=15km
Δt=450 sec



20 Dec. 2008
Simulation 36h comparison on the last 24h

Observations from CloudSat/Calipso

Calipso + CloudSat

=

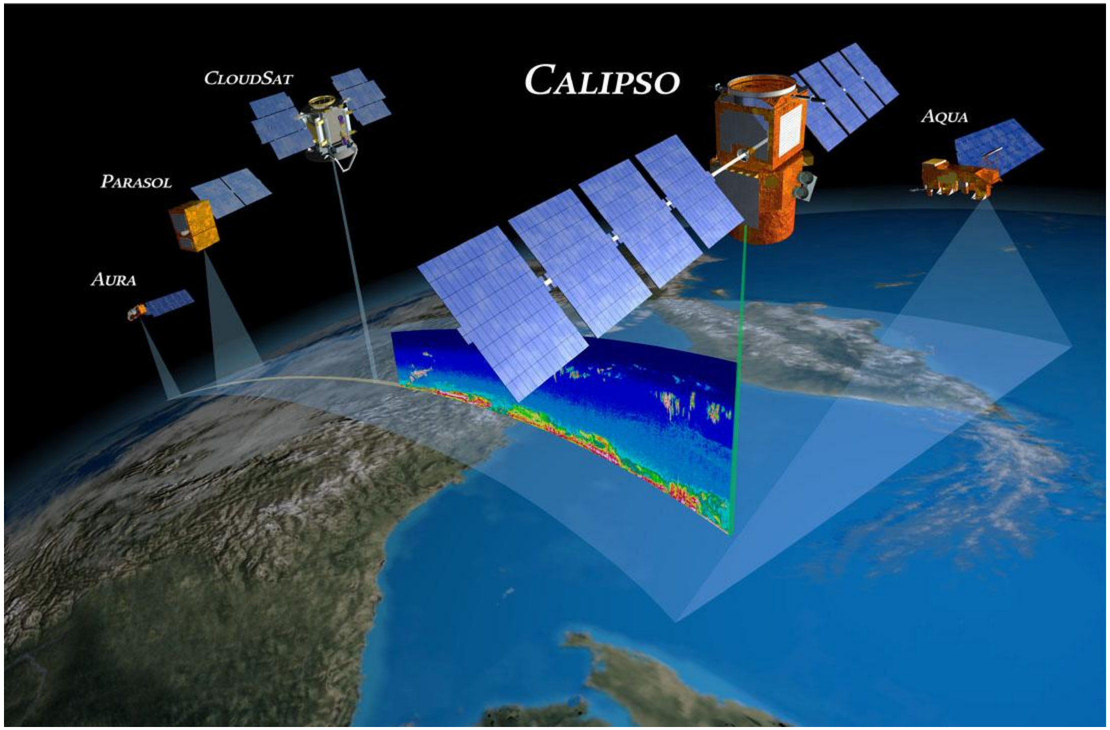
LiDAR + RaDAR + Modis (IR)

=

DARDAR CLOUD products

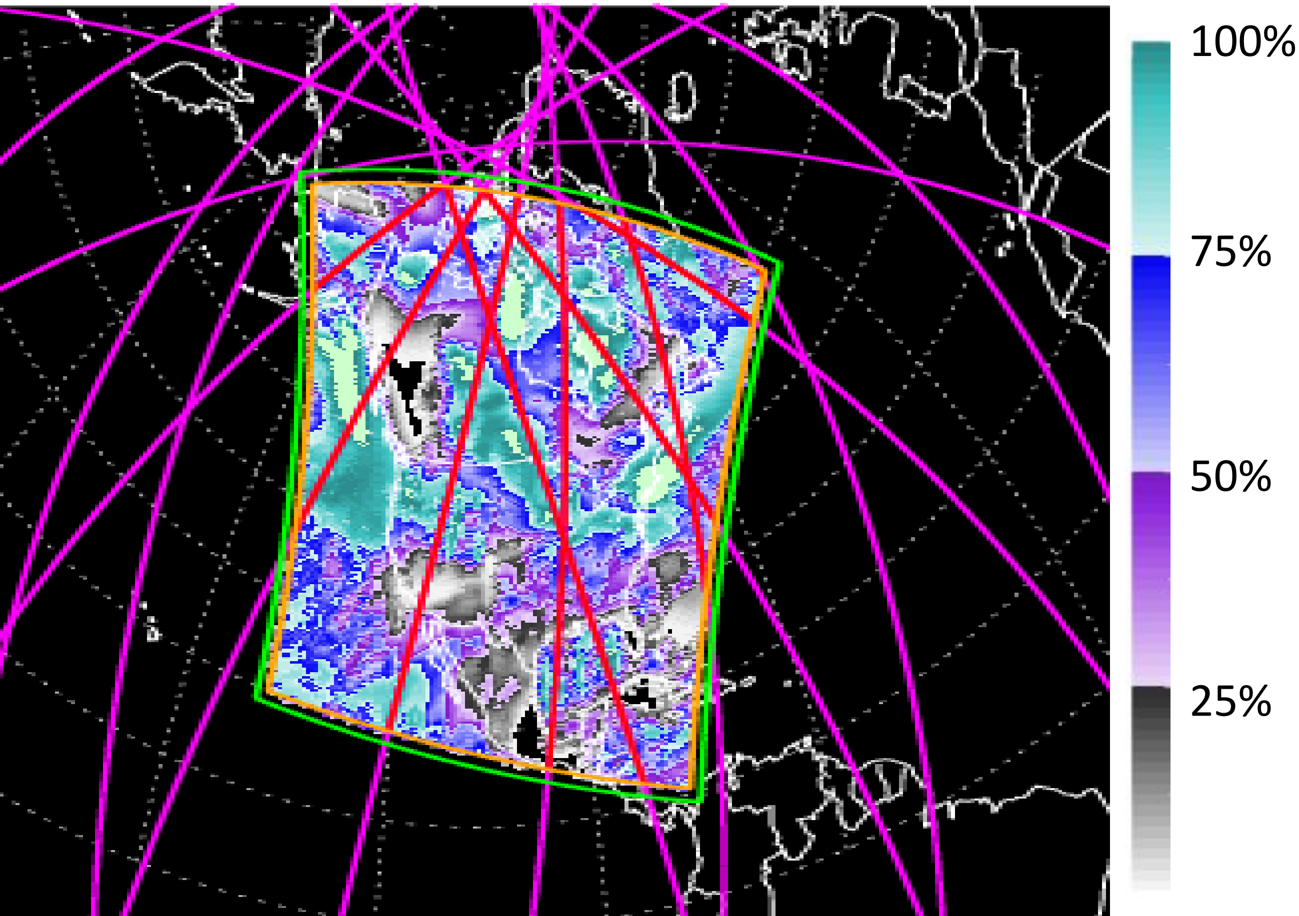
(J. Delanoe, R. Hogan, 2008, 2010)

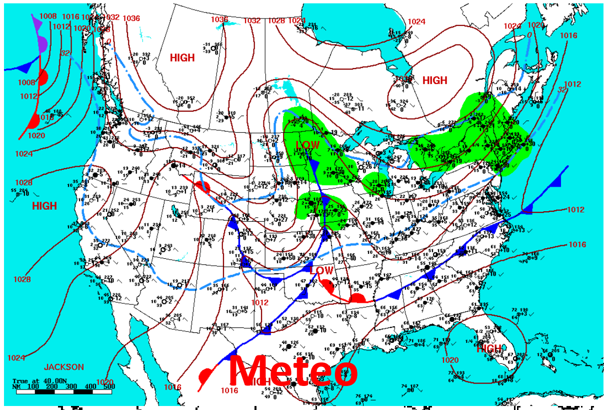
vertical profiles of cloud masks,
IWC, effective radius,
extinction, optical thickness, ...
with horizontal resolution of 1.1km
vertical resolution of 60m

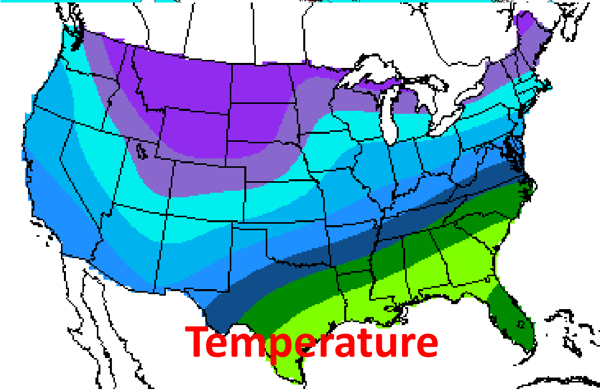


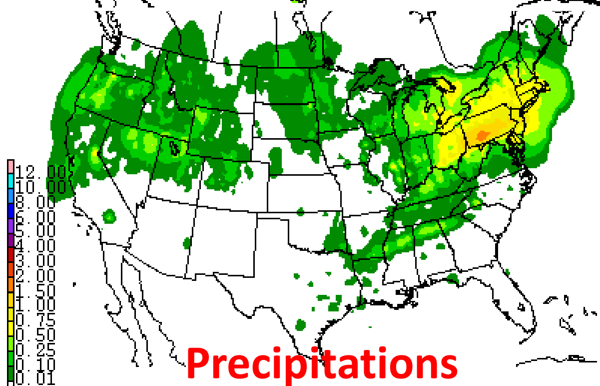
Intercomparison Case Study 20th December 2008

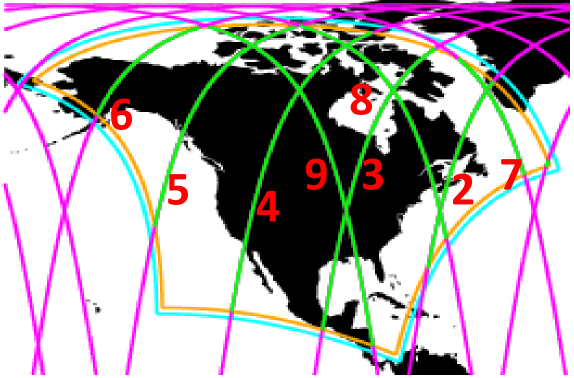
Daily Mean Non-Convective Cloud Cover (NARR)



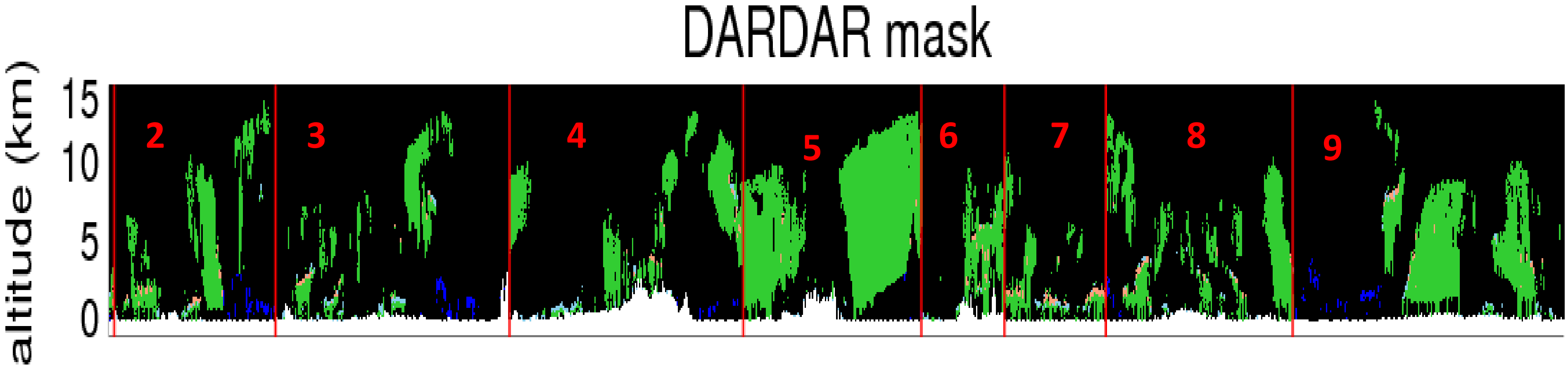






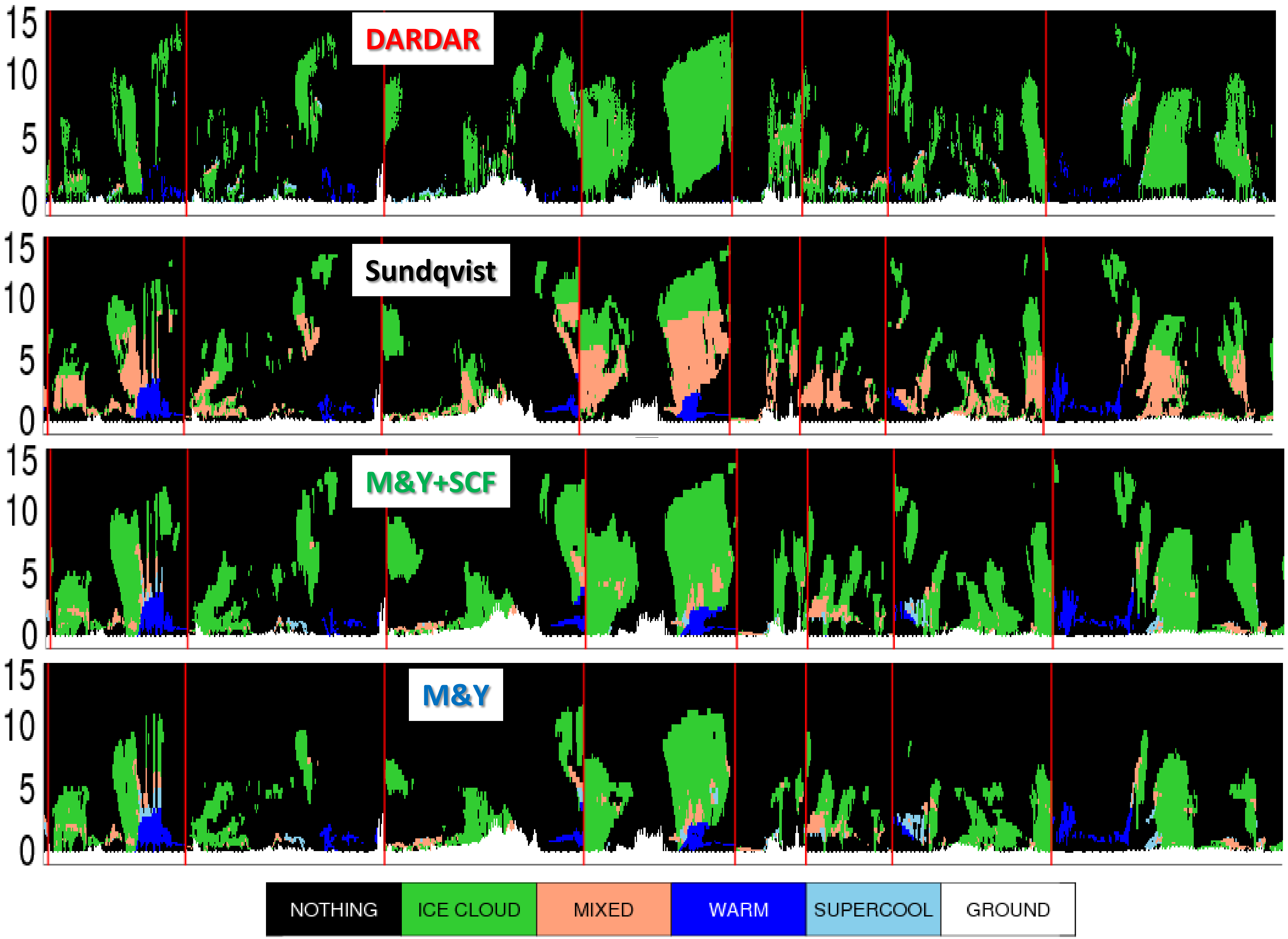


Intercomparison

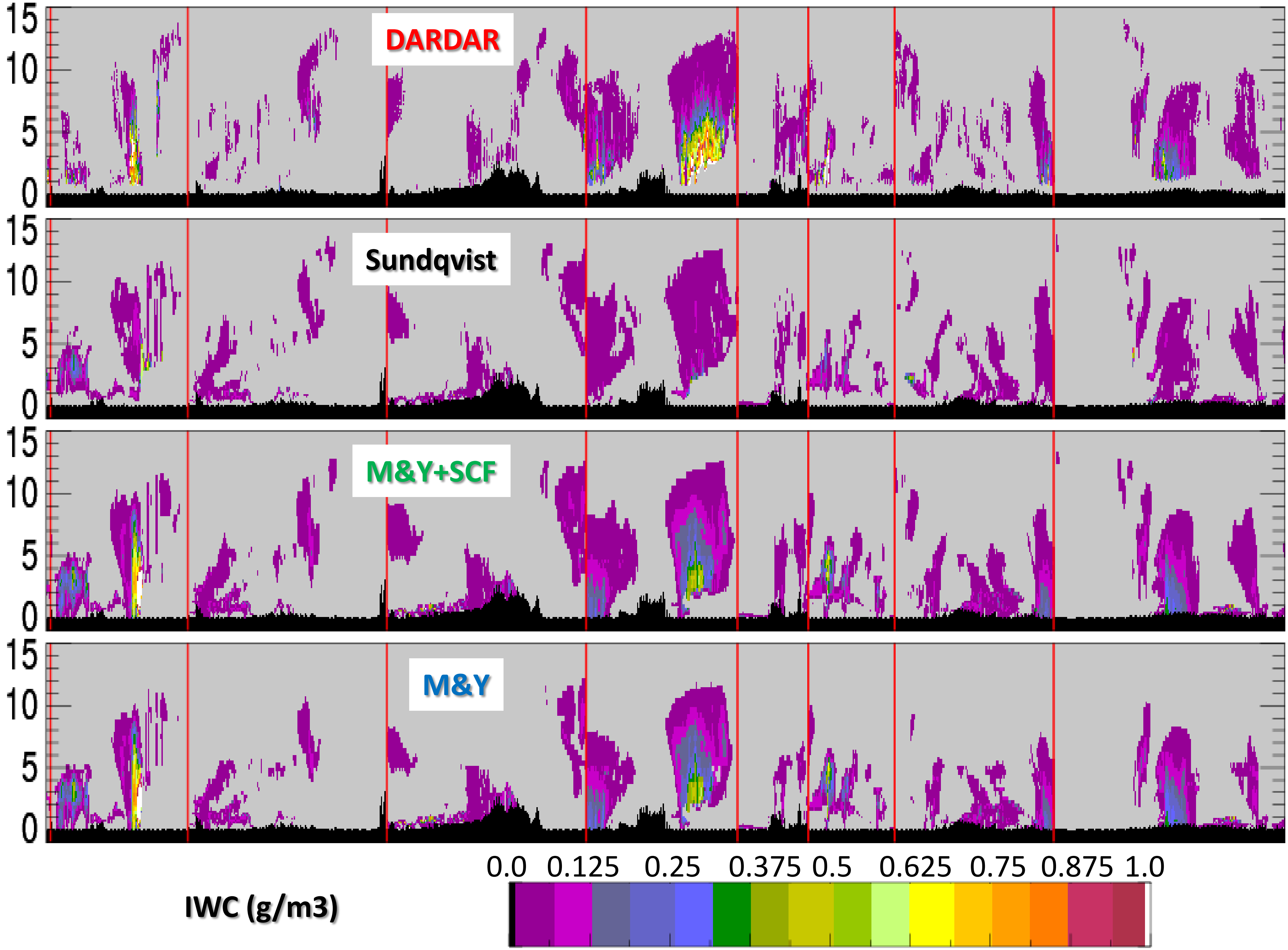


Each DARDAR profile of each overpass is compared to the closest profile in space and time from the GEM simulations.

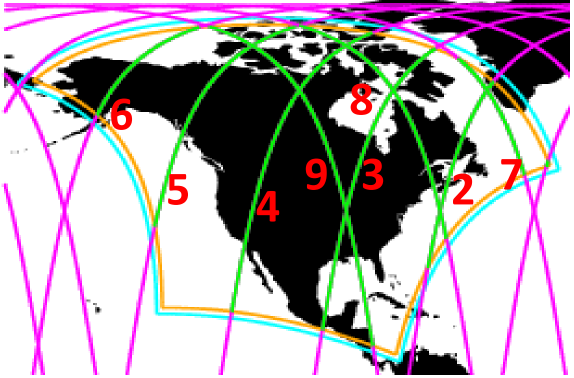
NOTHING	ICE CLOUD	MIXED	WARM	SUPERCOOL	GROUND



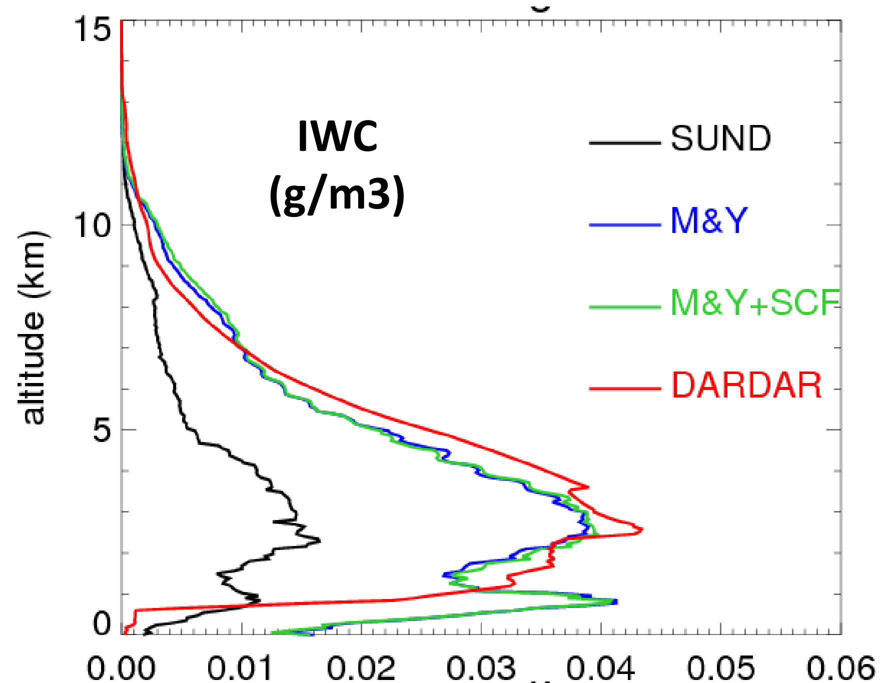
DARDAR
Sundqvist
M&Y+SCF
M&Y
15
10
5
0
NOTHING
ICE CLOUD
MIXED
WARM
SUPERCOOL
GROUND

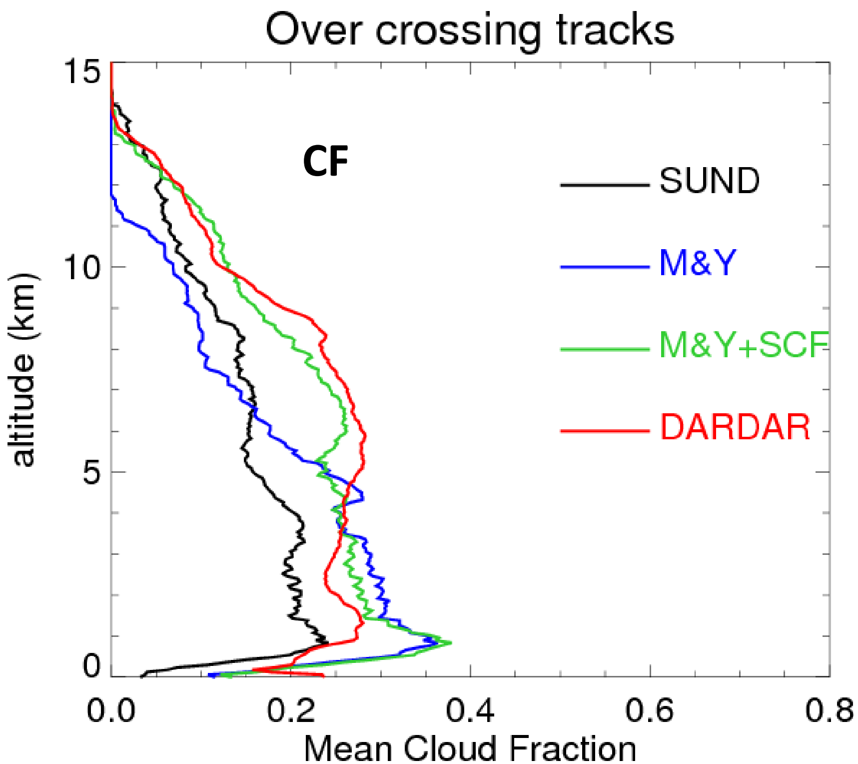


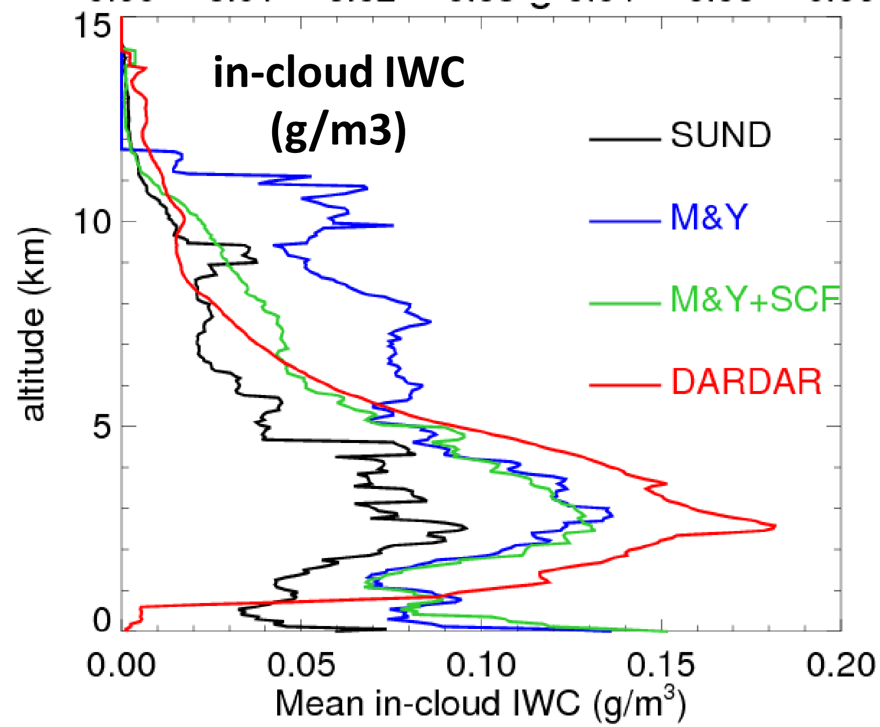
DARDAR
Sundqvist
M&Y+SCF
M&Y
15
10
5
0
0.0
0.125
0.25
0.375
0.5
0.625
0.75
0.875
1.0
IWC (g/m3)

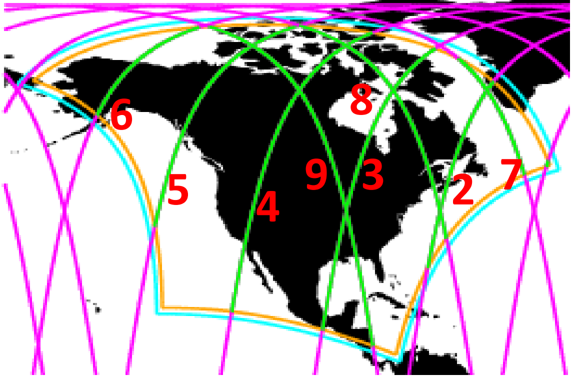


All Clouds Intercomparison





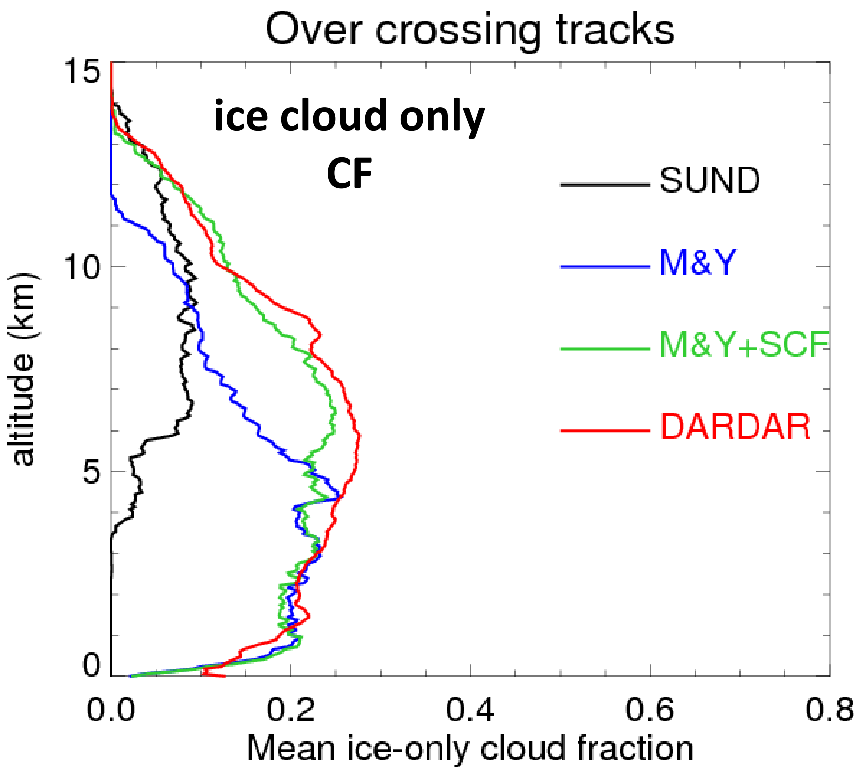


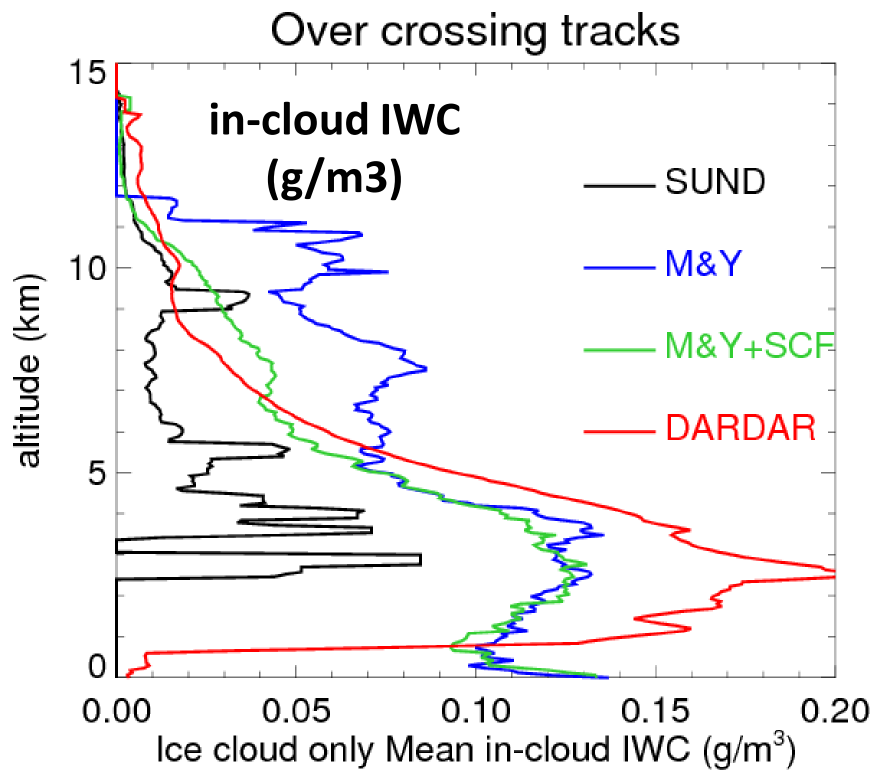


Ice Clouds

excluding mixed and liquid clouds

Intercomparison





Subgrid Cloud Fraction Scheme

- Simple and Purely Diagnostic
- Allows easy implementation in multi-moment multi-class microphysics model
- Allows supersaturation in ice clouds (frequent)
- Allows direct tuning of SCF via relative humidity threshold value U_{00}
- Allows tuning of subgrid precipitation fraction via overlap assumptions (affects precip. production)
- Shows clear improvement of simulated cloud fraction
- Shows clear improvement of in-cloud IWC

Significant step toward the use of 2-moment multi-class cloud scheme in NWP with a large range of resolutions.

3. Development of unified microphysics for coarser resolution to about 40 km

b) Consolidation of hydrometeor categories

- adding prognostic elements for hydrometeor categories
- consolidate the number of hydrometeor categories
- e.g. consolidate graupel and hail categories by predicting graupel density

GRAUPEL – a heavily rimed ice crystal

(to the extent that it is no longer possible to identify the original crystal habit shape)

GRAUPEL as a <u>single category</u> is problematic:

- large range of observed densities (ρ_g) (200-900 kg m^{-3})
- large range of observed fall speeds (2-30 m s^{-1})*
- moment dependencies on ρ_g

•(for high densities/fall speeds, would be <u>*hail*</u> in nature – can we consolidate graupel and hail categories?)

To address problems associated with a single category graupel:

Allow variable graupel density * by predicting the bulk volume mixing ratio B_g in the MY 2-moment scheme

* Connolly et al. (2005) [QJRMS]

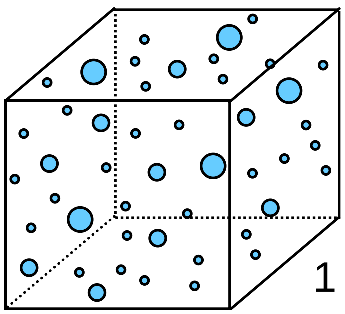
Mansell et al. (2010) [*JAS*]

Milbrandt and Morrison (2012) [*JAS, accepted*]

DESCRIPTION OF METHOD

Standard 2-moment scheme:

- q_g (mass mixing ratio); kg kg^{-1}
- N_g (number mixing ratio); # kg^{-1}



New prognostic variable:

- B_g (bulk volume mixing ratio); m^3 kg^{-1} → $\rho_g = \frac{q_g}{B_g}$

New prognostic equation:

$$\frac{\partial B_g}{dt} = \left(\frac{\partial B_g}{\partial t}\right)_{\text{advection}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{diffusion}} + \left(\frac{\partial \rho_g}{dt}\frac{q_g}{\rho_g^2}\right)_{\text{micro}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{sedimentation}}$$

DYNAMICS (advection + diffusion)

MICROPHYSICS SCHEME (micro + sedimentation)

Details in Milbrandt and Morrison (2012, JAS, accepted)

DESCRIPTION OF METHOD – Sedimentation

$$\frac{\partial B_g}{dt} = \left(\frac{\partial B_g}{\partial t}\right)_{\text{advection}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{diffusion}} + \left(\frac{\partial \rho_g}{dt}\frac{q_g}{\rho_g^2}\right)_{\text{micro}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{sedimentation}}$$

Sedimentation of B_g:

- Mass-weighted fall speed, $\overline{V}_g$ (function of PSD and fall speed parameters)
- Compute* fall speeds $V(\rho_g, D_g)$ off-line (for range of ρ_g and D_g):

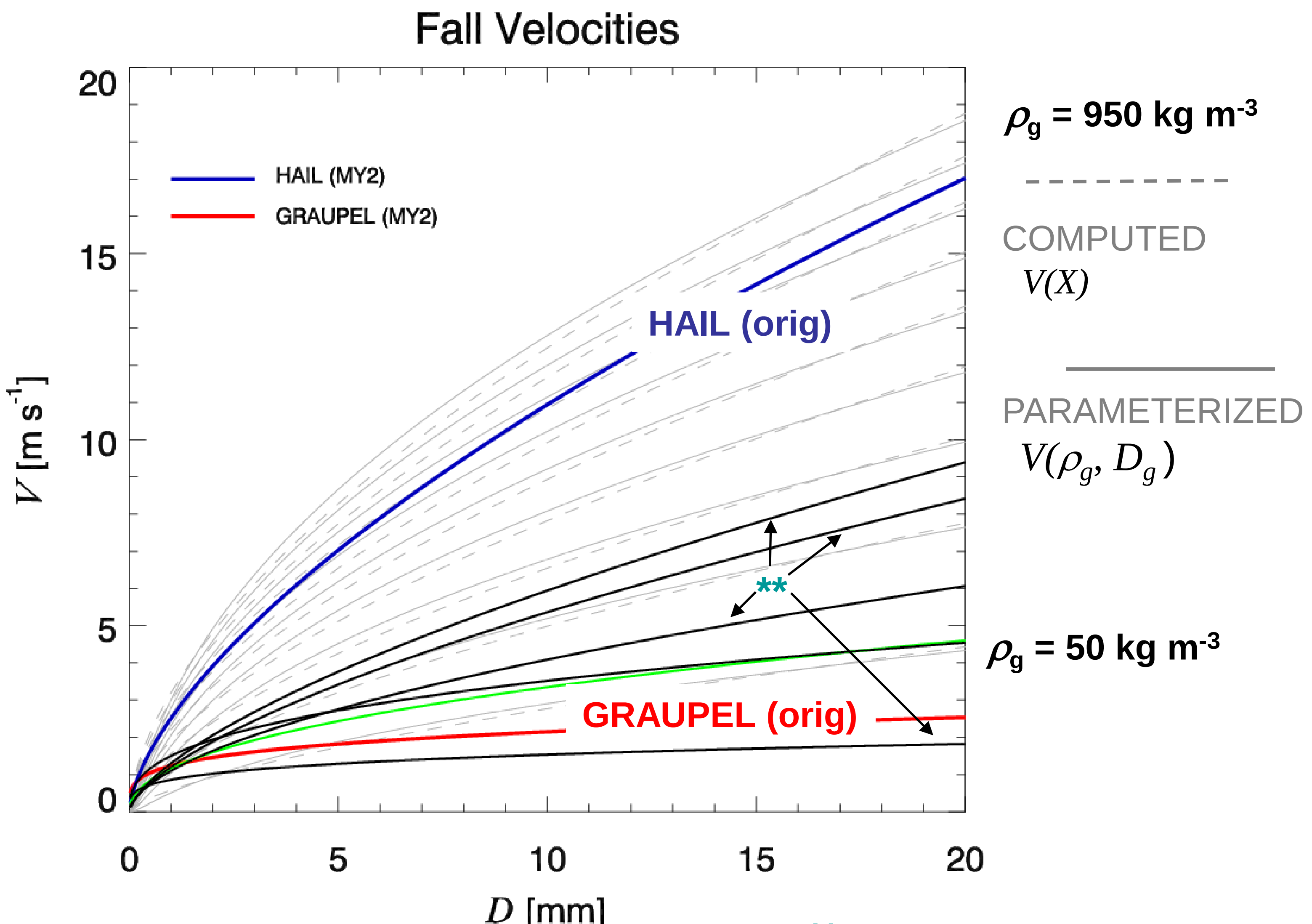
$$V_g = f(X); \quad X = \frac{4\rho_g g \rho_a D^3}{3\eta^2}$$ (Best number)

- Fall speed parameter are parameterized $a_g = f(\rho_g)$ and $b_g = f(\rho_g)$

$$V_g = a_g D^{b_g}$$

* Following: Mitchell and Heymsfield (2005)

DESCRIPTION OF METHOD – Sedimentation



** Other graupel types from Locatelli and Hobbs (1974)

DESCRIPTION OF METHOD – Microphysical Processes

$$\frac{\partial B_g}{dt} = \left(\frac{\partial B_g}{\partial t}\right)_{\text{advection}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{diffusion}} + \left(\frac{\partial \rho_g}{dt}\frac{q_g}{\rho_g^2}\right)_{\text{micro}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{sedimentation}}$$

$$\frac{d\rho_g}{dt}_{micro} = \text{INITIATION + RIMING + DIFFUSION + MELTING}$$

DESCRIPTION OF METHOD – Microphysical Processes

$$\frac{\partial B_g}{dt} = \left(\frac{\partial B_g}{\partial t}\right)_{\text{advection}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{diffusion}} + \left(\frac{\partial \rho_g}{dt}\frac{q_g}{\rho_g^2}\right)_{\text{micro}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{sedimentation}}$$

$$\frac{d\rho_g}{dt}_{micro}$$ = **INITIATION** + RIMING + DIFFUSION + MELTING

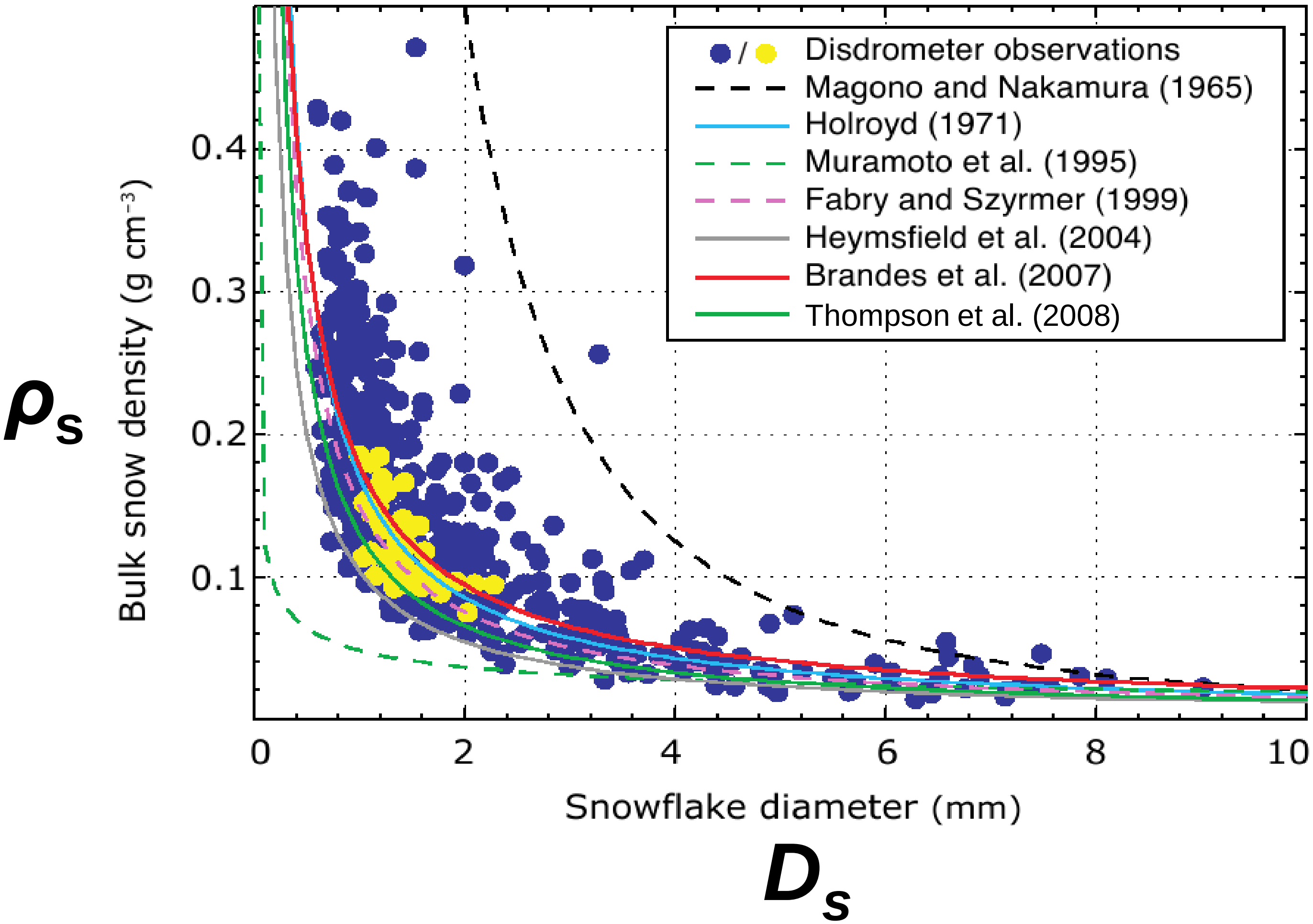
INITIATION:

- From contact freezing: ρ_{g_init} = mass-weighted mean ρ_x and ρ_y
- From snow: $\rho_{g\_init} = \rho_s(D_s)$

DESCRIPTION OF METHOD – Microphysical Processes

INITIATION:

- From snow: $\boldsymbol{\rho_{g\_init} = \rho_s(D_s)}$



(Can be exploited to predict **solid-liquid ratio**; see Milbrandt et al. (2012), *MWR)*

DESCRIPTION OF METHOD – Microphysical Processes

$$\frac{\partial B_g}{dt} = \left(\frac{\partial B_g}{\partial t}\right)_{\text{advection}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{diffusion}} + \left(\frac{\partial \rho_g}{dt}\frac{q_g}{\rho_g^2}\right)_{\text{micro}} + \left(\frac{\partial B_g}{\partial t}\right)_{\text{sedimentation}}$$

$$\frac{d\rho_g}{dt}_{micro}$$ = INITIATION + **<u>RIMING</u>** + DIFFUSION + MELTING

RIMING:

$$* \quad \rho_{rime} = f\left(\frac{0.5 D_{drop} V_{impact}}{T_{sfc}}\right)$$

$$D_{drop} = \left(\frac{6\rho q_c}{\pi \rho_L N_c}\right)^{\frac{1}{3}}$$

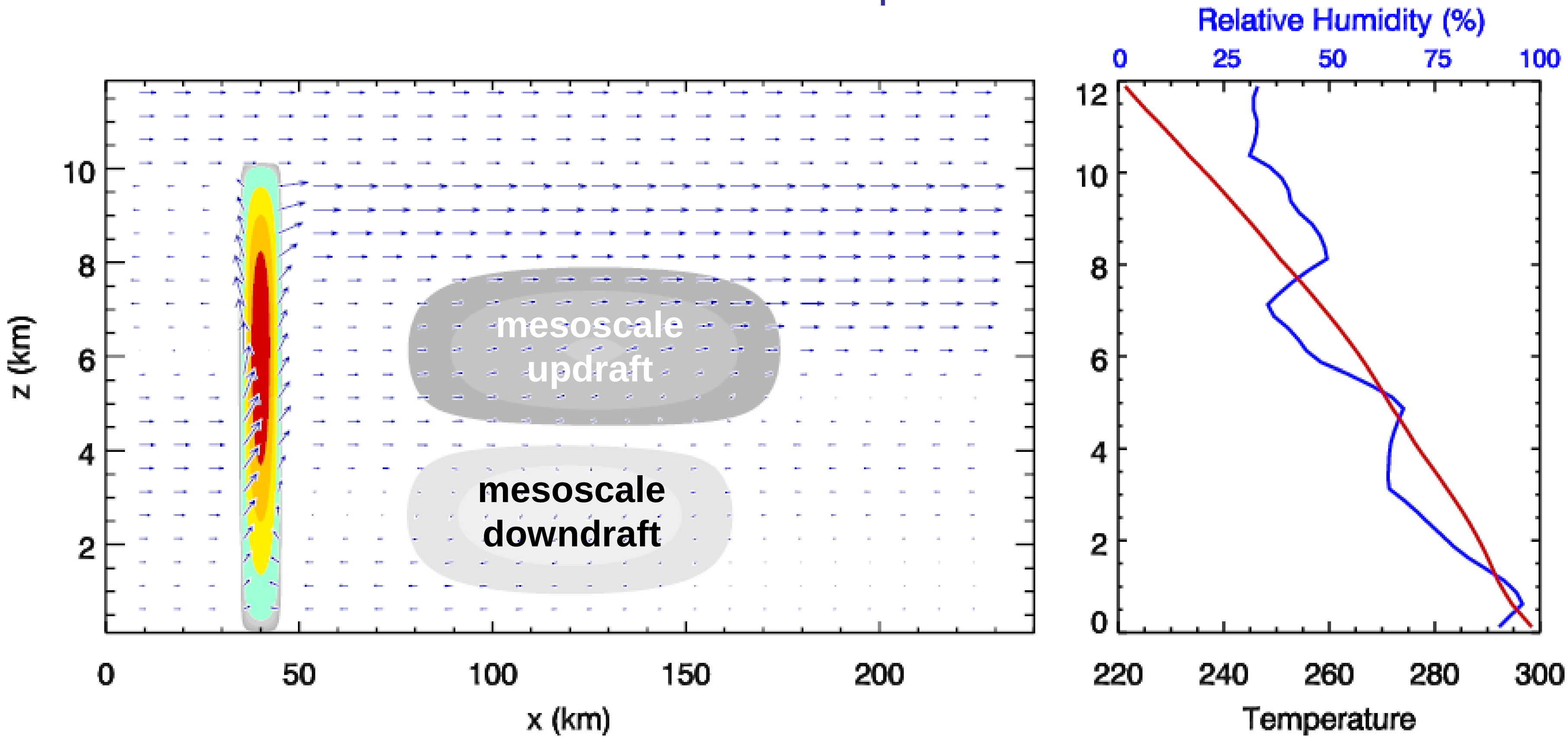
$$V_{impact} = \left|V(D_g) - V(D_{drop})\right|$$

- Dependence <u>size</u> of drops (cloud and rain) and graupel
- Thus, exploits 2-moment

* Cober and List (1993), *JAS*

2D Simulation – Set-up

- Idealized 2D kinematic model*
- Interfaced with modified 2-moment Milbrandt-Yau scheme**
- Prescribed flow field and convective updraft

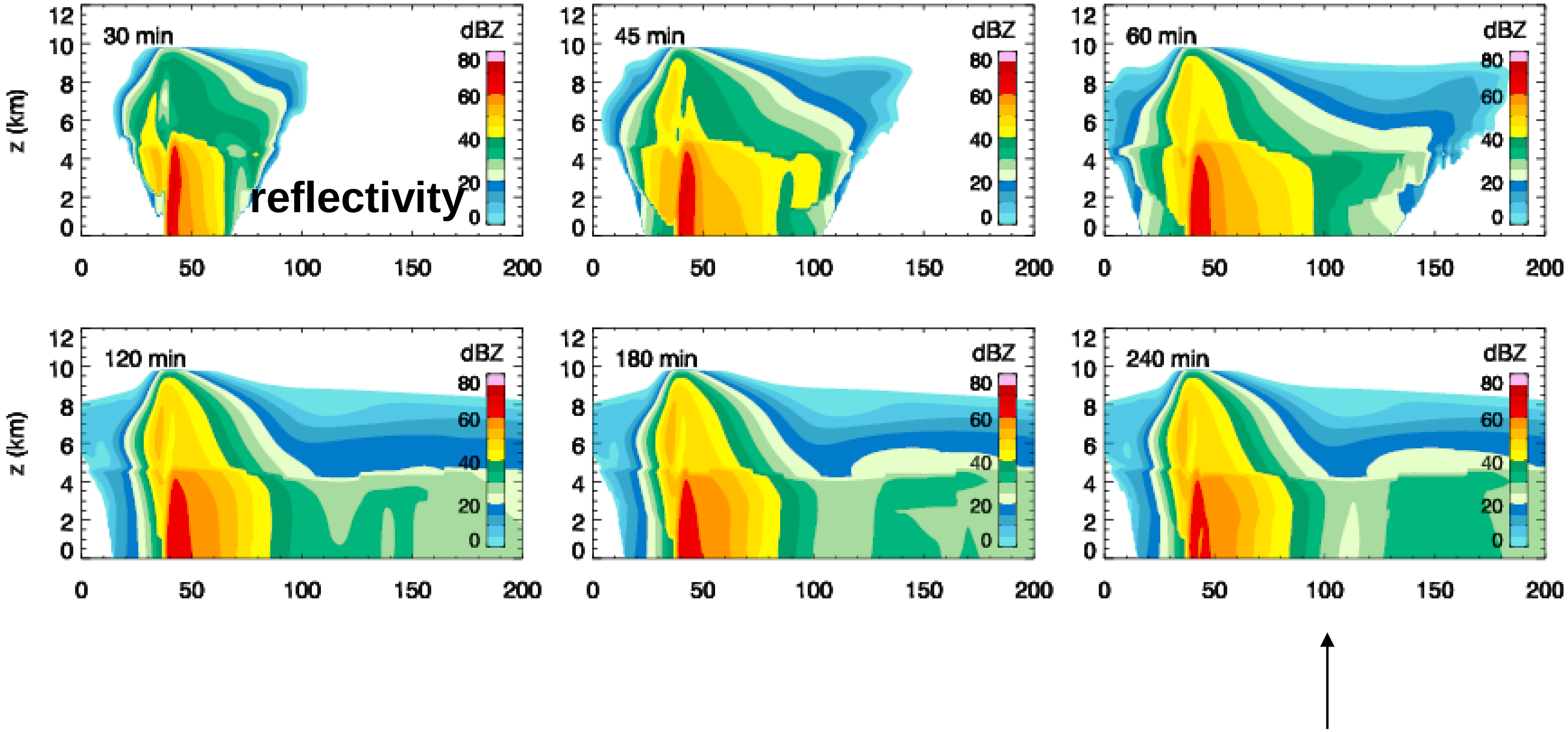


* Szumowski et al. (1998)

** Milbrandt and Yau (2005)

2D Simulation: *w_peak* = 20 m s^{-1}

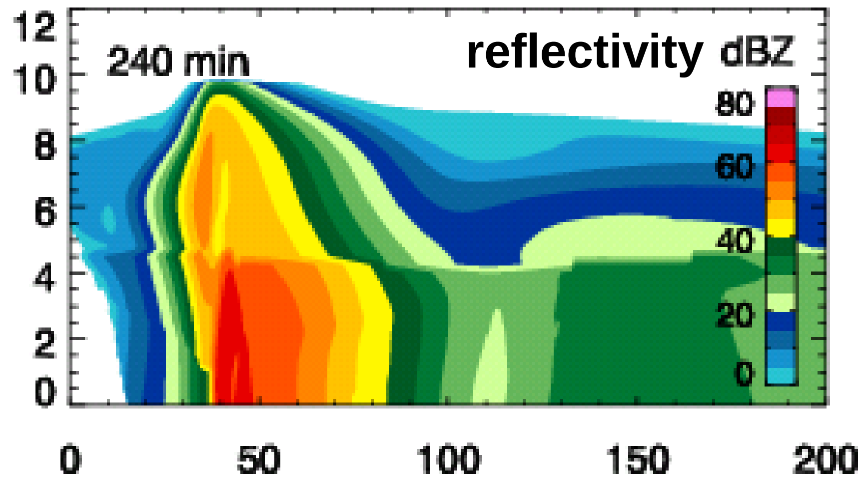
GRAUPEL ONLY - Prognostic ρ_g



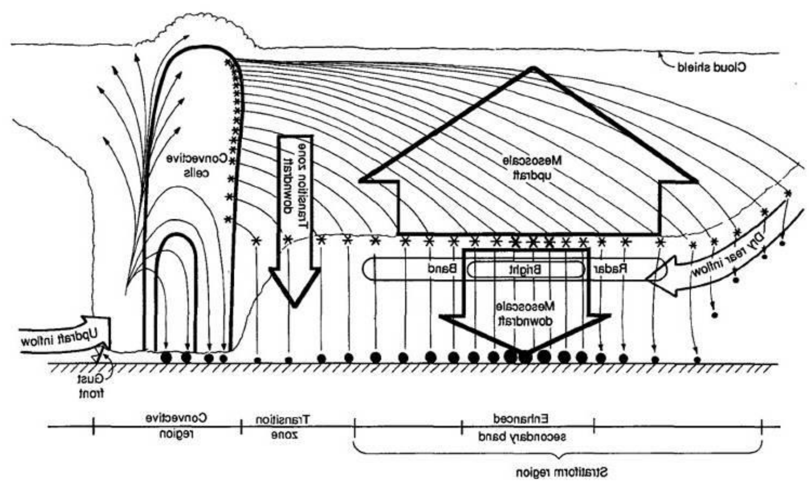
Steady state is reached

2D Simulation: *w_peak* = 20 m s^{-1}

Steady-state 2D simulation:

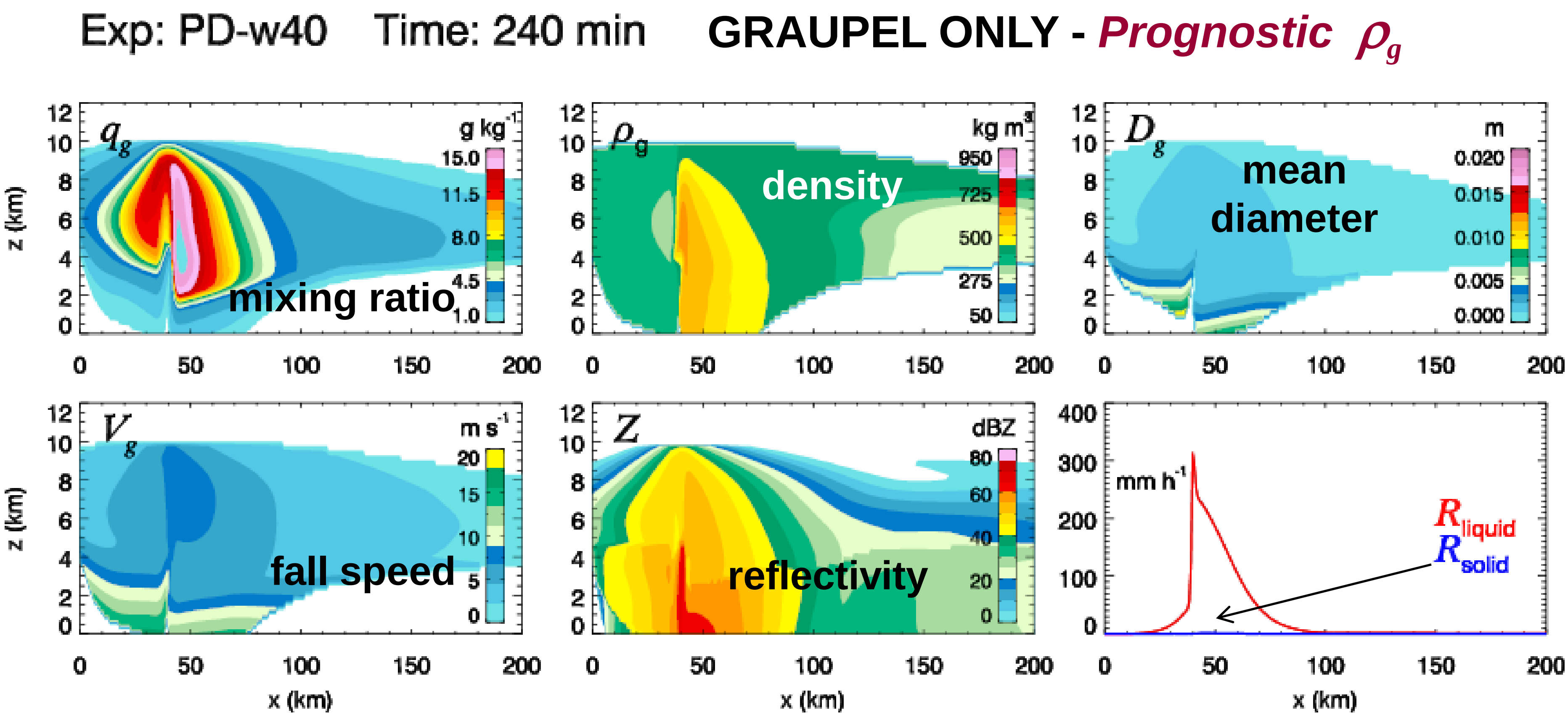


Conceptual model* of squall line:



* Biggerstaff and Houze (1991)

w_peak = 40 m s^{-1} *t* = 240 min

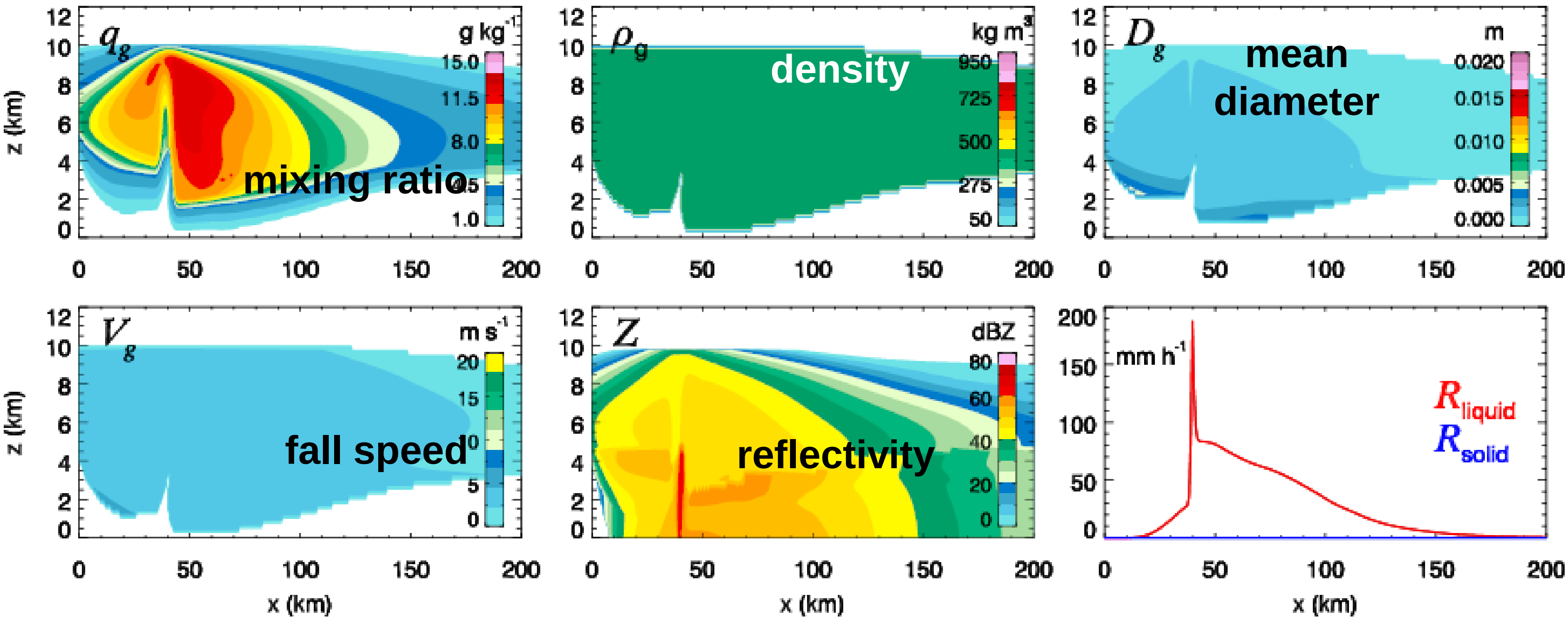


High density rimed-ice at surface
"hail" can still form from frozen water drops

w_peak = 40 m s^{-1}

t = 240 min

Exp: FF-w40   Time: 240 min   **GRAUPEL ONLY - *Fixed ρ_g 400 kg/m3***

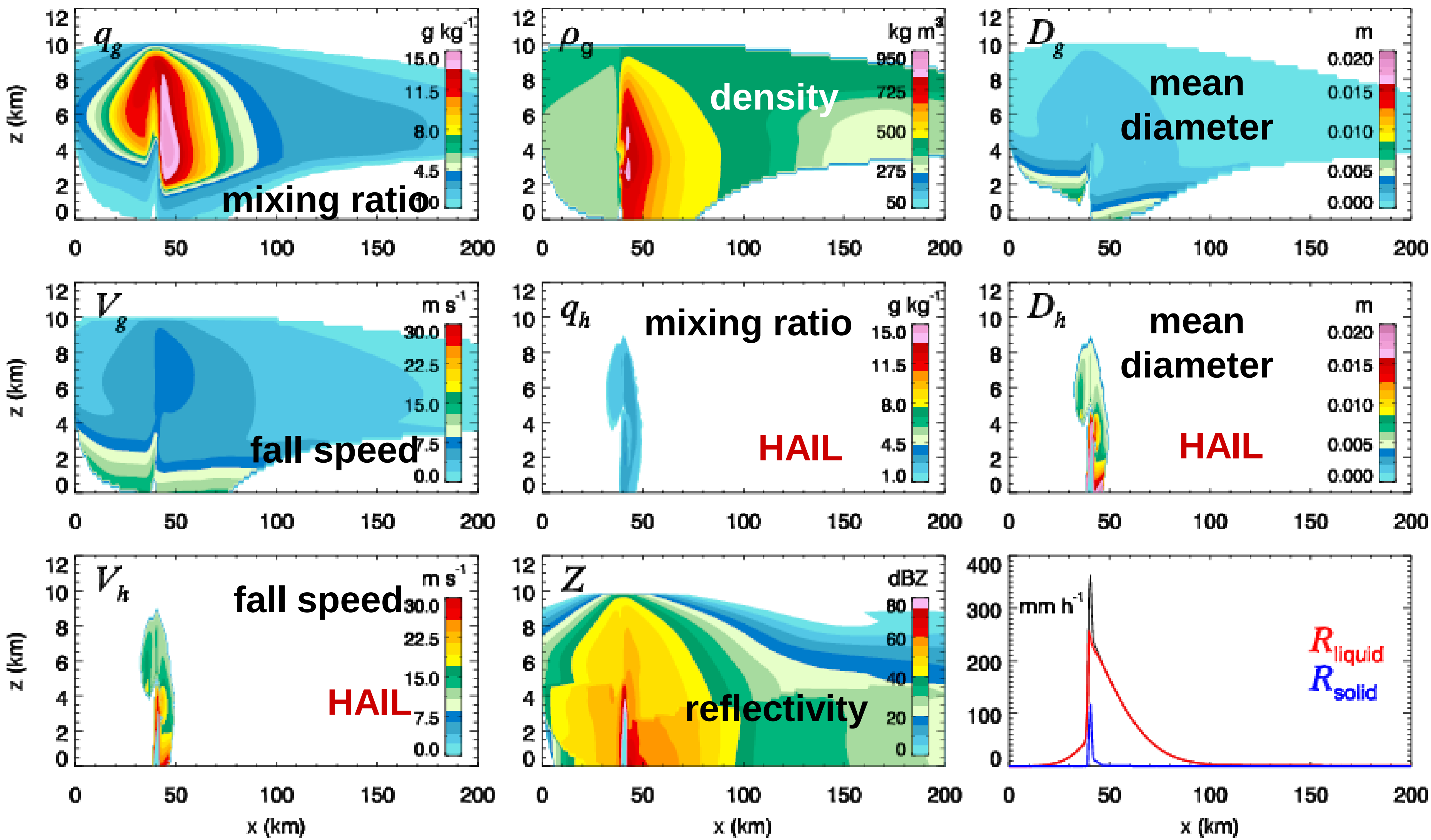


Smaller fall speed, graupel mass less in convective region, more in stratiform region, greater total precip. in stratiform region.
No graupel at surface, all melted

w_peak = 40 m s^{-1}

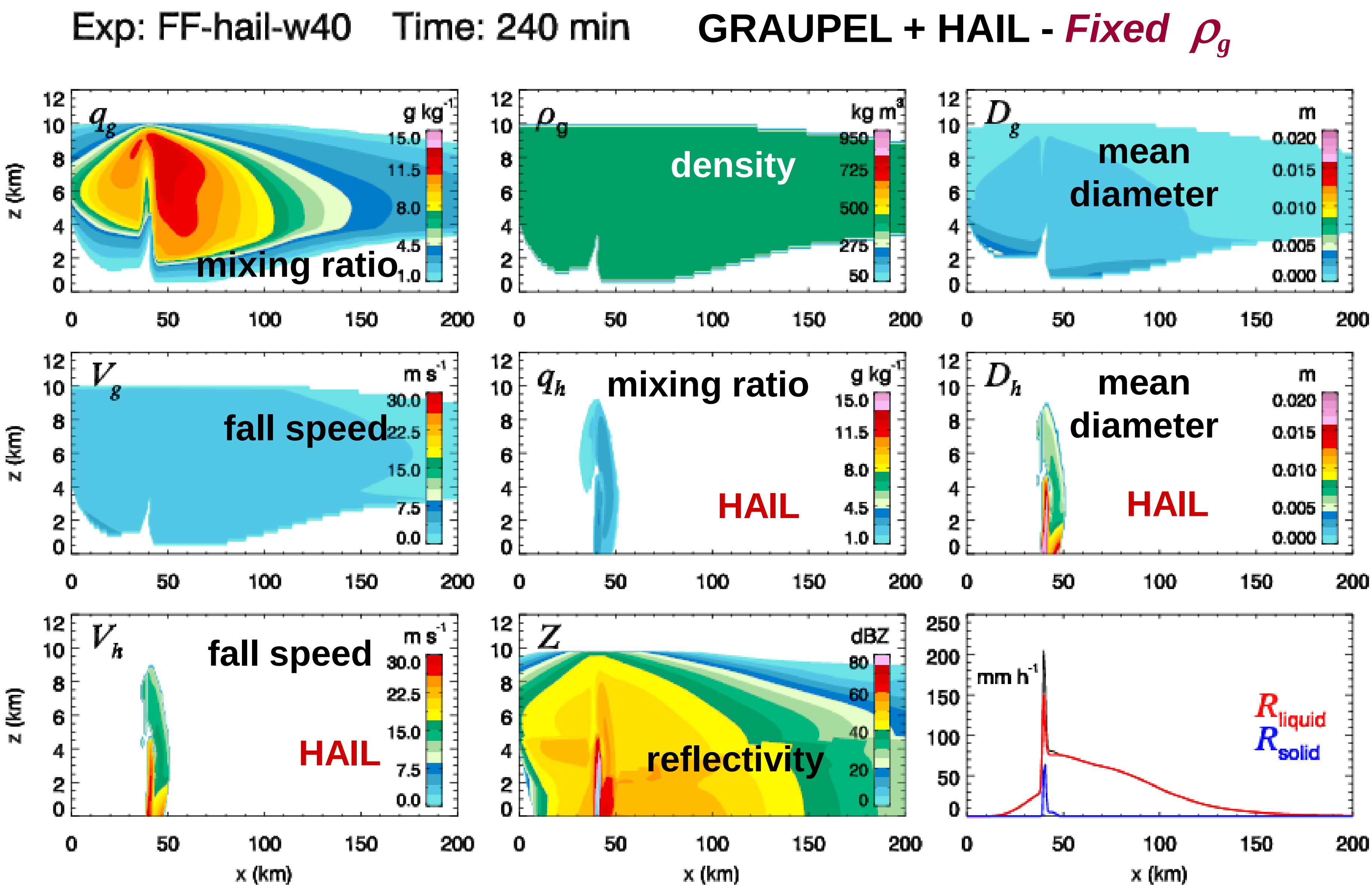
t = 240 min

Exp: PD-hail-w40 Time: 240 min **GRAUPEL + HAIL - *Prognostic* ρ_g**



w_peak = 40 m s^{-1}

t = 240 min



Similar differences as in no hail case, but with hail category, solid precip. at surface

- With prognostic ρ_g, a 2-moment bulk scheme can simulate a wide range of situations using a single rimed-ice category

- Need tests in 3D dynamical model to see if storm evolution and structure is realistic using single category prognostic density scheme because of latent heat feedback and water loading.

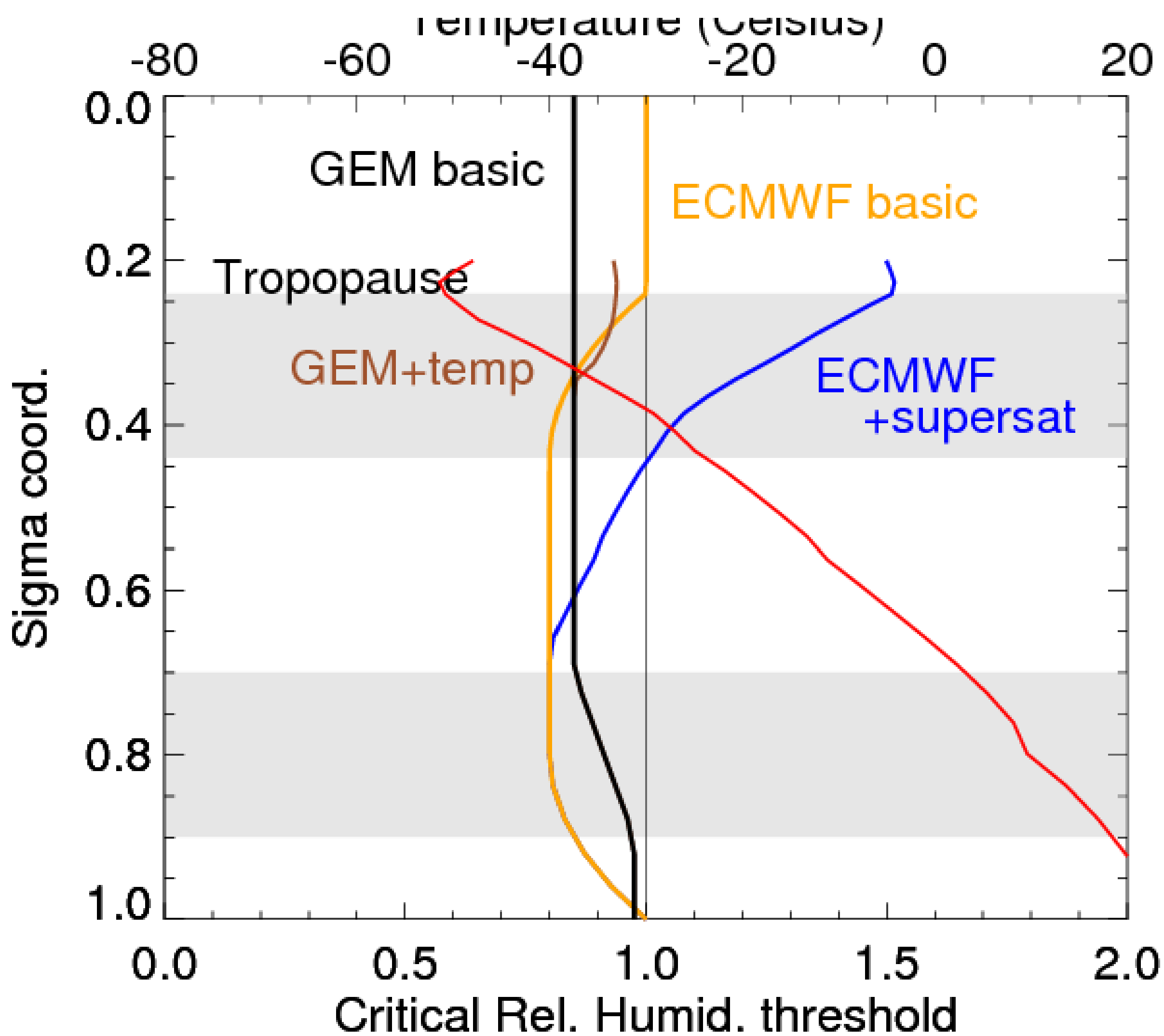
4. Future research

a) Testing of sub-grid scale cloud and precipitation fractions to about 40 km

b) Consolidation of ice crystal and snow categories

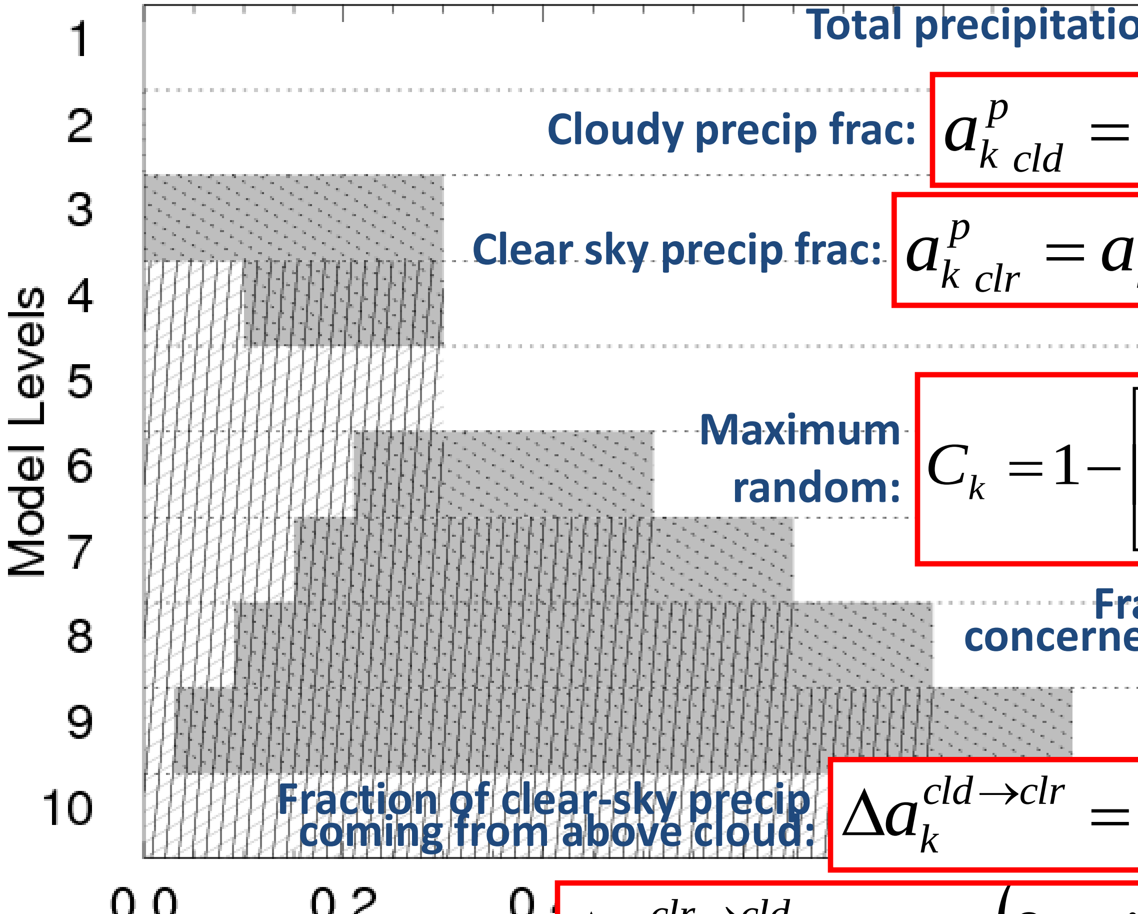
c) Better treatment of sub-grid scale vertical motion and sedimentation

d) Improve microphysics in convective parameterization scheme and proper interface between convection, microphysics, and radiation


Temperature (Celsius)
-80
-60
-40
-20
0
20
0.0
0.2
0.4
0.6
0.8
1.0
Sigma coord.
GEM basic
ECMWF basic
Tropopause
GEM+temp
ECMWF
+supersat
0.0
0.5
1.0
1.5
2.0
Critical Rel. Humid. threshold

3. THE SUBGRID PRECIPITATION FRACTION

How to find the clear sky precipitation fraction and the cloudy precipitation fraction



Total precipitation fraction: $$a_k^p = a_{k\ clr}^p + a_{k\ cld}^p$$

Cloudy precip frac: $$a_{k\ cld}^p = a_{k-1} + \Delta a_k^{clr \to cld} - \Delta a_k^{cld \to clr}$$

Clear sky precip frac: $$a_{k\ clr}^p = a_{k-1clr}^p - \Delta a_k^{clr \to cld} + \Delta a_k^{cld \to clr}$$

Maximum random: $$C_k = 1 - \left[(1 - C_{k-1}) \frac{1 - \max(a_k, a_{k-1})}{1 - \min(a_{k-1}, 1 - \delta)} \right]$$

Fraction never concerned by precip: $$\Delta C_k = C_k - C_{k-1}$$

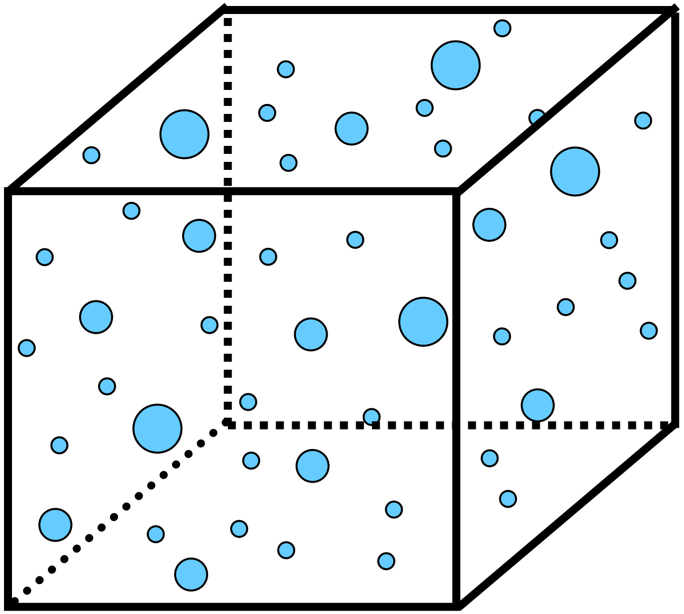
Fraction of clear-sky precip coming from above cloud: $$\Delta a_k^{cld \to clr} = a_{k-1} - \min(a_k - \Delta C_k, a_{k-1})$$

Fraction of clear-sky precip coming from clear-sky: $$\Delta a_k^{clr \to cld} = \max\left(0, \min\left(a_{k-1\ clr}^p, a_k - \Delta C_k - a_{k-1}\right)\right)$$

BACKGROUND

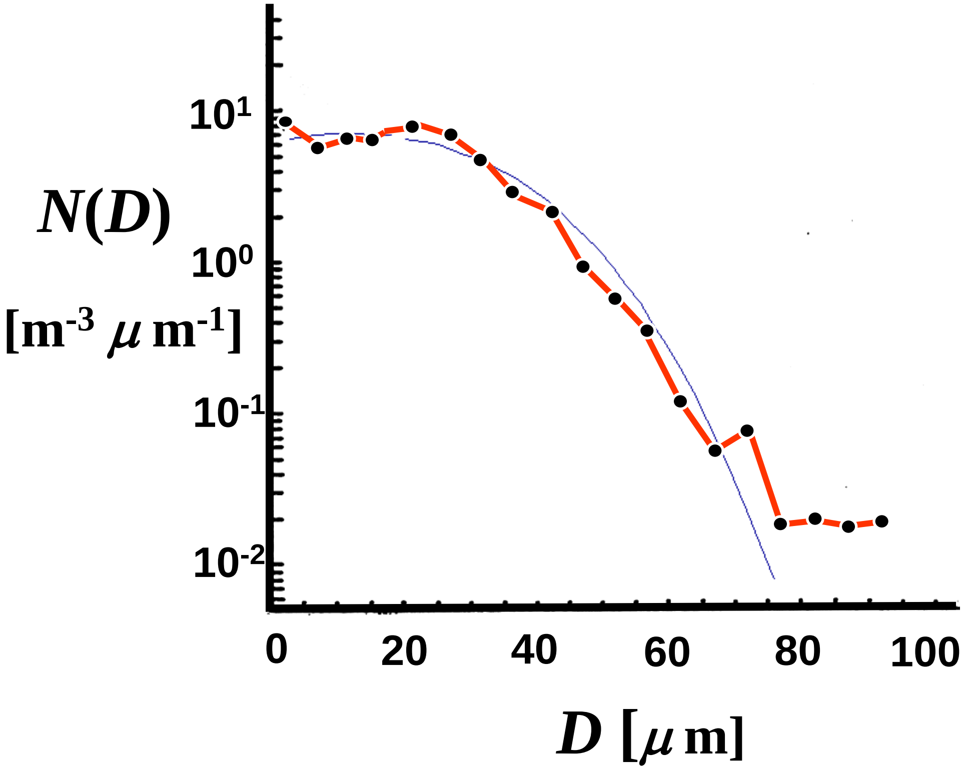
- Connolly et al. (2005) introduced idea of predicting the bulk volume mixing ratio B_g
  - 2-moment graupel category (q_g, N_g, and now B_g)
  - assumed rime density of 500 kg m^{-3}
  - fixed graupel fall speed parameters
- Straka and Mansell (2005) used computed rime density (Heymsfield and Pflaum 1985)
  - distributed over three 1-moment graupel categories (with different fixed densities and fall speed parameters)
- Mansell et al. (2010) merged prognostic B_g and computed rime density
  - a single 2-moment graupel category (q_g, N_g, and now B_g)
  - diagnosed rime density (Heymsfield and Pflaum 1985)
  - variable fall speed parameters
- Milbrandt and Morrison (2012, JAS, in press) added prognostic B_g to a 2-moment graupel category

Representing the size spectrum



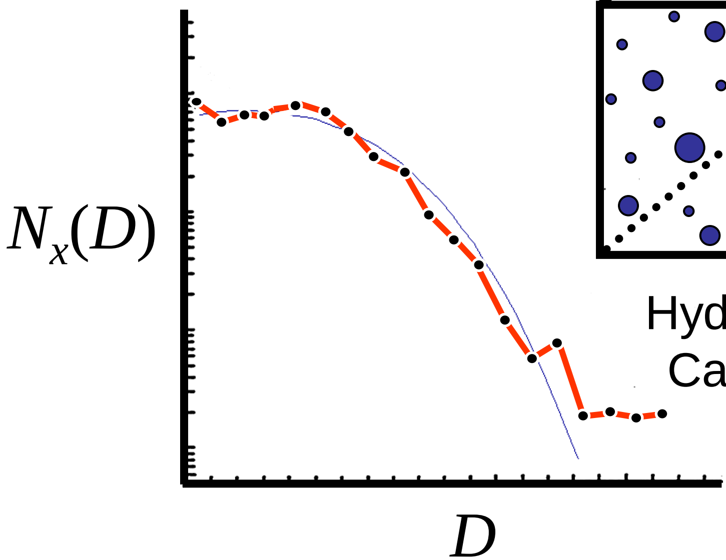
1 m^3

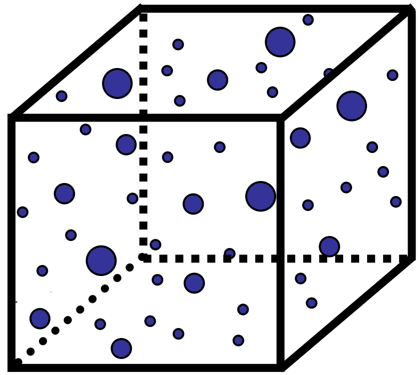
ANAYLTICAL FUNCTION



BULK METHOD

BULK METHOD





Hydrometeor
Category x

Total number concentration, N_{Tx}

$$N_{Tx} \equiv \int_0^\infty N_x(D)dD = M_x(0)$$

Mass mixing ratio, q_x

$$q_x \equiv \frac{c_x}{\rho}\int_0^\infty D^3 N_x(D)dD = \frac{c_x}{\rho} M_x(3),$$

$$where\ m_x(D) = c_x D^3,\ \rho\ is\ air\ density$$

Radar reflectivity factor, Z_x

$$Z_x \equiv \int_0^\infty D^6 N_x(D)dD = M_x(6)$$

***Size Distribution Function*:**

$$N_x(D) = N_{0x} D^{\alpha_x} e^{-\lambda_x D}$$

p^{th} moment:

$$M_x(p) \equiv \int_0^\infty D^p N_x(D)dD = N_{0x}\frac{\Gamma(1+\alpha_x+p)}{\lambda_x^{p+1+\alpha_x}}$$

BULK METHOD

Predict evolution of specific moment(s)

e.g. q_x, N_{Tx}, ...

↓

Implies prediction of evolution of parameters

i.e. N_{0x}, λ_x, ...

Total number concentration, N_{Tx}

$$N_{Tx} \equiv \int_0^\infty N_x(D)dD = M_x(0)$$

Mass mixing ratio, q_x

$$q_x \equiv \frac{c_x}{\rho}\int_0^\infty D^3 N_x(D)dD = \frac{c_x}{\rho}M_x(3),$$

$$where\ m_x(D) = c_x D^3, \rho = air\ density$$

Radar reflectivity factor, Z_x

$$Z_x \equiv \int_0^\infty D^6 N_x(D)dD = M_x(6)$$

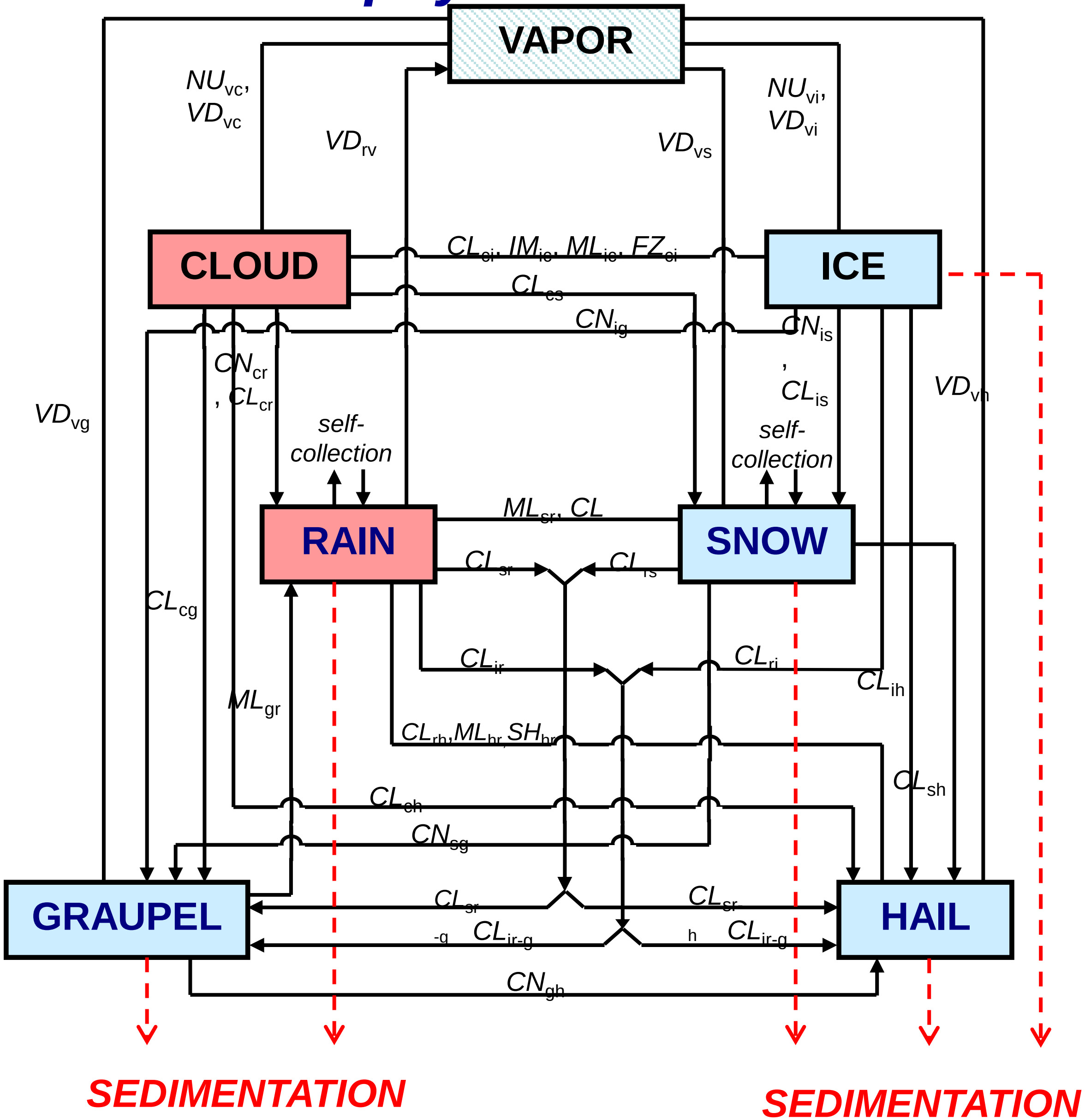
***Size Distribution Function*:**

$$N_x(D) = N_{0x} D^{\alpha_x} e^{-\lambda_x D}$$

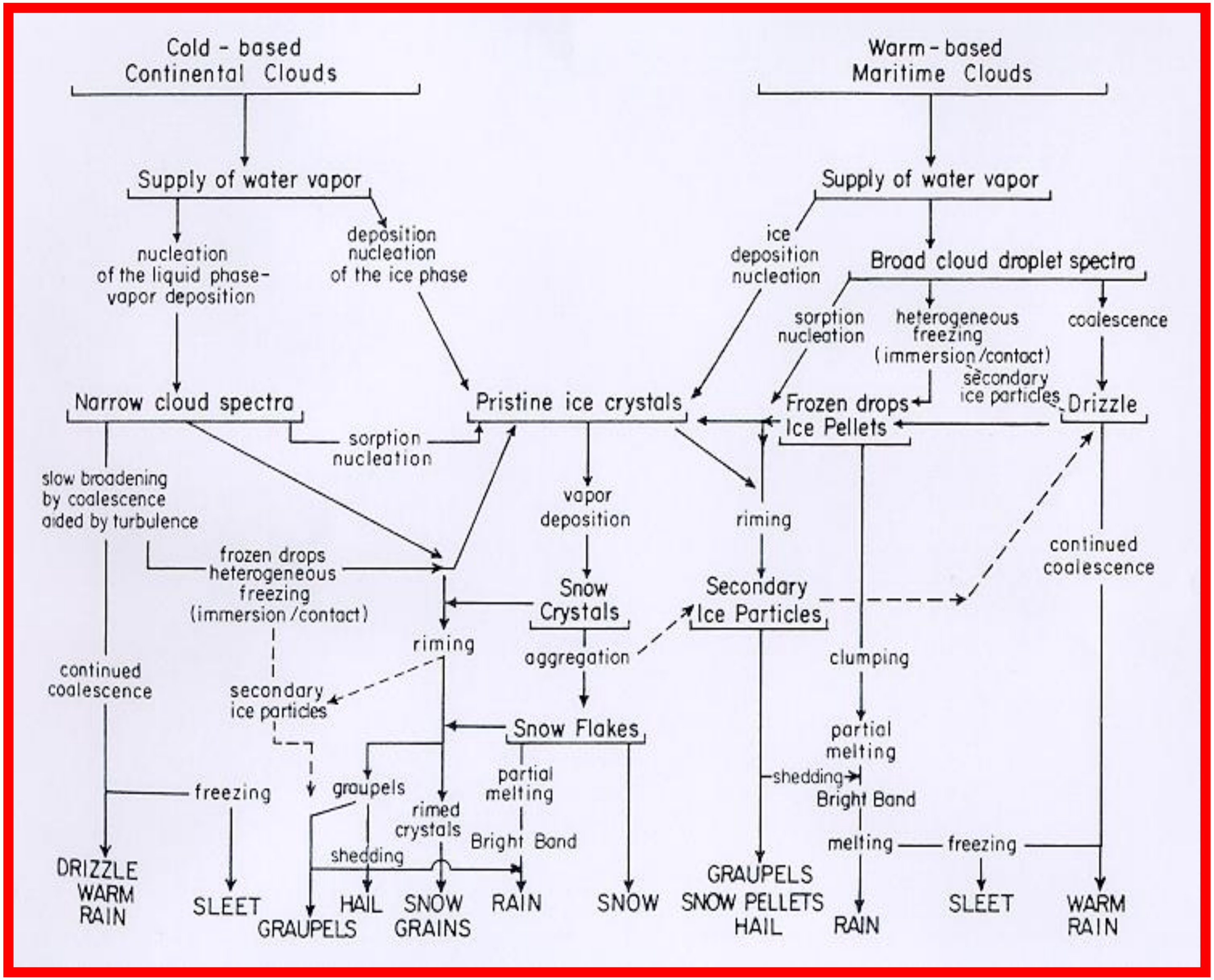
p^{th} moment:

$$M_x(p) \equiv \int_0^\infty D^p N_x(D)dD = N_{0x}\frac{\Gamma(1+\alpha_x+p)}{\lambda_x^{p+1+\alpha_x}}$$

The Six-Category Multi-moment Microphysics Scheme:



Cloud Microphysical Processes



Precipitation types from microphysics :

RN1 – Liquid Drizzle
RN2 – Liquid Rain
FR1 – Freezing Drizzle
FR2 – Freezing Rain
SN1 – Ice Crystals
SN2 – Snow
SN3 – Graupel (snow pellets)
PE1 – Ice Pellets (re-frozen rain)
PE2 – Hail (total)
PE2L – Large Hail