UNIVERSITA' DI TORINO
Dipartimento di Fisica

FREAK WAVES: BEYOND THE BREATHER SOLUTIONS OF NLS EQUATION

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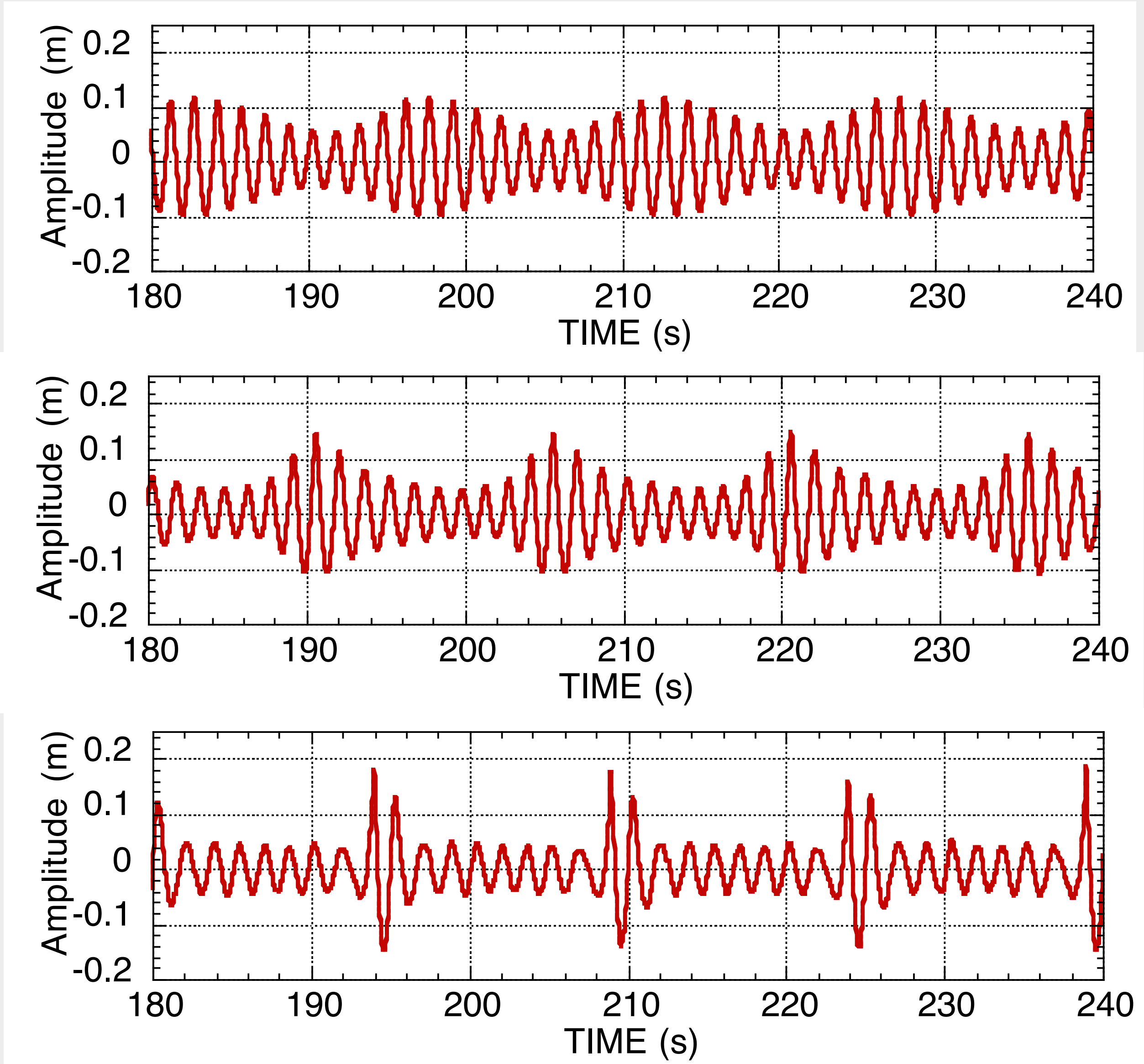
OUTLINE OF THE PRESENTATION

1) Quick review on modulational instability and breather solutions of the NLS equation

2) New approach on studying the modulational instability using Navier-Stokes equation

3) The case of the Louis-Majesty accident: crossing seas conditions

MODULATIONAL INSTABILITY: Experimental result



BREATHERS: EXACT SOLUTION OF THE NLS

N. Akhmediev,et al. (1987)

$$A(x,t) = A_0 \exp\left[-i\beta A_0^2 t\right]\left(\frac{\sqrt{2}\tilde{\nu}^2\cosh[\Omega t] - i\sqrt{2}\tilde{\sigma}\sinh[\Omega t)]}{\sqrt{2}\cosh[\Omega t] - \sqrt{2-\tilde{\nu}^2}\cos[(k_0/N)x]} - 1\right)$$

Two remarks:

1) $$A(x, t \to -\infty) = A_0 \exp(i\phi)(1 + \delta\cos[(k_0/N)x])$$

2) The solution depends on steepness and *N*

MAXIMUM AMPLITUDE

$$\frac{A_{max}}{A_0} = 1 + 2\sqrt{1 - \left(\frac{1}{2\sqrt{2}\varepsilon N}\right)^2}$$

1) It depends on the product εN

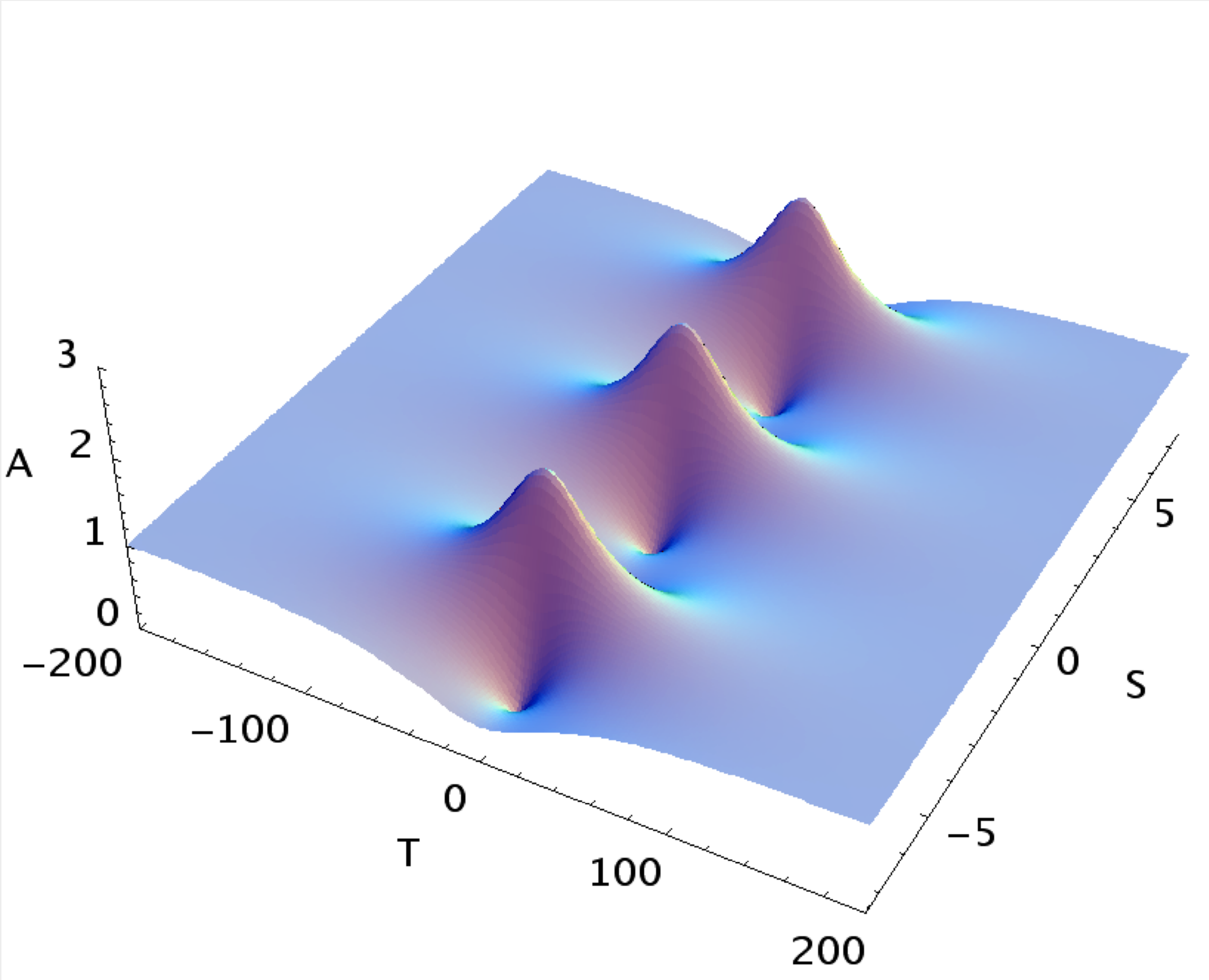
2) Maximum amplitude is 3 -> The Peregrine solution

Such solutions have been tested experimentally in a number of wave tanks and fully nonlinear computations

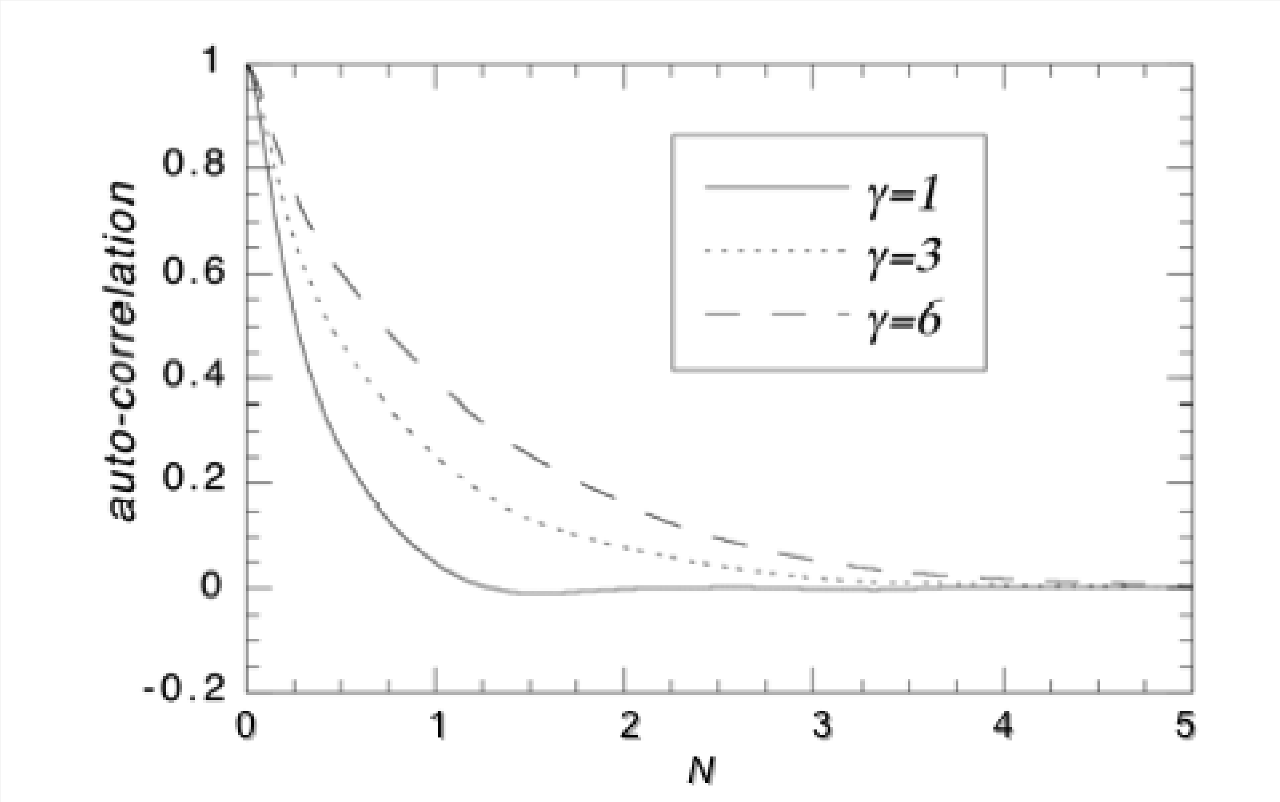
THE AKHMEDIEV SOLUTION

N=5

ε=0.1



OCEAN WAVES ARE CHARACTERIZED BY JONSWAP SPECTRUM



Example: N=3, ε= 0.1 – Wave group is stable

➔BREATHERS ARE RARE OBJECTS

BASED ON THIS BASIC IDEA OF MODULATIONAL INSTABILITY, AFTER YEARS OF THEORETICAL WORK, NUMERICAL SIMULATIONS AND EXPERIMENTAL WORK, THE FOLLOWING QUANTITY is NOW COMPUTED OPERATIONALLY AT THE E.C.M.W.F.:

$$kurtosis = 3 + \frac{\pi}{\sqrt{3}} BFI_{2D}^2 + 18\varepsilon^2$$

NORMAL VALUE

FREE MODES

BOUND MODES

$$BFI_{2D} = \frac{BFI}{\sqrt{1 + \alpha\,\Delta\theta^2 / (\Delta\omega / \omega_0)^2}}$$

α is a fitting constant

(Mori et al. JPO 2011)

MODULATIONAL INSTABILITY FROM NAVIER-STOKES SIMULATIONS

The numerical method has been developed by A. Iafrati (Iafrati 2010, JFM)

Initial condition:

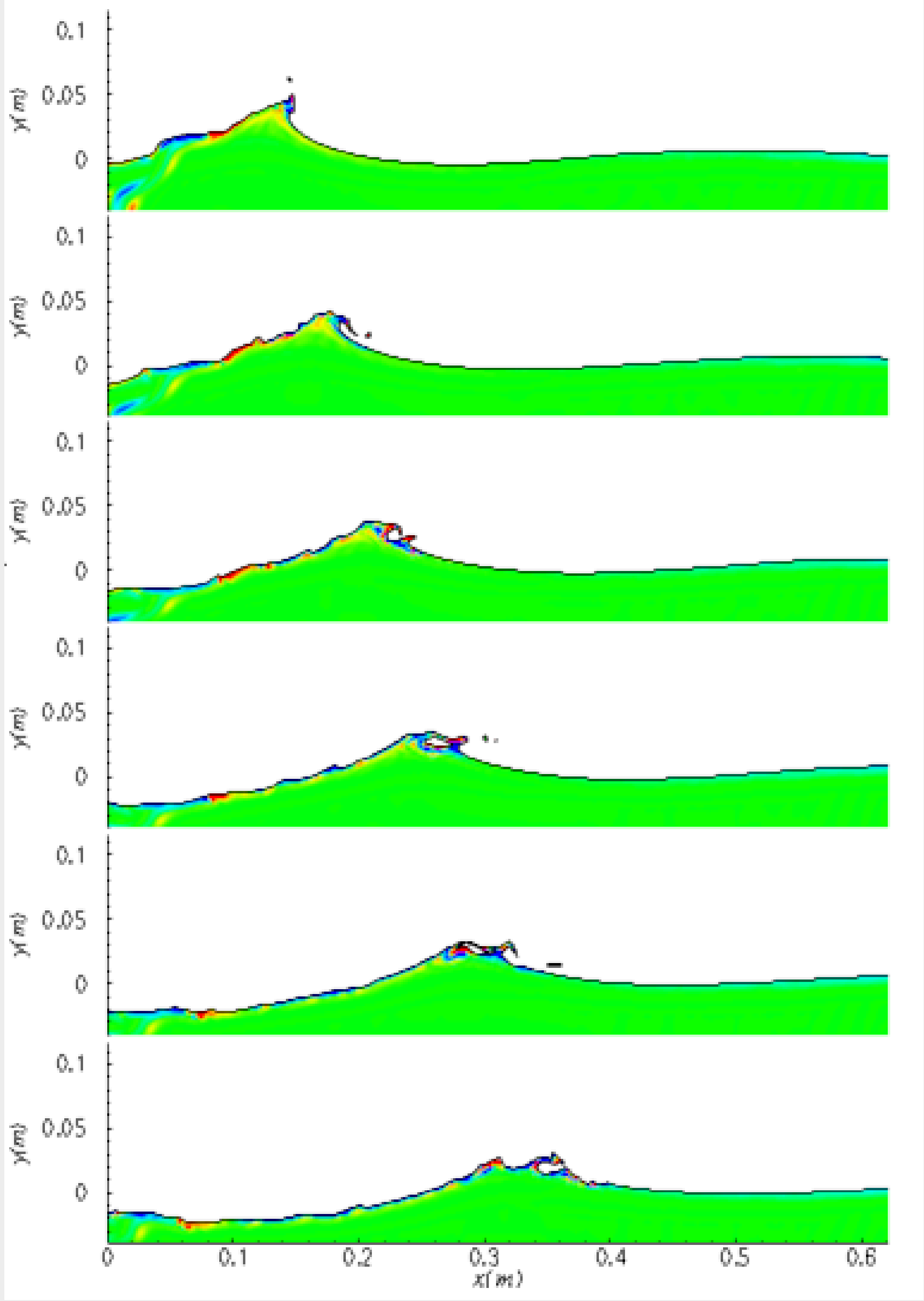
$$\eta(x,t=0) = A_0\cos(k_0x) + A_1\cos((k_0+K)x) + A_1\cos((k_0-Kx))$$

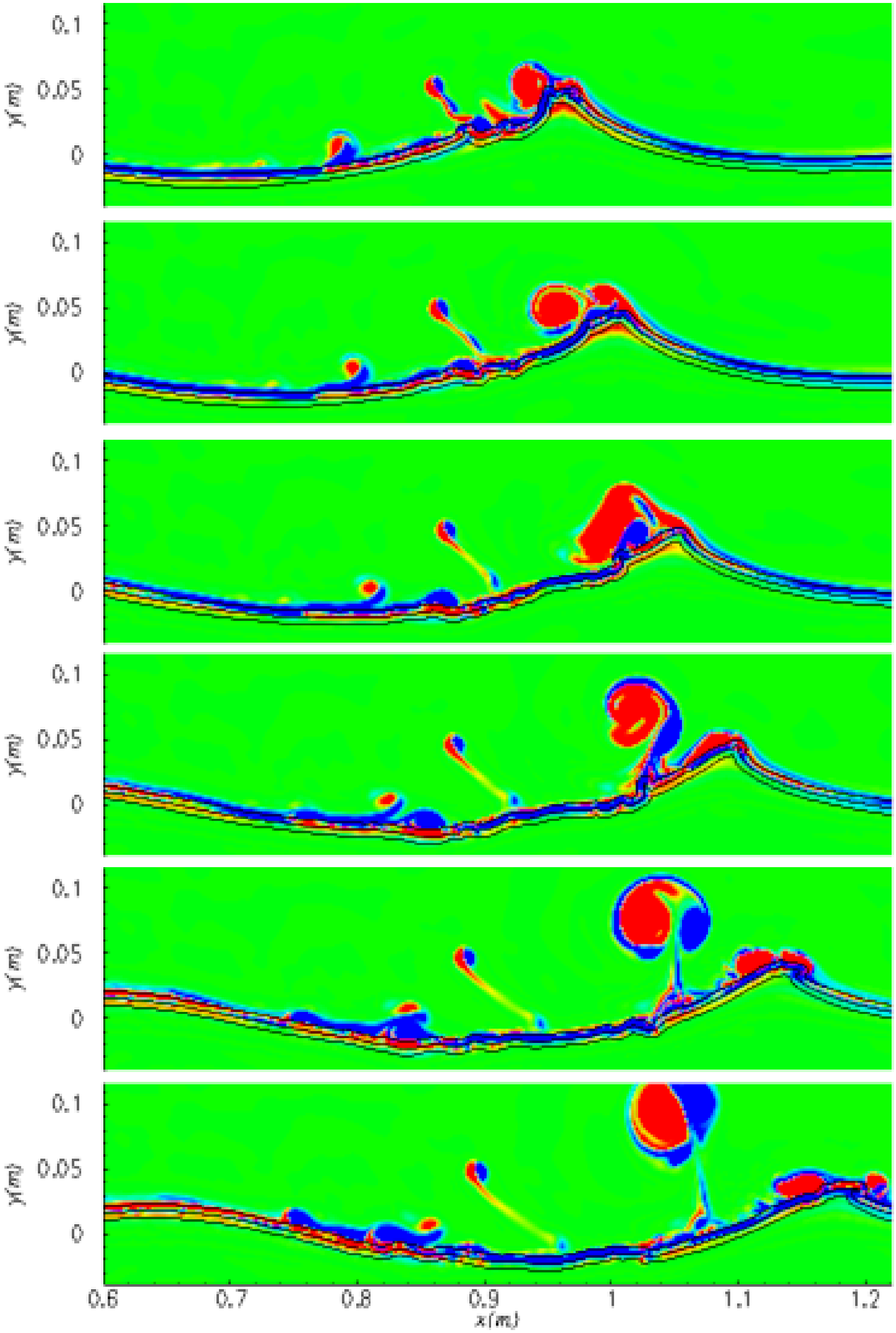
$\lambda_0 = 0.60$ m $K = k_0/5$

$\varepsilon_0 = k_0A_0$ is varied from 0.1 to 0.18

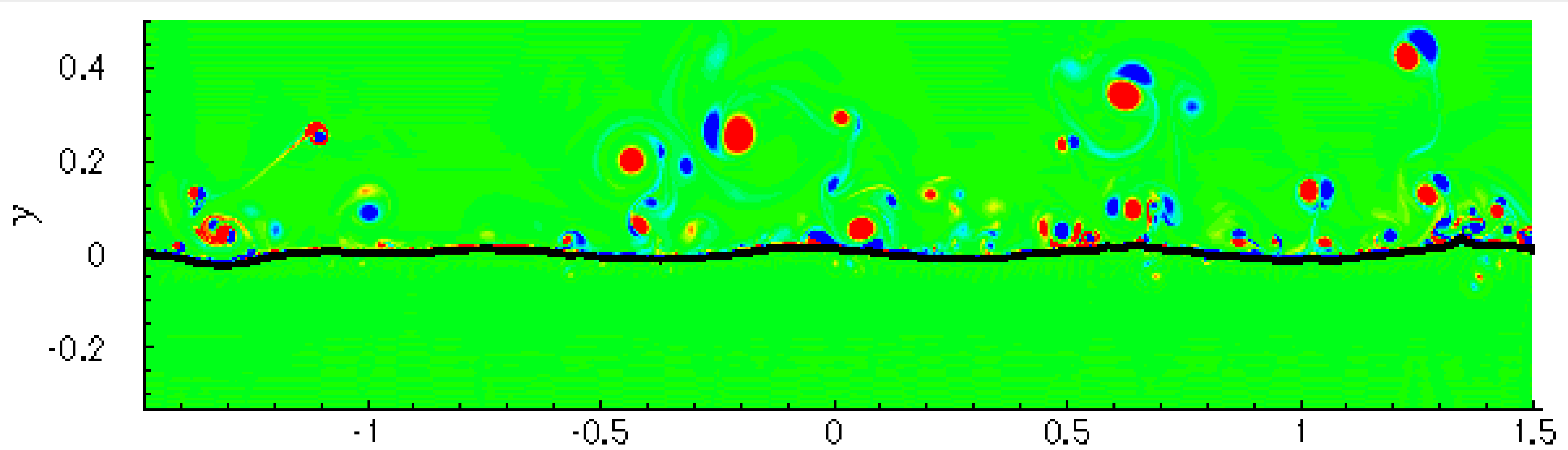
Computational domain:

horizontal dimension of 5 λ_0 and vertical of 4.33 λ_0


0.1
0.05
0
y(m)
0
0.1
0.2
0.3
0.4
0.5
0.6
x(m)



0.1
0.05
0
y(m)
0.6
0.7
0.8
0.9
1
1.1
1.2
x(m)

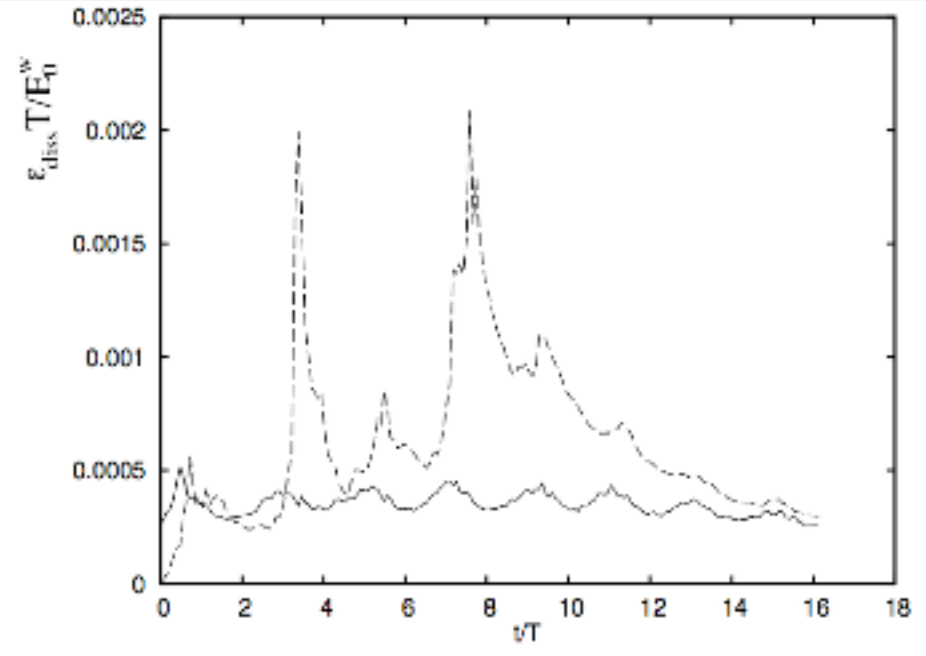


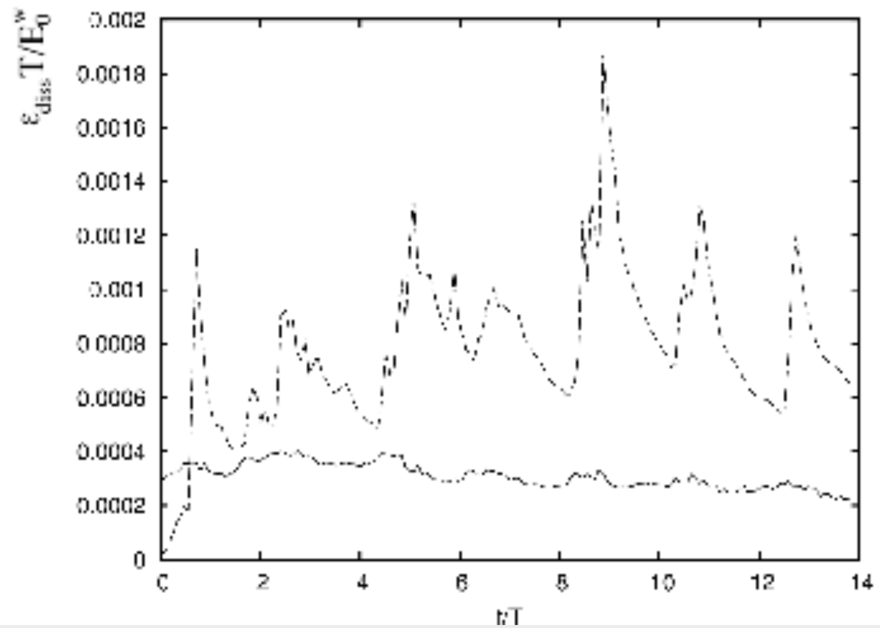
0.4
0.2
y
0
-0.2
-1
-0.5
0
0.5
1
1.5

DISSIPATION

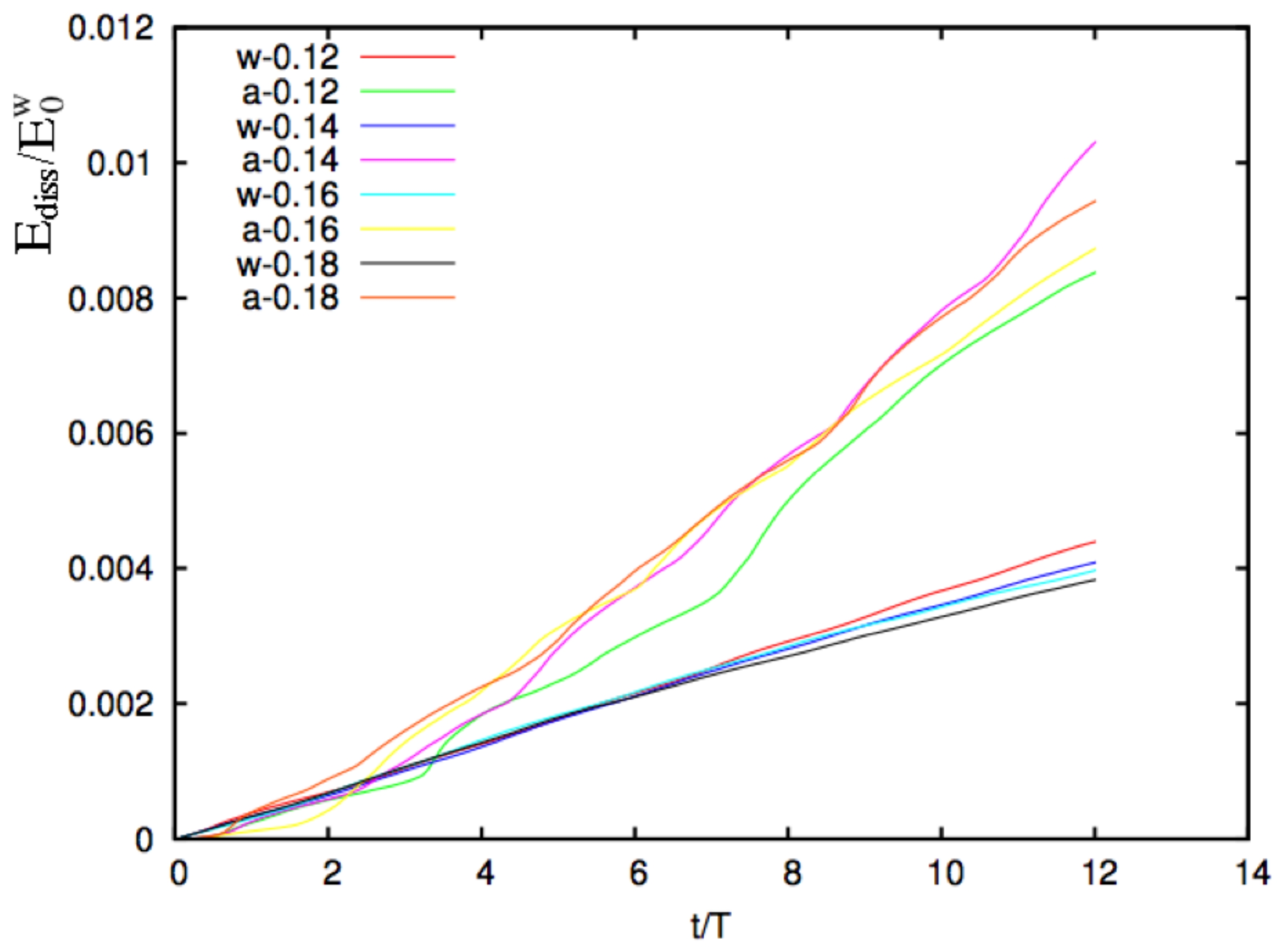
$$\varepsilon^{w}_{diss}(t) = \int_{d\geq 0} 2\mu e_{ij}\frac{\partial u_i}{\partial x_j}\,\mathrm{d}x\,\mathrm{d}y$$

$$\varepsilon^{a}_{diss}(t) = \int_{d<0} 2\mu e_{ij}\frac{\partial u_i}{\partial x_j}\,\mathrm{d}x\,\mathrm{d}y$$





$$E_{diss}(t) = \int_0^t \varepsilon_{diss}(t')dt'$$



CONCLUSIONS

1) Modulational instability does not imply always rogue waves

2) Wave breaking due to modulational instability may result in a dissipation of energy larger in the air than in the water

3) During the breaking process, dipoles are formed

4) Dipoles can reach the height of the wave length

Rogue waves in crossing seas: The Louis Majesty accident

THE ACCIDENT

On March 03, 2010 at 15:20 the Louis Majesty has been hit by a wave at deck n. 5 which is 16 m from the undisturbed sea-level

The wave broke the glass windshields in the forward section on deck five

Cavaleri et al. JGR 2012

THE FORECASTING MODEL

The wave fields are the result of the forecasting of Nettuno model from the Italian National Meteorological Service

Resolution of the meteorological model: 7 km

Resolution of the wave model in space (WAM): $1/20^0$

Spectral Resolution of the wave model:

number of frequencies: 30
number of directions: 36

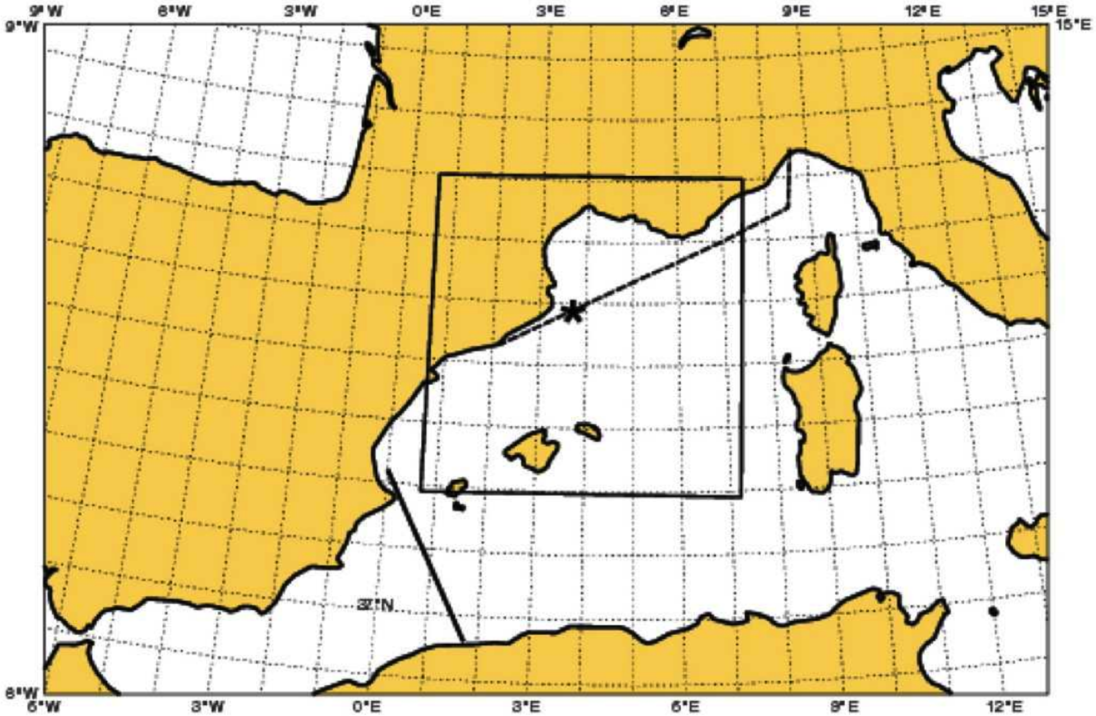


Figure 1. Western Mediterranean Sea. The enclosed area corresponds to the one considered in Figure 2. The line in the lower-left part shows the only satellite pass at a time close to the accident. Accident (star) and Begur buoy (dot) positions are also shown. The dash line is an indication of the expected route of the ship

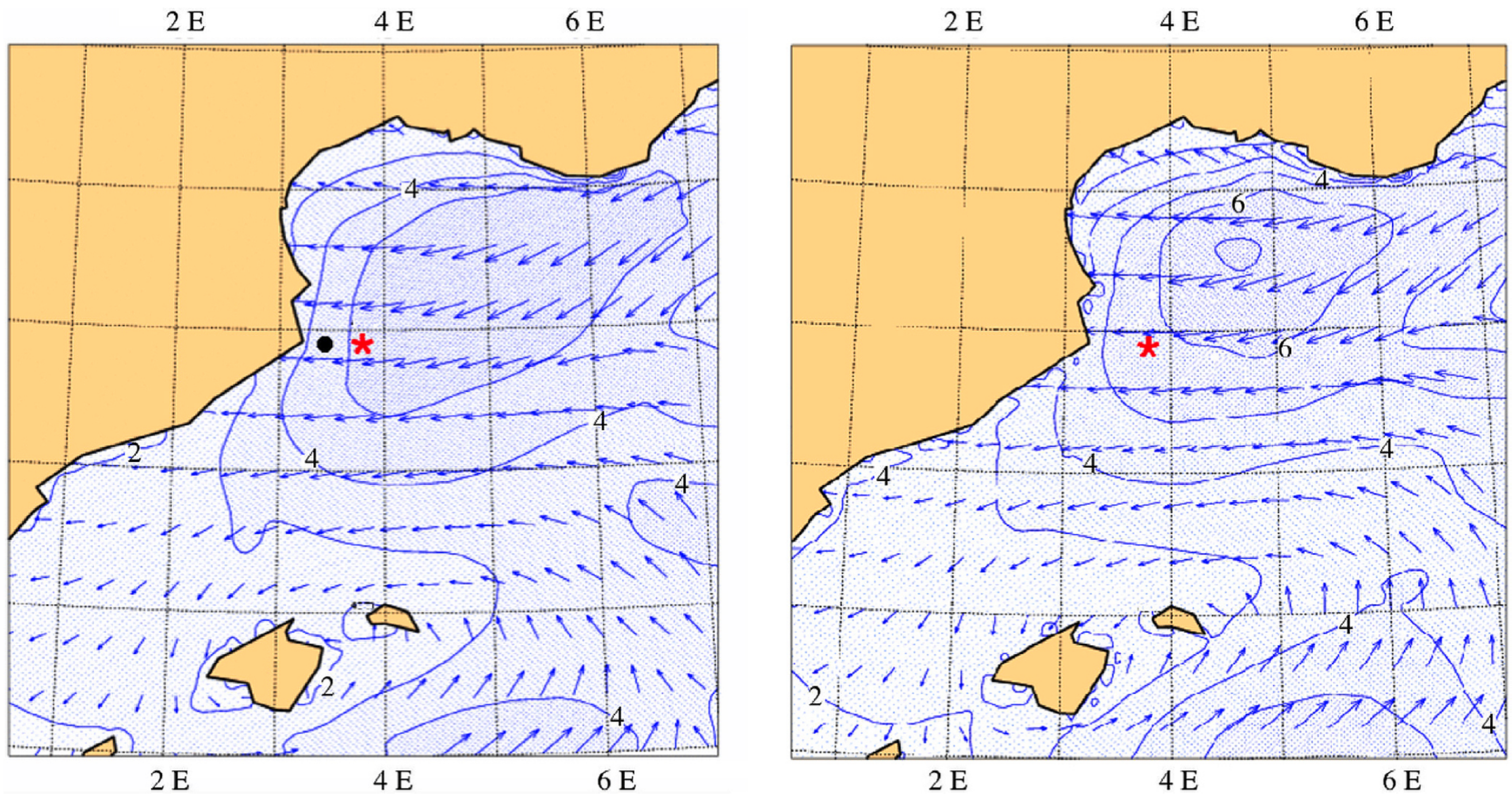


Figure 2. Significant wave height field at (left) 14:00 and (right) 15:00 UTC, 3 March 2010. Isolines at 1 m interval. The arrows indicate the mean wave direction, and their length is proportional to the significant wave height. The area, shown in Figure 1, spans 1°E–7°E, 39°N–44°N. The grid is shown at 1° intervals. The star points to the ship location at the time of the accident. The dot in Figure 2 (left) indicates the position of the Begur buoy.

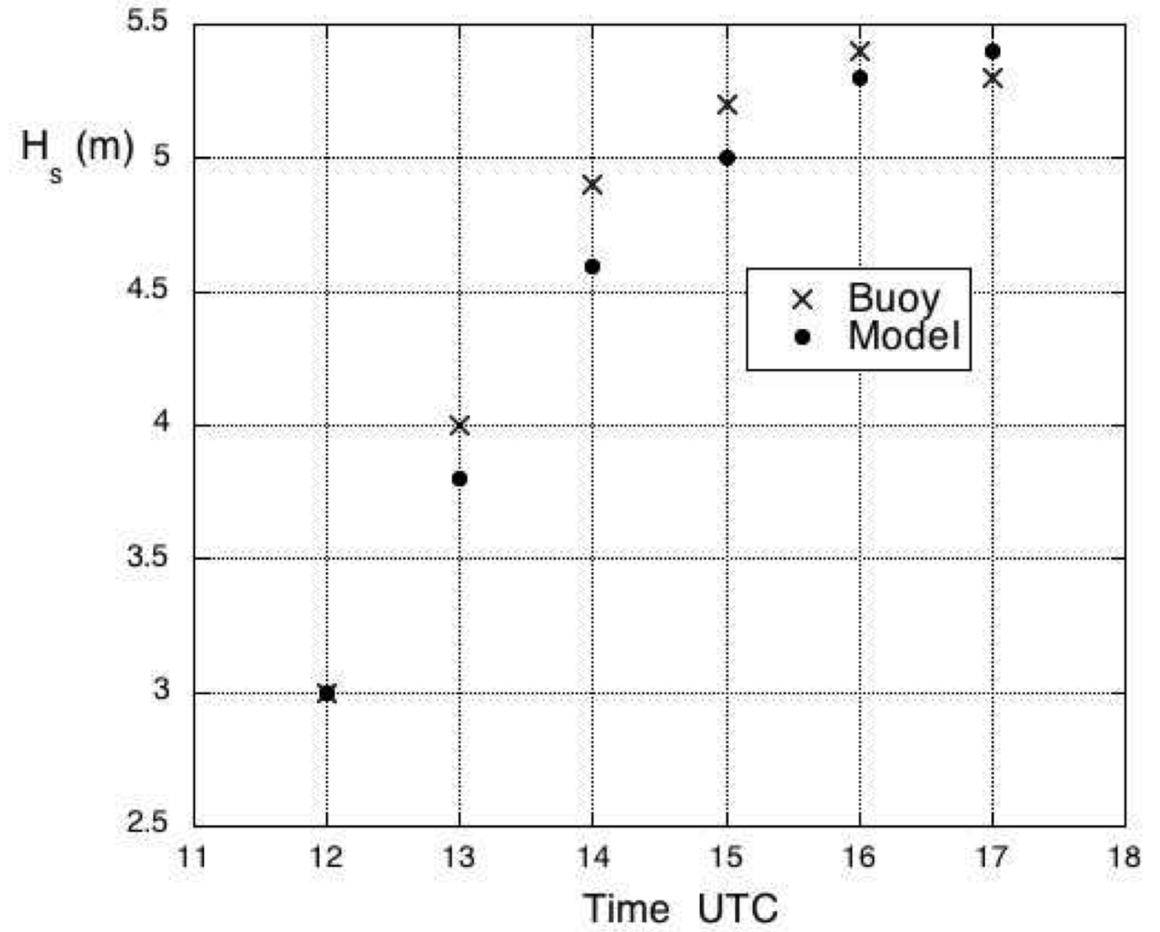
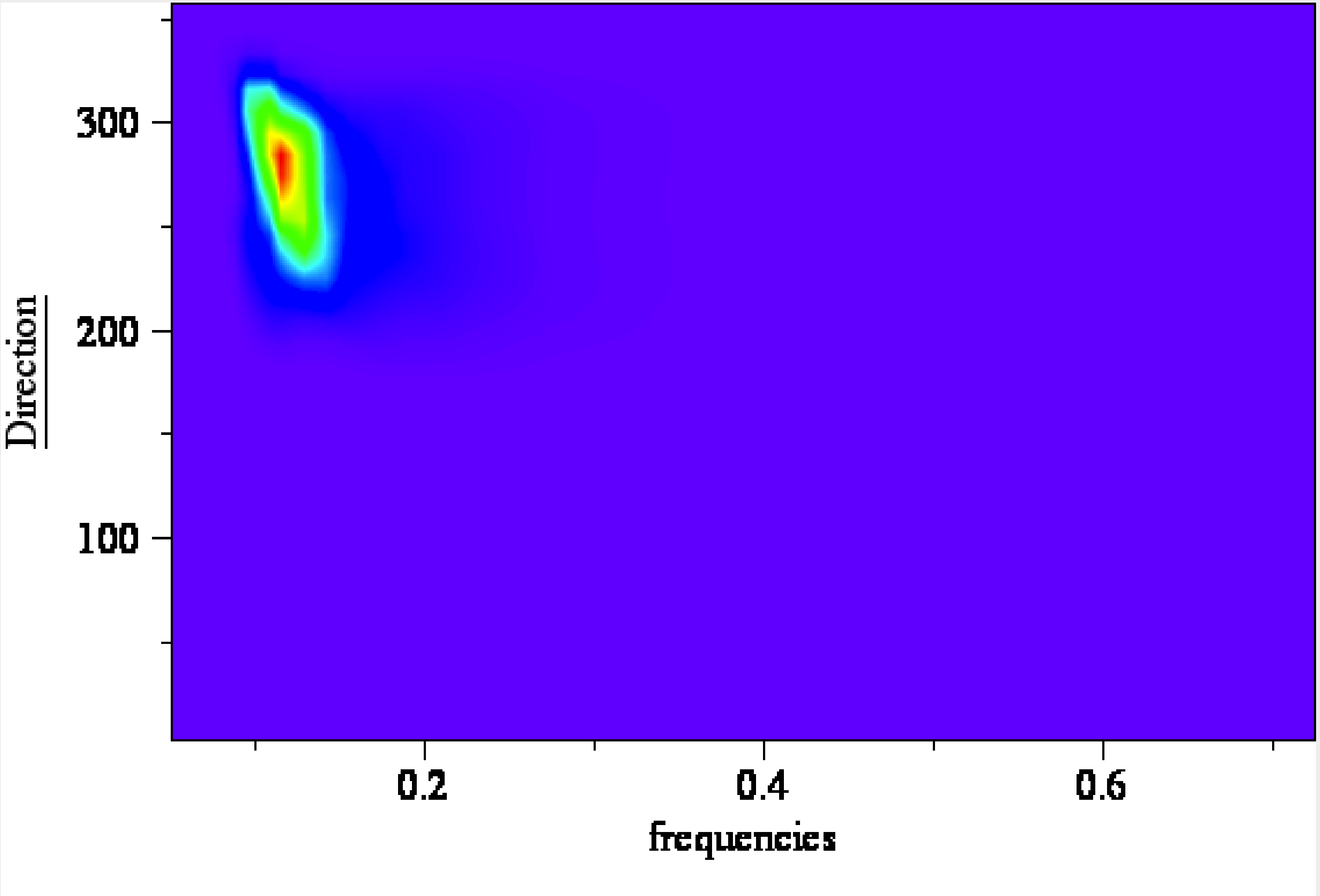


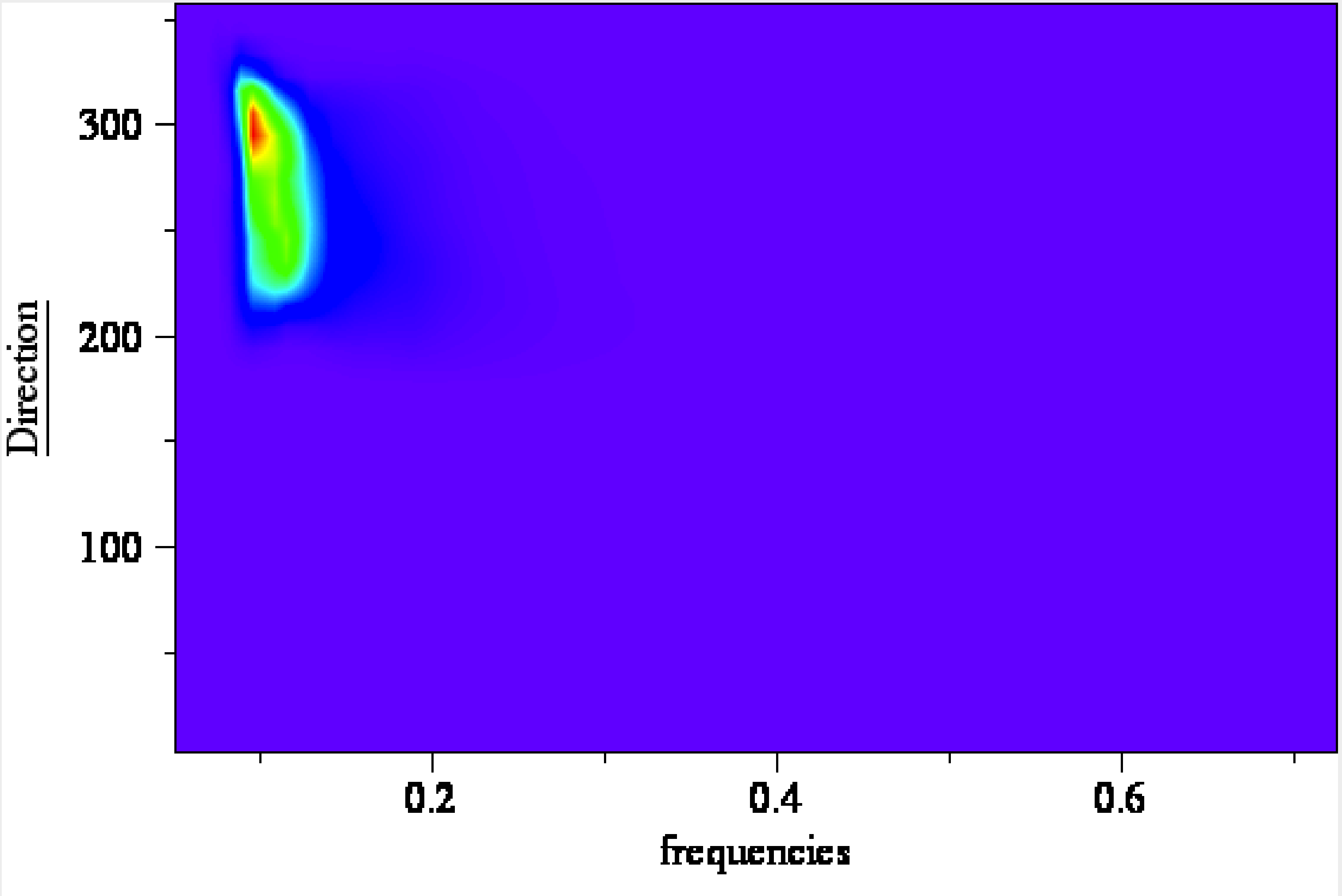
Figure 3. Significant wave heights measured on 3 March 2010 at the Begur buoy, crosses, (see Figure 2) and corresponding model values, circles.

DIRECTIONAL SPECTRUM

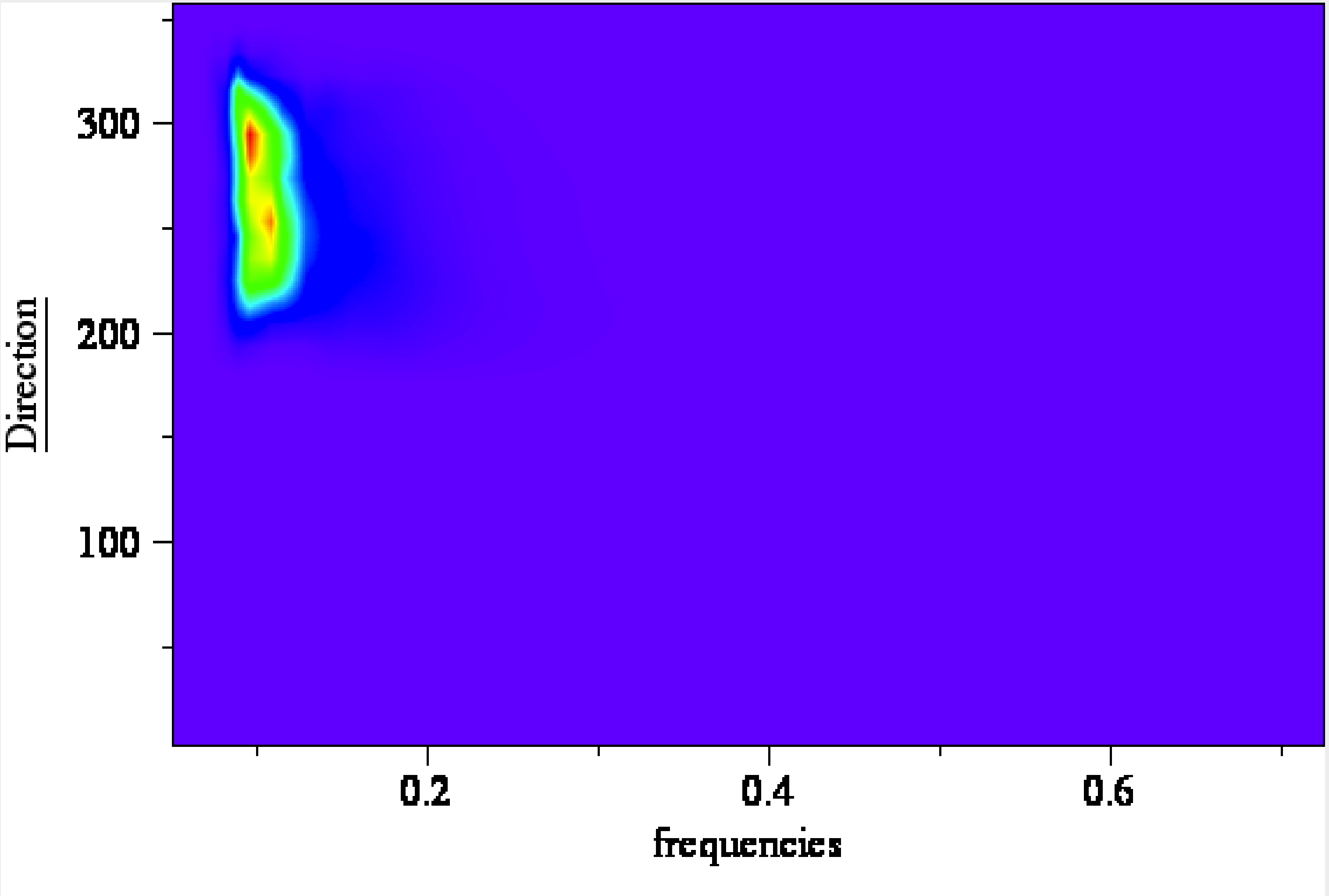
Time 12:00



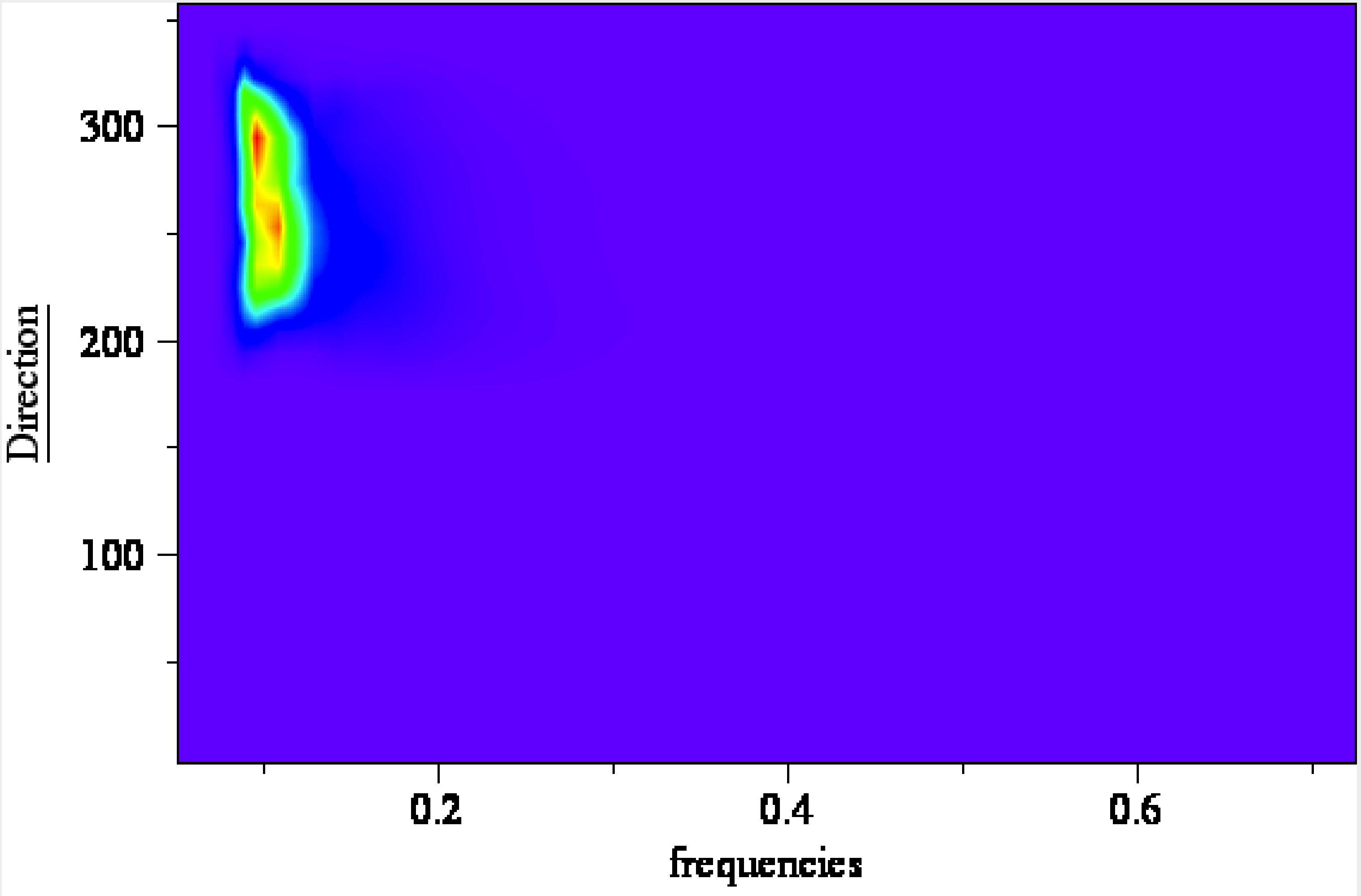
DIRECTIONAL SPECTRUM
Time 13:00



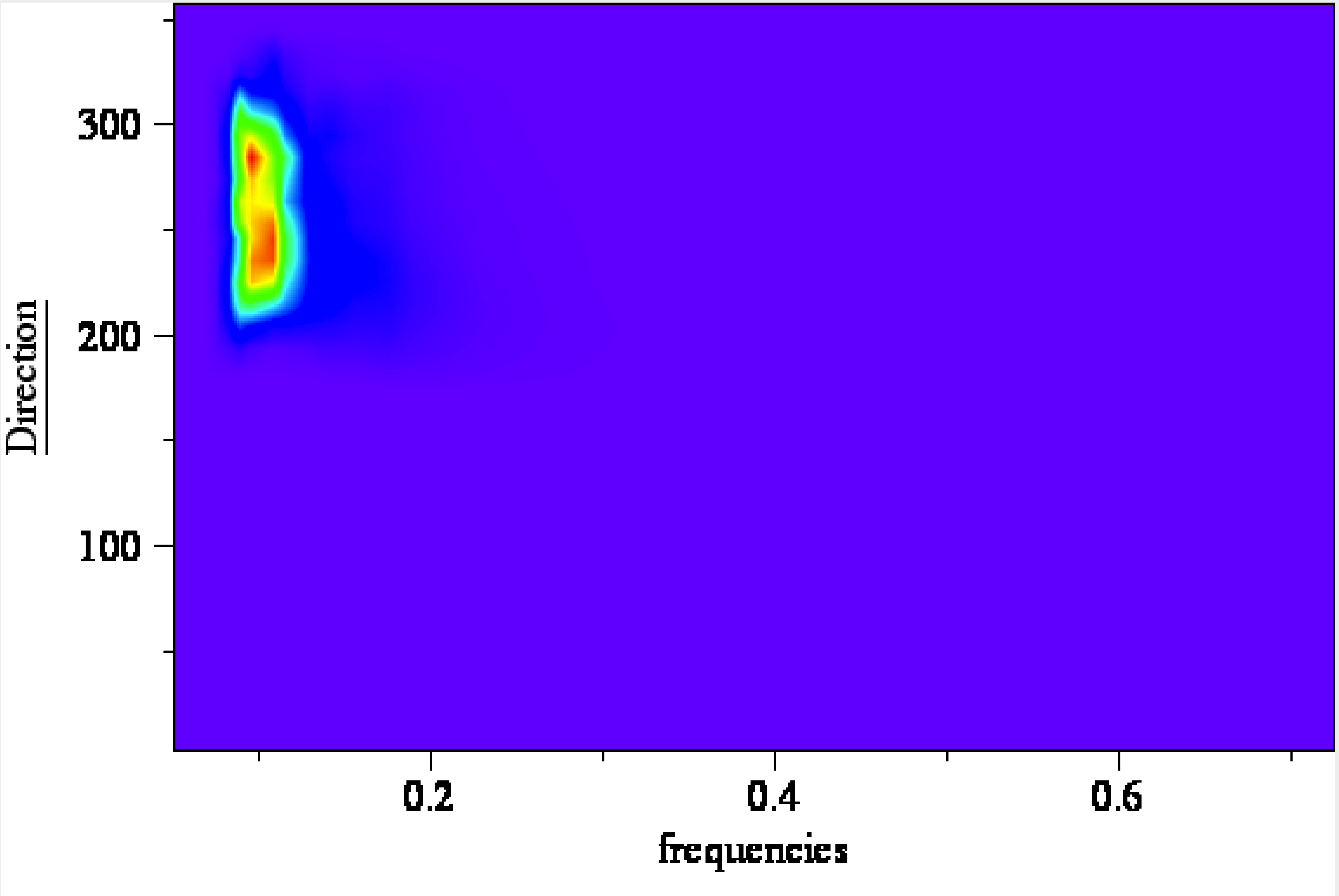
THE DIRECTIONAL SPECTRA
Time 14:00



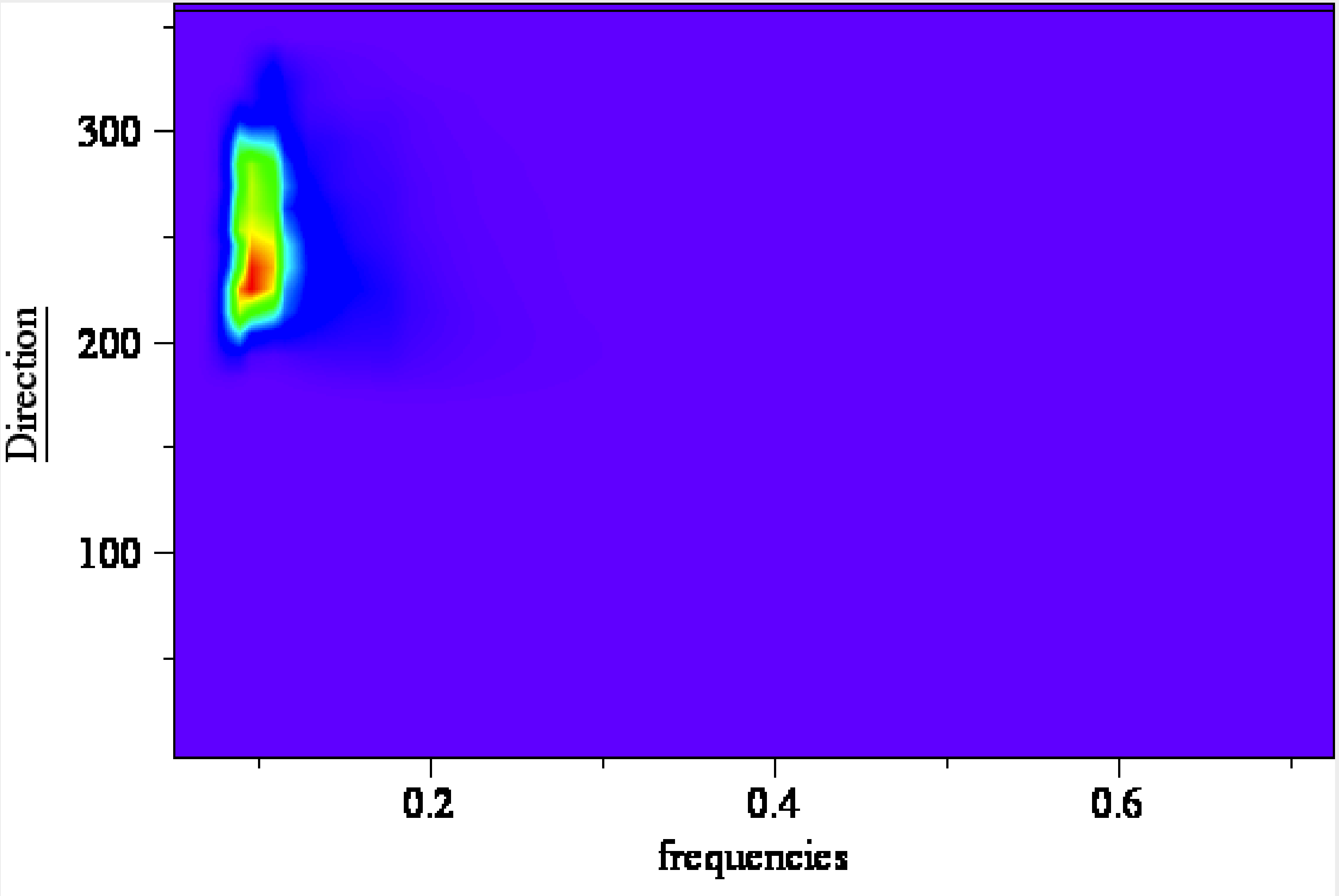
DIRECTIONAL SPECTRUM
Time 15:00



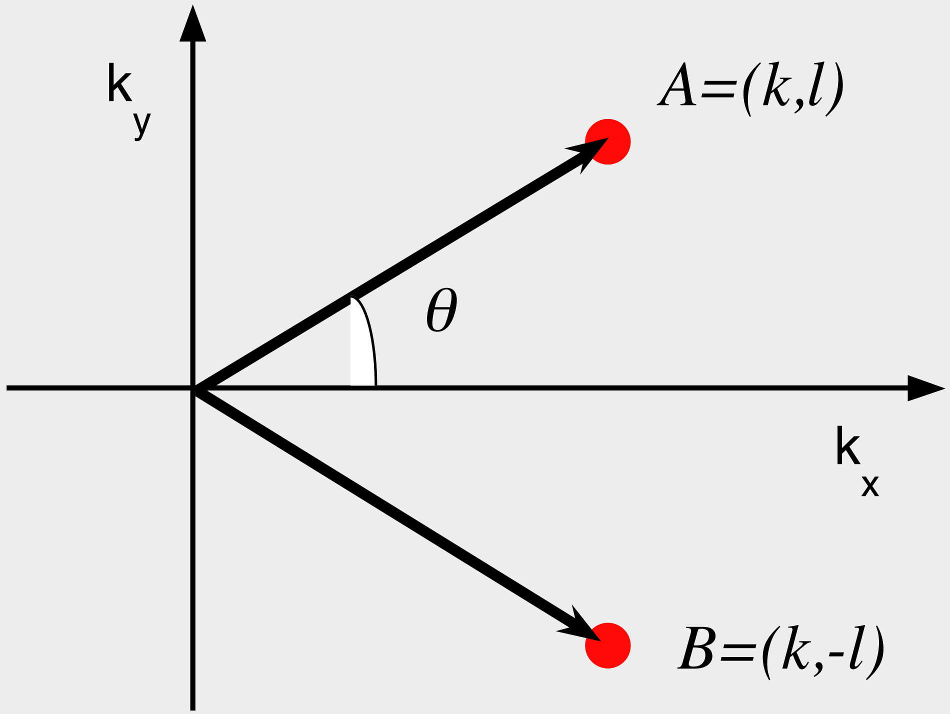
THE DIRECTIONAL SPECTRA
Time 16:00



THE DIRECTIONAL SPECTRUM
Time 17:00



CROSSING SEAS: THE SIMPLEST CASE



COUPLED NONLINEAR SHRODINGER EQUATION

Zakharov equation

$$i\frac{\partial b_o}{\partial t} = \omega_o b_o + \int T_{0,1,2,3} b_1^* b_2 b_3 \delta\left(\mathrm{k}_o + \mathrm{k}_1 - \mathrm{k}_2 - \mathrm{k}_3\right) d\mathrm{k}_1 d\mathrm{k}_2 d\mathrm{k}_3$$

- consider the following decomposition

$$b(\boldsymbol{k}) = A(\boldsymbol{k} - \boldsymbol{k}_A)e^{-i\omega(k_A)t} + B(\boldsymbol{k} - \boldsymbol{k}_B)e^{-i\omega(k_B)t}$$

with $\boldsymbol{k}_A = (k, l) \qquad \boldsymbol{k}_B = (k, -l)$

- suppose that both spectral distribution are narrow banded

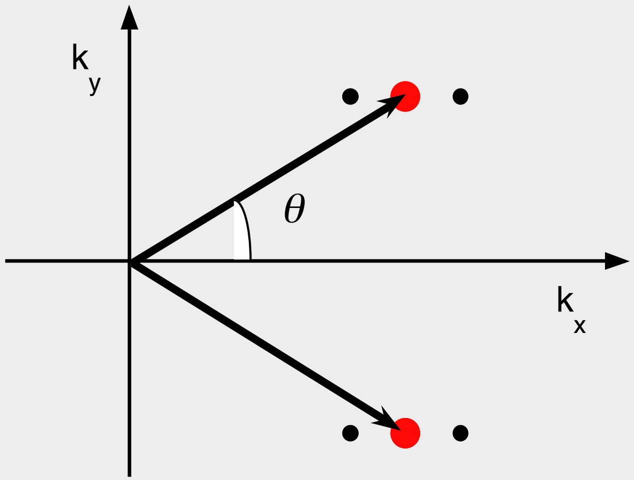
COUPLED NLS EQUATIONS

$$\frac{\partial A}{\partial t}+C_x\frac{\partial A}{\partial x}+C_y\frac{\partial A}{\partial y}-i\left[\alpha\frac{\partial^2 A}{\partial x^2}+\beta\frac{\partial^2 A}{\partial y^2}-\gamma\frac{\partial^2 A}{\partial x\partial y}\right]+i\left[\xi|A|^2+2\zeta|B|^2\right]A=0$$

$$\frac{\partial B}{\partial t}+C_x\frac{\partial B}{\partial x}-C_y\frac{\partial B}{\partial y}-i\left[\alpha\frac{\partial^2 B}{\partial x^2}+\beta\frac{\partial^2 B}{\partial y^2}+\gamma\frac{\partial^2 B}{\partial x\partial y}\right]+i\left[\xi|B|^2+2\zeta|A|^2\right]B=0$$

Coefficients are a function of *k* and *l*

- Consider perturbations only a function of k_x



$$\frac{\partial A}{\partial t} - i\alpha \frac{\partial^2 A}{\partial x^2} + i\left[\xi|A|^2 + 2\zeta|B|^2\right]A = 0$$

$$\frac{\partial B}{\partial t} - i\alpha \frac{\partial^2 B}{\partial x^2} + i\left[\xi|B|^2 + 2\zeta|A|^2\right]B = 0$$

AKHMEDIEV BREATHER SOLUTION

Look for a solution of the form:

$$A(x,t) = c_1\psi(x,t) \qquad B(x,t) = c_2\psi(x,t)Exp[i\delta]$$

For the Louis Majestic case $c_1 \approx c_2$

$\psi(x,t)$ **satisfies a standard NLS equation**

Parameters of the solution

$$\psi_0 = 1.7 \text{ m}$$

$$f_A = f_B = 0.1\ Hz$$

$$\varepsilon_A = \varepsilon_B = 0.07$$

$$N = 4$$

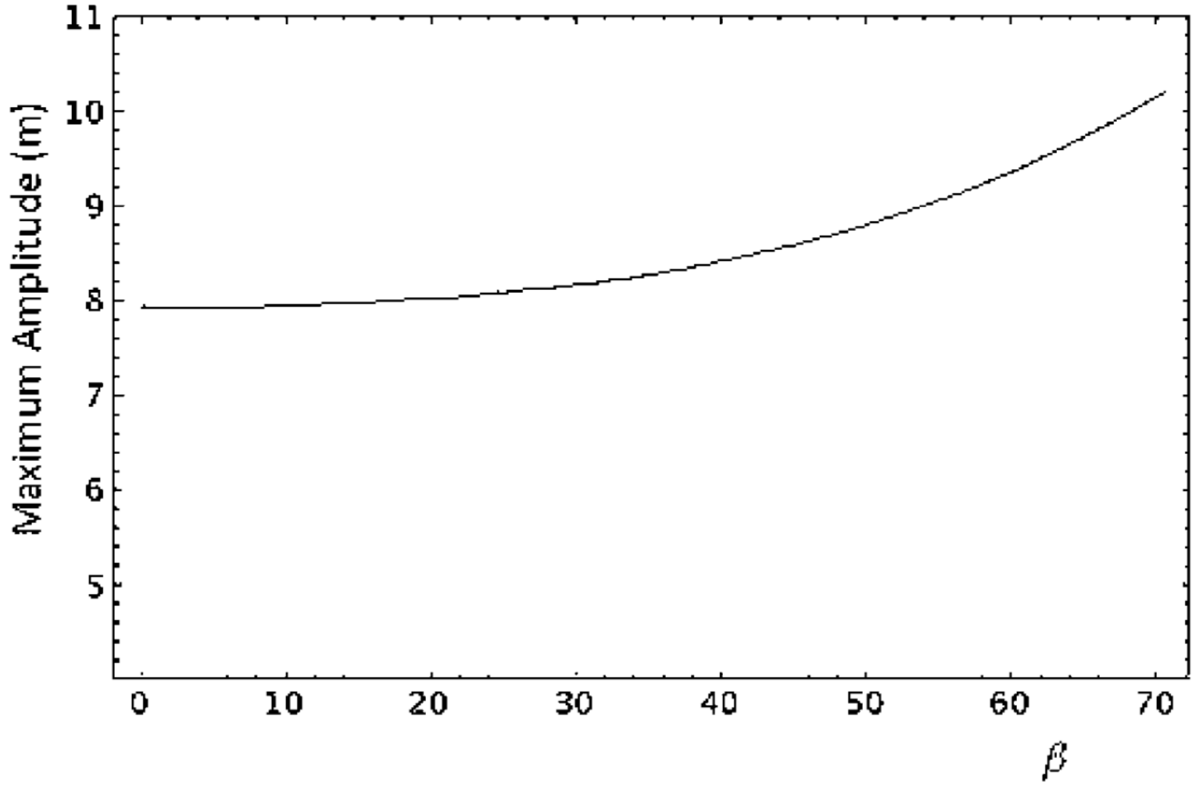


Figure 7. The maximum crest amplitude of the Akhmediev breather for the Louis Majesty sea state conditions as a function of the angle between the two wave systems.

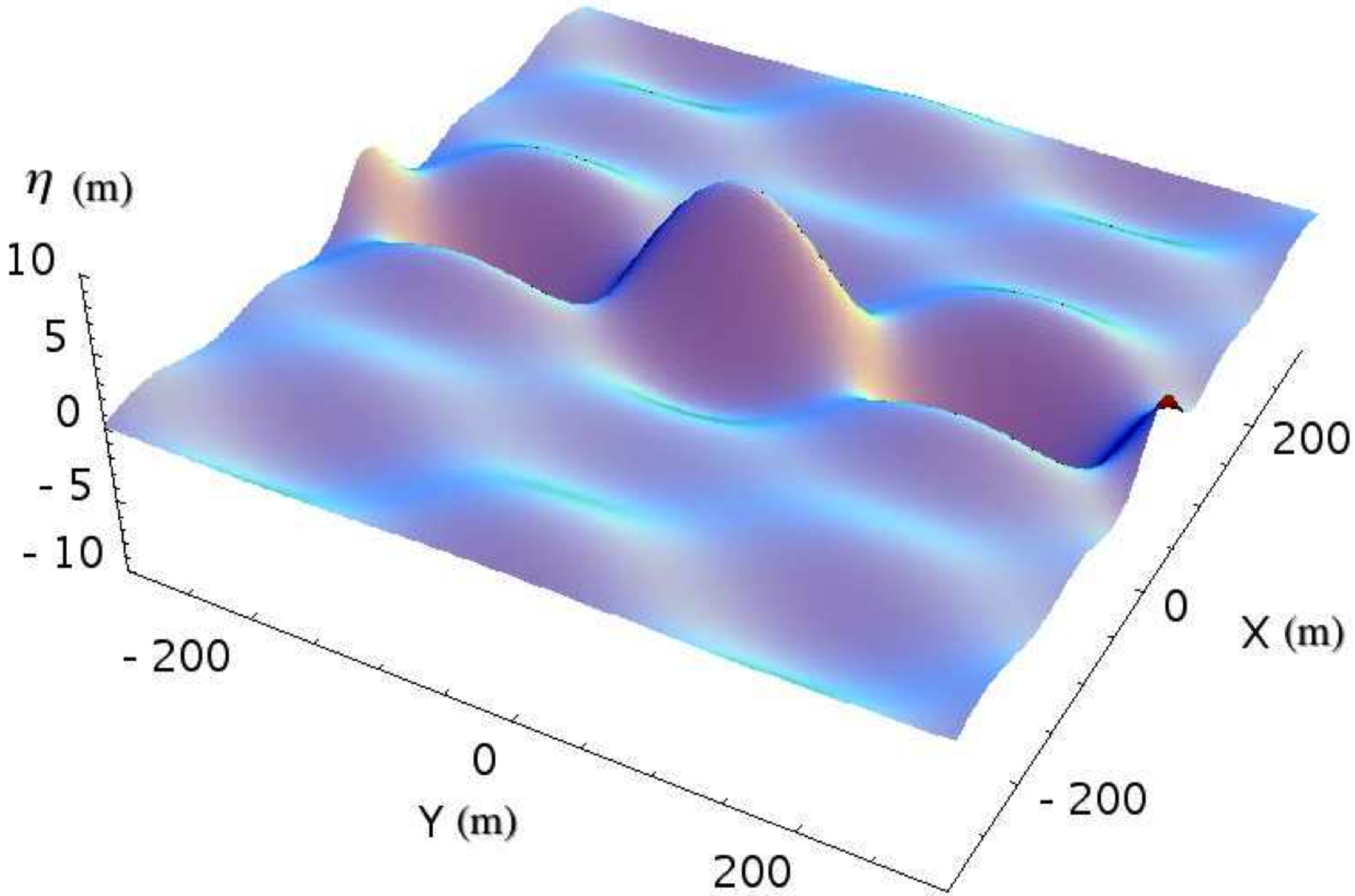


Figure 8. The surface elevation corresponding to the breather solution with $\beta = 50^o$.

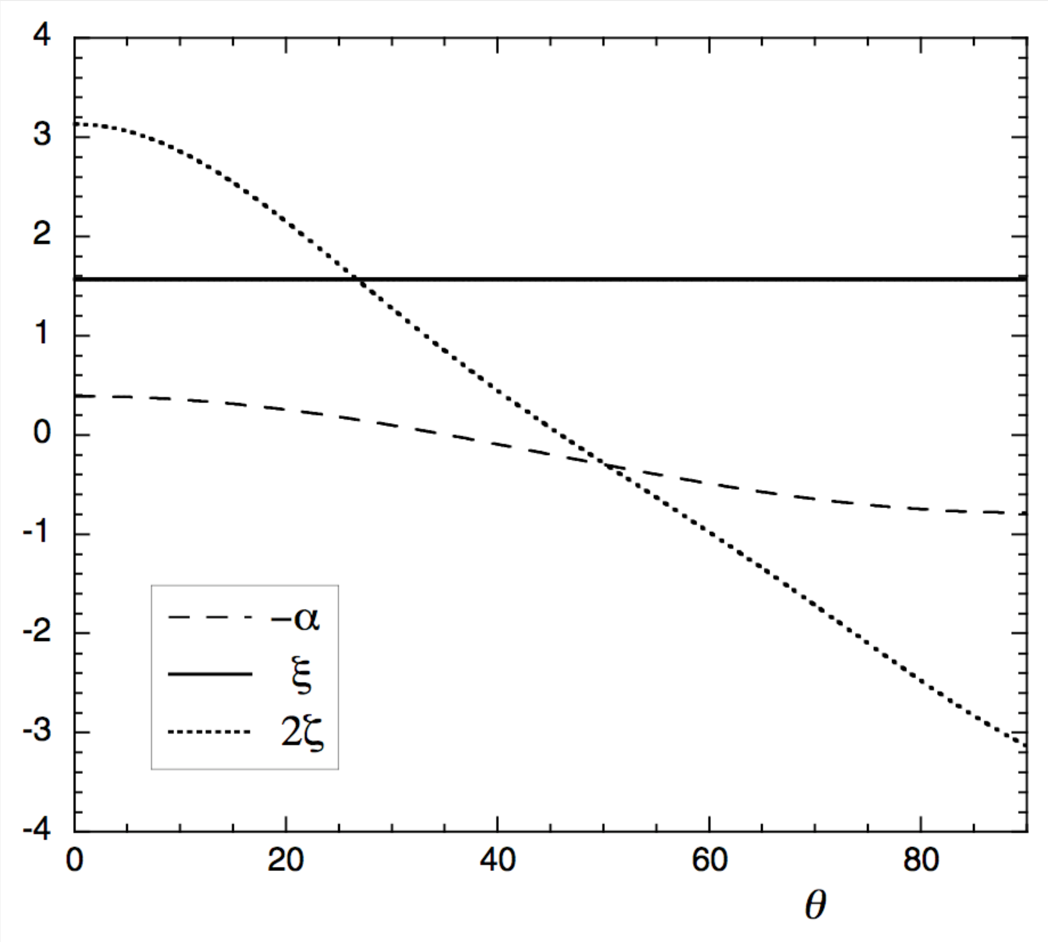
SUMMARY OF THE RESULTS

- For θ <35.3^0, dispersive and both nonlinear terms have the same sign

- The ratio between nonlinearity and dispersion becomes larger as θ approaches 35.3^0 (this is valid for both self-interaction and cross-interaction nonlinearity)

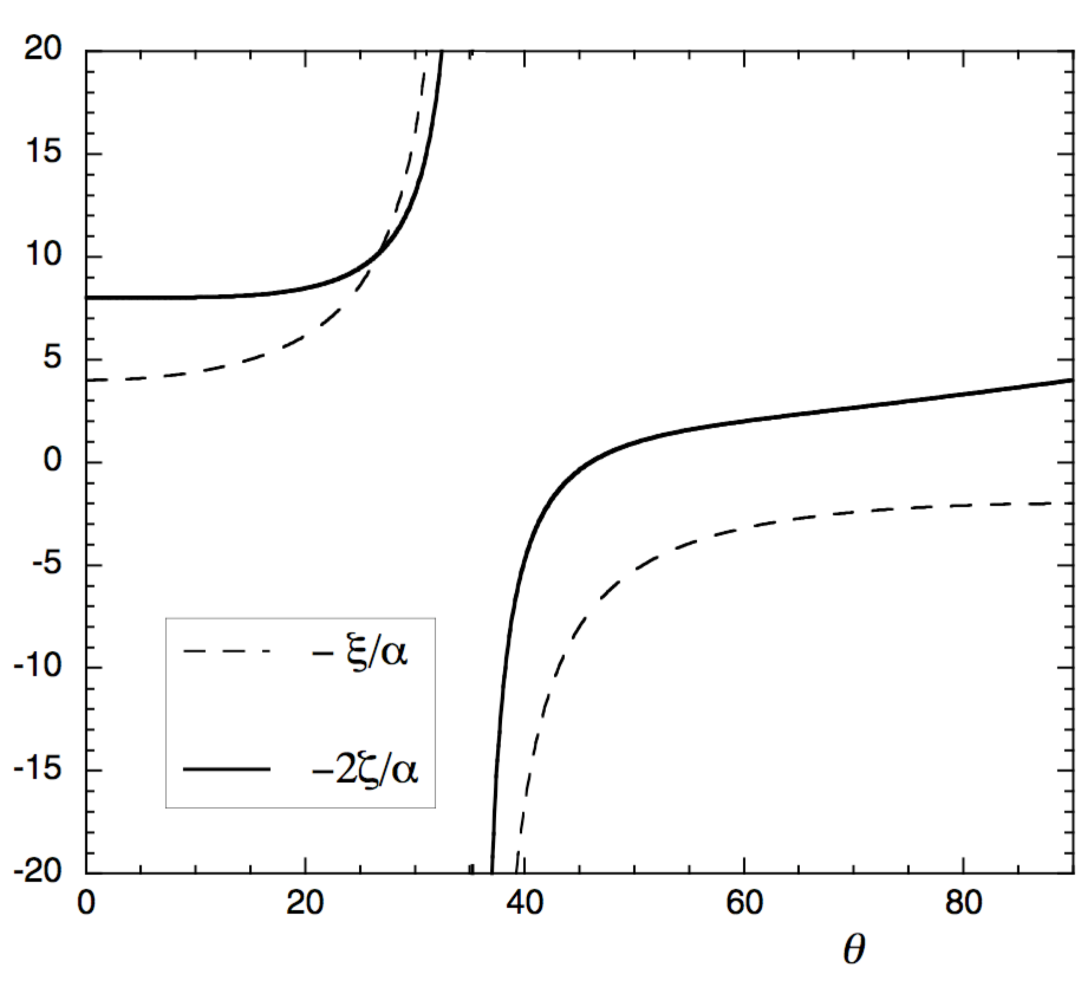
- The cross-interaction nonlinearity is stronger than the self Interaction one for angles between 0^0 and 26.7^0

ANALYSIS OF THE COEFFICIENTS



α= dispersive term
ξ= self-interaction term
ζ=cross-interaction term

ANALYSIS OF THE COEFFICIENTS



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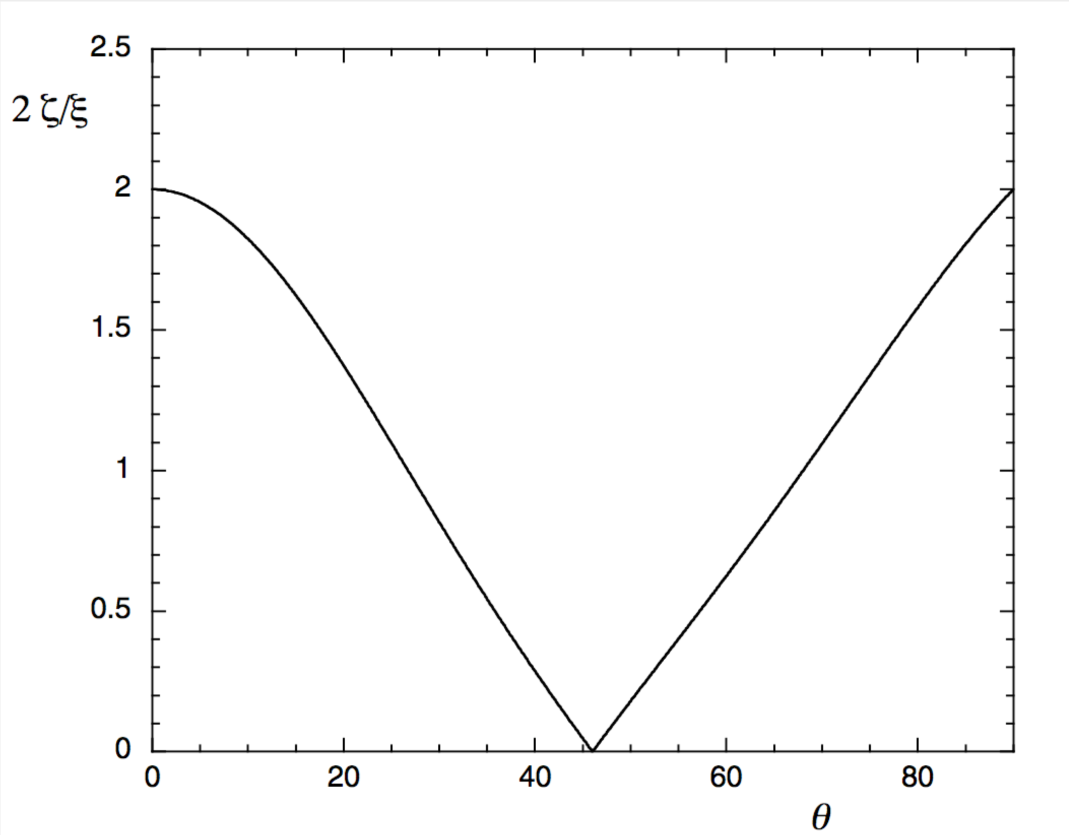
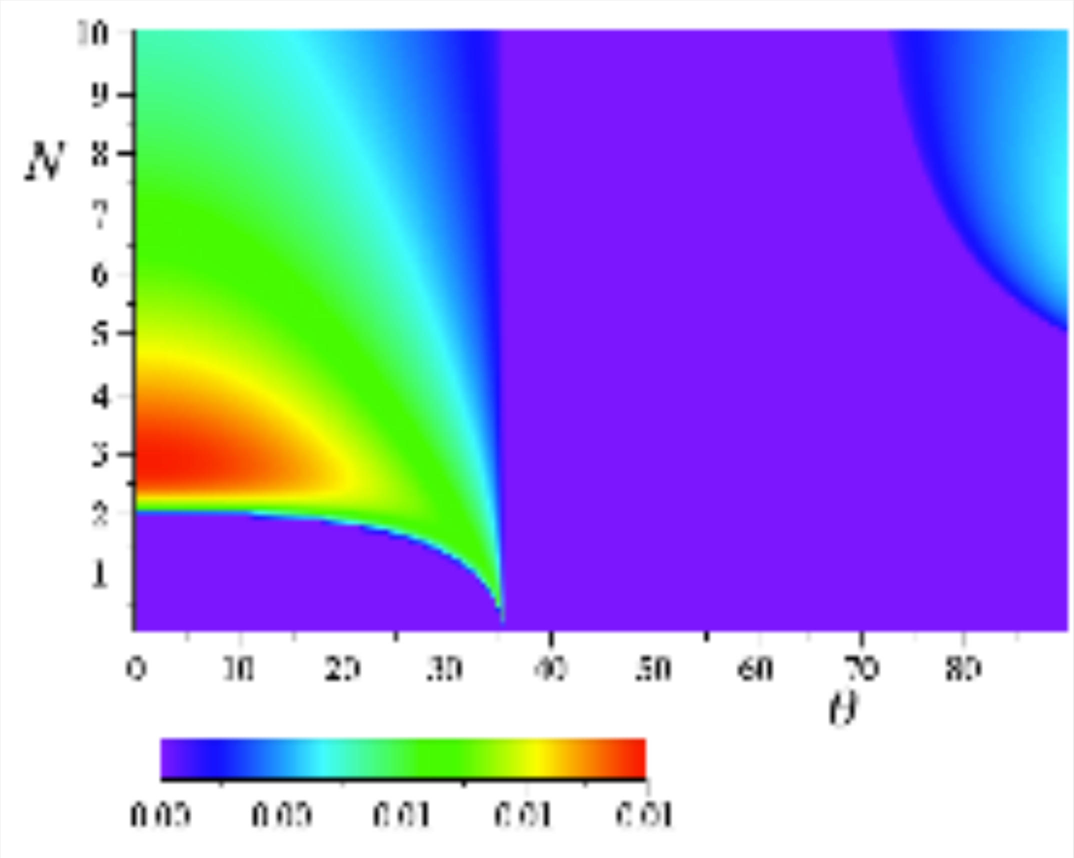
ANALYSIS OF THE COEFFICIENTS



α= dispersive term
ξ= self-interaction term
ζ=cross-interaction term

DISPERSION RELATION FOR PERTURBATION

$$\Omega = \sqrt{\alpha K^2\left[2(\xi + 2\zeta)A_0^2 + \alpha K^2\right]}$$



AMPLIFICATION FACTOR FOR BREATHER SOLUTIONS

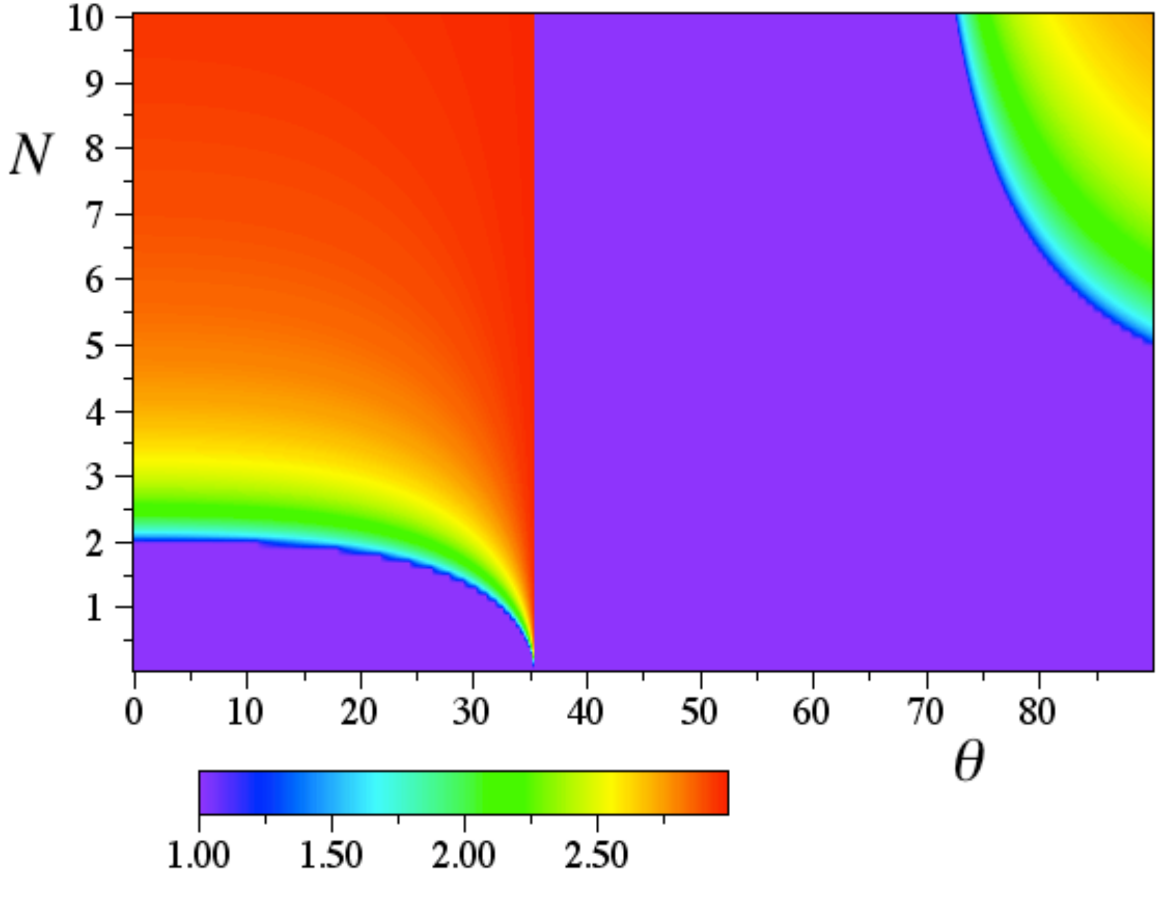
$$\frac{A_{\max}}{A_0} = 1 + 2\left[1 - \left(\sqrt{\frac{\alpha}{\xi + 2\zeta}}\,\frac{\kappa^2}{\varepsilon N}\right)^2\right]^{1/2}$$

N = number of waves under the envelope

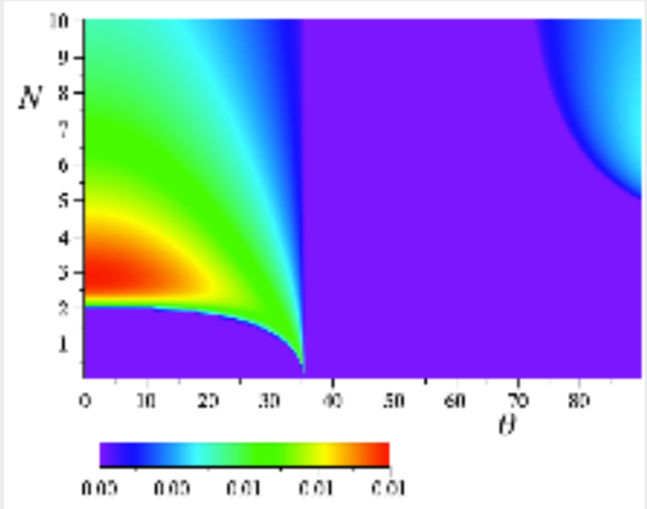
ε = initial steepness

κ = modulus of the wave number

AMPLIFICATION FACTOR



GROWTH RATE



SUMMARY OF THE RESULTS:

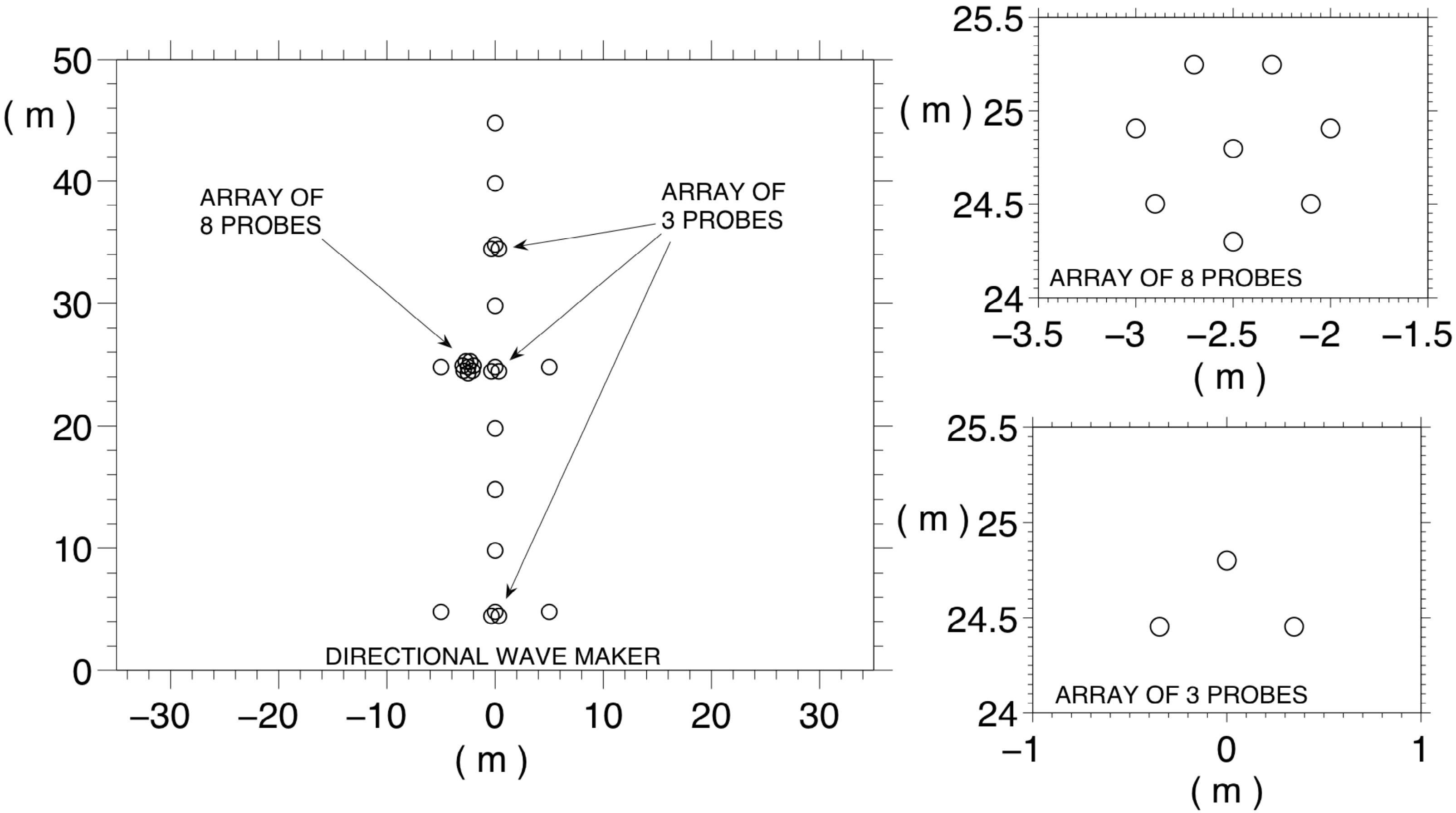
- The maximum amplification is for θ –>35.3^0 and large N
- The maximum growth rate is for $\theta=0^0$ and N≈3

CONSIDERATIONS:

Extreme waves are the result of a maximum amplification factor in a reasonable time scale

WE EXPECT LARGE EXTREME WAVE ACTIVITY AT ANGLES OF $\theta\approx20^0$- 30^0

EXPERIMENTS: MARINTEK FACILITY



DESCRIPTION OF THE EXPERIMENT

SUM OF TWO JONSWAP SPECTRA:

$$E(\omega,\theta) = E_1(\omega,\theta) + E_2(\omega,\theta)$$

with

$$E_1(\omega,\theta) = \frac{\alpha g^2}{\omega^5} Exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right]\gamma^{Exp\left[-(\omega-\omega_p)^2/(2\sigma^2\omega_p^2)\right]}\delta(\theta-\theta_0)$$

$$E_2(\omega,\theta) = \frac{\alpha g^2}{\omega^5} Exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right]\gamma^{Exp\left[-(\omega-\omega_p)^2/(2\sigma^2\omega_p^2)\right]}\delta(\theta+\theta_0)$$

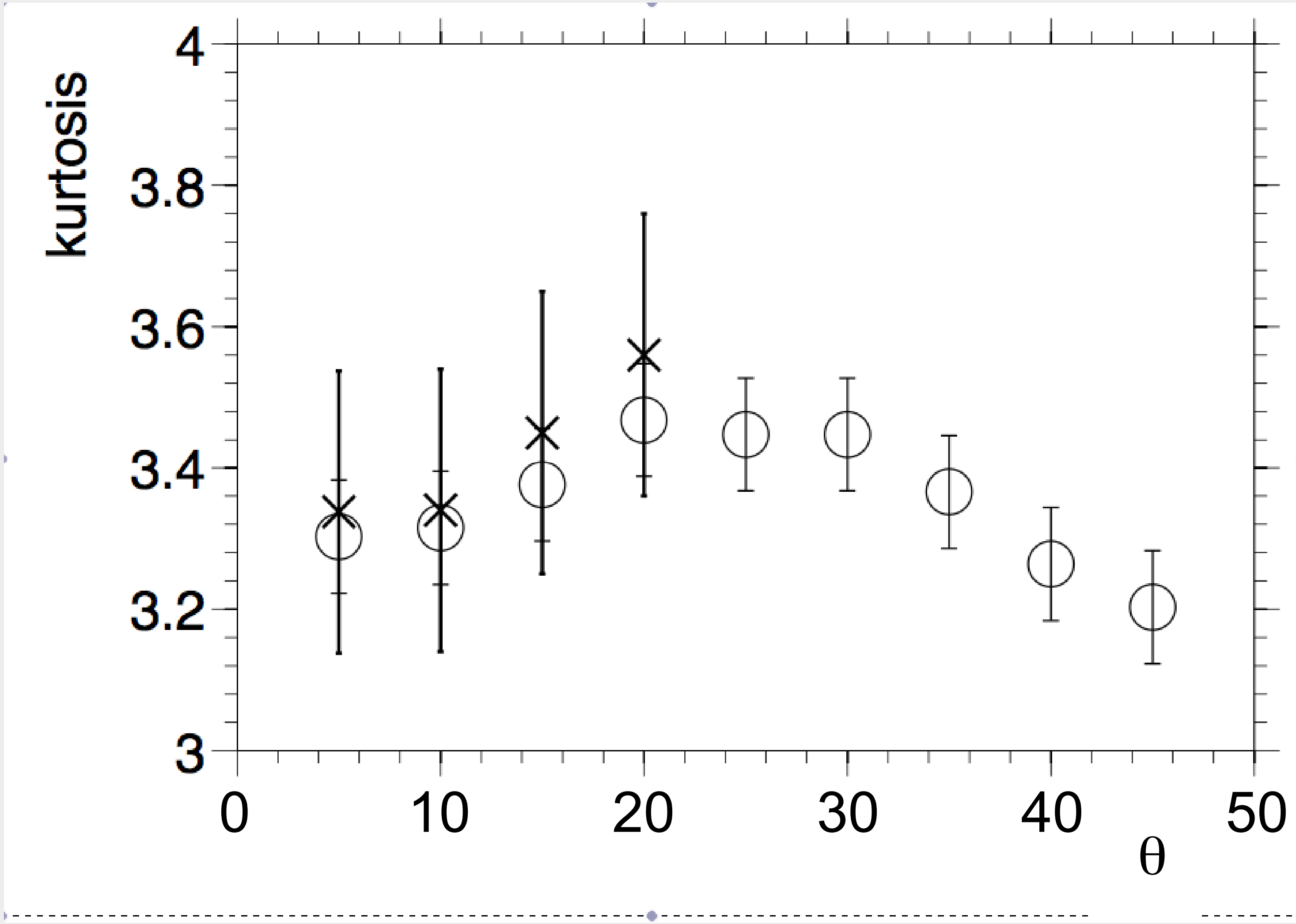
NUMERICAL SIMULATIONS

- HIGHER ORDER SPECTRAL METHOD (THIRD ORDER IN NONLINEARITY)

- BOX PERIODIC IN x AND y COORDINATES

- INITIAL CONDITIONS PROVIDED BY TWO JONSWAP SPECTRA TRAVELLING AT AN ANGLE

RESULTS ON MAXIMUM KURTOSIS

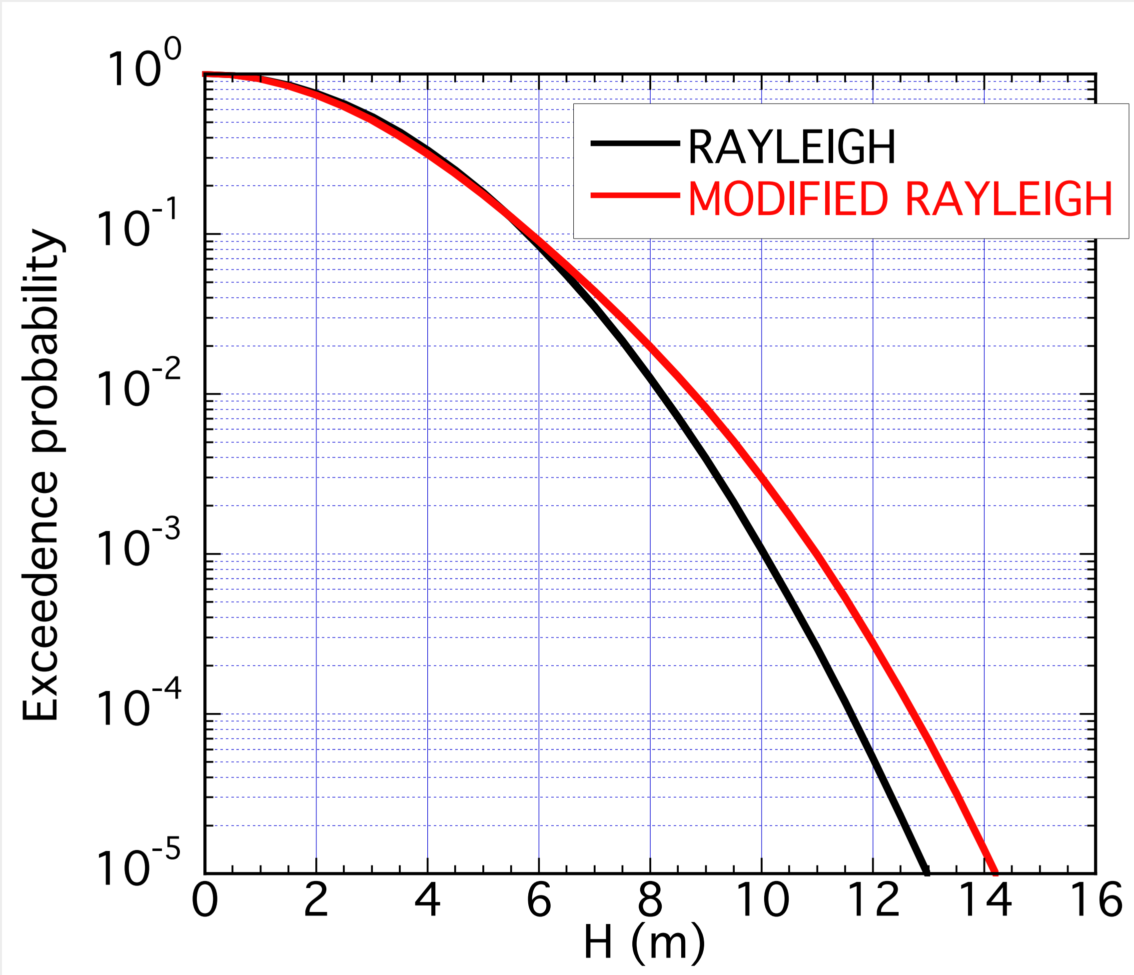


PROBABILITY OF EXCEEDENCE AT THE TIME OF THE ACCIDENT

Linear case:

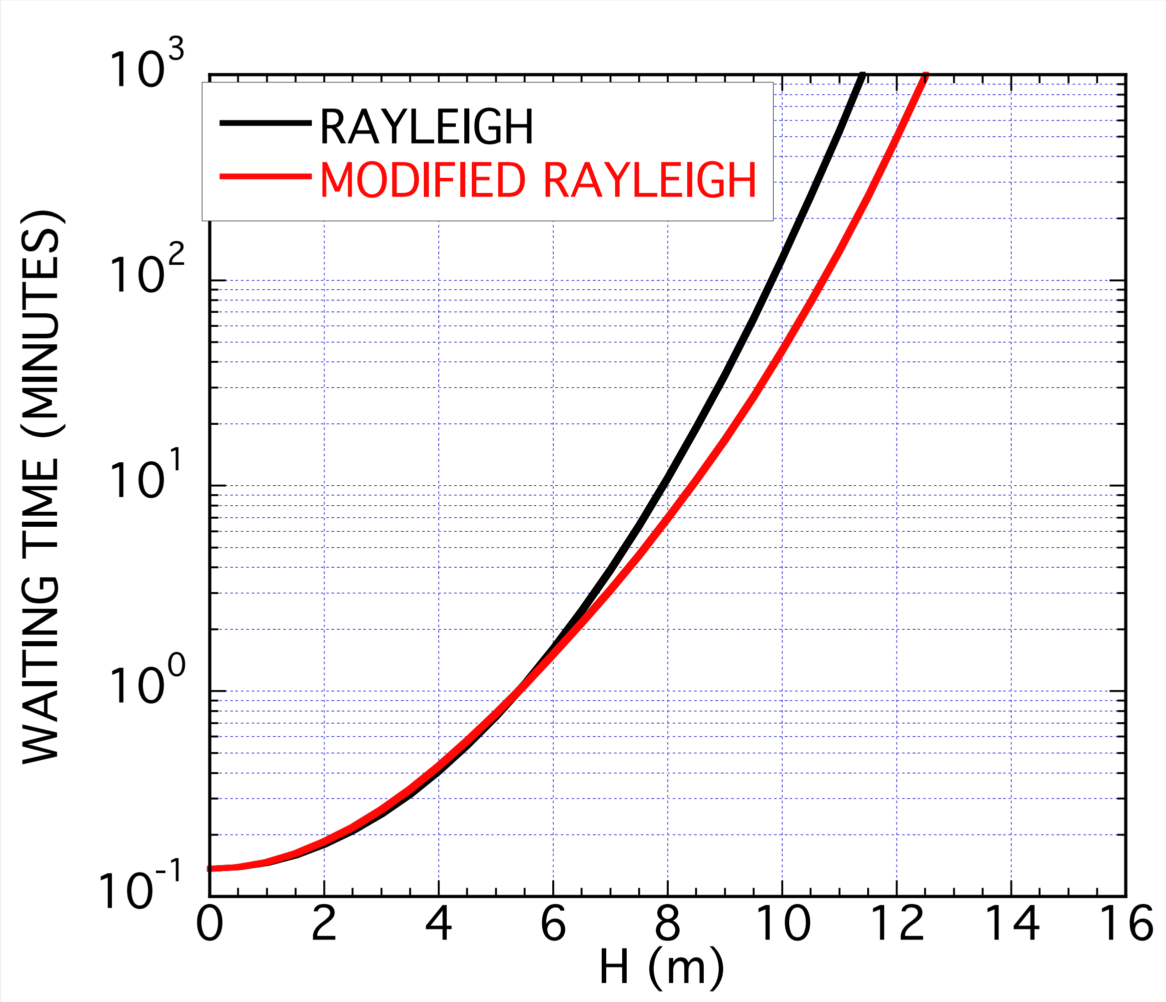
$$P_H(H) = e^{-\frac{1}{8}H^2}$$

PROBABILITY OF EXCEEDENCE AT THE TIME AND PLACE OF THE ACCIDENT



Hs=5.11 m

WAITING TIME AT THE TIME AND PLACE OF THE ACCIDENT



Hs=5.11 m

in standard conditions $\sigma = 0.073$ N m^{-1}. In the two-fluid modelling, the densities of air and water are the standard ones, $\rho_w = 1000$ kg m^{-3} and $\rho_a = 1.25$ kg m^{-3}. The values of the dynamic viscosities in water and air are $\mu_w = 10^{-3}$ kg m^{-1} s^{-1} and $\mu_a = 1.8\,10^{-5}$ kg m^{-1} s^{-1}, respectively.

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Limitations of the computation

The present work represents the first attempt to approach the problem of modulational instability starting from the Navier-Stokes equation. Results seems to be very encouraging. Clearly, it has a number of limitation due to the heaviness of the computation. Probably the most important one is that we have assumed that waves are long crested and the fluid domain is 2-dimensional. In reality we expect that 3D effect can take place and dipoles and vortices can become unstable. It will be just a matter of time to develop the 3D version of the NS code and verify our finding in a more realistic context. A second important limitation is the lack of wind: our simulations correspond to the propagation of a steep swell and what would be the consequences of a turbulent wind on the generation of vorticity during breaking event is unknown.