Evaluating methods for representing model error using ensemble data assimilation

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Evaluating model error schemes

• **Using an EPS**
  – Spread/error consistency, probabilistic scores.
  – Hard to know whether improvement comes simply from reducing spread deficiency.

• **Using an EnKF**
  – Tougher test if multiplicative inflation used as baseline, since scheme must do more than increase variance.
  – Evolution of all errors in DA cycle (not just model error) must be represented. Model error may not be dominant.
Un(der)-represented error sources in an EnKF ensemble

- Model error
- Sampling error
- Observation error
- Boundary condition error
- Forward operator error

\[
\begin{align*}
M x_a \\
\frac{1}{N} \sum_{j=1}^{N} (N \ll \infty) \\
R \\
T(z = 0) \Rightarrow T_s \\
H x_b
\end{align*}
\]
Experiences with Env. Canada system
(Houtekamer, Mitchell and Deng, MWR July 2009)

• Operational EnKF tested with
  – Multiple parameterizations
  – SKEB (stochastic kinetic energy backscatter)
  – SPPT (stochastically perturbed physics tend)
  – Additive inflation (isotropic covariance structure)
  – Multi-physics plus additive inflation
Experiences with Env. Canada system  
(Houtekamer, Mitchell and Deng, MWR July 2009)

<table>
<thead>
<tr>
<th>configuration</th>
<th>O-F (energy norm)</th>
<th>Energy spread in ob space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additive inflation</td>
<td>3.1388</td>
<td>2.0622</td>
</tr>
<tr>
<td>Multi-physics</td>
<td>3.2978</td>
<td>1.2773</td>
</tr>
<tr>
<td>SKEB</td>
<td>3.4348</td>
<td>1.2671</td>
</tr>
<tr>
<td>SPPT</td>
<td>3.3899</td>
<td>1.1670</td>
</tr>
<tr>
<td>Multi-physics + add. Infln.</td>
<td>3.0846</td>
<td>2.1335</td>
</tr>
<tr>
<td>SKEB + SPPT</td>
<td>3.3352</td>
<td>1.3608</td>
</tr>
<tr>
<td>SKEB+SPPT+Mult-physics +rescaled additive infln.</td>
<td>3.0940</td>
<td>2.1092</td>
</tr>
</tbody>
</table>

• Biggest impact from ad-hoc additive inflation.
• Addition of multi-physics improves assimilation slightly.
• SPPT and SKEB have less impact (tuned for EPS?, model error not dominant?)
Motivation for simple model expts

• Sources of assimilation error can be controlled (obs. error know perfectly, sampling error can be controlled).

• Access to truth aids in diagnostics, tuning of model error schemes.

• Can run lots of experiments.
2-level PE model on a sphere

- 2-level PE model on a sphere (Lee and Held, 1993 with parameters as in Hamill and Whitaker, 2010).
- 511 12-hourly obs of geopotential height at sonde locations (error = 10 m)
  - 20 member ensemble, serial deterministic (i.e. square-root) EnKF.
  - 1000 assimilation cycles, 3500 km localization (none in vertical)
- Truth from T42 nature run, assimilation with T31 model. Only sources of DA error are model error and sampling error.
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Methods for representing model error

*Multiplicative Inflation*

• Simple constant inflation not suitable when observing network and dynamics vary in space and/or time.

• Both sampling error and model error are expected to be a larger fraction of the total background error where observations have a larger impact (where $\sigma_b/\sigma_a$ is large).

• “relaxation” inflation is stronger where ensemble variance is reduced by the assimilation of observations.
Methods for representing model error

**Multiplicative Inflation**

- Relaxation to prior perturbations (RTPP, Zhang et al, 2005)
  \[ x'_{i}^{a} \leftarrow (1 - \alpha)x'_{i}^{a} + \alpha x'_{i}^{b} \]

- Relaxation to prior spread (RTPS)
  \[ \sigma^{a} \leftarrow (1 - \alpha)\sigma^{a} + \alpha\sigma^{b} \]

  which implies
  \[ x'_{i}^{a} \leftarrow x'_{i}^{a} \sqrt{\alpha \frac{\sigma^{b} - \sigma^{a}}{\sigma^{a}}} + 1 \]

- Both inflate more where observations have a strong tendency to reduce ensemble variance.
Methods for representing model error

**Multiplicative Inflation**

- Relaxation to prior spread works best, is less sensitive to choice of relaxation parameter.
- Jeff Anderson’s Bayesian adaptive inflation method performs similarly.
Methods for representing model error

Additive Inflation

• Add random samples from a specified distribution to each ensemble member after the analysis step.

• Env. Canada uses random samples of isotropic 3DVar covariance matrix.

• Here we use a dataset of 12-h forecast errors with the T31 model in which the initial conditions are perfect (T31 truncated states from the T42 nature run).
Methods for representing model error

**Additive Inflation**

- Additive inflation alone outperforms multiplicative inflation alone (compare values y-axis to values along x-axis)
- A combination of both is better than either alone.
- Multiplicative and additive inflation representing different error sources in the DA cycle?
Methods for representing model error

**Additive Inflation**

- Using random samples of actual model error is unrealistic.
- Instead, try random samples of 12-h differences from T31 run.
- Not quite as good as using actual model error, but still an improvement over multiplicative inflation alone.
- Additive inflation alone still better than multiplicative inflation alone (compare values along x and y axes).
Perfect Model results (Additive + Multiplicative Inflation)

• 6000 km localization, min error 4 times lower.
• When model error is absent, multiplicative inflation alone outperforms combination of add + mult inflation.
• Suggests that multiplicative inflation is better at capturing DA-related (i.e. sampling) error.
Large ensemble results (Additive + Multiplicative Inflation)

- 200 instead of 20 members, with model error. Min error reduced from 8.7 to 7.7.
- When sampling error is reduced, additive inflation alone outperforms combination of add + mult inflation.
- Suggests that additive inflation is better at capturing model-related errors.
Methods for representing model error

Stochastic Kinetic Energy Backscatter

- Insufficient resolution causes KE spectra to fall off too rapidly – missing upscale transfer to resolved scales.
Methods for representing model error

**Stochastic Kinetic Energy Backscatter**

- Algorithm described in Shutts (2005), Berner et al (2009)
  - Random streamfunction patterns generated by a AR1 process with specified decay timescale (same for all wavenumbers), and variance specified as a function of wavenumber (isotropic spatial correlation).
  - The laplacian of the random pattern multiplied by the hyperdiffusion KE dissipation becomes an extra forcing term in the vorticity equation.
  - Tunable parameters: total variance injected ($\sigma$), decay time scale ($\tau$), power law for wavenumber spectrum ($p$).
Methods for representing model error

Stochastic Kinetic Energy Backscatter

- Adding SKEB to T31 model makes KE spectrum look similar to T42 model.

SKEB parameters: $p=0$, $\sigma=15$, $\tau=6-h$
Methods for representing model error

**Stochastic Kinetic Energy Backscatter**

- A combination of SKEB and multiplicative inflation works better than either alone.
- SKEB alone comparable to multiplicative inflation alone (compare values along x and y axes).
- Results are slightly inferior to those obtained using additive + multiplicative inflation.
- y-axis is amplitude of random pattern ($\sigma$) – results do not change much if $p$ (power law) or time-scale ($\tau$) are varied.
Single-Ob Increments

Background Mean – solid black, $T$ increment for $T$ ob at black dot – colors 10,000 km localization

Perfect Model (100 members)

Mult Inf Only

Mult+Add Inf

Mult Inf+SKEB
Conclusions

• Improving background-error covariances in an EnKF is a tough test for a model error scheme.

• Multiplicative inflation and stochastic physics/additive inflation sample different sources of error in the DA
  – One samples model error, is not sensitive to the observation network.
  – One is sensitive to the observation network, samples other errors in the DA (sampling error, mis-specification of obs error, errors in forward operator etc.)
  – Combination of both works better than either alone (when there are both sources of error).

• Any of these methods can only do so much – improving the forecast model will usually have a larger impact on the data assimilation.
Reference

• sampling error largest where $\sigma_b/\sigma_a$ is large (Sacher and Bartello 2008 MWR, 1640-1654).
• model error is a larger fraction of background error in regions of dense/accurate obs where $\sigma_b/\sigma_a$ is large (Daley and Menard 1993 MWR, 1554-1565).
• adaptively estimated inflation looks like $\sigma_b/\sigma_a$ (Anderson et al 2009 BAMS, 1283-1296, Fig. 13).
• Additive inflation does most of the work in the Env Canada EnKF (Houtekamer, Mitchell and Deng, MWR July 2009).
Methods for representing model error

**Evolved Additive Inflation**

- Adding the additive noise to the previous ensemble mean analysis, evolving forward 12-h, then add the resulting perturbations to the current analysis is an improvement (following Hamill and Whitaker, 2010, MWR, 117-131)

- Flow-dependence gained by “conditioning” the perturbations to the current dynamics helps a little.
Methods for representing model error

**Evolved Additive Inflation**

- Using random samples of actual model error is unrealistic.
- Instead, try random samples of 12-h differences from T31 run.
- Samples are added to the previous analysis mean, then evolved forward 12-h, and recentered around the current analysis mean (following Hamill and Whitaker, 2010, MWR, 117-131)
- Not quite as good as using actual model error, but still an improvement over multiplicative inflation alone.
- Additive inflation alone still better than multiplicative inflation alone (compare values along x and y axes).
Experiences with Env. Canada system
(Houtekamer, Mitchell and Deng, MWR July 2009)

• Most of spread comes from additive inflation.
• Multi-physics adds some variance, esp in lower trop.
• SPPT and SKEB do not provide much variability to background ensemble.
• Are other sources of assimilation error (non-model) dominant, or do stochastic schemes need to be developed/tuned specifically for DA systems?