Stochastic tendency perturbations for NWP ensembles

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Workshop on Representing Model Uncertainty in numerical weather prediction (NWP) models and in climate models

Acknowledgements: Glenn Shutts, Martin Steinheimer & Peter Bechtold
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5 Summary
Estimating model error statistics

Truth versus (unperturbed) model mismatches over interval $\Delta t$

- mismatches $x_f - x_t$ are state vectors
- spatial, multi-variate and temporal correlations matter
- error will be a function of the initial state
Estimating model error statistics

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Estimating model error statistics (II)

estimate of truth versus model mismatches over interval $\Delta t$
Estimating model error statistics (II)

estimate of truth versus model mismatches over interval $\Delta t$
other estimate of truth versus model mismatches over interval $\Delta t$
Estimating model error covariances

Observable are $G_a = \langle (x_f - x_a)(x_f - x_a)^T \rangle$ and $G_o = \langle (Hx_f - y)(Hx_f - y)^T \rangle$ under some simplifying assumptions (linearity, temporally uncorrelated errors) we expect

$$G_o = HMAM^TH^T + HQH^T + R \quad \text{and} \quad G_a = MAM^T + Q + A$$
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IF, initial uncertainty $A$ (and $R$) precisely known, then

$$Q = G_a - MAM^T - A \quad \text{ and } \quad HQH^T = \ldots$$

yields the model error covariance $Q$. Vice versa, errors in $A$ (and $R$) will alias into errors of our estimate of $Q$ ($HQH^T$).
Estimating model error covariances

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under some simplifying assumptions (linearity, temporally uncorrelated errors) we expect

$$G_o = H M M^T H^T + H Q H^T + R \quad \text{and} \quad G_a = M A M^T + Q + A$$

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The analys is error covariance $A$ depends on the assimilation technique $\alpha$, $H$, $R$ and $Q$. Thus, we have

$$G_a = M A(\alpha, H, R, Q) M^T + Q + A(\alpha, H, R, Q) \quad (1)$$

→ a nontrivial inverse problem!
Ambiguity between initial uncertainty and model uncertainty

Without constraining the estimate of $A$ (completely) by data assimilation, both the representation of initial uncertainties ($A$) and tendency perturbations ($Q$) need to be set for an ensemble forecasting system.
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- However, setting $A$ and $Q$ for ensemble forecasts may be an under-determined problem!
- If the estimate of $A$ is “too small” (ie. the ensemble variance due to initial uncertainty represented by $A$ is lower than the error variance), “larger” $Q$ can compensate.
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- Consider for instance $\alpha \tilde{A}$ and $\beta \tilde{Q}$ for two estimates of analysis error covariance and model error covariance. Can we determine unambiguously $(\alpha, \beta)$ for a NWP ensemble?
A-Q-Ambiguity (an example with the EPS)

u850hPa, Northern Extra-tropics

spread_em, rmse_em
2010041300-2010050200 (20)

RMS

fc-step (d)

0 1 2 3 4 5

A, no Q in FC
4A, no Q in FC
A, Q on in FC

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A-Q-Ambiguity (an example with the EPS)

u850hPa, Northern Extra-tropics
Continuous Ranked Probability Score
2010041300-2010050200 (20)

CRPS vs. fc-step (d)

- A, no Q in FC
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u850hPa, Northern Extra-tropics
ContinuousIgnoranceScoreGaussian, ContinuousIgnoranceScoreGaussianClimate
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Model uncertainty representation at ECMWF

Status quo

The EPS uses

- Stochastically Perturbed Parameterization Tendencies (SPPT) a.k.a. stochastic physics
- Stochastic Kinetic Energy Backscatter (SKEB)

Work is in progress to make the representation of model uncertainties in the nonlinear forecasts in EPS and EDA consistent.
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The trajectory and the nonlinear forecast of the perturbed members of the EDA (Ensemble of 4D-Vars) use SPPT only.

- Work is in progress to make the representation of model uncertainties in the nonlinear forecasts in EPS and EDA consistent
- Full consistency requires more → weak-constraint 4D-Var
Stochastically Perturbed Parameterization Tendencies (SPPT)

- Physics tendencies $P$ perturbed by $\Delta P = rP$, with $r$ a random pattern
- Improved version of the original SPPT scheme (stochastic physics, Buizza, Miller & Palmer (1999))

- 2D Random pattern $r$ uses AR-1 processes in spectral space and is smooth in space and time (instead of $10^\circ \times 10^\circ$ tiles changing every 6 time steps)
- Three components with different correlation scales: 6 h, 3 d, 30 d and 500 km, 1000 km, 2000 km with standard deviations of 0.52, 0.18, 0.06, respectively
- Gaussian distribution, truncated at $\pm 2\sigma$ (instead of uniform distr.)
- Same pattern $r$ for $T$, $q$, $u$, $v$ (instead of an independent pattern for each variable)

See Tech Memo 598, Palmer et al. (2009) for more details.
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Multi-variate uniform versus univariate Gaussian

Probability to be within interquartile range for all four variables is $\frac{1}{16}$.

Probability to perturb at least one of the four variables in excess of $0.92$ of the maximum perturbation amplitude is $0.5 = (1 - 2 \times 0.08)^4$. 

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Tendency pert and the frequency of heavy precipitation

- multi-variate uniform distribution of \((u, v, T, q)\) ten. perturbations
- uni-variate Gaussian tendency perturbations

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**Graphs:**

1. **n.hem**
   - Multi-var. unit.
   - Uni-var. norm.

2. **tropics**
   - Multi-var. unit.
   - Uni-var. norm.

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NWP ensembles

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SPPT pattern

- 500 km
  - 6 h

- 1000 km
  - 3 d

- 2000 km
  - 30 d
Stochastic Kinetic Energy Backscatter
SKEB

- Rationale: A fraction of the dissipated energy is backscattered upscale and acts as streamfunction forcing for the resolved-scale flow (Shutts and Palmer 2004, Shutts 2005, Berner et al. 2009)

- **Streamfunction forcing** = \([bD]^{1/2} F(x, t)\),
where \(b, D, F\) denote the backscatter ratio, the (smoothed) total dissipation rate and the 3-dim evolving pattern, respectively
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- **Total dissipation rate:** sum of
  - “numerical” KE dissipation by numerical diffusion + interpolation in semi-Lagrangian advection
  - dissipation from orographic gravity wave drag parameterization
  - an estimate of the deep convective KE production

- **Boundary layer dissipation** is omitted

see also Tech Memo 598, Palmer et al. (2009) for further details
SKEB forcing

- $F$ uses AR-1 processes in spectral space with random vertical phase shifts
- decorrelation time of pattern $F$ is set to 7 h
- structure of pattern constrained by results from coarse-graining studies with T1279 IFS and CRM
Recent operational implementations affecting the EPS

- **35r3**: revised SPPT (1-scale)
- **36r2**: EDA pert. replace evolved SVs; SV pertn ampl. reduced by 10%
- **36r4**: SV pertn amplitude reduced by 50%; 3-scale SPPT & SKEB
- **36r1**: resolution increased to T639/319
Recent operational implementations affecting the EPS

Note, EDA uses the 1-scale version of SPPT (as implemented in 35r3 in the EPS)

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Note, EDA uses the 1-scale version of SPPT (as implemented in 35r3 in the EPS)
Impact of tendency perturbations on the EPS
for a fixed representation of initial uncertainties

- initial perturbations as used since 36r4
  - EDA perturbations instead of evolved SV perturbations
  - 50% reduced amplitude of initial SV perturbations
- T639, 50 member
- cycle 36r2
- 6 different tendency perturbations
  - no ten. perturbations
  - original SPPT, BMP99 (Buizza, Miller & Palmer, 1999)
  - single-scale SPPT (SPPT1 as implemented in 35r3)
  - three-scale SPPT (SPPT3 as implemented in 36r4)
  - stochastic kinetic energy backscatter (SKEB)
  - SPPT3+SKEB
Ensemble standard deviation (no symbols), EM RMSE (+)

v850hPa, Northern Mid-latitudes
spread_em, rmse_em
2008081012-2009122812 (40)

RMS

fc-step (d)
Ignorance score (=Logarithmic score)

\[ \text{ClgnS} = -\log(p_{fc}(y)); \text{ the smaller the better} \]

v850hPa, Northern Mid-latitudes

ContinuousIgnoranceScoreGaussian

2008081012-2009122812 (40)

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Spread-reliability of 500 hPa height — 20°–90°N

Jan 2010 configuration versus Nov 2010 configuration

- T639, 50 member
- cycle 36r2

Major improvement of a long-standing deficiency

Which change of the EPS configuration is responsible?
Spread-reliability of 500 hPa height — 20°–90°N

Impact of halved SV perturbation amplitude

- Main improvement from reduced SV perturbation amplitude
- Probabilistic skill of $0.5 \times SV$ is inferior to 36R2 configuration
- smaller contribution from 36R1 $\rightarrow$ 36R2
- upgraded tendency perturbations prevent underdispersion
Kalman filter and model uncertainty

see Daley & Menard (1993)

- variance evolution in the Kalman filter:

  forecast step \[ P^f = M P^a M^T + Q \] (2)

  analysis step \[ P^a = (I - K H) P^f, \] where (3)

  gain matrix \[ K = P^f H^T (H P^f H^T + R)^{-1} \] (4)
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- Equivalence between 4D-Var and a Kalman smoother
- Many ensemble assimilation techniques aim at approximating the Extended Kalman filter
Kalman filter and model uncertainty
see Daley & Menard (1993)

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\text{forecast step} \quad P^f = M P^a M^T + Q \quad (2)
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\]

- Equivalence between 4D-Var and a Kalman smoother
- Many ensemble assimilation techniques aim at approximating the Extended Kalman filter
- What is the impact of model uncertainty in the simplest possible KF?
- DM93 studied properties of the KF with stationary \( R M Q H \) for the case where all matrices can be diagonalized simultaneously
  \( \Rightarrow \) independent KF’s, each provides the analysis for one scalar variable
Sensitivity to model error variance

Stationary Kalman filter for a scalar variable

slow pertrowth
frequent obsns

\[ MM^T = 1.011 \]

fast pertrowth
infrequent obsns

\[ MM^T = 2.000 \]

\[ \text{all variances normalized by } R = \sigma^2 \]

\[ P^a \quad P^f \quad MP^a MT \quad Q \]
Impact of representing model uncertainties in EDA and EPS on ensemble forecasts

- 3 EDA experiments; (10 member, T399):
  - no tendency perturbations
  - SPPT
  - SPPT+SKEB

- 5 EPS experiments (20 member, T639):

<table>
<thead>
<tr>
<th>pertn. in EDA</th>
<th>perturbation in EPS</th>
</tr>
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<tbody>
<tr>
<td>None</td>
<td>None</td>
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<tr>
<td>None</td>
<td>Off-Off</td>
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<tr>
<td>SPPT+SKEB</td>
<td>Off-ON</td>
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<td>SPPT-ON</td>
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</tbody>
</table>

- no SV perturbations
- EDA perturbations defined with respect to EDA mean
- analysis uncertainties accounted for in verification

- 20 cases in April/May 2010
- cycle 36r4
- see also earlier results in Sec. 3 of Tech Memo 598, Palmer et al. (2009)
Ensemble standard deviation (no symbols), EM RMSE (+)

500 hPa geopotential — 35°N–65°N
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Continuous Ignorance Score

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The smaller the better
Continuous Ignorance Score

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Spread-reliability: $t = 48 \text{ h}$
Northern mid-latitudes $35^\circ \text{N}–65^\circ \text{N}$

- **500 hPa geopotential height**
  - RMS spread (m)
  - RMS error (m)
- **850 hPa meridional wind**
  - RMS spread (m/s)
  - RMS error (m/s)
Spread-reliability: $t = 48$ h
Northern mid-latitudes 35°N–65°N

500 hPa geopotential height

850 hPa meridional wind
Spread-reliability: meridional wind $t = 48\ h$

Tropics $20^\circ S$–$20^\circ N$
Spread-reliability: meridional wind \( t = 48 \text{h} \)
Tropics 20°S–20°N

![Graph showing RMS spread and error for 850 hPa and 200 hPa](image)

- 850 hPa
- 200 hPa

- RMS spread (m/s)
- RMS error (m/s)

- On-On
- Sppt-On
- Off-On
- On-Off
- Off-Off
Conclusions

- Stochastic tendency perturbations used in the operational ECMWF ensembles contribute significantly to ensemble spread and improve probabilistic skill.
- Improved ensemble forecast variances and improved probabilistic skill through a combination of:
  - introduction of EDA perturbations
  - reduced amplitude for SV perturbations
  - more active representation of model uncertainties

Diagnostics may need to be improved to distinguish well different representations of model uncertainty.

Model uncertainty contributes to initial uncertainty wherever a short-range forecast is used as prior information. A consistent representation of model uncertainty in data assimilation and forecast can help to better constrain the formulation of model uncertainty.
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- Not having precise estimates of initial error covariances hampers diagnostic of the characteristics of (random) model tendency errors

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- Model uncertainty contributes to initial uncertainty wherever a short-range forecast is used as prior information. A consistent representation of model uncertainty in data assimilation and forecast can help to better constrain the formulation of model uncertainty.
Plans

- Compare operational schemes with more basic tendency perturbations. For instance, additive noise, e.g. from scaled tendencies constructed from a tendency archive, (e.g. YOTC data).
- Diagnose tendency differences from different models started from the same initial conditions (resolution, different parameters, different parameterization schemes, ...). What is the nature of the *random component* of the differences?
- Develop improved diagnostics that permit to evaluate better the realism of different tendency perturbations.