Nonhydrostatic Modeling with NICAM

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Contents

- **Numerical method on NICAM Dynamical core**
  - **Horizontal discretization**
    - Icosahedral grid
      - Modified by spring dynamics (Tomita et al. 2001, 2002 JCP)
  - **Nonhydrostatic framework**
    - Mass and total energy conservation
      - (Satoh 2002, 2003 MWR)

- **Computational strategy**
  - Domain decomposition in parallel computer
  - Example of computational performance

- **Problems**
  - Current problems
  - Future problems

- **Summary**
NICAM project

NICAM project (~2000)

Aim:
- Construction of GCRM for climate simulation
  - Suitable to the future computer environment
    » Massively parallel supercomputer
    » First target: Earth Simulator 1
      40TFLOPS in peak
      640 computer nodes

Strategy:
- Horizontal discretization:
  - Use grid of icosahedral grid
    (quasi-homogeneous)
  - Anyway, 2nd accuracy over the globe!
- Dynamics:
  - Non-hydrostatic
    » Mass & total energy conservation
- Physics:
  - Import from MIROC (one of Japan IPCC models)
    except for microphysics.
Horizontal discretization
Grid arrangement

- **Arakawa A-grid type**
  - **Velocity, mass**
    - triangular vertices
  - **Control volume**
    - Connection of center of triangles
      - Hexagon
      - Pentagon at the icosahedral vertices

- **Advantage**
  - **Easy to implement**
  - **no computational mode**
    - Same number of grid points for vel. and mass

- **Disadvantage**
  - **Non-physical 2-grid scale structure**
    - E.g. bad geostrophic adjustment
**Horizontal differential operator**

- **e.g. Divergence**
  1. Vector: given at $P_i$
     \[ u(P_i) \]
  2. Interpolation of $u$ at $Q_i$
     \[ u(Q_i) \approx \frac{\alpha u(P_0) + \beta u(P_i) + \gamma u(P_{1+\text{mod}(i,6)})}{\alpha + \beta + \gamma} \]
  3. Gauss theorem
     \[ \nabla \cdot u(P_0) \approx \frac{1}{A(P_0)} \sum_{i=1}^{6} b_i \frac{u(Q_i) + u(Q_{1+\text{mod}(i,6)})}{2} \cdot n_i \]

**2nd order accuracy?**
- NO

→ Allocation points is not gravitational center (default grid)
Reconstruction of grid by spring dynamics

To reduce the grid-noise

1. STD-grid :
   Generated by the recursive grid division.

2. SPRING DYNAMICS :
   Connection of gridpoints by springs

\[
\sum_{i=1}^{6} k(d_i - \bar{d})e_i - \alpha \mathbf{w}_0 = M \frac{d\mathbf{w}_0}{dt}
\]

\[
\mathbf{w}_0 = \frac{dr_0}{dt}
\]

SPR-grid:
   Solve the spring dynamics
   ➔ The system calms down to the static balance
Modified Icosahedral Grid (2)

Gravitational-Centered Relocation

To make the accuracy of numerical operators higher

1. SPR-grid:
   Generated by the spring dynamics.

2. CV:
   Defined by connecting the GC of triangle elements.

3. SPR-GC-grid:
   The grid points are moved to the GC of CV.

The 2nd order accuracy of numerical operator is perfectly guaranteed at all of grid points.
Improvement of accuracy of operator

- **Test function**

- **Error norm**
  - **Global error**
    \[
    l_2(x) = \frac{\int [(x(\lambda, \theta) - x^+_T(\lambda, \theta))^2]^{1/2}}{\int [x^+_T(\lambda, \theta)^2]^{1/2}}
    \]
  - **Local error**
    \[
    l_\infty(x) = \frac{\max_{all \lambda, \theta} |x(\lambda, \theta) - x^+_T(\lambda, \theta)|}{\max_{all \lambda, \theta} |x^+_T(\lambda, \theta)|}
    \]

<table>
<thead>
<tr>
<th></th>
<th>STD-grid</th>
<th>SPR-GC-grid</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>L_2 norm</strong></td>
<td>Almost 2nd-order(●)</td>
<td>Perfect 2nd-order(○)</td>
</tr>
<tr>
<td><strong>l_inf norm</strong></td>
<td>Not 2nd order(▲)</td>
<td>Perfect 2nd-order(△)</td>
</tr>
</tbody>
</table>
Nonhydrostatic framework
Design of our non-hydrostatic modeling

- **Governing equation**
  - **Full compressible system**
    - Acoustic wave → Planetary wave
  - **Flux form**
    - Finite Volume Method
    - Conservation of mass and energy
  - **Deep atmosphere**
    - Including all metrics terms and Coriolis terms

- **Solver**
  - **Split explicit method**
    - Slow mode: Large time step
    - Fast mode: small time step
  - **HEVI (Horizontal Explicit & Vertical Implicit)**
    - 1D-Helmholtz equation
 Governing Equations

\[ \begin{align*}
\text{L.H.S. : FAST MODE} & \Rightarrow \text{R.H.S. : SLOW MODE} \\
\frac{\partial}{\partial t} R + \nabla_h \cdot \frac{V_h}{\gamma} + \frac{\partial}{\partial \xi} \left( \frac{W}{G^{1/2}} + G^3 \cdot \frac{V_h}{\gamma} \right) &= 0 \\
\frac{\partial}{\partial t} V_h + \nabla_h \frac{P}{\gamma} + \frac{\partial}{\partial \xi} \left( G^3 \frac{P}{\gamma} \right) &= \text{ADV}_h + F_{\text{Coriolis}} \\
\frac{\partial}{\partial t} W + \gamma^2 \frac{\partial}{\partial \xi} \left( \frac{P}{G^{1/2} \gamma^2} \right) + Rg &= \text{ADV}_z + F_{\text{Coriolis}} \\
\frac{\partial}{\partial t} E + \nabla_h \cdot \left( h \frac{V_h}{\gamma} \right) + \frac{\partial}{\partial \xi} \left[ h \left( \frac{W}{G^{1/2}} + G^3 \cdot \frac{V_h}{\gamma} \right) \right] \\
- \frac{V_h}{R} \left[ \nabla_h \frac{P}{\gamma} + \frac{\partial}{\partial \xi} \left( G^3 \frac{P}{\gamma} \right) \right] - \frac{W}{R} \gamma^2 \frac{\partial}{\partial \xi} \left( \frac{P}{G^{1/2} \gamma^2} \right) + Wg &= Q_{\text{heat}}
\end{align*} \]

**Prognostic variables**
- density
- horizontal momentum \( v_h = \gamma^2 G^{1/2} \rho v_h \)
- vertical momentum \( W = \gamma^2 G^{1/2} \rho w \)
- internal energy \( E = \gamma^2 G^{1/2} \rho e_{\text{in}} \)

**Metrics**
- \( R = \gamma^2 G^{1/2} \rho \)
- \( G^{1/2} = \left( \frac{\partial z}{\partial \xi} \right)_{x,y} \)
- \( G^3 = \left( \nabla_h \xi \right)_z \)
- \( \xi = \frac{H(z-z_s)}{H-z_s} \)
Temporal Scheme (in the case of RK2)

Assumption: the variable at $t=A$ is known.

1. 1st step:
   - Integration of the prog. var. by using $S(A)$ from $A$ to $B$.
   - Obtain the tentative values at $t=B$.
   - Obtain the slow mode tendency $S(B)$ at $t=B$.

2. 2nd step:
   - Returning to $A$, Integration of the prog.var. from $A$ to $C$ by using $S(B)$.

→ Obtain the variables at $t=C$
In small step integration, there are 3 steps:

1. **Horizontal Explicit Step**
   - Update of horizontal momentum

2. **Vertical Implicit Step**
   - Updates of vertical momentum and density.

3. **Energy Correction Step**
   - Update of energy

### Horizontal Explicit Step

- Horizontal momentum is updated explicitly by

\[
V_h^{t+(n+1)\Delta t} = V_h^{t+n\Delta t} + \Delta t \left( -\nabla_h \frac{P}{\gamma} - \frac{\partial}{\partial \xi} \left( G^3 \frac{P}{\gamma} \right)^{t+n\Delta t} + \left( \frac{\partial V_h}{\partial t} \right)_{\text{slow mode}} \right)
\]

**Fast mode**

**Slow mode**

- Given
Small Step Integration (2)

**Vertical Implicit Step**

- The equations of R, W, and E can be written as:

\[
\frac{R(t+(n+1)\Delta\tau) - R(t+n\Delta\tau)}{\Delta\tau} + \frac{\partial}{\partial\xi} \left( \frac{W(t+(n+1)\Delta\tau)}{G^{1/2}} \right) = G_R \quad (6)
\]

\[
\frac{W(t+(n+1)\Delta\tau) - W(t+n\Delta\tau)}{\Delta\tau} + \gamma^2 \frac{\partial}{\partial\xi} \left( \frac{P(t+(n+1)\Delta\tau)}{G^{1/2}\gamma^2} \right) + R(t+(n+1)\Delta\tau) g = G_z \quad (7)
\]

\[
\frac{P(t+(n+1)\Delta\tau) - P(t+n\Delta\tau)}{\Delta\tau} + \frac{\partial}{\partial\xi} \left[ \left( \frac{W(t+(n+1)\Delta\tau)}{G^{1/2}} \right) c_{s}^{2t+n\Delta\tau} \right] + \frac{R_d}{C_V} W(t+(n+1)\Delta\tau) \tilde{g} = \frac{R_d}{C_V} G_E \quad (8)
\]

- Coupling Eqs.(6), (7), and (8), we can obtain the 1D-Helmholtz equation for W:

\[
\frac{W(t+(n+1)\Delta\tau)}{\gamma^2} - \frac{\partial}{\partial\xi} \left[ \frac{1}{G^{1/2}\gamma^2} \frac{\partial}{\partial\xi} \left( \Delta^2 c_{s}^{2t+n\Delta\tau} \frac{W(t+(n+1)\Delta\tau)}{G^{1/2}} \right) \right] - \left[ \frac{\partial}{\partial\xi} \left( \Delta^2 \frac{R_d}{C_V} \tilde{g} \frac{W(t+(n+1)\Delta\tau)}{G^{1/2}\gamma^2} \right) \right] + \Delta^2 \frac{g}{\gamma^2} \frac{\partial}{\partial\xi} \left( \frac{W(t+(n+1)\Delta\tau)}{G^{1/2}} \right) = \text{R.H.S. (source term)} \quad (9)
\]

- Eq.(9) $\rightarrow$ W
- Eq.(6) $\rightarrow$ R
- Eq.(8) $\rightarrow$ E
Small Step Integration (3)

- **Energy Correction Step**
  
  \[ (\text{Total eng.}) = (\text{Internal eng.}) + (\text{Kinetic eng.}) + (\text{Potential eng.}) \]
  
  - We consider the equation of total energy

  \[
  \frac{\partial}{\partial t} E_{\text{total}} + \nabla_h \cdot \left[ (h + k + \Phi) \frac{V_h}{\gamma} \right] + \frac{\partial}{\partial \xi} \left[ (h + k + \Phi) \left( \frac{W}{G^{1/2}} + G^3 \cdot \frac{V_h}{\gamma} \right) \right] = 0 \tag{10}
  \]

  where \( E_{\text{total}} = \rho \gamma^2 G^{1/2} (e_{in} + k + \Phi) \)

  - Additionally, Eq.(10) is solved as

  \[
  E^{t+(n+1)\Delta \tau}_{\text{total}} = E^{t+n\Delta \tau}_{\text{total}} - \Delta \tau \left[ \nabla_h \cdot \left[ (h + k + \Phi) \frac{V_h}{\gamma} \right] + \frac{\partial}{\partial \xi} \left[ (h + k + \Phi) \left( \frac{W}{G^{1/2}} + G^3 \cdot \frac{V_h}{\gamma} \right) \right] \right]^{t+(n+1)\Delta \tau}
  \]

  - Written by a flux form.

  - The kinetic energy and potential energy: \( \rightarrow \) known by previous step.

  - Recalculate the internal energy:

  \[
  E^{t+(n+1)\Delta \tau} = E^{t+(n+1)\Delta \tau}_{\text{total}} - \rho^{t+(n+1)\Delta \tau} \gamma^2 G^{1/2} \left( k^{t+(n+1)\Delta \tau} + \Phi \right)
  \]
Large Step Integration

Large step tendency has 2 main parts:
1. Coriolis term
   - Formulated straightforward.
2. Advection term
   - We should take some care to this term because of curvature of the earth

Advection of momentum
- Use of a cartesian coordinate in which the origin is the center of the earth.
- The advection term of $V_h$ and $W$ is calculated as follows.
  1. Construct the 3-dimensional momentum $V$ using $V_h$ and $W$.
  2. Express this vector as 3 components as $(V_1, V_2, V_3)$ in a fixed coordinate.
     - These components are scalars.
  3. Obtain a vector which contains 3 divergences as its components.
     $\rightarrow \left( \nabla \cdot v_1 V, \nabla \cdot v_2 V, \nabla \cdot v_3 V \right)$ where $v_i = V_i / \left( G^{1/2} \gamma^2 \rho \right)$
  4. Split again to a horizontal vector and a vertical component.
     $\rightarrow ADV_h, ADV_z$
Computational strategy and performance
Computational strategy (1)

- **Domain decomposition**
  1. By connecting two neighboring icosahedral triangles, 10 rectangles are constructed. (rlevel-0)
  2. For each of rectangles, 4 sub-rectangles are generated by connecting the diagonal mid-points. (rlevel-1)
  3. The process is repeated. (rlevel-n)
Example ( rlevel-1 )
- # of region : 40
- # of process : 10
- Situation:
  - Polar region: Less computation
  - Equatorial region: much computation
- Each process
  - manage same color regions
  - Cover from the polar region and equatorial region.

Avoid the load imbalance
Computational strategy (3)

- **Structure in one region**
  - Icosahedral grid
    - Unstructured grid?
  - Treatment as structured grid
    - Fortran 2D array
    - Vectorized efficiently!

- **2D array → 1D array**
  - Higher vector operation length
Computational Performance (1)

- **Computational performance** Depend on…
  - Computer architecture, degree of code tuning…..

- **Performance on the old Earth Simulator**

  - **Earth Simulator**
    - Massively parallel super-computer based on NEC SX-6 architecture.
      - 640 computational nodes.
      - 8 vector-processors in each of nodes.
      - Peak performance of 1CPU : 8GFLOPS
      - Total peak performance : 8X8X640 = 40TFLOPS
      - Crossbar network

  - Target simulations for the measurement
    - 1 day simulation of Held & Suarez dynamical core experiment
Computational Performance (2)

- Scalability of our model (NICAM) --- strong scaling

**Configuration**
- Horizontal resolution: glevel-8
- 30km resolution
- Vertical layers: 100
- Computer nodes: increases from 10 to 80.

**Results**
- Green: ideal speed-up line
- Red: actual speed-up line
### Computational Performance (3)

**Performance against the horizontal resolution --- weak scaling**

The elapse time should increase by a factor of 2.

<table>
<thead>
<tr>
<th>g level (grid intv.)</th>
<th>Number of PNs (peak performance)</th>
<th>Elapse Time [sec]</th>
<th>Average Time [msec]</th>
<th>GFLOPS (ratio to peak[%])</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 (120km)</td>
<td>5 (320GFLOPS)</td>
<td>48.6</td>
<td>169</td>
<td>140 (43.8)</td>
</tr>
<tr>
<td>7 (60km)</td>
<td>20 (1280GFLOPS)</td>
<td>97.4</td>
<td>169</td>
<td>558 (43.6)</td>
</tr>
<tr>
<td>8 (30km)</td>
<td>80 (5120GFLOPS)</td>
<td>195</td>
<td>169</td>
<td>2229 (43.5)</td>
</tr>
<tr>
<td>9 (15km)</td>
<td>320 (20480GFLOPS)</td>
<td>390</td>
<td>169</td>
<td>8916</td>
</tr>
</tbody>
</table>

**Configuration**

As the grid level increases,

- # of gridpoints : X 4
- # of CPUs         : X 4
- Time intv.        : 1/2

**Results**

The elapse time increases by a factor of 2.
Problems & subjects
Current problems in NICAM (1)

Numerical problem

3.5km mesh run: sometimes, crash!
at the steep mountain area (e.g. Tibetan plateau)

- Possible cause
  - The CFL condition in the vertical direction?
    » Reduction of time step or application of vertical implicit method?
  - The large Horizontal-PGF error in the terrain-following coordinate.
    » If the horizontal resolution increases more and more, ....

- Reconsideration of vertical discretization from the terrain-following coordinate to height basis coordinate.
  - Vertical adaptive mesh for the PBL scheme.
Current problems in NICAM (2)

Climatology bias
- found at the Athena project
  - IFS: hydrostatic with c.p.
    - TL2047
    - 7km mesh

NICAM Bias:
- Excessive precipitation @ south Indian ocean @ SPCZ
- Little precipitation @ storm track area in NH @ western pacific ocean
- Almost same situation as the 14km run.
  → independent of resolution.

→ basically, physical scheme problem!

Dirmeyer et al. (2010, JCLI submitted)
Future subjects

- **Beyond a simple global cloud-system resolving**
  - Cloud resolving approach has advantages over the convetional approach.
    - Explicit representation of cold pool dynamics
    - Well capture the organization of cloud dynamics
      - meso-scale cloud system, CC, SCC, MJO and so on.

- **However, ...... climate simulation?**
  - Physics is not still sufficient!

- **Radiation-microphysics coupling with aerosol process is a key!**

  - **CURRENT :**
    - Microphysics: one or two moment bulk method
    - Radiation: prescribed or assumed diameter of cloud particle
  
  - **FUTURE :**
    - Microphysics: spectral method as regard to the size distribution
    - Aerosol: spectral method
    - Radiation: estimate the optical depth of cloud and aerosol by tight coupling with microphysics and aerosol models.

→ **Locality is very important!**
  Ocean / Land high latitude/ mid latitude/ tropics
Near future problem (computer side)

- Exa-FLOPS is coming soon!
  - Inner node:
    - Many-core /Many-socket → Memory bandwidth problem!! Bandwidth per core is very narrow (less than 0.1?).
      - Disadvantage for gridpoint method
        » Load/store of memory occurs frequently.
        » Short computation
      - But, welcome for complicated physics?
        » Calculation is dominated over the memory load/store.
    - Hybrid architecture with CPU and GPU?
      - Complicated programming?
  - Outer node:
    - Communication is dominated
      - Network topology, speed itself
    - Parallel IO
  - Coding problem:
    - What is the standard language?
      - OpenCL, new Fortran?
Summary

- **NICAM dynamical core**
  - Icosahedral A-grid
    - with grid modification by spring dynamics etc.
  - Conservative nonhydrostatic scheme
    - Mass & total mass
    - CWC
  - Time scheme
    - Split explicit scheme with HEVI for the small step solver.

- **Problem**
  - Numerically,
    - Steep mountain problem!
    - Need to change from terrain-following approach to height-base approach for vertical discretization.
  - Physically,
    - Precipitation bias still exists.
      - This can be solved by many tuning runs on K-computer (10 PFLOPS) within 1 or 2 years

- **Up to date, the parallel efficiency of NICAM is quite good.**
  - There is a lot of computer-side problems towards the next-generation supercomputer (ExaFLOPS)
    - Need to construct the model, considering the future environment.