

Non-hydrostatic modelling at the Met Office

Terry Davies Dynamics Research

© Crown copyright Met Office ECMWF Non-hydrostatic Workshop



Plan

• History

- Equations and approximations
- New Dynamics
- Results
- ENDGame and conclusions



History 1

- The first Met Office non-hydrostatic model was developed by Tapp and White (QJRMS 1976).
- Height-based terrain-following coordinates introduced by Carpenter (QJRMS 1979)
- Physical parametrizations, Golding (Bound. Layer Meteor. 1987)
- Used for process studies (Haar case study; Ballard et al Mon. Wea. Rev. 1992)
- Operational trials from Oct. 1984 but was its usefulness was limited by its coarse (15km) horizontal resolution and its lack of a state-of-the-art data assimilation system.
- Golding (Meteor. Atmos. Phys. 1992) upgraded the model to use a 2 time-level Semi-Lagrangian scheme but overall this had little impact on the model efficiency since the use of larger time steps was limited by lateral boundary errors.



History 2

- In the 1992 development of this non-hydrostatic model was stopped and the first (hydrostatic) Unified Model (UM, Cullen and Davies, QJRMS 1991) was used to run the UK Mesoscale Model configuration at 16km resolution.
- The UM was developed to be used for all production models in NWP and climate and its main requirement, in addition to being efficient, was to (formally) conserve mass; required for long (centuries) climate runs.
- Split-explicit scheme (Heun advection and forward-backward) similar to that used in operational NWP at the Met Office since the mid-1970's (Lax-Wendroff and forward-backward). Conserved mass as well as mass-weighted potential temperature and moisture.
- Used the deep atmosphere equations White and Bromley (1995) to reduce approximation to the full equations.



Fully-compressible, deep atmosphere equations

$$\frac{D_r u}{Dt} - \frac{uv}{r} \tan \phi - 2\Omega \sin \phi v + \frac{uw}{r} + 2\Omega \cos \phi w + \frac{1}{\rho r \cos \phi} \frac{\partial p}{\partial \lambda} = F_u,$$
$$\frac{D_r v}{Dt} + \frac{u^2}{r} \tan \phi + 2\Omega \sin \phi u + \frac{vw}{r} + \frac{1}{\rho r} \frac{\partial p}{\partial \phi} = F_v,$$
$$\frac{D_r w}{Dt} - \frac{u^2 + v^2}{r} - 2\Omega \cos \phi u + g + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0,$$

where

$$\frac{D_{r}}{Dt} = \frac{\partial}{\partial t} + \frac{u}{r} \frac{\partial}{\partial \lambda} + \frac{v}{r} \frac{\partial}{\partial \phi} + w \frac{\partial}{\partial r},$$

and

$$\nabla_{\boldsymbol{r}} \cdot \mathbf{u} = \frac{1}{\boldsymbol{r} \cos \phi} \left[\frac{\partial u}{\partial \lambda} + \frac{\partial \left(v \cos \phi \right)}{\partial \phi} \right] + \frac{1}{\boldsymbol{r}^2} \frac{\partial \left(\boldsymbol{r}^2 w \right)}{\partial r}.$$
$$\frac{D_{\boldsymbol{r}} \rho}{Dt} + \rho \nabla_{\boldsymbol{r}} \cdot \mathbf{u} = 0, \qquad \qquad \frac{D_{\boldsymbol{r}} \theta}{Dt} = F_{\theta}, \qquad \qquad p = R\rho T$$

© Crown Copyright 2010 EWGLAM, Exeter



Equation sets and switches

- Shallow atmosphere approximation consists of replacing r by a, the mean radius of the Earth and removing the terms in red, i.e. 4 metric terms and the $\cos \phi$ Coriolis terms. If only some of the red terms are retained then the equation sets do not possess appropriate conservation laws for energy and potential vorticity nor the axial angular momentum principle (White and Bromley, QJRMS 1995).
- Hydrostatic approximation merely removes the $\frac{D_r w}{Dt}$ term.
- Shallow atmosphere with the hydrostatic approximation are the (hydrostatic) primitive equations.
- These are the only approximations which satisfy the conservation laws for energy and potential vorticity and the axial angular momentum principle.
- The full equations make the spherical geopotential approximation. To incorporate height and latitudinal variations to gravity requires spheroidal geometry (White, Staniforth and Wood, QJRMS 2008).



Equation sets and switches

$$\frac{Du}{Dt} + c_p \left(\bar{\theta} + \delta_E \theta'\right) \frac{\partial \pi'}{\partial x} - fv = 0$$

$$\delta_V \frac{Dw}{Dt} + c_p \left(\bar{\theta} + \delta_E \theta'\right) \frac{\partial \pi'}{\partial z} + (1 - \delta_B) c_p \frac{d\bar{\theta}}{dz} \pi' - \frac{g}{\bar{\theta}} \theta' = 0$$

$$\frac{\delta_A}{\bar{\pi}} \left\{ \left(\frac{1 - \kappa}{\kappa}\right) \frac{D}{Dt} + \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right\} \pi' + \frac{\partial u}{\partial x} + \left(\frac{\partial}{\partial z} + \frac{\delta_C}{\bar{\rho}} \frac{d\bar{\rho}}{dz} + \frac{\delta_D}{\bar{\theta}} \frac{d\bar{\theta}}{dz} \right) w = 0$$

Equation set	Switches					
	δ_V	δ_A	δ_B	δ_C	δ_D	δ_E
Fully compressible	1	1	1	1	1	1
Hydrostatic	0	1	1	1	1	1
Pseudo-incompressible (Durran)	1	0	1	1	1	1
Anelastic(Wilhelmson-Ogura)	1	0	1	1	0	0
Anelastic(Lipps-Hemler)	1	0	0	1	0	0
Boussinesq	1	0	1	0	0	0



Approximation recommendations summary

- Davies, Staniforth, Wood and Thuburn, QJRMS 2003.
- Anelastic and Boussinesq equations are not suitable for NWP or climate modelling at any scale because they distort Rossby modes. They are suitable for process studies with shallow vertical scales.
- Pseudo-incompressible equations may be viable at small scales but not at large scales.
- Hydrostatic equations are good for large horizontal scales (but the equations are not hyperbolic, Oliger and A. Sundström SIAM 1978).
- See also Cullen, Acta Numerica 2007.



Physics-dynamics coupling

- SRNWP workshops in Toulouse 1997, Prague 1999. McDonald 1998 ECMWF seminars. SLAVEPP.
- Dubal, Wood and Staniforth, Mon. Wea. Rev., 2004, 2005.
- Difference equations using large time steps should reproduce steady-states.
- Large time steps require vertical diffusion (boundary layer + convection) to be treated sequentially after all other processes (sequential splitting).
- Parallel-split needs small time steps.



New dynamics, design criteria, 1993

- Cullen et al 1997, Davies et al ECMWF seminar 1998, QJRMS 2005.
- Improve UM balance to improve coupling with physics and data assimilation and to minimize noise problems.
- Use more accurate advection scheme and height-based terrain-following vertical coordinates.
- Use more efficient schemes permitting longer time steps.
- Use semi-implicit time stepping, semi-Lagrangian advection, C-grid staggering in horizontal and Charney-Phillips staggering in the vertical.
- Eulerian treatment of continuity equation (for dry density) to ensure conservation of mass.
- No extraction of reference state and no artificial diffusion.
- 3-d variable coefficient Helmholtz equation derived from linearised form of equation of state.



New dynamics overview

The governing equations can be written generically as

$$\frac{D\mathbf{X}}{Dt} = \mathbf{L}(\mathbf{x}, t, \mathbf{X}) + \mathbf{N}(\mathbf{x}, t, \mathbf{X}) + \mathbf{S}_1(\mathbf{x}, t, \mathbf{X}) + \mathbf{S}_2(\mathbf{x}, t, \mathbf{X}),$$
(1)

where **X** is a vector of the prognostic variables, **L** terms linear in **X**, **N** terms nonlinear in **X** and $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla$.

 \mathbf{S}_1 - "slow" physical source terms - radiation, gravity wave drag, microphysics, large scale precipitation

 \mathbf{S}_2 - "fast" physical source terms - convection and boundary layer (together with associated surface and cloud processes)

A target 2TL SISL discretization is given by

$$\frac{\mathbf{X}^{n+1} - \mathbf{X}_d^n}{\Delta t} = (1 - \alpha)(\mathbf{L} + \mathbf{N} + \mathbf{S}_1 + \mathbf{S}_2)_d^n + \alpha(\mathbf{L} + \mathbf{N} + \mathbf{S}_1 + \mathbf{S}_2)^{n+1}.$$
 (2)



New dynamics predictor-corrector scheme

$$\mathbf{X}^{(1)} = \mathbf{X}^n_d + (1 - \alpha)\Delta t(\mathbf{L} + \mathbf{N})^n_d + \Delta t(\mathbf{S}_1)^n_d + \alpha \Delta t(\mathbf{L} + \mathbf{N})^n, \quad (3)$$

$$\mathbf{X}^{(2)} = \mathbf{X}^{(1)} + \Delta t \mathbf{S}_2(\mathbf{X}^n, \mathbf{X}^{(1)}, \mathbf{X}^{(2)}),$$
(4)

$$\mathbf{X}^{(3)} - \alpha \Delta t \mathbf{L}^{(3)} = \mathbf{X}^{(2)} + \alpha \Delta t (\mathbf{N}^* - \mathbf{N}^n - \mathbf{L}^n),$$
(5)

where $\mathbf{X}^{(1)}$ is the first predicted value, $\mathbf{X}^{(2)}$ is the predicted value after the "fast" physics processes and $\mathbf{X}^{(3)}$ the final predicted value.

 $\mathbf{L}^{(3)} \equiv \mathbf{L}(\mathbf{X}^{(3)})$ and \mathbf{N}^* is an estimate of the non-linear terms at t^{n+1} .

 $\mathbf{X}^{n+1} \equiv \mathbf{X}^{(3)}$, i.e. it is given by the final corrector.

"Slow" physical processes evaluated in parallel using data at time-level n only .

"Fast" physical processes are evaluated sequentially, i.e. $\mathbf{S}_2(\mathbf{X}^n, \mathbf{X}^{(1)}, \mathbf{X}^{(2)})$



Iterative time-stepping

- The predictor-corrector scheme would have been iterated if acceptable results could not have been achieved, Bénard (2003).
- Iterative time-stepping made an option a few years ago, Diamantakis et al (2007) QJRMS.
- Adding one iteration effectively repeats the dynamics and "fast" physics (60% increase).
- However, interpolated winds rather than extrapolated winds can be used after first iteration. This increases robustness and usually (iterative) solver needs fewer iterations.
- In sets of case studies, increase in skill was achieved at lower cost than increasing resolution.



Test problems to verify design choices

- Shallow water test cases (Williamson et al J. Comp. Phys. 1992)
- Idealised Eady-wave (Nakamura and Held, JAS, 1989)
- A simulation of fog formation (Golding Mon. Wea. Rev. 1993)
- Vertically propagating sound waves (Golding Meteor. Atmos. Phys. 1992)
- Density currents (Straka et al 1993)
- 2d flows over hill and steady flow over cosine hill.
- Convective bubble (Robert 1993)
- Dynamical core tests (Held and Suarez, 1994)



Real data tests CRAY t3e

- Global case studies at N216L30 (432*325, 60km, 40km top) showed improvements to cross-polar flow and need for some polar filtering to reduce solver iterations.
- 3 year AMIP climate tests at N48L38 (96*73) showed improved stratocumulus sheets west of California, Peru and South West Africa)
- Built into UM system and coupled with 3DVAR for pre-operational trials for nearly 1 year.
- Global model N216L38 operational in August 2002.
- UK Mesoscale model 15km operational in October 2002,
- NAE introduced December 2002.
- NEC SX6/8 Spring 2004. 4DVAR October 2004.
- HadGEM1 (N96L38) spin-up run started May 2004.



New dynamics operations

- NAE resolution increased to 12km February 2005.
- \bullet Global N320L50, 640*481, 40km 63km top, December 2005.
- UK4, 288*360, 4km, 38 levels quasi-operational April 2005 (after Boscastle storm, August 2004).
- UK4 70 levels, 40km top, November 2007.
- HiGEM (N144L38), NuGEM (N216L38), HadGEM3(N96L85)
- IBM Power 6 spring 2009.
- UKV, 744*928, 1.5km, variable resolution to 4km around edges, operational summer 2009.
- Global 70 levels 80km top, November 2009.
- Global N512L70, 1024*769, 25km February 2010.







Measuring the skill

From a sample of 40 forecasts of convective rainfall events

4-hour accumulations T+2 to T+6

Starting from same 12-km fields. No additional data assimilation at 1 km.



Scale-selective verification methodology Roberts and Lean, MWR, 2008



25th June 2007





A showery day (25th August 2005)

Sat IR and NIMRDD precip

DLR and Surface Rain Rate (mm/hr)



Observed : Satellite IR and Radar rainfall rates

1.5 km Model : Outgoing Long-wa∨e Radiation and rainfall rates Satellite (Visible) MODIS

Crown copyright Met Office



Convection-permitting scales

- Little predictive skill near the grid scale.
- For quantities with high spatial variability (e.g. larger rainfall rates) forecast information needs aggregating (in space and time) if it is to be useful. (e.g. neighbourhood methods, EPS).
- Larger vertical velocities are forced by diabatic processes.
- Grid-scale is the minimum size of explicit updraughts.
- Excessive vertical velocities (grid-point storms) are common.
- Need (additional) mixing and/or (convective) adjustment to account for sub-grid scale plumes.
- Do not know whether explicit convection reduces systematic errors and/or improves variability at larger scales (e.g. MJO).



ENDGame and conclusions

- Eulerian treatment of continuity equation is a source of weak instability which is controlled by off-centring.
- To conserve mass (and tracers) use SLICE (Semi-Lagrangian Inherently Conserving and Efficient), Zerroukat et al (2009) QJRMS.
- Remove individual off-centring, extract reference profile and use iterative time-stepping.
- Fully-interpolating tri-cubic Lagrange or spline scheme (New dynamics uses non-interpolating scheme for potential temperature θ).
- Coriolis terms discretized to improve Rossby mode dispersion.
- v-at-the-poles (New dynamics has scalars are at poles).