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Cloud and precipitation overlap in simplified scattering microwave radiative transfer

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Abstract

Scattering radiative transfer calculations are used in the all-sky assimilation of microwave imager observations. For accurate simulations it is important to treat the cloud and precipitation overlap carefully. The current approximation for the cloud and precipitation fraction is not fully consistent with the representation of moist physics in the Integrated Forecasting System (IFS). The impact of going from the current representation to one that is more consistent with the moist physics is examined. Compared to reference radiative transfer simulations using the Independent Column Approach, rms errors are typically reduced. However, microwave imager first guess (FG) departures are not significantly affected. This is explained by the fact that FG departures are dominated by systematic model biases and by forecast errors (e.g. the inability to simulate cloud and rain in exactly the right place and time). Despite the lack of impact on the FG departures, it is still hoped to implement this change operationally, and to include it as an optional feature in the RTTOV-10 release.

1 Introduction

With the move to all-sky assimilation of microwave imagers at cycle 35r2, the scattering radiative transfer code (RTTOV-SCATT, Bauer et al., 2006) was upgraded to version 9.3, which allowed the use of an improved cloud and rain treatment (Geer et al., 2009). There had previously been biases in tropical rain and cloud areas of up to 10 K in some channels. The revised cloud treatment removed these biases, which would have been difficult to correct using the current version of Variational Bias Correction(VarBC) and might have prevented the move to all-sky assimilation. However, the improved approach had a simplified treatment of the precipitation fraction. This study examines whether further improvements can be made by correcting this and by making the treatment more consistent with the moist physics in the model.

2 Cloud overlap approaches

In the Integrated Forecasting System (IFS), performance limitations mean that RTTOV-SCATT can only be used to make one scattering radiative transfer simulation per observation. RTTOV-SCATT calculates the allsky brightness temperature T_{allsky} , which depends on the overall cloud fraction C_{av} as follows:

$$T_{allsky} = (1 - C_{av})T_{clear} + C_{av}T_{cloudy}.$$
(1)

 T_{clear} and T_{cloudy} are the brightness temperatures of a clear column, for which the cost of radiative transfer is low, and a cloudy sub-column, which requires a much slower scattering calculation. Originally, rather than use an average cloud fraction in Eq. 1, RTTOV-SCATT used the maximum cloud fraction in the profile. However, it was found that this caused large positive biases in areas of tropical convection. Microwave brightness temperatures (TBs) are strongly affected by the heavy rain in tropical convective systems. However, these systems typically occupy only a small part of a grid box. In contrast, the outflow cloud associated with tropical convection often covers the whole grid-box but generally has little impact on TBs. Because microwave radiative transfer is non-linear (the 'beamfilling effect', e.g. Kummerow, 1998), using the maximum cloud fraction caused a substantial over-estimate of the influence of the convective areas on the grid-box average T_{allsky} , and hence produced TBs that were too high.

To avoid such problems, the revised approach introduced with v9.3 (Geer et al., 2009) calculates an average

Table 1: Cloud overlap approaches to be tested.

Name	Cloud overlap
2R	RTTOV-SCATT with original cloud fraction.
2N	RTTOV-SCATT with new cloud/precip fraction.
20IC	ICA with original cloud fraction.
20ICN	ICA with new cloud/precip fraction.

cloud fraction over the whole profile, weighting by the total hydrometeor amount:

$$C_{av} = \frac{\sum_i (l_i + i_i + r_i + s_i) C_i \Delta z_i}{\sum_i (l_i + i_i + r_i + s_i) \Delta z_i}.$$
(2)

Here, l_i , i_i , r_i , s_i represent the cloud liquid, cloud ice, rain and snow densities in kg m⁻³ and Δz_i is the layer thickness in m. C_i is the cloud fraction at each layer in the model. It is implicitly assumed that the cloud fraction applies also to the rain and snow amounts. However, in addition to the cloud fraction, the moist physics schemes in the IFS actually have separate large-scale and convective precipitation fractions. The convective precipitation fraction is always assumed to be 0.05. Large-scale precipitation initially has the same fraction as the cloud that produces it, but as it falls this may be modified by the input of precipitation at lower levels from clouds with different fractions and overlaps (Jakob and Klein, 2000). Hence, for consistency within the IFS, it would be better to use this information.

We will refer to the v9.3 cloud fraction approach (eq. 2) as '2R', with '2' indicating the number of independent columns used in the radiative transfer (one clear; one cloudy). A new approach taking into account the IFS precipitation fraction will be referred to as '2N', and calculates C_{av} as follows:

$$C_{av} = \frac{\sum_{i} \left((l_{i} + i_{i})C_{i} + (r_{i}^{c} + s_{i}^{c})C^{c} + (r_{i}^{l} + s_{i}^{l})C_{i}^{l} \right) \Delta z_{i}}{\sum_{i} (l_{i} + i_{i} + r_{i}^{c} + s^{c} + r_{i}^{l} + s^{l}) \Delta z_{i}}.$$
(3)

Here, C^c is the convective precipitation fraction, which is set to 0.05 to reflect the assumptions in the convective moist physics in the IFS. C_i^l is the large-scale precipitation fraction at layer *i*. Profiles of C_i^l and C_i are taken from the output of the IFS's large-scale moist physics parametrization. The precipitation profile has been split into convective and large-scale components respectively: r_i^c and r_i^l for rain and s_i^c and s_i^l for snow. Again, these are taken separately from the outputs of the convective and large-scale moist physics parametrizations.

To assess the differences between the 2R and 2N approaches, we will compare to observations and to reference simulations. We will use the independent column approximation (ICA) as a reference model. This represents sub-grid variability in radiative transfer simulations, by dividing the grid box (or satellite footprint) into a set of independent sub-columns with radiative transfer done separately for each. Cloud distributions in each sub-column are usually assigned according to an overlap scheme. Within each layer of a sub-column, cloud is either wholly present or wholly absent. Here, we use 20 equal sized sub-columns, each simulated separately using RTTOV-SCATT. O'Dell et al. (2007) performed a similar study to this one and showed the ICA was accurate enough to use as a reference.

With the ICA, the cloud and precipitation overlap is defined by how we distribute the cloud and precipitation profiles into the different sub-columns. We use two different approaches here. The first, '20IC', allocates cloud according to maximum-random overlap (e.g. Geleyn and Hollingsworth, 1979), with precipitation allowed to fall out of cloud into non-cloudy areas beneath. This approach was used as the reference model in Geer et al.

(2009). However, no distinction is made between convective and large-scale precipitation. Hence, to be more representative of the IFS, a second approach is used here which treats convective precipitation as occupying just 0.05 of the grid box. Large scale precipitation is treated with the same maximum-random overlap approach as in 20IC. This second approach is labelled '20ICN'. Table 1 summarises the different simulations made in this study.

3 Experiment setup

Using the framework of all-sky 4D-Var assimilation of microwave imagers, a passive monitoring experiment was used to simulate Special Sensor Microwave/Imager (SSM/I) radiances using first guess (FG) forecasts as input. Initial conditions for each forecast came from the analysis of the cycle 35r2 esuite. Forecasts were done at T799 horizontal resolution and were generated for a period of 18 days, from 11 to 28 February 2009.

Forward radiative transfer simulations were made at every model grid point and time-step where an SSM/I observation (from the satellite DMSP F-13 only) was available within 10 km of the grid point centre. The SSM/I field of view is 70 km by 45 km at 19 GHz, compared to a model grid box size of 25 km by 25 km at T799. The model timestep is 15 minutes. SSM/I field of view and model grid-boxes will be considered to be equivalent, and this is an assumption that has worked well for operational rain and cloud assimilation at ECMWF.

All-sky assimilation is limited to areas equatorward of 60°N and 60°S, and ocean only, avoiding land and sea-ice surfaces. High latitudes are excluded because the atmospheric signal gets small and the dominance of frozen, rather than liquid, cloud and precipitation makes radiative transfer difficult. Overall, a set of 1.1 million model profiles, radiative transfer simulations and colocated SSM/I observations was available. This is the set of data used when statistics are binned by latitude and longitude, and shown as maps.

To calculate 'global' statistics, SSM/I observations were further restricted to latitudes between 30° N and 45° S, because poleward of this, biases exist that would have degraded the results. However, the main effects of changing the cloud overlap are limited to the tropical and subtropical region anyway, which will be evident from the map plots. This restriction on latitude reduces the set of observations to 776,000. This is then further divided into two samples. The first sample is representative of rainy and heavy cloud areas, and will be referred to for convenience simply as 'rainy'. This sample contains all profiles where either the 2R simulations or the observations show a 37 GHz polarisation difference of less than 40 K (e.g. Petty, 1994). This selects 60,500 profiles. The second sample contains the other 715,500 profiles, and is representative of lighter cloud and clear sky conditions.

4 Results

4.1 Cloud fractions

Figure 1a shows the cloud fraction currently used inside RTTOV-SCATT, meaned over the 18-day experiment period, and taking in all-sky situations, whether clear, cloudy or rainy. Fractions above 0.5 are typical of the midlatitude storm-tracks and the subtropical marine stratocumulus regions such as the one off the west coast of South Africa. It may be surprising to see the very low cloud fractions (< 0.2) in the inter-tropical convergence zone (ITCZ). This reflects the fact that tropical convection is scattered in time and space, and that the majority of the total hydrometeor amount in a grid box is found inside the convective cores, which generally occupy only a small fraction of the area of a grid box. This is not to say that outflow cloud, for example, does not completely

a



Figure 1: (a) Average cloud fraction using the original approach, binned at SSM/I observation locations over 11 - 28th February 2009; (b) Change in cloud fraction: New approach minus original.

cover the sky. Instead, the vertically-weighted and averaged cloud fraction (Eq. 2) used by RTTOV-SCATT is intended to use an averaged cloud fraction that is representative of the most radiatively important clouds and precipitation.

Figure 1b shows the difference between the new vertically-averaged cloud fraction (Eq. 3) and the current one. There are two competing effects:

- In cases where there was both convective and large-scale cloud at certain levels, the original approach would implicitly take the convective precipitation fraction to be that of the large-scale cloud. Using an explicit convective precipitation fraction of 0.05, which is typically smaller than that of any large-scale cloud, overall decreases the vertically averaged cloud/precipitation fraction.
- The large scale precipitation fraction will, however, be larger now than it was before, because the precipitation retains the (typically relatively large) fraction of the cloud that it fell from, rather than taking on the (possibly much lower) cloud fraction of the air that it falls into.



Figure 2: Mean change in TB in simulations of SSM/I channel 19v: (a) 20ICA simulations, new approach (20ICN) minus original (20IC); (b) RTTOV-SCATT simulations, new approach (2N) minus original (2R)

The dominant effect is to increase the cloud fraction, but this net increase is relatively small: of order 0.05 in the ITCZ, and 0.1 in the midlatitude storm tracks.

Figure 2a shows the difference in channel 19v brightness temperatures (TBs) simulated by the 20IC and 20ICN approaches. In both cases, large-scale precipitation has been assigned according to maximum-random overlap, so what the figure shows is just the impact of restricting the convective precipitation fraction to 0.05. Given the non-linear dependence of microwave TBs on rain amounts, this would be expected to reduce TBs, and indeed it does, but the effect is really quite small, with a maximum 2 K in some parts of the ITCZ. The results are similar for other channels, although with slightly varying magnitudes of the patterns of difference.

Fig. 2b compares the 2R and 2N approaches and shows the combined impact of the net restriction of the convective precipitation fraction and the general expansion of the large-scale precipitation fraction. Expanding the large-scale fraction would generally increase TBs via the beamfilling effect. The overall effect on RTTOV-SCATT TBs is almost completely neutral. This suggests that the effect on TB of the changes to the convective fraction balance that of the large-scale fraction. This is in apparent contrast to the net increase in the average precipitation fraction (Fig. 1b). This is likely because a change to a convection-dominated profile, with typically very heavy amounts of precipitation and cloud, has a large influence on TB. In contrast, a profile dominated by large-scale precipitation may well only have very light rain and cloud, which has a smaller influence on TB.



Figure 3: a) Root mean square and b) mean errors in 2R (light grey) and 2N (white) simulated TBs, as compared to the 20ICN simulations, for the rainy sky sample.



Figure 4: As Fig. 3 but showing the clear and cloudy sky sample.

4.2 Independent column calculations as truth

Using the 20ICN simulations as truth, mean and RMS errors were calculated for the 2R and 2N simulations. These are presented, by channel, in Fig. 3 for the 'rainy' sample and Fig. 4 for the clear and cloudy sample. The RTTOV-SCATT two-column approximation (whether 2R or 2N) has RMS errors of 1 K to 6 K in rainy skies and 0.5 K to 1 K in clear and cloudy skies. As discussed in Geer et al. (2009), these errors are considered an acceptable trade-off, given the much greater computational speed of the two-column approach. O'Dell et al. (2007) have demonstrated approaches that do substantially better, but which require at least one extra scattering calculation to be made, hence increasing the computational cost by roughly 100%.

In general, 2N simulation errors are lower than those from 2R. This is what we should expect, since 2N and 20ICN both start from the same description of cloud and precipitation, with separate fractions for large-scale and convective precipitation. In contrast, 2R assumes the cloud fraction can be used to describe the precipitation fraction too. The difference between 2R and 2N in Figs. 3 and 4 give an idea as to the size of the errors resulting from this assumption.

Channel	Bias b [K]
19v	2.58
19h	1.35
22v	3.87
37v	-0.09
37h	0.74
85v	0.01
85h	1.23

Table 2: SSM/I observation minus ECMWF 20ICN first guess, meaned over 45°S to 30°N and 11 to 28 February 2009.

A feature in both Figs. 3b and 4b is the trend from marginally positive biases at low frequencies (e.g. 19 GHz) to generally negative biases at higher frequencies (e.g. 85 GHz). It is possible that this is associated with the increasing influence of snow scattering with frequency. The cloud fraction used in Eq. 2 is constant with frequency, but for better accuracy it could be varied to take account of the differing optical properties of the atmosphere at different frequencies (Geer et al., 2009). While desirable, such a change would require substantial technical modifications to RTTOV-SCATT.

There are also a few exceptions to the general pattern that 2N has smaller errors than 2R. The main ones are the 37 GHz v and h channels in heavy cloud or rainy conditions, where 2N shows slightly larger errors than 2R. The use of just a single scattering column simulation is always going to be imprecise. For example, it involves trading off the effects of the confined convective rain against those of the large-scale cloud and rain occupying much of the grid box. These channels are particularly sensitive to cloud, and it is likely that the trade-off is not perfect here. (Again, making the effective cloud fraction vary appropriately with frequency might help reduce errors here.) However, overall, the revised overlap simulations (2N) would be preferable to the original (2R).

4.3 Observations as truth

This section examines observation minus forecast (or 'departure') statistics for the different cloud overlap schemes. It is usual in data assimilation to find systematic biases between model and observations. Here, rather than apply the usual VarBC bias correction, we wanted to study the 'raw' biases between model and observations. However, we still wanted to remove the global mean bias from the calculations, assuming that this is likely to do with problems in the radiative transfer modelling which are equally likely to be found in clear or cloudy skies. Examples would be biases in surface emissivity calculations, imperfectly known instrument spectral response functions, or errors in the spectroscopy databases for WV lines.

The global mean bias, $b = \overline{T_{obs} - T_{sim}}$, has been calculated from the observed and simulated TBs T_{obs} and T_{sim} , including all observations in the region from 45°S to 30°N. Here, T_{sim} is taken from our reference 20ICN simulations. Results are calculated separately for each channel and are given in Table 2.

We remove these biases to prevent them inflating our rms statistics, so that in the following figures we are looking at the statistics of bias-corrected departures, d_k , as follows:

$$d_k = T_{obs}^k - b - T_{sim}^k.$$

$$\tag{4}$$

Here, k is an index for a single observation. The correction does no more than remove the 'global' offset between ECMWF and observed TBs.



Figure 5: a) Root mean square and b) mean bias-corrected departures in 2R (light grey), 20ICA (white), 2N (dark grey), 20ICN (stripes) simulated TBs, as compared to the SSM/I observations, for the rainy-sky sample

Figures 5 and 6 show the departure statistics for rainy and clear/cloudy samples respectively. Departure mean and rms statistics are typically 2-3 times larger for the rainy sample, indicating both the difficulty of predicting rain in the correct place, and the strong sensitivity of brightness temperature to rain and heavy cloud. However, it is very hard to find any significant differences between any of the schemes we have tested.

Figure 7 shows the spatial mean of channel 19v departures using either the 2R or 2N approaches. Again, it is difficult to tell the difference, but it is very obvious that the departures are affected by numerous regional biases of up to 3K in magnitude. In most cases, the likely candidate would be forecast model error, i.e. biases in the modelled cloud and precipitation. We are currently working to identify the causes of these biases and hopefully fix them, but we cannot yet say for certain what causes them. It is worth first looking at the channel 19v rainy-sample rms errors compared to 20ICN, which reduce from roughly 3.0 K to 2.6 K when going from 2R to 2N (Fig. 3a). In contrast, comparing to SSM/I observations, rms departures are more than 10 K (Fig. 5a). It appears that model errors, both systematic and random, dominate the comparisons with SSM/I, and even if the revision to the cloud overlap scheme does make an improvement, it is lost amongst the other sources of error.

4.4 Sensitivity to convective precipitation fraction

The IFS moist physics assumes that convection occurs in 0.05 of the grid box. This is a fairly arbitrary assumption, and it is worth testing the sensitivity of the revised cloud fraction to this. Hence, we ran another set of simulations setting $C^c = 0.1$ in Eq. 3. The differences in the vertically-averaged cloud fraction (Fig. 8) are extremely small (typically 0.01) compared to those between the original and revised cloud and precipitation approaches (typically 0.07; see Fig. 1b). These increases in the vertically averaged cloud fraction cause slightly larger TBs (Fig. 9), via the beamfilling effect. As might be expected, however, the effect on TB simulations is rather small. Cloud and precipitation overlap in microwave radiative transfer



Figure 6: As Fig. 5 but for the clear and cloudy sky sample

5 Conclusion

Scattering radiative transfer calculations are used in the all-sky assimilation of microwave imager observations, which are sensitive to the sea surface, moisture, cloud and precipitation. However, performance limitations mean that only one scattering radiative transfer simulation can be made per observation. Hence, for accurate simulations it is important to treat the cloud and precipitation overlap carefully. A previous upgrade to the cloud overlap in the radiative transfer code (RTTOV-SCATT) produced unambiguous improvements in the quality of our radiative transfer simulations, reducing FG departure biases substantially in rainy areas (Geer et al., 2009). However, the upgraded cloud overlap was not fully consistent with the way cloud and precipitation overlap is represented in the moist physics of the IFS. In particular, precipitation was assumed to be confined to the area given by the local layer's cloud fraction. However, precipitation has typically fallen from higher in the atmosphere, so its fraction may be very different from that of the local cloud layer.

In the IFS, there are actually three different cloud or precipitation fractions. First, the 'cloud fraction' applies to large-scale cloud. Second, convective precipitation is assumed to take place in a fixed 0.05 fraction of the grid box area. Third, large-scale precipitation initially has the same fraction as the cloud that produces it, but as it falls this may be modified by the input of precipitation at lower levels from clouds with different fractions and overlaps. ¹ Good practice would suggest that the representation of cloud in the radiative transfer should be as consistent as possible with that in the moist physics. ²

We explored the impact of going from the current representation of cloud and precipitation in RTTOV-SCATT

¹It is worth remarking that there should actually be a fourth fraction, applying to convective cloud, but this cloud does not explicitly exist in the IFS; the only 'convective' cloud that is explicitly represented is the detrainment into the surrounding environment, but this is actually represented by the large-scale cloud scheme. This may be a problem for rain and cloud-affected microwave simulations, but it is a problem to be dealt with another time.

²However, there is not yet a consistent overlap approach through the IFS. The radiation scheme (Morcrette et al., 2008) and the diagnostic computation of column cloud amounts use generalised cloud overlap (Räisänen et al., 2004). Precipitation overlap in the large-scale moist physics scheme uses Geleyn and Hollingsworth (1979) maximum-random overlap. However, due to a 'feature' in the ECMWF implementation of maximum-random overlap, the difference between this and generalised overlap is surprisingly small. See Forbes (2008) for information.



Figure 7: Mean SSM/I channel 19v FG departures using (a) 2R and (b) 2N simulations.

('2R') to one that is consistent with the moist physics ('2N'). Compared to reference radiative transfer simulations using the Independent Column Approach, rms errors are typically reduced, for example going from 3.0 Kto 2.6 K in rainy areas in channel 19v. However, SSM/I FG departures are not significantly affected. This is explained by the fact that FG departures are dominated by systematic model biases and by forecast errors (e.g. the inability to simulate cloud and rain in exactly the right place and time). For rainy channel 19v observations, this may be equivalent to about 10 K in brightness temperature, and against this it would be hard to see an improvement of 0.4 K in the simulations, even if all that theoretical improvement were to occur in practice too.

Despite the lack of impact on the FG departures, we still hope to implement this change operationally. It is sensible to keep the treatments of cloud in the forecast model and in the radiative transfer as consistent as possible. RTTOV-SCATT has been modified in such a way that the user may now, if required, determine for themselves the vertically-weighted average cloud fraction used in the scattering radiative transfer, based on their own cloud and precipitation overlap. This feature may be beneficial for other users of RTTOV-SCATT, such as the Met Office, and it will be included in the RTTOV-10 release.

For the future, the most promising areas to further improve the accuracy of cloud and precipitation overlap in RTTOV-SCATT are:

- As discussed by Geer et al. (2009), to make the effective cloud fraction (e.g. Eq. 3) vary with frequency, taking into account the changes in vertical weighting function and radiative influence of rain, cloud and snow.
- When computer performance allows, to move to an O'Dell et al. (2007) approach, which is much more



Figure 8: Change in cloud fraction: New approach using 0.1 convective precipitation fraction minus new approach using 0.05.



Figure 9: Mean change in TB in 20ICA simulations of SSM/I channel 19v: New approach using 0.1 convective precipitation fraction minus new approach using 0.05.

accurate than the current two-column approach (with independent column simulations as truth) but roughly 100% slower.

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