Diagnostics of model numerical cores

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Introduction

♦ **Reductionism**: Assume that essential flow characteristics remain unchanged when the domain, the forcings or the equations are changed. Typically, a complete physical similarity is precluded, but an approximate dynamic similarity can be of substantial practical importance (*Buckingham, 1914*).

♦ Often a hierarchy of numerical model setups with gradually increased complexity is used.

♦ Diagnostics include the comparison with analytic solutions, with large-eddy or global-scale simulation benchmarks, or with geophysical laboratory experiments and direct numerical simulations (DNS) thereof.
Hierarchy of reduced models

- Single-column model (1-D, model physics)
- X-Z slice model (2-D, multi-layer, physics and/or dynamics)
- Shallow water model (2-D, single layer, model dynamics)
- The full numerical model (3-D, multi-layer, physics and/or dynamics)
  - Limited-area domain simulations
  - Reduced-size planet simulations
  - Held-Suarez “model climate” simulations
  - Aquaplanet “model climate” simulations
  - DNS/LES of a laboratory experiment
What is the model dynamical core?

- Historically, the “dynamical” (dry) part of the model without any diabatic forcings (i.e. physics).

- However, in NWP with increased resolution O(0.1-10km) more and more of the physics (e.g. gravity waves, moist processes, convection) are resolved and it becomes increasingly ambiguous if a parametrized (subgrid-scale) term should be computed at all and if it should be computed in the “physics” interface or the “dynamics”.
The shallow water model

\[
\frac{\partial \mathbf{v}_h}{\partial t} + \mathbf{v}_h \cdot \nabla \mathbf{v}_h + f \mathbf{k} \times \mathbf{v}_h + g \nabla h = 0
\]

\[
\frac{\partial h}{\partial t} + \mathbf{v}_h \cdot \nabla h + h \nabla \cdot \mathbf{v}_h = 0
\]

♦ Justification as a reduced model (Taylor, 1936; Matsuno, 1966)

♦ Published test cases Williamson et. al., JCP 102, p. 211-224 (1992)

♦ Example 1: Advection of cos-bell hill over the pole

♦ Example 2: The Rossby-Haurwitz wave and the influence of the lunar gravitational potential
Example 1: Advection of Cosine bell over the pole (alpha=90 degrees)

IFS-SHW

\[ h_0 = 1000 \text{m} \]

Initial

After 12 days == 1 rotation

Difference
Example 1: Advection of Cosine bell over the pole (alpha=90 degrees)

Error measures graphed as a function of time or as a function of resolution

\[
\begin{align*}
    l_1(h) &= \frac{I[|h(\lambda, \theta) - h_T(\lambda, \theta)|]}{I[|h_T(\lambda, \theta)|]} \\
    l_2(h) &= \sqrt{\frac{I[(h(\lambda, \theta) - h_T(\lambda, \theta))^2]}{I[h_T(\lambda, \theta)^2]}} \\
    l_\infty(h) &= \max_{\lambda, \theta} \frac{|h(\lambda, \theta) - h_T(\lambda, \theta)|}{\max_{\lambda, \theta} |h_T(\lambda, \theta)|}
\end{align*}
\]

Example: Semi-implicit spectral element shallow water model on the cubed sphere
Example 2: The Rossby-Haurwitz wave ... and the influence of the Moon

♦ Rossby-Haurwitz waves (here wavenumber 4) are steadily propagating solutions of the fully nonlinear nondivergent barotropic vorticity equation on the sphere (Haurwitz, 1940)
**Example 2: The Rossby-Haurwitz wave ... and the influence of the Moon**

- The analytic Rossby-Haurwitz wave initial condition is assumed to evolve similarly steady with the shallow water equations.

- The Rossby-Haurwitz wave is found to be unstable for wavenumbers \( \geq 4 \) (Lorenz 1972, Hoskins, 1973, Thuburn and Li, 2000)

- Is the gravitational pull of the Moon sufficient to trigger such an instability of the Rossby-Haurwitz wave?

\[
\begin{align*}
\frac{du}{dt} &= \ldots - \frac{1}{a \cos \theta} \frac{\partial}{\partial \lambda} \Omega_{\text{tide}} \\
\frac{dv}{dt} &= \ldots - \frac{1}{a} \frac{\partial}{\partial \theta} \Omega_{\text{tide}}
\end{align*}
\]

(e.g. Chapman and Lindzen 1970)

**Animation:**

(e.g. Chapman and Lindzen 1970)
Example 2: The Rossby-Haurwitz wave ... and the influence of the Moon

Does the lunar gravitational potential matter for medium-range weather forecast?

\[
\bar{s} = \frac{1}{K} \sum_{k=1}^{K} \sigma_s (x_{control}^k - x_{moon}^k)
\]

number of forecast cases

spatial standard deviation of the forecast difference

(see Jung and Vitart, MWR, Vol.106, 2006 for details)
Local- and synoptic-scale simulations on the sphere ...

The size of the computational domain is reduced without changing the depth or the vertical structure of the atmosphere by changing the radius \( (a < a_{\text{Earth}}) \)

Scale analysis for NH local-scale problems

\[
\frac{du}{dt} = 2\Omega(v \sin \phi - \delta w \cos \phi) + \frac{uv}{\Gamma a} \tan \phi - \delta \frac{uw}{\Gamma a} - \frac{1}{\rho \Gamma a \cos \phi} \frac{\partial p}{\partial \lambda}
\]

\[
\frac{dv}{dt} = -2\Omega u \sin \phi - \frac{u^2}{\Gamma a} \tan \phi - \delta \frac{vw}{\Gamma a} - \frac{1}{\rho \Gamma a} \frac{\partial p}{\partial \phi}
\]

| \(|U^2/L|\) | \(|f_0 U|\) | \(|f_0 W|\) | \(|U^2/a|\) | \(|UW/a|\) | \(|\Delta p/\rho L|\) |
|---|---|---|---|---|---|
| 10^{-1} | 10^{-3} | 10^{-4} | 10^{-3} | 10^{-4} | 10^{-1} |

\(a \approx 100\text{km}\)

\[
\frac{dw}{dt} = \delta 2\Omega u \cos \phi + \delta \frac{u^2 + v^2}{\Gamma a} - \frac{1}{\rho} \frac{\partial p'}{\partial r} - g \frac{\rho'}{\rho}
\]

| \(|UW/L|\) | \(|f_0 U|\) | \(|U^2/a|\) | \(|dP'/\rho H|\) | \(|N^2 H|\) |
|---|---|---|---|---|
| 10^{-2} | 10^{-3} | 10^{-3} | 10^{-2} | 10^{-2} |

\(a \approx 100\text{km}\)
Test-bed for NH effects

- 3D global simulations, without the prohibitive cost, when resolving non-hydrostatic effects.
- Study the influence of the model formulation and/or various numerical choices on selected wave-types in three dimensions.
- Use of the established vertical discretization and/or physical parameterization packages.
- Use of the existing optimized 3D code framework.
Description of experiments

♦ All IFS experiments conducted in reduced-grid $T_L 159$ or $T_L 255$ configuration (320 or 512 gridpoints along equator).

♦ Comparison with analytic solutions and with the all-scale research code EULAG on the sphere.
Hydrostatic approximation

\[ \frac{dw}{dt} - \frac{1}{\rho} \frac{\partial p}{\partial z} - g \rightarrow \frac{1}{\rho} \frac{\partial p}{\partial z} = -g \]

Filters vertically propagating acoustic waves

\( w \) is obtained \textit{diagnostically} from the continuity equation.
Acoustic waves

- The nonhydrostatic, fully elastic model admits the propagation of sound waves
- Comparison against analytic solution
- Impact of the semi-implicit formulation on the propagation
A spherical acoustic wave in a stratified (isothermal) atmosphere

analytical solution from (Landau and Lifshitz, Fluid Mechanics, §70.)
A sound wave in which the distribution of density, velocity and other flow variables only depends on the distance from some point.

\[ C_0 = 340 \text{m/s} \]
A spherical acoustic wave in a stratified (isothermal) atmosphere

analytical solution in dashed line

- t=0
- t=50s
- t=50s
- t=50s

explicit, dt=0.01s

explicit, dt=0.01s

Semi-implicit, dt=1s

Semi-implicit, dt=10s

NH-IFS T_L 159L91

Nils Wedi, ECMWF 2009  Slide 18
Gravity waves

♦ Propagation
♦ Drag
Quasi two-dimensional orographic flow with linear vertical shear

The figures illustrate the correct horizontal (NH) and the (incorrect) vertical (H) propagation of gravity waves in this case (Keller, 1994). Shown is vertical velocity.
The critical level effect on linear and non-linear flow past a three-dimensional hill

\[ Ri = 1 \]

(see Grubisic and Smolarkiewicz, 1997)

\[ Ri = \left( \frac{N z_c}{U_0} \right)^2, \hat{h} = \frac{h_0 N}{U_0} \]

\[ U(z) = U_0 (1 - z/z_c) \]
The critical level effect on linear and non-linear flow past a three-dimensional hill

\[ \frac{U_0}{NL_\lambda} = 1 \]

EULAG

H-IFS

NH-IFS
Mountain drag

$U_0 / N L_\lambda = 0.2$

$U_0 / N L_\lambda = 1$
Convective motion (3D bubble test)

Hydrostatic-IFS after 1000s

warm

cold

Neutral stratification

NH-IFS

EULAG

0s

1000s
Convective motion (3D bubble test)

1800s

2400s

NH-IFS

EULAG
Held-Suarez “model climate” simulations

Held and Suarez (BAMS, 1994)

\[ \frac{d\nu}{dt} = ... - k_v \nu \] \hspace{1cm} \text{Augment frictional/heating terms}

\[ \frac{dT}{dt} = ... - k_T (T - T_{eq}) \]

\[ T_{eq} = \max \left\{ 200K, [315K - (\Delta T)_y \sin^2 \phi - (\Delta \theta)_z \log \left( \frac{p}{p_0} \right) \cos^2 \phi] \left( \frac{p}{p_0} \right)^\kappa \right\} \]

meridional temperature gradient pole - equator

vertical temperature gradient at equator
Held-Suarez simulations / Rossby waves

- Zonal mean diagnostics
- Temporal anomalies
- Kinetic energy spectra
- Wavenumber frequency spectra
Held-Suarez climate

\[ a = a_{Earth}/10 \]

\[ \Omega = 10 \times \Omega_{Earth} \]

zonal-mean

zonal wind U
Held-Suarez climate

\[ a = \frac{a_{\text{Earth}}}{10} \]

\[ \Omega = 10 \times \Omega_{\text{Earth}} \]
Held-Suarez climate \( a = a_{\text{Earth}}/10 \)

Temporal anomalies of zonal wind

Averaged 30°N-50°N

IFS

EULAG
Held-Suarez climate

wavenumber-frequency diagrams of zonal wind

e.g. Rossby wave dispersion:

$$\omega_R \approx \frac{2\Omega N^2 m}{n(n+1)N^2 + f^2 a^2 (\delta_A N^2 / c_s^2 + k_z^2 + \Gamma_A^2)}$$
The influence of the shallow vs deep atmosphere model formulation

\[ \Omega = 20 \times \Omega_{Earth} \quad a = a_{Earth}/20 \]

**IFS** Held-Suarez simulation with a shallow hydrostatic model

*Simmons and Burridge 1981*
The influence of the shallow vs deep atmosphere model formulation

\[ \Omega = 20 \times \Omega_{Earth} \quad a = a_{Earth}/20 \]

IFS Held-Suarez simulation with a deep hydrostatic model following *White and Bromley, 1995*
The influence of the shallow vs deep atmosphere model formulation

\[ \Omega = 20 \times \Omega_{Earth} \quad a = a_{Earth}/20 \]

IFS Held-Suarez simulation with a deep non-hydrostatic elastic model following Wood and Staniforth 2003
The influence of the shallow vs deep atmosphere model formulation

$$\Omega = 20 \times \Omega_{Earth} \quad a=a_{Earth}/20$$

**EULAG** Held-Suarez simulation with a deep non-hydrostatic anelastic model, Wedi and Smolarkiewicz, 2009
Held-Suarez Tropics

IFS - Temporal anomalies of velocity potential

600hPa

200hPa
5-year Held-Suarez (dry) simulations: (temporal) anomaly of velocity potential 200hPa

Eastward propagation with periods of 39-66 days!
Aquaplanets

Neale, R.B. and B.J.Hoskins, 2000a:

♦ Different SST distributions force the flow

♦ 3 year “model climate”

♦ APE intercomparison project with an ATLAS of many different models/parameters in preparation

http://www.met.reading.ac.uk/~mike/APE/
Courtesy of D. Williamson
Courtesy of D. Williamson
Tropical Wave Spectra (control) Courtesly of Mike Blackburn

Log-power of symmetric modes of equatorial precipitation (10°N - 10°S average)
Tropical Wave Spectra (ECMWF)  Courtesy of Mike Blackburn

- Recent model changes to the convection parametrization (cy32r3, November 2007, Bechtold et al. 2008)
- Precipitation organised strongly in equatorial waves
Aquaplanet control: precipitation

(Courtesy of Peter Bechtold)

Note the unrealistically high zonally averaged total precipitation rates (mm/day) in 29r2.

The theoretical phase speed for Kelvin waves is 15-20 m/s, here we roughly see 18 m/s in 32r3.
Aquaplanet control: Hadley cell and CAPE
(Courtesy of Thomas Jung)

\[
\text{CAPE} = \int_{z_1}^{z_n} g \left( \frac{T_{\text{parcel}} - T_{\text{env}}}{T_{\text{env}}} \right) \, dz
\]
Higher resolution models

average $5^\circ$N-$5^\circ$S

pre-HadGAM1
N96 L38
1.25° x 1.875°

IFS Cy29r2
$T_L$159 L60
~125km grid

GME
Icosahedral L31
~100km grid

NICAM
Icosahedral L54
7km grid

non-hydrostatic
no convective param.

Courtesy of D. Williamson
Tropical Variability (precipitation)  
average 5ºN-5ºS  

Courtesy of D. Williamson
Variability of organized convection

- The formation of organized convection is sensitive to explicit or implicit viscosity in under-resolved simulations.

- The sensitivity is consistent with linear theory adapted to include the effect of anisotropy of viscosity (horizontal vs. vertical) at moderate Rayleigh number.


- It suggests a careful control of the effective numerical viscosity in the numerical core! (such as the dependence of the truncation error on the derivatives of the flow variables rather than the flow variables themselves).
DNS of laboratory experiment: The QBO

- The laboratory experiment shows the pure gravity wave acceleration of the zonal mean zonal flow.
- The setup represents a stringent test of the model numerical core.
The stratospheric QBO

- westward
+ eastward

(unfiltered) ERA40 data (Uppala et al, 2005)
The laboratory experiment of Plumb and McEwan

- The principal mechanism of the QBO was demonstrated in the laboratory
  Plumb and McEwan, J. Atmos. Sci. 35 1827-1839 (1978)

- University of Kyoto

Animation:  

(Wedi and Smolarkiewicz, J. Atmos. Sci., 2006)
Time – height cross section of the mean flow $U$ in a 3D simulation

Animation
Schematic description of the QBO laboratory analogue

(a) Waves propagating right

\[ S = -8 \]

(b) Waves propagating right

\[ S = +8 \]

(c) Waves propagating left

(d) Waves propagating left

Waves propagating right

Critical layer progresses downward

Wave interference

Waves propagating left

Critical layer progresses downward

Wave interference

\[ +U \] filtering

\[ -U \] filtering
**Viscous simulation**

**Eulerian**

**semi-Lagrangian**

![Graphs showing viscosity simulation in Eulerian and semi-Lagrangian methods.](image)
Local Richardson number analysis

\[ Ri = -\frac{g}{\rho} \frac{\partial \rho}{\partial z} \frac{1}{\left(\frac{\partial u}{\partial z}\right)^2} \]

Regions of wave overturning and breaking
Inviscid simulation

Eulerian
St=0.36, n=640
Top absorber

Eulerian
St=0.25, n=384
Top rigid, freeslip

semi-Lagrangian
St=0.25, n=384
Top rigid, freeslip
Numerical realisability of wave driven flows

Influence on the period and the vertical extent of the resulting zonal mean zonal flow changes

- Lower horizontal resolution results in increasing period (\(~16\) points per horizontal wavelength still overestimates the period by 20-30%)
- Lower vertical resolution results in decreasing period and earlier onset of flow reversal as dynamic or convective instabilities develop instantly rather than previously described wave-wave mean flow interaction (\(\text{need} \sim 10-15\) points per vertical wavelength, \(<5\) no oscillation observed)
- First or second order accurate (e.g. rapid mean flow reversals with 1\(^{\text{st}}\) order upwind scheme)
- A low accuracy of pressure solver may result in spurious tendencies with a magnitude similar to physical buoyancy perturbations and are due to the truncation error of the Eulerian scheme; equally explicit vs. implicit formulation of the thermodynamic equation results in distorted longer mean flow oscillation (explicit may be improved by increasing the vertical resolution)
- Choice of advection scheme (flux-form Eulerian more accurate)
- Upper boundaries and stratification changes (may catalyze the onset of flow reversal; also in 2D Boussinesq experiments due to wave reflection, in atmospheric conditions also changing wave momentum flux with non-Boussinesq amplification of gravity waves)
Summary

- Introduced the concept of reductionism and a hierarchy of reduced models with gradually increased complexity to test a numerical model core but even more so to test the model assumptions and our understanding!
Additional slides
Conclusions

♦ Suitable for testing 3D non-hydrostatic developments in an existing hydrostatic technical framework with minimal effort and computational cost.

♦ Reduced-size planet experiments offer exciting new avenues for research with complex NWP models, while further testing the code for potential future applications.

see also Q. J. R. Meteorol. Soc. 135: 469-484 (2009) for more details!
<table>
<thead>
<tr>
<th><strong>IFS</strong> <a href="http://www.ecmwf.int/research">www.ecmwf.int/research</a></th>
<th><strong>EULAG</strong> <a href="http://box.mmm.ucar.edu/eulag">http://box.mmm.ucar.edu/eulag</a></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time integration is semi-implicit or iterative-centred-implicit (ICI), coupled to a two-time-level semi-Lagrangian advection on a reduced Gaussian grid, using the spectral transform method and a direct solver for the resulting Helmholtz equation.</td>
<td>Time integration is an implicit, two-time-level, flux-form Eulerian or semi-Lagrangian fluid solver on a regular grid, with an iterative, preconditioned conjugate residual algorithm for solving the resulting elliptic pressure equation.</td>
</tr>
<tr>
<td>Terrain-following, mass-based coordinate</td>
<td>Terrain-following, height coordinate</td>
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<td>Fully compressible</td>
<td>Anelastic/Boussinesq/other</td>
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<td>Spectral horizontal representation with spherical harmonics basis functions; finite-difference (NH) / finite element (H) vertical discretization</td>
<td>Non-staggered/staggered gridpoint representation, finite-differences</td>
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<tr>
<td>Deep/shallow atmosphere</td>
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Cost-effectiveness

- The cost of a resolution of 1km globally, $O(\sim T_L 20000)$, is approximately $300 \times T_L 2047$ (approx. 10 km) and currently not affordable.

- However, reducing the radius by a factor 10 (even accounting for a corresponding reduction in time-step) reduces the problem size to make 1km globally affordable now, which in return opens up intriguing opportunities for research in the future on initial and boundary conditions for small planets... ?!
A multiphase example (Sylvie Malardel)

\[ \rho = \rho_d + \rho_v + \rho_c + \rho_i + \rho_r + \ldots \]
\[ c_v = (1 - q_v - q_c - q_i - q_r)c_{vd} + q_v c_{vv} + (q_c + q_r)c_{liq} + (q_i + \ldots)c_{sol} \]
\[ c_p = (1 - q_v - q_c - q_i - q_r)c_{pd} + q_v c_{pv} + (q_c + q_r)c_{liq} + (q_i + \ldots)c_{sol} \]
\[ R = R_d - (R_v - R_d)q_v - R_d (q_c + q_i + q_r + \ldots) \]

\[ p = \rho RT \]

Hydrostatic Model

\[ \frac{Dv}{Dt} = \ldots \]
\[ \frac{D}{Dt} q_{(k=v,c,i,r,\ldots)} = P_{phys,q_k} \]
\[ \frac{DT}{Dt} - \frac{RT}{c_p} \frac{Dp_{hyd}}{Dt} = \frac{Q}{c_p} = P_{phys,T} \]
\[ \frac{Dp_{hyd}}{Dt} = -\int \nabla_p \cdot v \]
A multiphase example (Sylvie Malardel)

Nonhydrostatic Model
(note the additional prognostic equations for pressure and vertical velocity)

Physics or Dynamics?

currently set to 0 in IFS
Held-Suarez: Time variance of stationary zonal mean zonal flow

$T_y=60K$

$T_y=80K$

$29r^2$

mf_tm_uu
(temporal) anomaly of velocity potential 200hPa

Aquaplanet Control

Full IFS ‘climate’ simulation
Aqua-planet experiments with (a) flat, (b) peaked and (c) constant SST distribution.

(5-day low-pass filtered)