Another look at Proper Scoring Rules

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Outline

- Some history of proper scores
- Theory and experiment – localized ensemble scores
- Is properness important?
Definition of “proper”

- A proper scoring rule is a score for which a forecaster obtains a best score value by forecasting according to his/her true beliefs.
- A strictly proper scoring rule results in best score value only if the forecaster forecasts according to his/her true beliefs.
Some history

- Most scores for probability forecasts of categorical variables proved to be proper in late 1960s and early 70s by Murphy and Stael von Holstein (several papers)
  - Brier score, rank probability score
  - Ratio skill scores asymptotically proper, but often improper in the way calculated
  - Linear probability error is improper
- Recent revival of interest in proper scores for verification of ensemble distributions
  - E.g. Gneiting and Raftery, paper accepted by J. Amer Statistical Assn.
  - Comprehensive theoretical review: Scores for pdfs which are non-linear are proper, linear scores are not.
Results from Gneiting and Raftery, 07

Inflation factor of standard deviation for several scores

Based on U Wash ensemble forecasts for temperature at 160 stations.
Proper Experiment

- Two scores:
  - Probability score (Wilson et al, 1999)
  - Ignorance score (Roulston and Smith, 2002)

\[ \text{IGN} = -\log_2(P(T=T_o +/- 1)) \]

\[ P(T_{obs} \mid T_{eps}) = \int_{T-\Delta T}^{T+\Delta T} f(T_{eps})dT \]
Proper Experiment (2)

- Data:
  - 90 days temperature ensemble forecasts, 16 members;
    209 Canadian stations (~18000 cases)
- Assumed normal distribution
- Two methods:
  - multiplied sd by factors up to 5 and divided by up to 20 + ensemble mean
  - Truncated tails of normal distribution, added to central part.
Proper experiment (3) - PS
Proper experiment (4)
Proper experiment (5)
Discussion - Is properness important?

• An alternative view:
  – Is the design of a proper score needed to offset the advantage of using prior knowledge about the variable being predicted?
  – Importance of properness vs. convenience and user-understanding of score.
  – Use of “nearly proper” scores.

• Next step
  – CRPS on same sample.
Thank you!