Stochastic Modelling of the Oceanic Eddies

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1 Introduction

It is widely recognized that the mesoscale oceanic eddies are capable of driving the large-scale oceanic currents. These eddies are generally not resolved in comprehensive oceanic General Circulation Models (GCMs), therefore the models are rather inaccurate. Developing efficient mathematical models of the eddies is, arguably, the most important problem facing the modern physical oceanography. The eddy effects can be roughly groupped into two classes: dynamic and kinematic. The dynamic effects are associated with divergence of the eddy potential vorticity (PV) or momentum fluxes. The kinematic effects are associated with transport of material quantities. Below, some new stochastic models for both kinds of effects are discussed. The main advantage of the proposed model is their ability to simulate ubiquitous non-diffusive and anti-diffusive oceanic eddy effects, which are a fundamental obstacle in ocean circulation and turbulence closures. The results are tested against the double-gyre, fluid-dynamic ocean model that explicitly resolves the eddies (e.g., Siegel et al. 2001).

2 Dynamic Effects

The random-forcing eddy model is developed in Berloff (2005a,b). The new diagnostic method is proposed, which is based on dynamical decomposition of the flow into the large-scale and eddy components. The method yields the time history of the eddy forcing, which can be used as additional, external forcing in the corresponding non-eddy-resolving model of the gyres. The main strength of this approach is in its dynamical consistency: the non-eddy-resolving solution driven by the eddy forcing history correctly approximates the original large-scale flow component. It is shown that statistical decompositions, which are based on space-time filtering diagnostics, are dynamically inconsistent. The diagnostics algorithm is formulated and tested, and the diagnosed eddies are analysed, both statistically and dynamically. It is argued that the main dynamic role of the eddies is to maintain the eastward-jet extension of the subtropical western boundary current. This is done largely by both the time-mean isopycnal-thickness flux and the eddy flux fluctuations. The fluctuations drive large-scale flow through the nonlinear rectification mechanism. The time mean relative-vorticity flux contributes mostly to the eastward jet meandering. Finally, eddy fluxes driven by both the eddies and the large-scale flow are found to be important. The latter is typically neglected in the analysis, but here it corresponds to important large-scale feedback on the eddies.

Enhancement of the eastward-jet extension of the subtropical western boundary current by the eddies is an anti-diffusive process, which can not be represented in terms of turbulent diffusion. However, it is shown that the eddy forcing history can be approximated as a space-time correlated, random-forcing process in such way that the non-eddy-resolving solution correctly approximates the eddy-resolving one. Thus, the random-forcing model can potentially replace the diffusion model, which is commonly used to parameterise eddy effects on the large-scale currents. The eddy forcing statistics are treated as spatially inhomogeneous but stationary, and the dynamical roles of space-time correlations and spatial inhomogeneities are systematically explored. The integral correlation time, oscillations of the space correlations, and inhomogeneity of the variance are found to be particularly important for the flow response.

Nonlinear rectification of the ocean circulation driven by random forcing, which simulates the effect of unresolved eddies, is studied in an idealized closed basin (Berloff 2005c). The results are based on the analysis of randomly forced solutions and linear eigenmodes. Depending on the forcing strength, two rectification regimes are found: zonal jets and isolated gyres. It is shown that both regimes are due to nonlinear interactions of resonant basin modes. In the zonal-jet regime, these interactions involve complex interplay between resonant baroclinic modes and some secondary modes. Both Rhines' scaling for zonal jets and prediction of gyres based on the maximum entropy argument are not confirmed.

Next, the role of mesoscale oceanic eddies is analyzed in a quasigeostrophic coupled ocean-atmosphere model operating at a large Reynolds number (Berloff et al. 2005). The dominant mode of variability in the model is due to oceanic response to transitions between two distinct wind-forcing regimes associated with intrinsic atmospheric dynamics. Ocean eddies affect the adjustment process via nonlinear rectification of the oceanic eastward jet in a way to create long-term sea surface temperature anomalies that effectively feed back on the atmospheric flow. These eddies are then parameterized in a coarse-resolution ocean model as a nonstationary random process, whose properties are derived from the control coupled simulation and depend on the ocean state. A coupled model with such a non-eddy-resolving ocean component simulates climatology and low-frequency variability of the eddy-resolving coupled solution. So far, this solution is the most comprehensive one: it contains stochastically represented eddies; it incorporates both ocean and atmosphere components; and parameters of its stochastic component are closed in terms of the large-scale flow states.

3 Kinematic Effects

The material-transport eddy models are developed in Berloff and McWilliams (2003, 2002) for simulating observed transports of material by turbulent flow in the presence of coherent fluid structures, and they use only few internal parameters characterizing particular type of turbulence. The analyses are based on ensembles of Lagrangian particle trajectories that can be interpreted as passive-tracer concentration. It is found that the transport by mesoscale eddies differs in many ways from the commonly used model of homogeneous, isotropic eddy diffusion. The single-particle dispersion, which describes the spreading process, is generally anisotropic and inhomogeneous, and in most places it is not diffusive during intermediate-time intervals after tracer release. In most of the basin and especially in the deep layers, sub-diffusive single-particle dispersion occurs due to long-time trapping of material by coherent structures such as vortices near the strong currents and planetary waves in the eastern part of the gyres. Super-diffusive dispersion behavior is found in the western part of the subtropical gyre and near the boundaries. Sub- and super-diffusion are associated with a strong first negative and second positive lobe, respectively, in the Lagrangian velocity autocorrelation function. The two-particle dispersion, which describes the mixing process, is characterized by initial exponential growth, and its exponent has strong geographical inhomogeneities, with faster rates in the upper western gyres.

A hierarchy of inhomogeneous, non-stationary stochastic models of material transport is formulated, and its properties are described. The transport models from the hierarchy sequence provide progressively more skillful simulations of the subgrid-scale transport by mesoscale eddies. The stochastic transport models yield random motion of individual passive particles, and the probability density function of the particle population may be interpreted as the concentration of a passive tracer. Performance of the models is evaluated by (a) estimating their parameters from Eulerian and Lagrangian statistics of a fluid-dynamic reference solution, (b) solving for the transport, and (c) comparing the stochastic and fluid-dynamic transports. The simplest, but least skillful, member of the hierarchy is the commonly used diffusion model, which is equivalent to the random-walk process for particle positions. The higher-order members of the hierarchy are the Markov-1 (a.k.a. Langevin or random acceleration), Markov-2, and Markov-3 models, which are jointly Markovian for particle position and its time derivatives. Each model in the hierarchy incorporates all features of the models below it. The Markov-1 model simulates short-time ballistic behavior associated with exponentially decaying Lagrangian velocity correlations, but on large times it is overly dispersive because it does not account for trapping of material by the coherent structures. The Markov-2 model brings in the capability to simulate intermediate-time, sub-diffusive spreading associated with such trappings and with both decaying and oscillating Lagrangian velocity correlations. The Markov-3 model is also capable of simulating intermediate-time, super-diffusive spreading associated with sustained particle drifts combined with the trapping phenomenon and with the related asymmetry of the decaying and oscillating Lagrangian velocity correlations. The mathematical formalism behind the stochastic-model hierarchy is based on the theory of autoregressive processes and on the derivation of the well-mixedness condition.

The hierarchy can be advanced by broadening the range of simulated motions and by allowing transitions from one type of motion to another through the stochastic randomization (Berloff et al. 2002). The randomization implies that the parameter is represented by a probability distribution rather than a fixed average value. The probability distribution represents different populations of mesoscale fluctuations co-existing within a geographical region. It is shown that the randomized model performs systematically better than the non-randomized one, although only modestly so in some transport measures. Other ideas for advancing the hierarchy may include accounting for correlations between different components of the velocity or acceleration vector (Reynolds 2002) and generalizing the hierarchy for particle pairs (Piterbarg 2001).

4 Summary

Stochastic models of mesoscale-eddy effects on the ocean circulation are a new approach, with tremendous potential. The main practical goal of this approach is to advance comprehensive ocean GCMs by incorporating eddy effects without actually resolving the eddies dynamically. In turn, the advanced GCMs will help to understand the climate variability and to make reliable forecasts. It is argued here that the optimal research strategy is based on dynamical decomposition and diagnostic analysis of eddy-resolving solutions. The corresponding deterministic eddy effects are both the starting point and the testing ground for their stochastic approximations. Finally, it is pointed out that the stochastic models have to be compatible with the large-scale dynamics and with intrinsic spatial inhomogeneity and nonstationarity of the oceanic turbulence.

5 References

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