Conservation issues

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Overview

• Why worry about conservation?
• Conserving Eulerian schemes
• Non-conserving semi-Lagrangian schemes
• A posteriori fixes
• Inherently conserving semi-Lagrangian schemes
• =>
Overview (continued)

- Cell-integrated schemes
- Cascade interpolation to the rescue!
- Some problems
- ECMWF plans
- Conclusions
Why worry about conservation?

• Mass conservation (e.g., in long integrations)
• Moisture (significant drift even in “dynamical core” experiments with semi-Lagrangian integration scheme)
• Other advected quantities (e.g. when chemistry is included)
Digression

A slightly heretical observation:

If the \textit{continuous} equations conserve $X$, then if the numerical scheme is \textit{accurate} it should conserve $X$ reasonably well.

A scheme which conserves $X$ exactly but is otherwise inaccurate is not very useful.
Conserving Eulerian schemes

e.g., shallow-water continuity equation:

\[
\frac{\partial \phi}{\partial t} = - \left\{ \frac{\partial}{\partial x} (\phi u) + \frac{\partial}{\partial y} (\phi v) \right\}
\]

C-grid (for example):

\[
\frac{\partial \phi}{\partial t} = - \left\{ \delta_x (\bar{\phi}_x u) + \delta_y (\bar{\phi}_y v) \right\}
\]

(Spectral: more or less automatic)
Problem (for some)

Eulerian integration schemes are *inefficient* compared with semi-Lagrangian schemes

BUT

In general, semi-Lagrangian schemes are *not* formally conserving.
Two ways to tackle the problem

(1) A posteriori fixes (compute the gain/loss of $X$ after each timestep, then restore it).

(2) Modify the semi-Lagrangian scheme so that it becomes inherently conserving.
A posteriori fixes (1)

How do we decide where to modify the new field of $X$ in order to restore conservation?

- We could simply add/subtract the same amount everywhere

- Better philosophy is to make adjustments in regions where we expect the original semi-Lagrangian solution to be most in error.
A posteriori fixes (2)

Priestley (MWR Feb 1993): adjustment depends on difference between linear and cubic interpolation.

Bermejo and Conde (MWR Feb 2002): similar but more sophisticated (& is proportional to the cube of the difference).

(Both combined with quasi-monotone version of the semi-Lagrangian scheme).
Cell-integrated schemes (1)

- An *inherently* conserving SL scheme:
- Instead of finding the departure point corresponding to each arrival gridpoint, find the departure points corresponding to the *corners of the cell* surrounding each arrival gridpoint
- Integrate over the “departure cell” (with assumed distribution)
- “Remap” (transport to “arrival cell”)
Cell-integrated schemes (2)

• Rancic (MWR July 1992)
• Laprise & Plante (MWR Feb 1995) – also downstream version
• Nair & Machenhauer (MWR March 2002) – on the sphere
• Lauritzen (PDEs on the Sphere 2004) – in three dimensions
Cell-integrated schemes (3)

- 1 dimension: OK
- 2 dimensions: complicated
- 3 dimensions: very complicated!
- (Complicated => expensive too)
- Is there a way out?
Cascade interpolation to the rescue! (1)

- In two dimensions \((x,y)\) with rectangular mesh
- First find the departure points as usual, then use them to construct “Lagrangian” mesh
- Find the points at which the Lagrangian \(Y\)-lines intersect the Eulerian \(x\)-lines
- Interpolate (1-dim) along the Eulerian \(x\)-lines
- Then interpolate (1-dim) along the Lagrangian \(Y\)-lines for the values at the departure points.
Cascade interpolation to the rescue! (2)

- Purser & Leslie (MWR Oct 1991) – cascade interpolation
- Leslie & Purser (MWR Aug 1995) – conservative version
- Nair, Côté & Staniforth:
  - (QJ, Jan 1999) – simpler version of cascade interpolation
  - (QJ, Apr. 1999) – extension to sphere
Cascade interpolation to the rescue! (3)

- Zerroukat, Wood & Staniforth:
- (QJ, Oct 2002) – added conservation ("SLICE")
- (QJ 2004, in press) – extension to the sphere
Some problems

- Spherical geometry ("engineering" needed near the pole for lat-long grid)
- Reduced grid for ECMWF model (no longer have "tensor product" grid)
- Distributed memory - communication
- Icosahedral grids - ???
ECMWF plans

- Diagnostics of non-conservation
- Try “a posteriori fix” – what difference does it make? (moisture, interaction with physics etc.)
- Try cascade interpolation (could go back to “non-reduced” lat-long grid for special applications)
Conclusions

• Semi-Lagrangian schemes can be made conservative (but it’s not easy)
• Choice between a posteriori fixes and inherently conserving versions
• Inherently conserving: cell-integrated or based on cascade interpolation
• Still some practical problems (sphere, reduced grid, …)