Recent developments in the vertical discretization of the ECMWF model with impact on the stratosphere and tropopause

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with help from many colleagues at ECMWF
Outline

- Finite-element discretization for the vertical
- Cubic spline interpolation in the vertical semi-Lagrangian advection
- Numerical instability during sudden stratospheric warming events at $T_L 511$
- Increase in vertical resolution (60 levels → 90 levels)
Finite-element (FE) discretization for the vertical

- We use cubic B-splines as basis functions with compact support (finite elements).

- No staggering of variables used. All (including pressure) are held on the same set of levels (full levels). (Good for semi-Lagrangian advection.)

- Only non-local operations are evaluated in FE space, products of variables are evaluated in physical space. (Similar to spectral transform method in the horizontal.)

- In the semi-Lagrangian version of the ECMWF model, the only non-local operations in the vertical are integrations (no derivatives). Therefore, we have derived the FE form only for the integration operator.
FE scheme: Integral operator in finite-element form

\[ F(x) = \int_{0}^{x} f(y)dy \]

Expanding \( f \) and \( F \) in terms of sets of linearly independent functions with compact support \( \{e_i\} \) and \( \{d_i\} \), respectively:

\[
\sum_{i=1}^{M} C_i d_i(x) = \sum_{i=1}^{N} c_i \int_{0}^{x} e_i(y)dy
\]

Using the Galerkin method with \( \{d_i\} \) as test functions

\[
\sum_{i=1}^{M} C_i \int_{0}^{1} d_j(x)d_i(x)dx = \sum_{i=1}^{N} c_i \int_{0}^{1} \left[ d_j(x) \int_{0}^{x} e_i(y)dy \right]dx, \quad j = 1, \ldots, M
\]

In matrix form:

\[
AC = Bc \quad \Leftrightarrow \quad C = A^{-1}Bc
\]

(integral in FE space)

Incorporating the transformation to finite-element space and back into the integral operator, i.e.

\[
c = S^{-1}f \quad \& \quad F = \widetilde{S}C
\]

\[ \Rightarrow \quad F = \widetilde{S}A^{-1}BS^{-1}f \]
FE scheme: Cubic B-splines as basis functions

No staggering of basis set \( \{d_i\} \) with respect to set \( \{e_i\} \) (good for semi-Lagrangian adv.)

Condition \( F(0)=0 \) enforced by incorporation into basis functions, i.e. \( d_i(0)=0 \) for all \( i \).
Basis functions \( d_0, d_1, d_2 \) computed by linear combination of \( e_{-1} \) with \( e_0, e_1, e_2 \), respectively.

Not restricted to regular spacing of nodes.
FE scheme: **Hat-functions (linear splines) as basis functions**

![Diagram of FE scheme with hat-functions as basis functions](image)
FE scheme: Accuracy

Test: numerical integration of $\sin(6\pi x)$, $x \in [0,1]$ for different resolutions with $N$ equidistantly spaced nodes

Reference = analytical integral $I_A$. Error = $\max\{(I_N-I_A)/I_A\}$ in %

<table>
<thead>
<tr>
<th></th>
<th>FD scheme</th>
<th>Linear FE</th>
<th>Cubic FE</th>
<th>Cubic collocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=60</td>
<td>0.82e+0</td>
<td>0.14e-2</td>
<td>0.90e-8</td>
<td>0.14e-2</td>
</tr>
<tr>
<td>N=120</td>
<td>0.21e+0</td>
<td>0.85e-4</td>
<td>0.31e-10</td>
<td>0.85e-4</td>
</tr>
<tr>
<td>estim. order</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>4</td>
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On nodes $O(h^{2(k+1)})$ where $k$ is the degree of the basis functions and $h$ the distance between nodes

Superconvergence
Benefits from the FE scheme

- FE scheme improves the treatment of the gravity wave terms and dampens the computational (zigzag) mode in the vertical present in finite-difference schemes with no staggering of winds and temperature (Lorenz grid).

=> Reduces the amplitude of grid-wave noise in the stratosphere.

![Graph showing the vertical Fourier spectrum of divergence in the tropical stratosphere, averaged from 10N to 10S.](attachment:image.png)

- FD
- cubic FE
- linear FE
Benefits from the FE scheme (2)

Improved vertical integration of the continuity equation leads to a more accurate vertical velocity for semi-Lagrangian advection. => improved tracer conservation
Cubic spline interpolation for the vertical in the semi-Lagrangian advection

Ozone conservation
in the ECMWF semi–Lagrangian model at TL159L60

change in global ozone mass [%]

FD scheme + cubic Lagrange interpolation in the vertical
FD scheme + cubic spline interpolation in the vertical
4-point cubic Lagrange int.
FD scheme + cubic FE scheme + cubic Lagrange interp. in the vertical
FD scheme + cubic FE scheme + cubic FE scheme

time [days]

cubic spline on \(O_3\) + cubic FE scheme

FE scheme
Numerical noise during sudden stratospheric warming (1)

Noise appears only in integrations at high horizontal resolution ($T_{L511}$).

It is highly predictable (up to 8 days ahead) suggesting that it is linked to a specific well-predicted feature of the large-scale flow.

Forecasts don’t fail, noise disappears again when flow pattern changes.
Numerical noise during sudden stratospheric warming (2)
Vertical trajectory calculation for semi-Lagrangian advection:

\[
\eta_A(t + \Delta t) = \eta_D(t) + \Delta t \frac{\dot{\eta}_A(t) + [2\dot{\eta}(t) - \dot{\eta}(t - \Delta t)]}{2}
\]

Smoothing of vertical velocity by least square fit through 4 surrounding points instead of just linear interpolation between 2 points. Done only for vertical velocity used in vertical trajectory calculation.
Numerical noise during sudden stratospheric warming (4)
Increase in vertical resolution

L60  L90

Location of the vertical levels and layer thickness in pressure for L60 and L90

60 levels  90 levels
L90: Fit to Radiosonde Temperatures in the Analysis

exp:edmp L90/L60 2002060100-2002061512(12)
TEMP-T Tropics
used T

STD.DEV

BIAS

L90 in black, L60 in red

exp - ref nobsexp

-5  249
-12 1595
-15 3410
-10 3984
+101 4193
+73 4515
+16 3407
+6 2963
+1 3230
+1 4467
-3 6394
+6 8521
+9 7847
-4 5473
+2 4811
Tropical Cold-Point Tropopause in L90 / L60

Averaged over the deep tropics [10S to 10N].
Analyses and radiosondes averaged in time from 20020601 to 20020615
Forecasts averaged over whole month of June.
L90: Impact on ozone conservation

![Graph showing the impact of different schemes on ozone conservation over time. The graph plots the change in global ozone mass [%] against time [days]. Different lines represent different schemes: L90, FE+spline, L90, FE, and L90. Each line shows a decrease in ozone mass over time, with the L90, FE+spline scheme showing the least decrease.]
Model top raised from 0.1hPa to 0.01hPa
L91: Comparison with CIRA86 Climatology for July

- **July, 70N**
- **July, Equator**
- **July, 70S**
L91: Comparison with CIRA86 Climatology for January

January, 70N

January, Equator

January, 70S
L91: Reduction in vertical velocity in the tropics

Vertical velocity (hPa s⁻¹)
19890101
level=ALL Region: -10.0/0.0/10.0/360.0°
Average: day 0.0 to 426.0

⇒ improved ozone in lower stratosphere

Ozone mass mixing ratio (mg kg⁻¹)
19890101
level=ALL Region: -10.0/0.0/10.0/360.0°
Average: day 0.0 to 180.0
Future work

♦ Based on the L90 or L91 model version, try to
  ♦ improve vertical transport in the stratosphere
    ➔ benefit for ozone assimilation and interactive ozone with radiation
  ♦ reduce large model errors near the stratopause
    ➔ less problems with assimilation of satellite data
♦ Continue work on the use of vertical spline interpolation in the semi-Lagrangian advection.
♦ Improve upper boundary condition (Nils Wedi).