Realism of sensitivity patterns

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Content of talk

- Define sensitivity patterns
- Define “Key analysis errors”
- Discuss links between the structure and realism of sensitivity patterns and data assimilation
  - Show vertical and horizontal structure of sensitivity patterns
  - Show links between sensitivity patterns and Eady index
  - Compare sensitivity patterns and sensitivity perturbed forecasts against observations
- Conclusions
Sensitivity method

Method developed at MeteoFrance/ECMWF primarily by Florence Rabier


- Use 48 hour forecast error as penalty term in the cost function
- Define a norm to enable calculation of the difference between two atmospheric states
- Use adjoint of tangent linear model to determine perturbation at initial time
The diagnostic function to be minimised:

\[ J = 0.5 < P(x^f_c - x^v_t), P(x^f_c - x^v_t) > \quad \text{or} \quad \]

\[ J = 0.5 < P(M(x_0) - x^v_t), P(M(x_0) - x^v_t) > \]

where \( P \) is the projection on the area \((30^\circ N; 90^\circ N)\)

\( M \) represents the non-linear model integrated for 48 hours (time \( t \))

\( x_t^v \) represents the verifying analysis valid at 48 hour forecast time (\( t \))

A norm is required to quantify the forecast error.

An often used definition is the square energy norm:

\[ < x, x > = 0.5 \int_0^1 \int \int_A (u^2 + v^2 + R \ln p_s)^2 + T^2 C_p / T_r dA(\partial p / \partial \eta) d\eta \]
The gradient of \( J \) at time \( t \) can be written as:

\[
\nabla J_t = P(x_t^{\text{fc}} - x_t^{\text{ver.ana}})
\]

If the tangent linear approximation is valid for 48 hours:

\[
\delta x_t = M(x_0 + \delta x_0) - M(x_0) \approx R \delta x_0
\]

where \( R \) represents the tangent linear model, it can be shown that

\[
\nabla J_0 = R^* P(x_t^{\text{fc}} - x_t^{\text{ver.ana}})
\]

where \( R^* \) represents the adjoint of the tangent linear model

\( \nabla J_0 \) is the sensitivity of the forecast error to the initial condition
Example of the gradient of $J$

$$\nabla J_t = P(x_t^{fc} - x_t^{ver.ana})$$

at time $t=48h$ for temperature at level 43 (650 hPa) on 12 UTC 3 January 2003

Black contours: Z500 hPa analysis valid at 12 UTC 3 January 2003
Sensitivity gradients at $t=0$ and $t=48$

\[ \nabla J_0 = R^* P(x_{fc}^t - x_{ver.ana}^t) \]

\[ \nabla J_t = P(x_{fc}^t - x_{ver.ana}^t) \]
This method only determines the gradient at initial time:

$$\nabla J_0 = \mathbf{R}^* \mathbf{P}(x_{t}^{fc} - x_t^{ver.ana})$$

A perturbation is found by trial-and-error, based on typical values for fastest growing singular vectors ($\lambda = 10 - 15$ times amplification in 48 hours).

It can be shown (Rabier et al. 1996 QJRMS) that a good perturbation estimate can be expected if:

$$\delta x_0 = -\alpha \nabla J_0 \approx -\frac{1}{\lambda^2} \nabla J_0 \quad \alpha \approx \left[ \frac{1}{15^2}; \frac{1}{10^2} \right] = [0.004; 0.01]$$

Adding such a perturbation to the initial analysis field in most cases improve the 2-5 day forecast - because information from observations during the first two forecast days is included.
“Key analysis errors”

- Klinker, Rabier and Gelaro “Estimation of key analysis errors using the adjoint technique” QJRMS (1998), 124, pp. 1909-1933

- Extended the sensitivity method so it could determine the perturbation step-size
- Performed a number of iterations to partially minimize the objective cost function
- Three iterations with the energy norm gave the best fit to observations and meteorologically reasonable perturbations
- These perturbations were called “Key analysis errors” because they were expected to describe the most important analysis errors
"Key analysis errors"

For the sensitivity gradient we previously defined:

\[ J = 0.5 < P(M(x_0) - x_t^{\text{ver.ana}}), P(M(x_0) - x_t^{\text{ver.ana}}) > \]

where \( P \) is the projection on the area (30°N;90°N)

\( x_t^{\text{ver.ana}} \) represents the verifying analysis valid at 48 hour forecast time (t)

This can be also be written as:

\[ J = 0.5 (M(x_0) - x_t^{\text{ver.ana}})^T A (M(x_0) - x_t^{\text{ver.ana}}) \]

where \( A \) is the matrix defining the inner product

including the projection on the area (30°N;90°N)

The first order approximation of cost function change with respect to increment is:

\[ \delta J = (R \| \delta x_0 \|)^T A (M(x_0) - x_t^{\text{ver.ana}}) \]

where \( R \) represents the tangent linear model
It can be shown (Klinker et al. 1998 QJRMS) that the maximum cost function change under the constraint \[ \| \delta x_0 \|_c^2 = N \] is:

\[
\delta x_0 = \frac{1}{2\lambda} \nabla J_c \quad \text{where} \quad \lambda^2 = \frac{1}{4N} \nabla J_c^T C \nabla J_c
\]
It can be shown (Klinker et al. 1998 QJRMS) that the maximum cost function change under the constraint $\|\delta x_0\|_c^2 = N$ is:

$$\delta x_0 = \frac{1}{2\lambda} \nabla J_c$$

where $\lambda^2 = \frac{1}{4N} \nabla J_c^T C \nabla J_c$

Ignore the mathematics!

The important things to note:

- An optimal step-size $\delta x_0$ can be determined
- The step-size depends on the choice of inner-product norm
- The spatial pattern depends also on the norm
- Validity of tangent linear approximation for 48 hours assumed
Layout of “key analysis error” calculations

Thanks! François

Climatologies of sensitive areas for short-term forecast errors over Europe

EUMETNET-EUCOS Study

TM 334 2001

G.J. Marseille and F. Bourtier
Layout of “key analysis error” calculations

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Operational analysis at time $t_0$

Initial analysis $x_0$

Updated analysis

Model integration full physics $F$

Forecast error

Cost gradient at $t_0$ $rac{\partial J}{\partial x_0}$

Adjoint model integration basic/improved physics

Cost gradient at $t_0 + 48$ $rac{\partial J}{\partial x_t}$

1st iteration: gradient fields

Scaling

Scaled gradients

Targeting area

3rd iteration: key analysis errors

Interpolation

Analysis update

Perturbed analysis

Model integration full physics

Perturbed forecast

Verifying analysis $x_t$

Climatologies of sensitive areas for short-term forecast errors over Europe

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TM 334 2001

G.J. Marseille and F. Bouttier
Key analysis errors – an example

Temperature perturbation at 650 hPa after 1 iteration (0.3 K contouring)

Temperature perturbation at 650 hPa after 2 iterations (0.3 K contouring)
Key analysis errors – an example

Temperature perturbation at 650 hPa after 1 iteration (0.3 K contouring)

Temperature perturbation at 650 hPa after 3 iterations = Key analysis errors
The energy norm and two other norms have been used in my study: the “approximate Hessian norm” and the $J_b$ norm.

The **Hessian approximation** used in the assimilation system is

$$H = B^{-1/2} \left( I + \sum_{i=1}^{L} (\mu_i - 1) w_i w_i^T \right) B^{-1/2}$$

where $w_i$ are the $L=100$ leading eigenvectors of the Hessian and $B$ is the background error covariance matrix.

The **$J_b$ norm** does not include any Hessian information, i.e. $L=0$ above, so:

$$H = B^{-1/2} (I) B^{-1/2} = B^{-1}$$
My sensitivity experiments

- T159/T159 4D-Var assimilations were performed for December 2001 and January 2002
- For the assimilation experiments “key analysis errors” were calculated daily based on respectively:
  - Energy norm sensitivities at 1200 UTC + 48 hours
  - Jb norm sensitivities at 0300 UTC + 48 hours
  - Hessian norm sensitivities at 0300 UTC + 48 hours
- The structure of the different sensitivity patterns were explored
- Short range (24 hour) forecasts which included comparison against good observations at proper time and location were run
- Observation statistics from these runs were used to investigate the realism of sensitivity patterns
Scores for control and sensitivity forecasts December 2001/January 2002

As expected:
The “key analysis error” modified analyses results in improved 2-7 day forecasts
Eady index and rms of Energy norm sensitivity temperatures. January 2002

Eady index

RMSE of Eady index based on analyses
Upper Level 300hPa, Lower Level 850hPa
Period valid from 2002010112 until 2002012912

Rms of energy norm sensitivity temperatures level 42 January 2002
Eady index and rms of Hessian norm sensitivity temperatures. January 2002

Eady index

RMSE of Eady index based on analyses
Upper Level 850hPa, Lower Level 925hPa
Period valid from 2002010100 UTC until 2002010112 UTC

Hessian norm sensitivities

ECMWF SV Anal VT: Tuesday 1 January 2002 03UTC Model Level 42 **temperature
rms of Jb and Hessian norm sensitivity temperatures. January 2002

Jb norm sensitivity

Hessian norm sensitivity
rms of energy norm and Hessian norm sensitivity temperatures. January 2002

- Energy norm sensitivity
- Hessian norm sensitivity
1 January 2002 case study

Eady index

Energy norm sensitivity
Temperature level 42
1 January 2002 case study

Jb norm sensitivity

Hessian norm sensitivity
1 January 2002 case study

Energy norm sensitivity

Hessian norm sensitivity
Analysis fields valid 1 January 2002

MSL pressure

Potential temperature
Model level 42
1 January 2002 Japan case study

Eady index

Energy norm
Sensitivity
Temperature
Level 42
1 January 2002 Japan case study

ECMWF SV Anal VT: Tuesday 1 January 2002 03UTC Model Level 42 temperature

Jb norm
Sensitivity
Temperature
Level 42

Hessian norm
Sensitivity
Temperature
Level 42
1 January 2002 Japan case study

Cross-sections for temperature sensitivity patterns

Energy norm  Jb norm  Hessian norm
1 January 2002 Japan case study

Cross-sections for vorticity sensitivity patterns

Energy norm  Jb norm  Hessian norm
Analysis field valid 1 January 2002

Potential temperature east-west cross section
Temperature and vorticity spectra

Temperature power spectra
Model level 42 for various sensitivity patterns

Vorticity power spectra
Model level 42 for various sensitivity patterns
Temperature and vorticity spectra profiles

Temperature spectra profiles for sensitivity patterns
Average values for January 2002

Vorticity spectra profiles for sensitivity patterns
Average values for January 2002
Temperature and vorticity sensitivity

Energy and Hessian patterns often differ a lot
Temperature and vorticity sensitivity

Energy and Hessian amplitudes often differ a lot.
Temperature sensitivity
Energy norm more linked to unstable regions

Energy norm
Temperature sensitivities

Eady index

Hessian norm
temperature sensitivities
Temperature sensitivity

Energy norm more linked to unstable regions
Temperature sensitivity

Energy and Hessian norm are sometimes very similar

Energy norm
Temperature sensitivities

Eady index

Hessian norm
temperature sensitivities
American profilers zonal wind component

Energy norm sensitivities. ~47000 obs./hour

R.m.s of (obs-background)

1 7 13 19 25

-2 -1 0 1 2 3 4

Standard analysis  Energy norm  Difference*50
American profilers zonal wind component

Energy norm sensitivities. ~47000 obs./hour

R.m.s. of background

Standard analysis  Energy norm  Difference*50
American profilers zonal wind component

Hessian norm sensitivities ~47000 obs/hour

R.m.s. of (obs-background)

Standard analysis  Energy norm  Difference*50
Hessian norm sensitivities ~47000 obs/hour

American profilers zonal wind component
American profilers meridional wind component

Red is control forecast
Black is sensitivity run

Energy norm sensitivity
Hessian norm sensitivity

22000-24000 observations/hour
QUIKscat wind speed

Red is control forecast
Black is sensitivity run
Up to 60000 observations/hour

Energy norm sensitivity
Hessian norm sensitivity
SSM/I wind speed

Red is control forecast
Black is sensitivity run
Up to 5000 observations/hour

Energy norm sensitivity
Hessian norm sensitivity
DRIBU surface pressure

**Red is control forecast**

**Black is sensitivity run**

200-320 observations/hour

Energy norm sensitivity

Hessian norm sensitivity
DRIBU wind speed

Red is control forecast
Black is sensitivity run

200-320 observations/hour

Energy norm sensitivity
Hessian norm sensitivity
SYNOP surface pressure

Red is control forecast
Black is sensitivity run
22000-96000 observations/hour

Energy norm sensitivity
Hessian norm sensitivity
Conclusions

• Sensitivity patterns depends very much on the norm used
• Energy norm sensitivities are smaller scale and often very different in structure than $J_b$ or Hessian norm sensitivities
• Energy norms are more closely associated with baroclinic regions than seen for $J_b$ or Hessian norms
• $J_b$ and Hessian norms give rather similar sensitivity patterns
• Forecasts from sensitivity pattern modified analyses are often further away from observations during the first 12 hours than is the case for the control forecasts
• From approximately 12 forecast hours and onwards the sensitivity forecasts are closer to the observations than is the case for the control forecast – as expected
• These results of relevance for: understanding poor Reduced Rank Kalman Filter performance, targeting, restructuring of observing systems and estimating the benefit of new satellite instruments