Wind forcing in a spectrum of waves

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1. Introduction

This paper brings together ideas developed over the last 10 years on the wind forcing of a spectrum of ocean waves. Three threads of research are brought together.

Firstly, our understanding of how wind forces a single sinusoidal wave component has developed substantially in recent years. It is now widely recognized that there are two fundamentally-different mechanisms at work, namely (i) the critical-layer mechanism (Miles, 1957) and (ii) the inner-region mechanism (Belcher and Hunt 1993), whereby turbulent stresses lead to a thickening of the boundary layer downwind of the crest of the waves, which then leads to an asymmetric pressure perturbation that drives wave growth. These processes were reviewed recently by Belcher and Hunt (1998) who conclude that the inner-region mechanism is dominant over for slow waves, which have $c/u_* < 10$, and fast waves, which have $c/u_* > 15$. However, there remains an interesting possibility that the critical-layer mechanism may be important for driving waves in wave groups (Belcher, Cohen and Hunt 1998). For the purposes of this paper, we focus on the inner-region mechanism and leave the question of the role of wave groups for future study.

The second thread to be woven together here is the concept of sheltering, which was first identified by Janssen (1982) and has recently been studied by Makin and Kudryavtsev (1999). Sheltering can be explained as follows. A wave exerts a net drag on the wind, and this drag is manifested as a thin layer of air close to the wavy surface that has a reduced turbulent momentum flux, when averaged over a wavelength. In stationary homogeneous conditions, the total average momentum flux must be constant with height. The difference between the total momentum flux and the turbulent momentum flux is carried by the organized motions associated with the flow over the wavy water surface, the so-called wave-induced stress. Now, according to the inner-region mechanism for wind generation of waves, it is the magnitude of the turbulent momentum flux that yields the strength of the driving of waves by wind. Hence waves of much shorter wavelength riding upon a longer wave are subjected to wind forcing mediated by the turbulent momentum flux that has been reduced by the wave induced stress of the longer wave. This process is sheltering. In the second part of this paper we illustrate sheltering with a simple example that can be studied in the laboratory and indicate how the process can be quantified.

The third thread to be woven together here is the concepts developed decently to understand the equilibrium range of wind wave spectra. It is observed (see e.g. Phillips 1977) that the high wavenumber (or frequency) tail of wind wave spectra develop into a local equilibrium, where at each wavenumber within the equilibrium range the local generation of wave energy or action is balanced by the local loss of wave energy or action. It is of great interest to understand the variation of this equilibrium range of the spectrum with wavenumber (or frequency) and its dependency on wind speed or total wind stress because this part of the spectrum largely determines the drag of the sea surface on the atmosphere and also because this part of the spectrum needs to be parameterized in practical wave forecast models such as the system operated at ECMWF. In the final part of this paper we indicate a how sheltering affects the dynamics of the equilibrium range.

2. Quantification of sheltering

Several laboratory studies (e.g. Mitsuyasu 1966, Phillips and Banner 1974, Donelan 1987) have shown that short wind-generated waves are suppressed by a train of long mechanically-generated waves. The energy in the short wind-generated waves reduces as the slope of the long wave increases. The explanation for this observation suggested here, and developed in detail by Chen and Belcher (2000), is that the turbulent momentum flux near the surface of the long wave is reduced by the drag exerted on the wind by the long wave via the sheltering mechanism. This in turn reduces the rate of wind forcing of the short waves, which means that the wind-generated short waves have less energy at each fetch.

The quantitative model for this process developed in Chen and Belcher (1999) is as follows. The wave-induced stress in the air flow exerted by the long wave is calculated using results from Belcher's (1999) extension of the calculations of turbulent boundary layer flow over slow and fast waves developed by Belcher and Hunt (1993) and Cohen and Belcher (1999) respectively. The turbulent momentum flux at the surface of the long wave is then the difference between the total wind stress minus the wave-induced stress. The energy of the short wind-generated waves is then calculated using this value for the turbulent momentum flux in the empirical fetch law of Mitsuyasu and Rikiishi (1978). This analysis then yields a simple formula for the ratio of the energy in the short wind-generated waves with and without the long mechanically-generated wave (Eq. (19) in Chen and Belcher 2000). The ratio of energy with and without the long wave reduces with slope of the long wave and with fetch, in accord with the experiments. Chen and Belcher show how the model agrees extremely well with the laboratory measurements.

These results serve to illustrate the concept of sheltering and to show that with current understanding sheltering can be quantified reliably. Full details can be found in Chen and Belcher (2000).

3. Sheltering in the equilibrium range of wind driven ocean waves

In this final section the idea of sheltering is applied to a whole spectrum of waves. In particular sheltering is integrated into Phillips (1985) model for the equilibrium range of wind wave. The resulting integral equation is solved analytically to yield formulae for the wavenumber and frequency spectra of ocean waves. Full details of this work are contained in Hara and Belcher (2001).

Hence, a new analytical model is developed for the equilibrium range of the spectrum of wind-forced ocean surface waves. Firstly, the existing model of Phillips (1985) does not satisfy overall momentum conservation at high winds. This constraint is satisfied by applying the concept of sheltering to the equilibrium range. Waves exert a drag on the air flow so that they support a fraction of the applied wind stress, which thus leaves a smaller turbulent stress near the surface to force growth of shorter wavelength waves. Formulation of the momentum budget accounting for this sheltering constrains the overall conservation of momentum and leads to a turbulent stress that is local in wavenumber space, which reduces as the wavenumber increases. This local turbulent stress then forces wind-induced wave growth. Following Phillips (1985), the wind sea is taken to be a superposition of linear waves, and equilibrium is maintained by a balance between the three sources and sinks of wave action.

These assumptions lead to analytical formulae for the local turbulent stress and the degree of saturation of waves, B, in the equilibrium range. We identify a sheltering wavenumber, k_s, over which the local turbulent stress is significantly reduced by longer waves. At low wavenumbers or at low winds, when $k \ll k_s$, the sheltering is weak and $B(k)$ has a similar form to the model of Phillips (1985). At higher wavenumbers or at higher winds, $k \gg k_s$, $B(k)$ makes a transition to being independent of wave number k. The additional constraint of conservation of momentum also yields a formula for the coefficient that appears in the solution for $B(k)$. These forms are consistent with the model of Belcher and Vassilicos (1997). The spectra for mature seas are calculated from the model and are shown to agree with field observations. In particular, our model predicts more realistic spectral levels toward the high wavenumber limit compared to the previous model of Phillips (1985).

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