CRITICAL LEVEL FILTERING OF OROGRAPHIC GRAVITY WAVES IN DIRECTIONAL WIND SHEAR

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Abstract

Most orographic gravity wave drag parametrizations employ a two-dimensionality assumption when describing the upward transmission of wave pseudo-momentum. The issue of wave breaking is then reduced to the computation of a saturation wave amplitude dependent on flow properties in the direction of the surface stress vector, and an actual wave amplitude based on the Eliassen-Palm theorem. The saturation amplitude (or equivalently, the saturation stress) then acts as an upper bound on the actual wave amplitude (or stress). When the wind component in the direction of the surface stress falls to zero at some height, so also does the saturation amplitude and all stationary wave energy must be absorbed beneath. However, if the wind direction changes with height, the Eliassen-Palm theorem does not hold in general since wavevectors at right angles to the flow are subject to critical level absorption.

Here, some analytic solutions and numerical model simulations are described which show the structure of the stationary gravity wave field when the wind backs continuously with height. It is shown that a component of the wave energy is effectively blown downwind of the mountain at any level and that the boundary of this 'asymptotic wake' is defined by the surface composed of all fluid particles (moving with the basic state wind) that have passed directly over the mountain top. The net pseudo-wave momentum propagating vertically above the mountain is slowly depleted by the selective filtering-out of individual wave components as they are advected along, and just beneath the asymptotic wake surface.

A simple adaption of the orographic gravity wave drag parametrization scheme used in the UK Meteorological Office's forecast model to account for this three-dimensional critical-level filtering process is described.

1. INTRODUCTION

The process of critical level absorption for orographically-forced gravity waves is usually associated with a surface along which the wind speed is zero and the wind direction changes discontinuously by 180 degrees. At this surface, linear theory suggests that wavevectors of *all* azimuthal orientations will experience critical level absorption and only a small proportion of the wave energy is transmitted (provided the Richardson number is greater than unity). Grubišić and Smolarkiewicz (1997) have examined the 3D linear problem of uni-directional constant-shear flow over an isolated hill in the presence of an elevated critical level. They show how the parabolic wave envelopes familiar in the constant wind solution widen on approaching the critical level. At the critical level, the horizontal wind perturbation becomes unbounded and wavebreaking would be expected in the unapproximated problem as wave energy piles up beneath the zero wind surface.

Such unbounded growth of energy does not occur for cases in which the wind direction changes continously with height and where the mountain is isolated (Shutts, 1997). In this situation, wave packets propagate up towards the height at which the horizontal projection of their phase lines is parallel to the flow direction. The relative group velocity tends to zero at this level in such a way that the wave packet takes infinitely long to get there and so it gets blown indefinitely far downwind away from the hill. Piling-up of wave energy is averted but even so it may be shown that the magnitude of vertical wind shear in the wave packet grows slowly downwind – ultimately leading to wave breakdown a long way downwind of the mountain.

The fact that the net vertical flux of horizontal momentum changes with height in flows which back or veer might be thought to contradict the Eliassen-Palm theorem (Eliassen and Palm, 1961) yet this, of course, is not the case because of the existence of critical level absorption at all levels where the wind turns (Broad, 1995). Note however that if the wind backs and then veers by a lesser degree at a higher level, the veering phase will not be accompanied by stress deposition since the associated wavevectors have already been removed in the backing phase (assuming that no wave energy can be transmitted through the critical level).

Current orographic gravity wave parametrization schemes, whether or not they treat mountain anisotropy, essentially make use of 2D linear, hydrostatic wave theory under the WKB assumption i.e. they assume that the wave field takes the form of a wavetrain with slowly-varying vertical wavelength dictated by local flow parameters. This precludes the representation of partial internal wave reflections or wave trapping (actually, the current scheme used in the UK Meteorological Office forecast model does make some crude allowance for these processes). On the basis of these same assumptions – together with the assumption that critical levels are perfect absorbers – it is straightforward to calculate the height-dependence of the total momentum flux vector when represented as a Fourier integral over all wavenumbers (Shutts, 1995). This lead to a proposed revision to that part of the gravity wave drag parametrization scheme that assumes the wave stress is height-independent if its magnitude is less than the saturation stress for that level. In the presence of wind backing or veering, part of the net momentum flux may be removed in accordance with the linear Fourier description of the wave field and a chosen anisotropic spectrum function for the orography. Unfortunately, this compromises the wave saturation concept (Lindzen, 1981) which is based on a monochromatic description of the wave field. Now, with wind turning, even an infinitesimal-amplitude localised wave disturbance is associated with some measure of wave breaking !

Whilst it is fairly straightforward to deduce that some kind of wave absorption always occurs in flows which back or veer with height, it is not so obvious how this would manifest itself. What after all is the form of the resulting stationary wave field and where does wavebreaking occur in the horizontal ? To some extent the latter question parallels that of trapped lee waves – what is their dissipation mechanism and how far downstream do they travel before dissipation occurs ? These issues are central to gravity wave drag parametrization since flow deceleration is experienced in regions of wave breaking.

In the middle atmosphere where gravity wave drag has a dominating effect on the global circulation, these critical level filtering effects assume an even greater importance for a number of reasons. Firstly, at these levels stationary gravity waves are probably no more important than travelling wave modes forced by any number of alternative physical processes (e.g. convection, shearing instability). Under these circumstances, the turning of the wind that is relevant is that perceived from the wave's reference frame and so even if the wind field was zonal, a meridionally-propagating localised wave disturbance would see a turning wind profile. Another consideration is that at these levels the saturated wave spectrum is more likely to be azimuthally isotropic than the major forced gravity waves of the troposphere.

Gravity wave drag parametrization schemes used in middle atmosphere 'climate models' are of a quite different nature to those used by the NWP modelling community since they do not directly involve forcing parameters such as the orographic sub-gridscale height variance. Instead they assume that enough gravity wave energy is generated by tropospheric sources to ensure wave saturation from the lower stratosphere upwards. Wave drag is then dictated by the selective removal of pseudo-momentum in a set of chosen azimuthal directions according to various nonlinear descriptions of the saturated gravity wave spectrum at the high vertical wavenumber end (e.g. the doppler spreading hypothesis of Hines, 1997). Since these parametrization schemes are typically initiated at heights between 10 and 20 km there is the interesting possibility of running two GWD schemes concurrently – one to represent the effects of orographically-forced waves and the other to represent a background spectrum of waves (e.g. McFarlane et al, 1997).

2. LINEAR THEORY

The structure of the stationary wave field in uniform, stratified flow above an isolated hill has been presented by Smith (1980) using linear theory in the hydrostatic limit. Gravity waves with a characteristic vertical wavelength of $2\pi U/N$ (where U is the uniform flow speed and N is the buoyancy frequency) are found immediately above the hill like the solution for a two-dimensional ridge except now the disturbance energy decays with height. This decay in the hydrostatic (non-dispersive limit) is caused by the 'filtering out' of wavevector contributions by advection parallel to their associated phase lines. That this should happen is clear from group velocity arguments.

Non-dimensionalizing the dispersion relation for hydrostatic gravity waves in a non-rotating system using N^{-1} as a time scale and U/N as a length scale gives:

$$\sigma = k - \frac{K}{m} \tag{1}$$

$$C_{gx} = \frac{\partial \sigma}{\partial k} = 1 - \frac{k}{Km} \tag{2}$$

$$C_{gy} = \frac{\partial \sigma}{\partial l} = -\frac{l}{Km} \tag{3}$$

and

$$C_{gz} = \frac{\partial \sigma}{\partial m} = \frac{K}{m^2} \tag{4}$$

where σ is the wave frequency, $K = (k^2 + l^2)^{1/2}$ and the wavevector (k, l, m) is associated with group velocity (C_{gx}, C_{gy}, C_{gz}).

Since $\sigma = 0$ for stationary waves, eq.(1) gives

$$m = \frac{K}{k} \tag{5}$$

which upon substituting into eqs.(2) and (3) gives:

$$(C_{gx}, C_{gy}) = \frac{l}{K^2}(l, -k). \tag{6}$$

If ψ is the angle made between the phase lines and the x-axis then

$$(k, l) = K(-\sin\psi, \cos\psi) \tag{7}$$

and eq.(6) becomes

$$(C_{gx}, C_{gy}) = \hat{r} \cos \psi \tag{8}$$

where $\hat{r}$ is the unit vector pointing in the direction along the phase lines rotated 90 degrees clockwise from the vector (k, l).

This merely states that the horizontal group velocity is just the component of the wind downstream along the phase lines. For wavevectors pointing in the x-direction, $\psi = \pi/2$ and the horizontal group velocity is zero. Therefore, for orographic ridges with axes lying at right angles to the flow direction, the group velocity vector is vertically upward and no spreading of wave energy occurs in the downstream direction. If a velocity component parallel to the ridge axis were added, this would have no effect on the resulting wave field.

An isolated hill generates a full azimuthal spectrum of gravity waves and so lateral dispersion of wave energy is inevitable as the individual wave components are advected parallel to their phase lines. Wave energy in the 'beam' above the hill leaks out by advection and this forms a paraboloidal surface about which the disturbance energy is concentrated (Fig. 1).

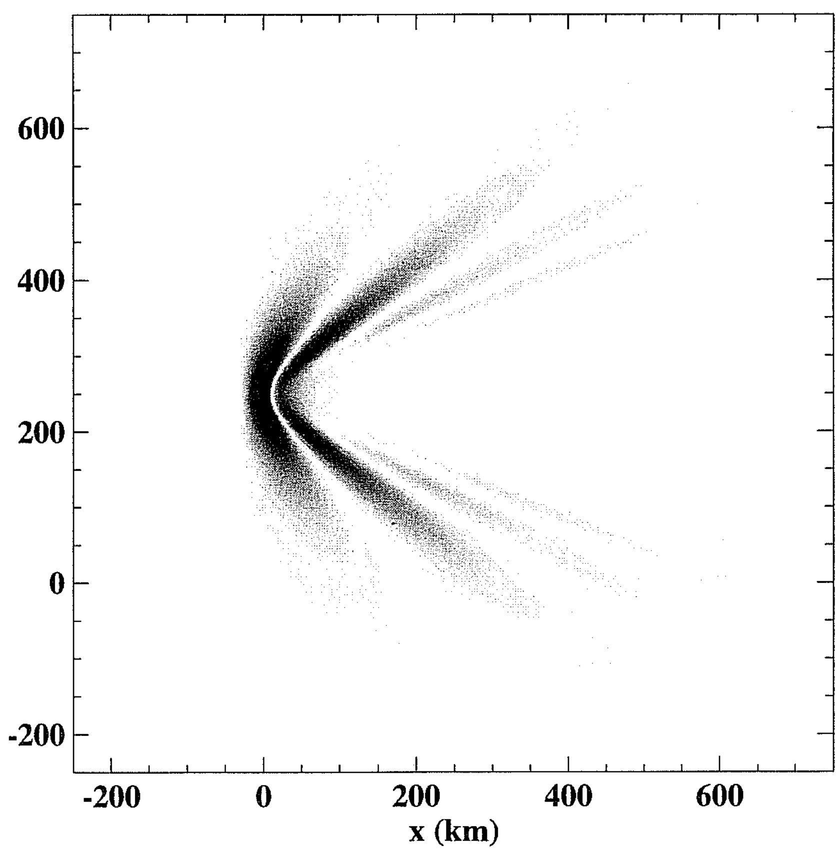

Fig. 1. *Typical horizontal wave pattern generated by uniform, hydrostatic flow over an isolated hill.*

At low levels, the parabolic wave locus is defined by waves whose phase lines are almost parallel to the direction of the flow and therefore whose vertical component of group velocity is small. Smith (1980) showed that these are associated with a 'singular wake' in which the vertical wavelength decreases progressively downstream and near the surface. Ultimately, the related increase in vertical wind shear would cause wave breakdown through shearing instability.

The behaviour of these wave modes is similar to those approaching a zero wind speed critical level where the vertical component of the group velocity tends to zero. However in this cspecial ase *all* wave modes get absorbed, not an infinitesimally small angular sector of the wave spectrum. If the wind vector rotates continuously in height then this critical level behaviour extends to all waves whose wavevectors lie in the azimuthal sector at right angles to the sector swept out by the wind vector (i.e. the sector formed by a 90 degree clockwise rotation of the sector mapped out by the wind vector). The singular wake (or asymptotic wake as it will be referred to hereafter) therefore effectively follows the wind vector as it rotates with height.

Shutts (1997) used ray-tracing arguments to study the asymptotic behaviour of the wave energy as wave packets approach their critical levels in a flow with a turning wind vector. Here is was shown that since the vertical group speed tends to zero on approaching a critical level, the wave packet is blown indefinitely far away from the hill and spreads laterally in the process. This results in the wave energy tending to zero at the critical level in contrast to the zero wind speed critical level of uni-directional flow where the energy increases indefinitely. Using wave action conservation arguments for a basic flow with wind vector $(U_0, \Lambda z)$ where U_0 and Λ are constants it was shown that the wave energy density was proportional to $z - z_c(\phi)$ where z_c is the critical level height dependent on the azimuth angle ϕ of the wind at that height.

The structure of the wave field at a distance from the hill can be found by the stationary phase technique. The details of the derivation are not presented here but can be found in Shutts (1997) : the main results will however be summarised now.

The solution is for the turning wind field defined above and for a flow with constant buoyancy frequency. The equation of the vertical displacement η is:

$$\frac{\partial^2 \eta}{\partial \zeta^2} + \frac{\mu^2}{\zeta^2}\eta = 0 \tag{9}$$

where ζ is a transformed vertical coordinate defined by:

$$\zeta = \frac{1}{1 + (\Lambda z \tan\phi)/U_0} \tag{10}$$

and $\mu = \beta/\sin\phi$ with $\beta = N/\Lambda$. The vertical coordinate η is defined so that the height range between the ground and z_c maps to the η interval 1 to ∞ when $-\pi/2 \leq \phi \leq 0$. Wave modes with $0 < \phi \leq \pi/2$ are associated with decreasing ζ with height, tending to zero as $z \longrightarrow \infty$.

If $h(r,\theta)$ is the height of a hill defined in polar coordinates r and θ, then the linearized lower boundary condition on $\eta(r,\theta,\zeta)$ is

$$\eta(r,\theta,1) = h(r,\theta) \tag{11}$$

and the upper boundary condition requires that individual wave modes are associated with pure upward energy propagation – either at their critical level or at infinite height.

For the bell-shaped hill given by:

$$h(r,\theta) = \frac{h_0}{(1 + r^2/a^2)^{3/2}} \tag{12}$$

where h_0 is the maximum height of the hill and a is a measure of its width, the stationary phase approximation for η is given by:

$$\eta(r_*,\theta,\zeta) \sim 2h_0 \frac{\zeta_0^{1/2} K_* \exp(-K_*)}{r_*} \cos(\mu_0 \log \zeta_0) \tag{13}$$

for positive ζ_0 where the asterisks denote non-dimensional variables e.g. $r_* = r/a$,

$$K_* = \frac{\beta}{r_* \cos\theta}\left[\frac{\zeta_0 - 1}{\sin\theta\cos\theta} - \tan\theta \log \zeta_0\right] \tag{14}$$

with μ_0 and ζ_0 given by:

$$\mu_0 = \beta/\cos\theta \tag{15}$$

and

$$\zeta_0 = \frac{1}{1 - \Lambda z \cot\theta / U_0} \tag{16}$$

Eq.(13) is not straightforward to interpret due to the dependencies of K_* and ζ_0 on z and θ. However, the factor $K_* \exp(-K_*)$ has a maximum of e^{-1} at $K_* = 1$ and wave modes close to this value dominate the amplitude function. This being the case, setting $K_* = 1$ in eq.(14), and noting that near the critical line ζ_0 becomes very large, we may deduce that ζ_0 is roughly proportional to r_* since the logarithmic term in ζ_0 is easily overwhelmed. Therefore, eq.(13) suggests that the magnitude of η is proportional to $r_*^{-1/2}$ in the asymptotic wake.

It is easily verified that as $\Lambda \to 0$ in eq.(13), the well-known constant wind formula

$$\eta \sim 2h_0 \frac{K_* \exp(-K_*)}{r_*} \cos\left(\frac{Nz}{U_0 \sin\theta}\right) \tag{17}$$

is recovered where K_* is now given by:

$$K_* = \frac{Nzx}{U_0 y^2}. \tag{18}$$

Notice that in this case, the above argument suggests $|\eta|$ decays as $1/r_*$ along the parabolic wave locus i.e. faster than in the asymptotic wake.

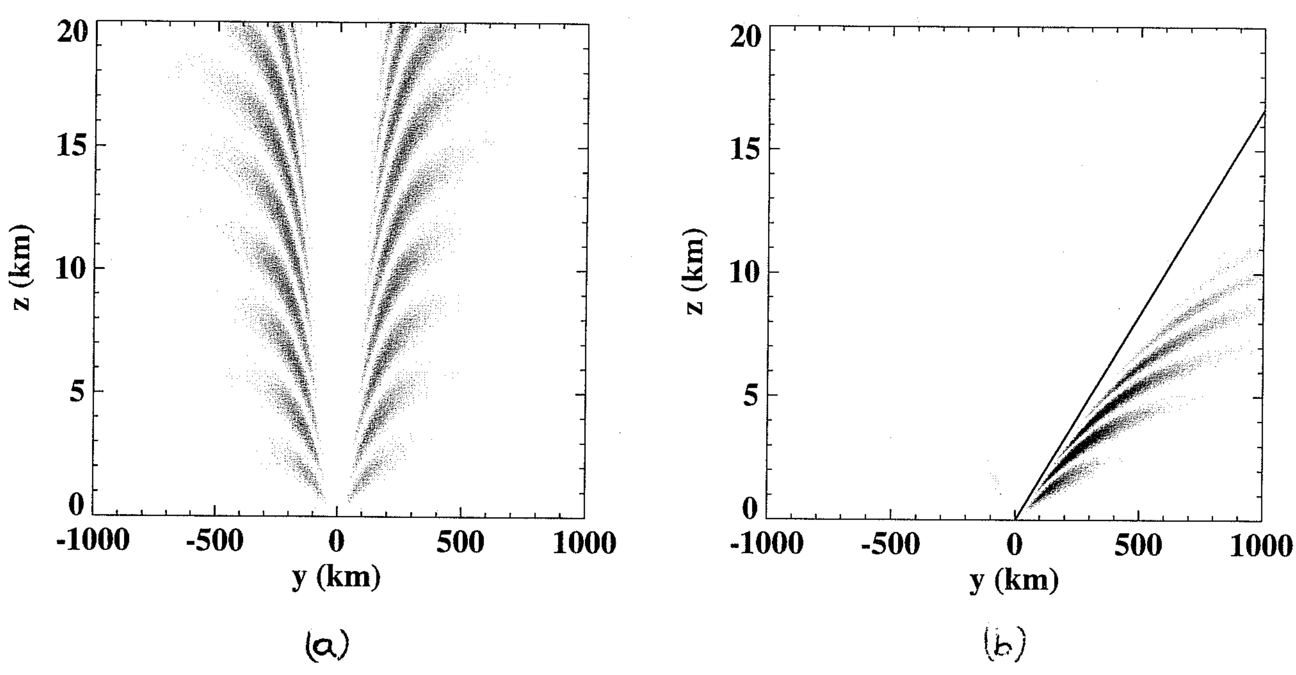

Fig. 2. *North-south vertical cross-section of $|u'|$ 200 km downwind of an isolated hill (1 km high and with width parameter a = 20 km) for (a) a uniform westerly flow of 10 ms^{-1} (b) a flow which backs uniformly with height from a surface westerly towards a southerly flow at large heights i.e. U = 10 ms^{-1} and $V = 3 \times 10^{-3} z$ ms^{-1} when z is in metres. The full linear greyscale range (white to black) represents the range 0 to 0.3376 ms^{-1} and the line indicates the asymptotic wake boundary.*

It is instructive to compare vertical $y - z$ cross-sections of the wave field downwind of the hill for the constant flow case and the wind-backing case. Figs. 2 (a) and (b) show u' (the perturbation wind in the x-direction) in a vertical plane at x=125 km for a mountain centred at $x = 0$ with $a = 20$ km. for these cases. Static compressibility of the basic density field is ignored here so the 'density amplification' effect is absent. The symmetry of the constant wind case is destroyed by the wind backing, reducing u' for negative y and increasing it for positive y. Wavevectors pointing into the fourth quadrant (in a Cartesian system centred on the mountain) see a decreasing wind component with height which causes shorter vertical wavelengths and larger horizontal wind fluctuations compared to the constant wind case. These modes are blown parallel to their phase lines into the *first quadrant* up until they reach their critical levels. The 'quiet zone' lying about the symmetry axis of the constant wind case now extends over a greater range of angles as the wind vector rotates anticlockwise with height.

Gravity wave drag parametrization requires a mathematical expression for the height variation of the vertical momentum flux, $\overline{\mathbf{V}'w'}$. This may be written as a Fourier integral over all horizontal wavevectors of solutions to the vertical structure equation for stationary internal gravity waves. If the wave motion is hydrostatic and partial internal reflections are ignored, it may be assumed that the surface momentum flux associated with each wave mode is the same as that for corresponding uniform wind problem based on the surface wind speed and direction. This assumption has frequently been employed in gravity wave drag parametrization even though reflection and wave trapping (through non-hydrostatic effects) are often important in reality. For orographic gravity waves of horizontal wavelength greater than about 30 km it may be a reasonable simplification to make.

The surface momentum flux obtained from the linearized problem of uniform flow over a piece of orography contained within a rectangular horizontal area of dimensions X and Y is given by:

$$\overline{\mathbf{V}'w'} = -2NU_0 \int_0^{\infty} \int_{-\pi/2}^{\pi/2} A(K,\phi)\cos(\phi - \chi_0)\mathbf{K}K\,dK\,d\phi \qquad (19)$$

with

$$A(K,\phi) = \frac{4\pi^2}{XY}|\hat{h}|^2 \qquad (20)$$

and where $\hat{h}(K, \phi)$ is the Fourier transform of the orographic height function $h(x, y)$, U_0 is the wind speed and χ_0 is the wind direction.

Under the assumption mentioned above, eq. (19) may be regarded as applicable to flows with slowly-varying wind speed and direction with height so that U_0 and χ_0 are now *surface* values and where the limits of azimuthal integration are modified to exclude the sector of wave directions for which critical level absorption has occurred below. For instance, if $\chi_0 = 0$ and the wind vector has rotated anticlockwise χ degrees from the x-axis at height z, then eq.(19) becomes

$$\overline{\mathbf{V}'w'}(z) = -2NU_0 \int_0^\infty \int_{-\pi/2+\chi}^{\pi/2} A(K, \phi) \cos\phi \mathbf{K} K dK d\phi. \tag{21}$$

3. NUMERICAL MODEL SIMULATIONS

In order to confirm some of the deductions made in the previous section concerning the form of the wave field and vertical momentum flux in turning flows, simulations with a non-hydrostatic numerical model were carried out. The model is that developed by T. Clark (Clark, 1977, 1979) which has been used successfully in numerous published studies (e.g. Clark and Gall, 1982; Clark and Miller, 1991). Whilst a full description of the model would be out of place here we note that it employs the anelastic approximation due to Lipps and Hemler (1982) and model variables are expanded about an idealized input profile. The terrain-following coordinate of Gal-Chen and Sommerville (1975) is used to represent mountains and has been proven for steep orographic gradients.

For the purposes of the experiments to be described here, the Coriolis parameter was set to zero along with all surface energy and momentum fluxes. All parametrizations involving water substance were switched-off and the relative humidity was effectively zero. Under these conditions the model flow follows a dry adiabatic process and there is free slip along the mountain surface. The model is set up with 106 points in x and y forming a regular grid of resolution 4 km; and 160 levels in the vertical separated by 75 m. The 'inner' gridpoints of a horizontal slice through the model define a square region of side 416 km. The domain depth is 12 km and the high vertical resolution is chosen to help resolve critical level processes.

Normally, the experiments are carried out with a damping layer in the stratosphere : here we choose a basic state wind field which rotates uniformly with height from westerly at the surface to easterly at the model top (i.e. the angular rotation of the wind is proportional to height). The wind speed is held constant at 10 ms^{-1}. In this configuration it has been shown that a damping layer is not necessary as all gravity waves suffer critical level absorption before reaching the model top. A bell-shaped hill is specified according to eq.(12) setting $h_0 = 100\ m$ and $a = 20$ km (a plan view of the orography is given in Fig. 3).

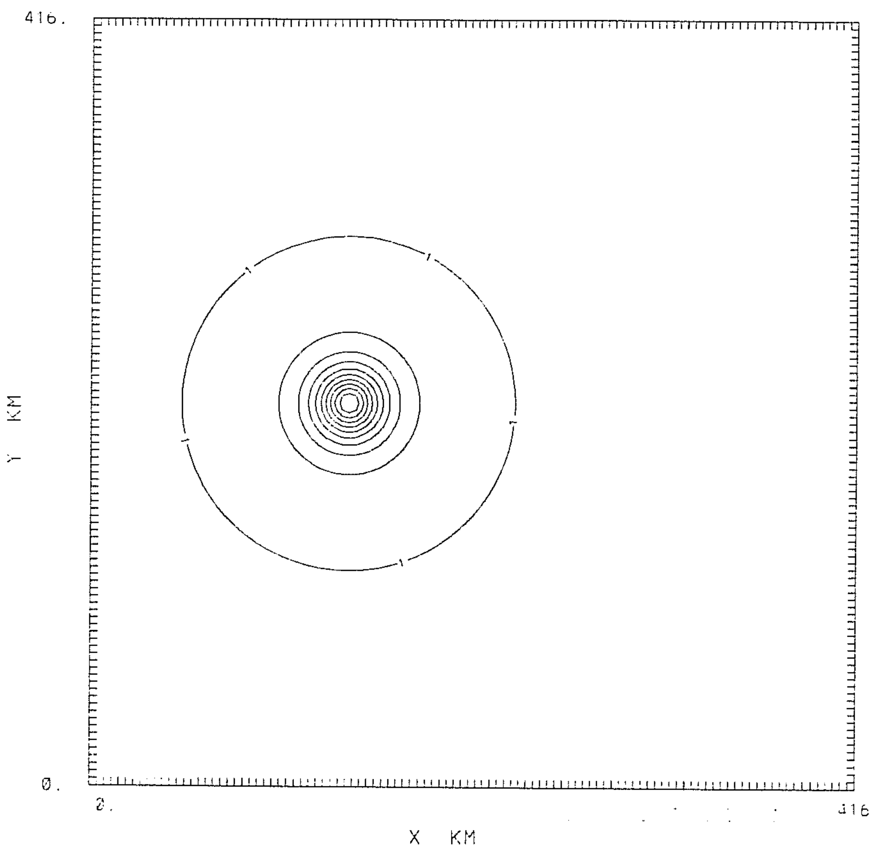

Fig. 3. *Orographic height field used in numerical model simulation. Contour interval: 10 m.*

With the above configuration, the model was integrated for 12 hours until a quasi-steady state was achieved. In view of the slow time-scales associated with the approach of wave packets to a critical level, the degree of steadiness was checked (and confirmed) against a 24 hour simulation. Figs. 4 (a) and (b) show the u and w fields plotted at a height of 3.68 km at the end of the 12 hour simulation. The trailing wake can be seen in the u field which extends out of the northern boundary of the domain. The wind direction indicated by the thick solid line extending out of the origin of the hill, marks the azimuthal limit of the quiet zone in the gravity wave field. At a distance from the hill, waves propagating into the sector defined by the x-axis and this line would have suffered critical level absorption before getting up to this level. For a hill of this size, the perturbation u in the wake is rather weak ($\sim 0.2\ ms^{-1}$). The w field resembles the corresponding solution for uniform flow except rotated by an angle smaller than that by which the wind has backed.

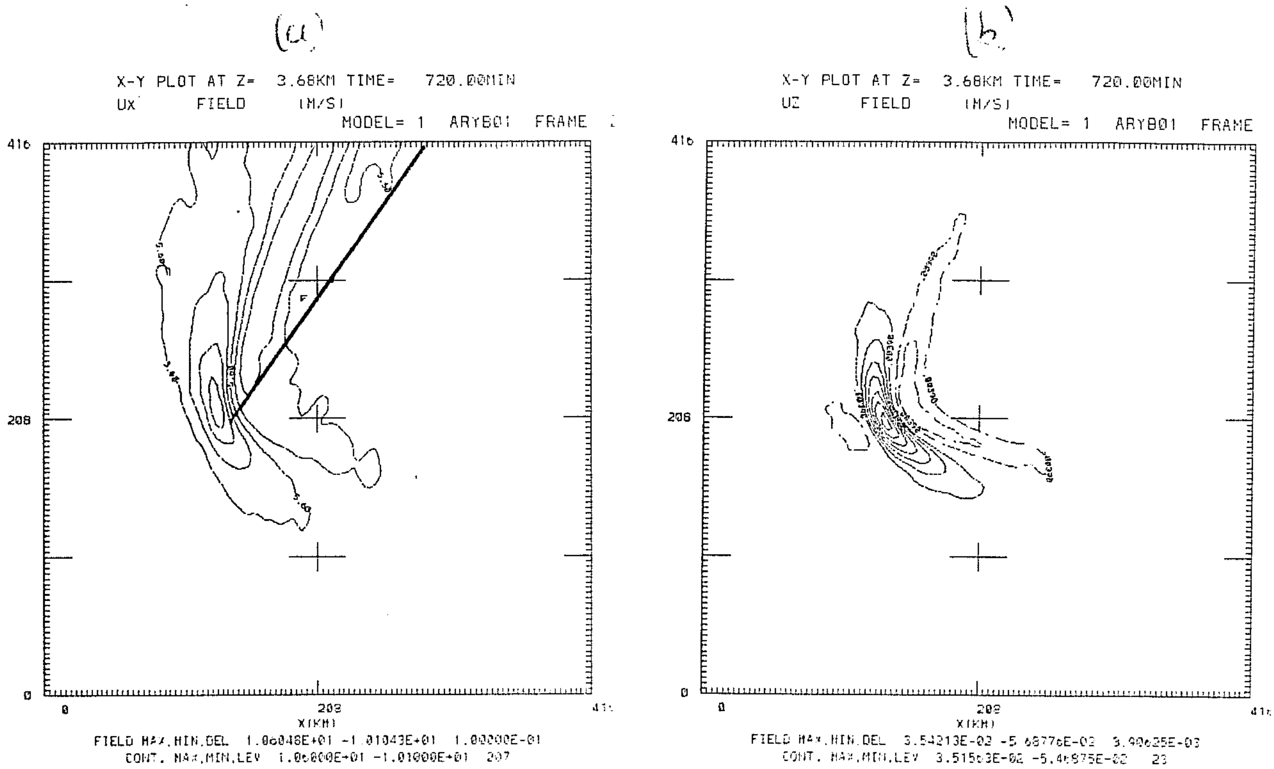

Fig. 4. *(a) u (b) w fields at a height of 3.68 km after 12 hours of integration for a basic state flow which backs with height. Contour intervals: 0.1 ms^{-1} and* 0.0039 *ms^{-1} respectively*

A comparison between the vertical momentum flux in the model and that determined from eq.(21) is shown in Fig. 5 (a). The values have been normalized against the analytical value for uniform flow over a bell-shaped hill i.e. $-\rho_0 N\pi h_0^2 a U_0/4XY$. The agreement between the two curves is quite good although the model's momentum flux decays more rapidly with height due to implicit diffusion in the finite difference representation of the flow.

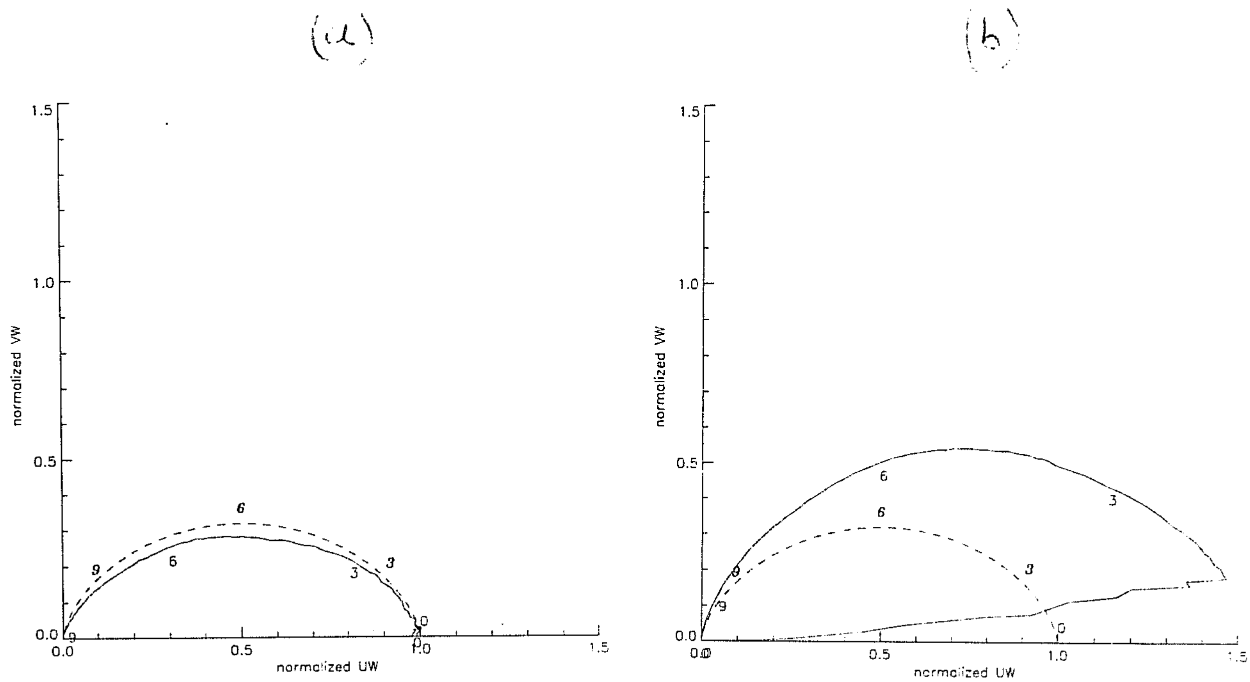

Fig. 5. *Polar plot of the normalized vertical momentum flux for (a) the 100 m hill and (b) a 1 km high hill. Analytical curves are given by the dashed lines and the numbers indicate heights in km.*

Two aspects of the momentum flux profile need to be considered with respect to the related two-dimensional problem where the hill is an infinite ridge lying parallel to the y-axis. Firstly, the direction of the momentum flux vector swings out to angle greater than 45 degrees – though its magnitude is rather small at this point. For a 2D ridge the direction of the momentum flux would not change with height. Secondly, the magnitude of the wave stress decreases steadily with height in the wind-turning case but would be height-independent in the 2D problem up until the critical level at the mid-level – where the wind component normal to the ridge vanishes and all of the stress would be deposited (according to linear theory). Gravity wave drag parametrizations based on a monochromatic description of the wave stress (like the above 2D case) would clearly be erroneous for a flow such as this.

It is instructive now to consider the effect of a bigger hill. Fig. 5 (b) shows the normalized vector wave stress for a bell-shaped hill of height 1 km. as simulated by the numerical model. Now the magnitude of the vertical momentum flux exceeds the linear value by a factor of about 1.5 near the surface and remains greater than the linear curve throughout. The tail-back of the model wave stress curve to zero, for heights below 1 km, is an artifact of the method of calculation below mountain top level. Note that the range of angles spanned by the momentum flux vector is now greater than that of linear theory (e.g. about 45 degrees at a height of 6 km).

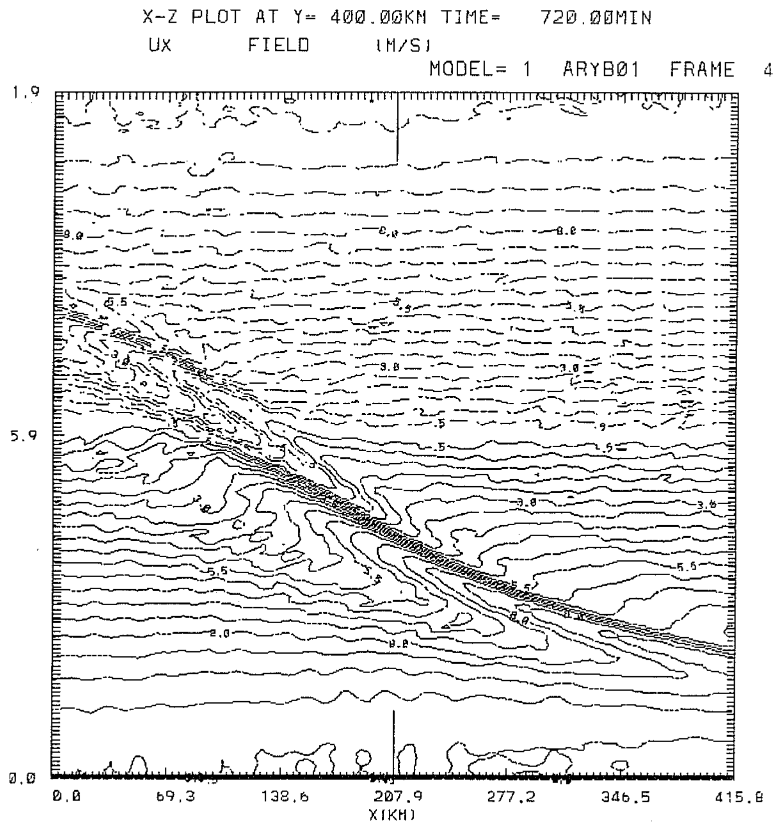

Fig. 6. *u on the northern domain boundary after 12 hours in the simulation of backing flow over a 1 km mountain. Contour interval: 0.5* ms^{-1}

The details of the quasi-steady flow in this case are too complicated to present here but Fig. 6 shows the form of the critical level wake in a vertical cross-section of u on the domain's northern boundary. The wake takes the form of an intense (but thin) shear layer or vortex sheet sloping from a height of 6 km on the western boundary to 2.2 km on the eastern boundary. A rough estimate of the vertical shear in the vortex sheet is $\sim 10^{-2}$ s^{-1} which is comparable with the buoyancy frequency and implies that the Richardson number is of order unity. Profiles of potential temperature show that the stability is weak in some parts of the wake and so the waves must be close to, or actually, breaking. Linear theory requires that the vertical wind shear in the wake increases downwind and so this vortex sheet is likely to persist for long distances and contain turbulence.

4. PARAMETRIZATION SCHEME

From the above discussion, it is clear that the vertical momentum flux is not constant in layers where the wind backs or veers (unless the wind vector has previously visited those azimuthal directions at lower levels). Analytic and numerical model simulations suggest that eq. (21) can be used to replace the Eliassen-Palm assumption in orographic gravity wave drag parametrization schemes i.e. the requirement that wave stress is constant with height in the absence of wave saturation. If a local Cartesian system is set up with the x-direction taken as the surface wind direction, then the momentum flux at any height may be written as:

$$\overline{\mathbf{V}'w'} = -2NU_0 \int_0^\infty \int_{-\pi/2+\chi_b}^{\pi/2+\chi_v} A(K,\phi)\cos\phi \mathbf{K} K\, dK\, d\phi. \tag{22}$$

where $\chi_b(z)$ is 'most backed' wind direction (measured as a positive angle from the x-axis) beneath height z and $\chi_v(z)$ is the 'most veered' direction and is of negative sign.

In order to use eq. (22) in a parametrization scheme, it is necessary to choose a convenient analytical form for $A(K,\phi)$ – the orographic height variance spectrum function. The UK Meteorological Office GWD parametrization scheme assumes the following definition for $A(K,\phi)$:

$$KA(K,\phi) = \left(\frac{K_0}{K}\right)^{3/2}[a\cos^2\phi + 2b\sin\phi\cos\phi + c\sin^2\phi] \tag{23}$$

with $b^2 < ac$ and $c > 0$ to ensure positivity (Gregory et al, 1997). The power law factor was motivated by the observation that the Earth's orographic spectrum does indeed appear to follow a power law though the exponent varies somewhat between about 1 and 1.7, depending on the mountain range (and possibly its geological age) (Bretherton, 1969). There is some redundancy in having *two* constants K_0 and a but they are included here for aesthetic considerations. The constants can be determined from the mean of the orographic gradient tensor $\mathcal{H}_{ij}$ given by:

$$\mathcal{H}_{ij} = \overline{\frac{\partial h}{\partial x_i}\frac{\partial h}{\partial x_j}}. \tag{24}$$

over a model grid box. Eq. (22) may also include other factors such as a mountain Froude number function to account for flow blocking. It may be applied up to the height where wave saturation (in the monochromatic sense envisaged by Lindzen, 1981) is diagnosed. To be consistent, the saturation stress τ_s is diagnosed using

$$\tau_s = \alpha\rho U_T^3/N \tag{25}$$

where U_τ is the component of the wind in the direction of the wave stress *at the current height*, ρ is the density and α is some constant of proportionality. Note that this definition is at odds with that adopted by Gregory et al (1997) who always resolve the wind in the direction of the surface stress on the basis that the wave field can be associated with an equivalent monochromatic wave field whose wavevector lies in the surface stress direction.

Within layers where the saturation stress magnitude (τ_s) is reached it is not obvious what the direction of the stress vector should do. However, it seems reasonable to assume that if the wind backs or veers, then the same considerations that apply in the linear limit should apply to the saturated wave field i.e. the azimuthal range of wavevectors carrying momentum is restricted to those that lie in the range $-\pi/2 + \chi_B$ to $\pi/2 + \chi_V$.

One of the computational benefits of representing directional wind shear in gravity wave drag parametrizations should be the vertical 'smearing' of flow deceleration increments, helping to avoid the unpleasant side-effects of 'drag delta-functions'.

5. DISCUSSION

One might question the importance of wind turning effects in NWP simulations on the grounds that large directional wind shear is not that common. Such a statement is difficult to support when the full range of atmospheric flow situations is considered. It is certainly true that the strong westerly winds in middle latitudes are usually accompanied by strong westerly thermal wind shear and therefore little wind direction change with height. In fact, thermal wind considerations demand that flows which back (veer) with height are accompanied by cold (warm) thermal advection (e.g. the passage of fronts). Where depressions track over mountain ranges one should expect to see large directional wind shear to the north of their centres, but perhaps the most obvious location for directional wind shear is in the boundary layer, particularly in stable flows past mountains. This latter case is just one aspect of the important problem of boundary layer-orographic gravity wave interaction however.

Beyond the troposphere, the wind direction may deviate quite substantially from the surface direction and wave momentum deposition could occur over deep layers. Regions where there are climatological zero wind speed critical level conditions (e.g. above the subtropical Trade Winds and in the summer, lower stratosphere) must also be favoured locations for strong directional wind shear. The effect of this will again be to smear the deceleration caused by stationary gravity waves in the vertical .

As mentioned in the Introduction, middle atmosphere GWD parametrization schemes are quite different in nature, addressing as they do the travelling wave spectrum. One might wonder to what extent the issues of saturated gravity wave spectra affect the orographic wave problem. As seen earlier, waves comprising the asymptotic wakes are essentially frozen into the flow until such times as they dissipate through convective or shearing instability; or until they find a synoptic flow environment that permits them to propagate vertically again. In our chosen flow, which backs uniformly from westerly to easterly, all of the wave vector contributions to the surface drag get dispersed laterally and trapped in the mean flow at their respective critical level heights until they eventually dissipate. The associated pseudo-momentum flux is scattered in wave packets whose energy spreads out in the horizontal – collapses in the vertical – and whose particle motions are eventually affected by the Coriolis force.

The interesting point here is that one often regards quasi-inertia gravity waves as carrying little vertical momentum flux since w is much smaller than those values found immediately above mountains. But in this 180 degree wind rotation problem, *all* of the wave pseudo-momentum finally appears in this form and covers a very large horizontal area. It suggests the possibility that a significant fraction of the vertical momentum flux generated by mountains (and other wave sources) may be exist in an omnipresent 'background' of low frequency inertia-gravity waves. Although the fluxes are locally small, their large areal extent might compensate to make the net flux worthy of parametrization. Vertical fine structure in the horizontal wind field, as seen in radiosonde ascents, may therefore represent important momentum transfer.

If this speculation is well-founded then it suggests that the type of GWD parametrization scheme used in middle atmosphere studies (based on a saturated spectrum of low frequency gravity waves e.g. Hines, 1997) should actually be used throughout a NWP model. What is required is a better appreciation of the processes that affect gravity waves with low vertical group speed that propagate a long way horizontally from mountains. The properties of wave groups travelling away from mountains will be affected by temporal variation in the flow (which may alter the absolute wave frequency measured from the ground) and spatial variations in the flow which change the wavevector. These considerations apply both to trapped and untrapped wave motion. It may be that horizontal and vertical wind shears in tropospheric flow are sufficiently effective in 'mopping-up' sluggish gravity waves that a saturated spectrum cannot be formed there.

Whatever the merits of parametrizing the background spectrum of low (intrinsic) frequency waves, study of the three-dimensional structure of orographic gravity waves in flows which turn with height appears to be an important step in improving our parametrization techniques.

6. ACKNOWLEDGEMENT

I wish to thank Dr Adrian Broad for his useful comments this paper.

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