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Effect of surface gravity waves on the heat flux

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EFFECT OF SURFACE GRAVITY WAVES ON HEAT FLUX

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1. INTRODUCTION

In the past decade evidence has been collected on the sea state dependence of the drag coefficient for momentum transfer. This was made possible because of an increased accuracy of the measurement of turbulent wind fluctuations, thereby obtaining a reliable estimate of the momentum flux $-<\delta u \delta w>$ (where δu, δw are the fluctuating parts of the horizontal and vertical component of the wind). All this was found to agree with theoretical work on the interaction of wind and waves (*Janssen*, 1982, 1989, 1991): the sea state dependence of the theoretical drag coefficient is in fair agreement with the HEXOS observations (*Janssen*, 1992).

The state of affairs is, however, less clear regarding the sea state dependence of heat and moisture flux. Observational evidence of wind and sea state dependence of the Dalton number C_q and the Stanton number C_m does not exist, not because these quantities have not been measured but because the measurement of heat flux $-<T' w'>$ and moisture flux $-<m' w'>$ is less accurate, giving such a large scatter that no reliable dependency may be inferred. An exception is perhaps the laboratory work of Ocampo-Torres et al (1994) which does show some wind and sea state dependence of the moisture flux. Regarding the sea state dependence of these fluxes it should be noted that extra complications arise because C_q and C_m only depend on the square root of the drag coefficient C_D. For heat exchange this may be seen as follows. Denoting by ΔT the air-sea temperature difference, one has

$$\Delta T = \frac{q_*}{\kappa u_*} \ln(z/z_T)$$

where z_T is a thermal roughness (usually taken to be constant), u_* is the friction velocity, $q_* = -<w'T'>$ and κ the von Karman constant.

The heat exchange coefficient C_q is given by

$$q_* = C_q\, U_{10}\, \Delta T_{10} \tag{1}$$

and, on elimination of ΔT_{10}, one finds

$$C_q = C_D^{1/2} \frac{\kappa}{\ln(10/z_T)} \tag{2}$$

where $C_D = (u_*/U_{10})^2$. Assuming for the moment that z_T is sea state independent, we see that C_q is, through C_D, sea state dependent as well, although the dependence is weaker because of the square root. Since it was not easy to determine the sea state dependence of C_D, it is clear that experimentally finding a sea state dependence of C_q is an even harder task.

All this assumes that z_T is independent of wind speed and sea state. Here I would like to investigate whether this is the case or not. To that end, I have extended the theory of wind-wave generation to include effects of (thermal) stratification. From previous work it is found that the mean flow is affected by the waves through a diffusion term

$$\frac{\partial}{\partial t} U_o = \frac{\partial}{\partial z} K(z) \frac{\partial}{\partial z} U_o + D_w \frac{\partial^2}{\partial z^2} U_o \tag{3}$$

where $K(z)$ denotes a turbulent eddy viscosity and D_w denotes the effect of surface gravity waves on the mean flow (hence D_w depends on the wave spectrum). In the so-called <u>passive scalar approximation</u> the evolution of the mean temperature T_o is found to be given as

$$\frac{\partial}{\partial t} T_o = \frac{\partial}{\partial z} \left\{ K(z) \frac{\partial}{\partial z} T_o + D_w \frac{\partial}{\partial z} T_o \right\} \tag{4}$$

where the turbulent eddy viscosity for momentum and temperature are assumed to be equal to each other.

With a passive scalar we mean a quantity that does not affect the dynamics of the flow to a significant extent. In the case of flow over gravity waves, it is known that the momentum flux is mainly determined by the input to the high-frequency waves. As for these waves the Richardson number $g\ \rho'_o/\rho_o/(U'_o)^2$ is very small, the growth rate of these waves is hardly affected by temperature effects (see *Janssen and Komen*, 1986) hence the wave diffusion coefficient D_w is mainly determined by the shear in the flow.

Next, using an approximation for D_w obtained from *Janssen* (1991), the steady state temperature equation is solved in order to investigate a possible dependence of the temperature roughness on the sea state. A brief discussion of our results is given.

It should be noted that this treatment may also be applied to the transport of other parameters, such as CO_2 density or moisture, across the air-sea interface as long as the passive scalar approximation holds.

2. EVOLUTION EQUATIONS FOR TEMPERATURE

We consider a two-dimensional plane parallel flow whose speed and density depend on height. Acceleration of gravity points in the negative z-direction and in equilibrium the air-sea interface is located at $z=0$ (cf Fig 1).

We consider gravity waves superimposed on this equilibrium and, in particular, we are interested in the effect of these waves on the mean flow and temperature distribution (as usual we assume the hydrostatic approximation hence $T'_o/T_o = -\rho'_o/\rho_o$).

For the waves we take the adiabatic equations with infinite sound speed. The rate of change of the mean flow is then determined by turbulence (which is parametrized by a mixing length hypothesis) and the wave-induced stress $-\langle\delta u \; \delta w\rangle$ where δu, δw are wave-induced velocities of air, hence

$$\frac{\partial}{\partial t} U_o = \frac{\partial}{\partial z}\left(K(z) \frac{\partial}{\partial z} U_o - \langle\delta u \; \delta w\rangle \right) . \tag{5}$$

Here, $K(z) = l^2 |\partial U_o/\partial z|$, $l = \kappa z$ and the wave-induced stress was obtained explicitly in *Janssen* (1982),

$$\frac{\partial}{\partial t} U_o = \frac{\partial}{\partial z}\left(K(z) \frac{\partial}{\partial z} U_o \right) + D_w \frac{\partial^2}{\partial z^2} U_o \tag{6}$$

where the wave-diffusion coefficient depends on the wave spectrum $\Phi(k)$,

$$D_w = \frac{\pi \; \omega^2 \; |\chi|^2}{|c - v_g|} \Phi(k) . \tag{7}$$

Here, $\omega = \sqrt{gk}$ is the angular frequency of the wave, k its wavenumber, $v_g = \partial\omega/\partial k$ is the group velocity and χ is the normalised vertical component of the wave-induced velocity which obeys the so-called Rayleigh equation. The wave number k in Eq (7) has to be expressed as a function of height through the resonance condition $U_0(z) = c$, with c the phase speed ω/k of the waves. Equation (6) tells us that the air flow at a certain height z changes in time owing to the resonant interaction of a water wave with the air flow. In this fashion there is an energy transfer from air flow U_0 to the water waves, resulting in a slowing down of the air flow. In other words, there is a possible sea-state dependence of the air flow.

In fact, detailed calculations using the complete quasi-linear approach of Eq (6) have already been reported by *Janssen* (1989), and a parametrization of the wave effect has been presented by *Janssen*

(1991). We shall use this latter approach to obtain a parametrization of the wave diffusion coefficient in the next section.

Here, we continue with a calculation of the effect of waves on the temperature distribution. In two dimensions the appropriate equation for the temperature reads

$$\frac{\partial}{\partial t} T + \frac{\partial}{\partial x}(uT) + \frac{\partial}{\partial z}(wT) = \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) \tag{8}$$

Separating fluctuations from mean quantities we have

$$\begin{aligned} &T=T_o+\delta T \\ &u=U_o+\delta u \\ &w=\delta w \end{aligned} \tag{9}$$

Inserting (9) in (8) and taking ensemble averages we obtain for a homogeneous sea

$$\frac{\partial}{\partial t} T_o = -\frac{\partial}{\partial z} <\delta w \ \delta T> + \frac{\partial}{\partial z} K \frac{\partial}{\partial z} T_o \tag{10}$$

while the (linear) equation for the temperature fluctuations becomes

$$\frac{\partial}{\partial t} \delta T + \frac{\partial}{\partial x}(\delta u \ T_o + U_o \ \delta T) + \frac{\partial}{\partial z}(\delta w \ T_o)=0 \ . \tag{11}$$

Here, in the spirit of Miles' theory we have neglected the effect of turbulence on the wave-induced temperature fluctuation. In addition, we have neglected terms in (11) which are quadratic in the fluctuations (quasi-linear theory).

In order to solve for the temperature fluctuation we perform the usual Fourier analysis,

$$\begin{aligned} \delta T &= \int dk(\delta \hat{T} \ e^{i\theta} + c.c.), \ \theta = kx - \omega t \\ \delta w &= \int dk(\delta \hat{w} \ e^{i\theta} + c.c.) \end{aligned} \tag{12}$$

to obtain from Eq (11) the following relation between $\delta\hat{T}$ and $\delta\hat{w}$,

$$\delta\hat{T} = \frac{i\delta\hat{w}}{kW} \frac{\partial T_o}{\partial z} \tag{13}$$

where $W=U_o-c$. The wave-induced transport of temperature then follows at once, as

$$q_w = -<\delta w \ \delta T> = -\int dk(\delta\hat{T} \ \delta\hat{w}^* + \delta\hat{w} \ \delta\hat{T}^*) \tag{14a}$$

$$= -\int dk \left[\frac{i|\delta\hat{w}|^2}{kW} \frac{\partial T_o}{\partial z} + c.c. \right] \tag{14b}$$

Note that (14b) has a singularity at $W=0$, corresponding to the resonant wave-mean flow interaction. For growing waves this singularity is interpreted as

$$\frac{1}{W} = \frac{P}{W} + \pi i \delta(W) \tag{15}$$

where the principle value term P/W gives a real contribution only. Therefore,

$$q_w = 2\pi \frac{\partial}{\partial z} T_o \int dk \frac{|\delta\hat{w}|^2}{k} \delta(W) \tag{16}$$

The final step is then to relate the wave-induced velocity with the surface elevation η; in linear approximation we have

$$\frac{\partial}{\partial t} \eta = w$$

hence

$$z=0: \ \delta\hat{w} = -i\omega\delta\hat{\eta}$$

This suggests that one can write for $z>0$

$$\delta\hat{w} = -i\omega\delta\hat{\eta} \ \chi$$

where χ is the normalised wave-induced velocity. Introducing then the wave number spectrum

$$\Phi(k) = 2|\delta\hat{\eta}|^2 \tag{17}$$

we obtain for the wave-induced temperature transport

$$q_w = D_w \frac{\partial}{\partial z} T_o \tag{18}$$

where D_w is identical to the diffusion coefficient for momentum.

Therefore, the mean temperature equation becomes

$$\frac{\partial}{\partial t} T_o = \frac{\partial}{\partial z} \left\{ K(z) \frac{\partial}{\partial z} T_o + D_w \frac{\partial}{\partial z} T_o \right\} \tag{19}$$

and in the next section we shall search for equilibrium solutions of (19).

3. PARAMETRIZATION OF THE EFFECT OF WAVES AND THE TEMPERATURE DISTRIBUTION

In order to make progress we need an approximate expression for the wave diffusion coefficient for $z \geq 0$. The starting point is the momentum balance in the steady state

$$\frac{\partial}{\partial z} \left\{ l^2 \left| \frac{\partial U_o}{\partial z} \right| \frac{\partial U_o}{\partial z} \right\} + D_w \frac{\partial^2}{\partial z^2} U_o = 0, \; l = \kappa z \tag{20}$$

Integrating once with respect to height we have

$$\tau_t + \tau_w = \tau \tag{21}$$

with $\tau = u_*^2$ the surface stress and τ_w the wave-induced stress. From *Janssen* (1991) we know that from $z \geq z_o$ a good approximation for the wind profile is

$$U_o(z) = \frac{u_*}{k} \ln\left(\frac{z + z_1}{z_o + z_1} \right), \; z \geq z_o \; . \tag{22}$$

Then, by means of the stress balance we obtain for the unknown roughness z_1

$$z_1 = z_o \left(\frac{1}{\sqrt{1-x}} - 1 \right), \; x = \tau_w / \tau \tag{23}$$

and τ_w is determined from the wave model.

It should be remarked that for large height one has for the wind profile

$$U_o(z) = \frac{u_*}{\kappa} \ln(z/z_2), \; z \gg z_2 \;. \tag{24}$$

with $z_2 = z_0 + z_1 = z_0/\sqrt{1-x}$.

Now, substitution of (22) into (20) gives as an expression for the wave diffusion coefficient

$$D_w = 2\kappa u_* \frac{z\, z_1}{z+z_1} \tag{25}$$

which is formally only valid for $z \geq z_o$. We also need the wave-diffusion coefficient in the range $0 \leq z < z_o$. Since (25) has the desired property that $D_w \to 0$ for $z \to 0$ (because the wave spectrum vanishes for high frequencies) we simply use (25) also for $0 \leq z < z_o$.

In order to be able to calculate the heat transport, we make the additional assumption that close to the sea surface this transport is determined by molecular conduction. To that end we introduce an additionaldiffusivity $\delta\nu_z$ (with δ an empirical constant) and in the steady state the equation for the temperature becomes (after integration over height z):

$$D_w \frac{\partial}{\partial z} T_o + \left(l^2 \frac{\partial U_o}{\partial z} + \delta\nu_z \right) \frac{\partial}{\partial z} T_o = q_* \tag{26}$$

hence

$$\frac{\partial}{\partial z} T_o = \frac{q_*}{\kappa u_*} \frac{(z+z_1)}{z_v z_1 + z(z_v + 2z_1) + z^2}, \; z_v = \frac{\delta\nu_z}{\kappa u_*} \;. \tag{27}$$

Solving this differential equation for $\Delta T = T_a - T_s$ with boundary conditions $\Delta T=0$ at $z=0$ gives

$$\Delta T = \frac{q_*}{\kappa u_* (x_- - x_+)} \left\{ (z_1 - x_+) \ln\left(1 + \frac{z}{x_+}\right) - (z_1 - x_-) \ln\left(1 + \frac{z}{x_-}\right) \right\} \tag{28}$$

where $x_\pm$ are the negative roots of the denominator of the RHS of Eq (27),

$$x_\pm = (z_1 + \tfrac{1}{2} z_v) \mp \{ z_1^2 + (\tfrac{1}{2} z_v)^2 \}^{1/2} \;. \tag{29}$$

Introducing the heat exchange coefficient C_{q_*} according to

$$q_* = C_{q_*} U_{10} \Delta T_{10}$$

we obtain

$$C_{q_*} = \frac{q_*}{U_{10} \Delta T_{10}} \tag{30}$$

where U_{10} follows from (24) and ΔT_{10} follows (28). Introducing then the drag coefficient C_D according to

$$\tau = C_D U_{10}^2 \rightarrow C_D = \left(\frac{\kappa}{\ln 10/z_2} \right)^2 \tag{31}$$

we finally arrive at

$$C_{q_*} = \frac{q_*}{u_* \Delta T_{10}} C_D^{1/2} . \tag{32}$$

In Fig 2 I have plotted C_{q_*} versus $C_D^{1/2}$ for different sea states and wind speeds. Remarkably, an approximate square root dependence of C_{q_*} on C_D is clearly seen, especially for large C_D (i.e. large winds). *De Cosmo et al* (1996) have obtained a similar result from the HEXOS data, although the scatter is quite big. In order to understand what is happening I use the following approximation for ΔT which is valid for small viscous roughness length z_v; then

$$x_+ \simeq \tfrac{1}{2} z_v, \; x_- \simeq 2 z_1$$

and the temperature profile becomes for large z

$$\Delta T = \frac{q_*}{2\kappa U_*} \left\{ \ln\left(\frac{z}{z_v} \right) + \ln\left(\frac{z}{z_1} \right) \right\} \tag{33}$$

and the heat exchange coefficient becomes

$$C_{q_*} = \frac{2\kappa C_D^{1/2}}{\ln\left(\dfrac{L}{z_v}\right) + \ln\left(\dfrac{L}{z_1}\right)} . \tag{34}$$

Taking fixed sea state and noting that $z_v \sim \dfrac{1}{u_*}$ and $z_1 \sim u_*^2$ we note that presumably the denominator of (34) is approximately constant (i.e. independent of u_*) and therefore C_{q_*} is approximately dependent on the square root of C_D.

Furthermore, in Fig 3 I have plotted C_{q_*} versus wind speed for different sea states. The temperature difference was 1° K. Clearly, there is a much weaker dependence of C_{q_*} on wind speed compared to the wind speed dependence of C_D. Also, C_{q_*} is less sensitive to the sea state. In this context it should be noted that very young waves with $\tau_w/\tau = 0.99$ hardly ever occur in nature and should therefore be regarded as an extreme case.

It is of interest to study the behaviour of the heat fluxes on a global scale. To that end the WAM model was run on analysed ECMWF winds which resulted in an estimate of the wave-induced stress. Hence, the roughness lengths for momentum and heat transfer are known. Fig 4 shows C_{q_*} versus wind speed, while Fig 5 shows the corresponding drag coefficient versus wind speed. Note that C_{q_*} shows hardly any wind speed dependence, in agreement with the HEXOS observations of *De Cosmo et al* (1996). However, the scatter is much less. This much reduced scatter is also evident in the plot for the drag coefficient C_D (Fig 5) and is probably related to the different time and length scales of the numerical simulation (6 hrs and 3°) and the HEXOS observations (averaging time is 1 hr). The longer time and length scale in the numerical simulation presumably leads to smoothing of the results for C_D, C_{q_*} and wind speed U_{10}.

Finally, it is noted that the HEXOS results regarding moisture and heat flux give almost identical mean relations between transport coefficients and wind speed. This is surprising since one would expect that spray would play a role in the process of moisture transfer. In the range of wind speeds observed during HEXOS (U_{10}<18m/s) this is apparently not the case, and therefore also for moisture transfer the passive scalar approximation seems appropriate. In Fig.4 I therefore have included the mean observed HEXOS relation for moisture flux (De Cosmo et al, 1996) and laboratory observations from Ocampo-Torres et al, 1994. While from the HEXOS observations no wind speed dependence of the Dalton number could be inferred, the laboratory observations do seem to suggest a systematic trend with wind speed. In this context it should be noted that the minimum in the observed Dalton number from the laboratory which occurs at about 3 m/s corresponds with the minimum wind speed needed for wave generation. For winds above 3 m/s a rather sudden increase in mean square slope of the waves was observed corresponding with an increase of Dalton number with wind speed. In this manner Ocampo-Torres et al, 1994, could establish a relation between moisture transfer and the sea state.

4. CONCLUSIONS

We have investigated the dependence of heat and moisture flux on the sea state. The sea state dependence is much less pronounced than for momentum transfer since both the Stanton number C_{q_*} and the Dalton number C_m depend on the square root of the drag coefficient.

Makin and Mastenbroek (1996) also made an attempt to model the impact of ocean waves on the heat flux by using a more sophisticated turbulence closure model. These authors claim that a mixing length model such as used in this note would result in a decrease of exchange coefficients for increasing wind speed and that a so-called two-equation eddy viscosity model is needed to obtain an increase with wind speed. The present results seem to indicate that a more sophisticated turbulence model is not required to obtain the desired trend with wind speed because the ocean waves also have, through the wave-diffusion coefficient D_w, a direct impact on heat exchange.

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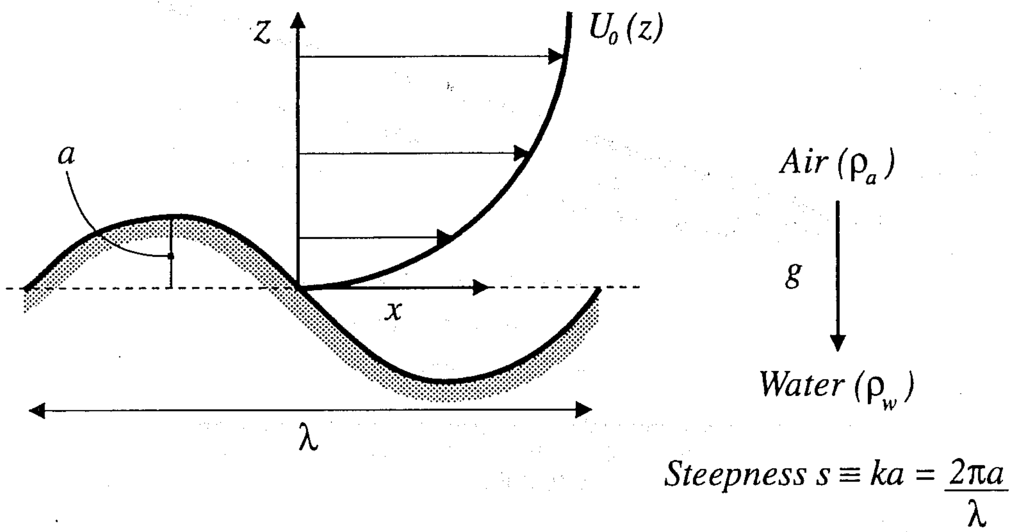

Fig 1   The problem

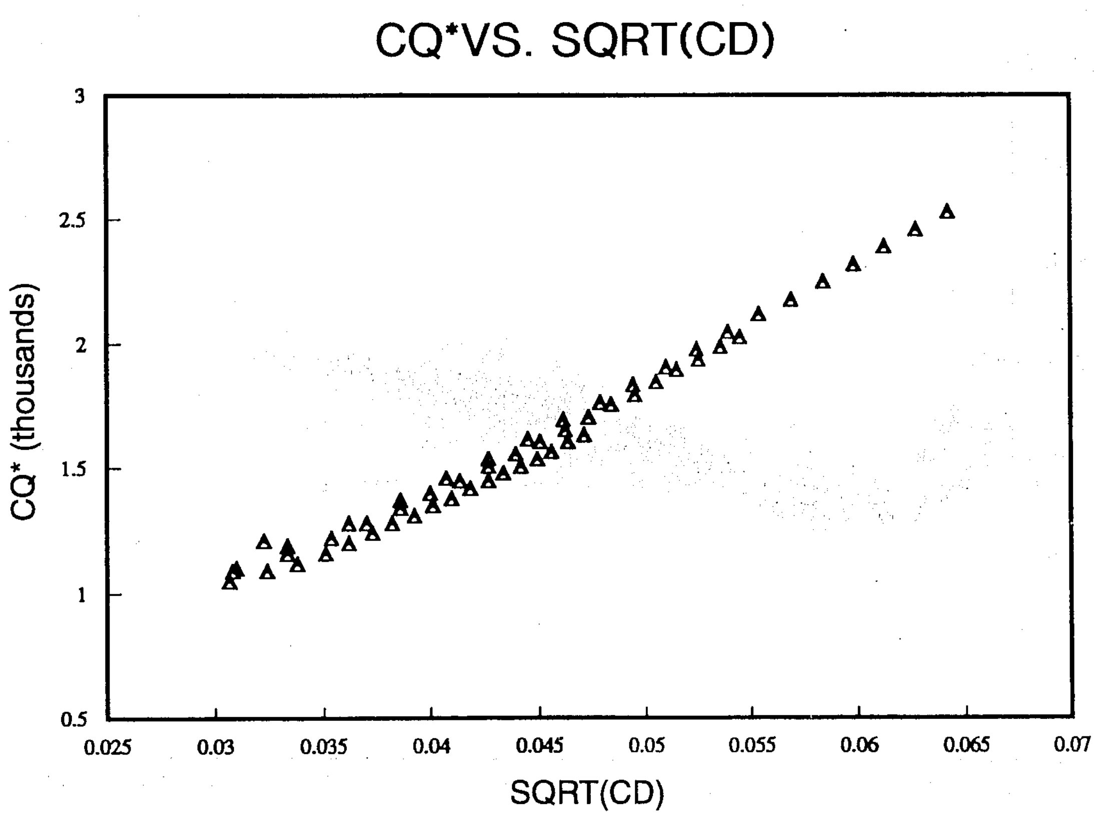

Fig. 2 Dependence of Stanton number C_q on the drag coefficient C_D.

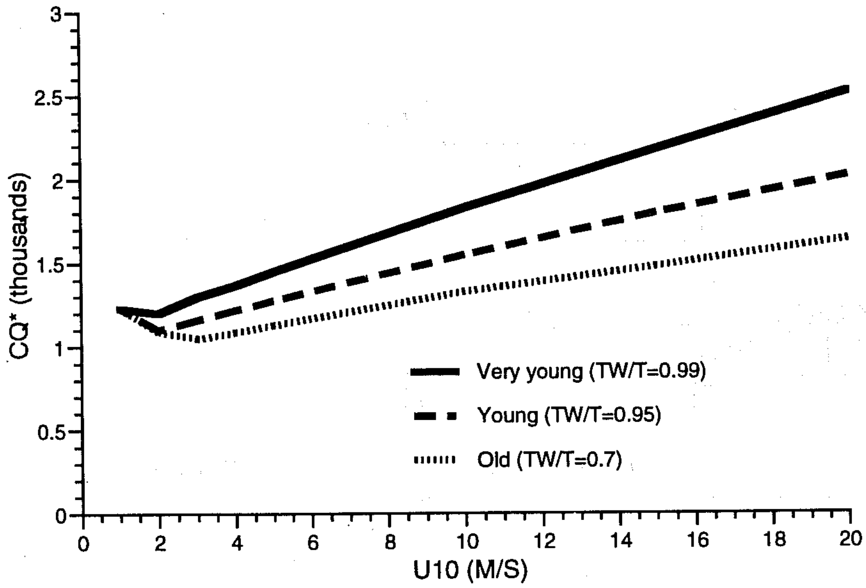

Fig. 3 Sea state dependence of Stanton number.

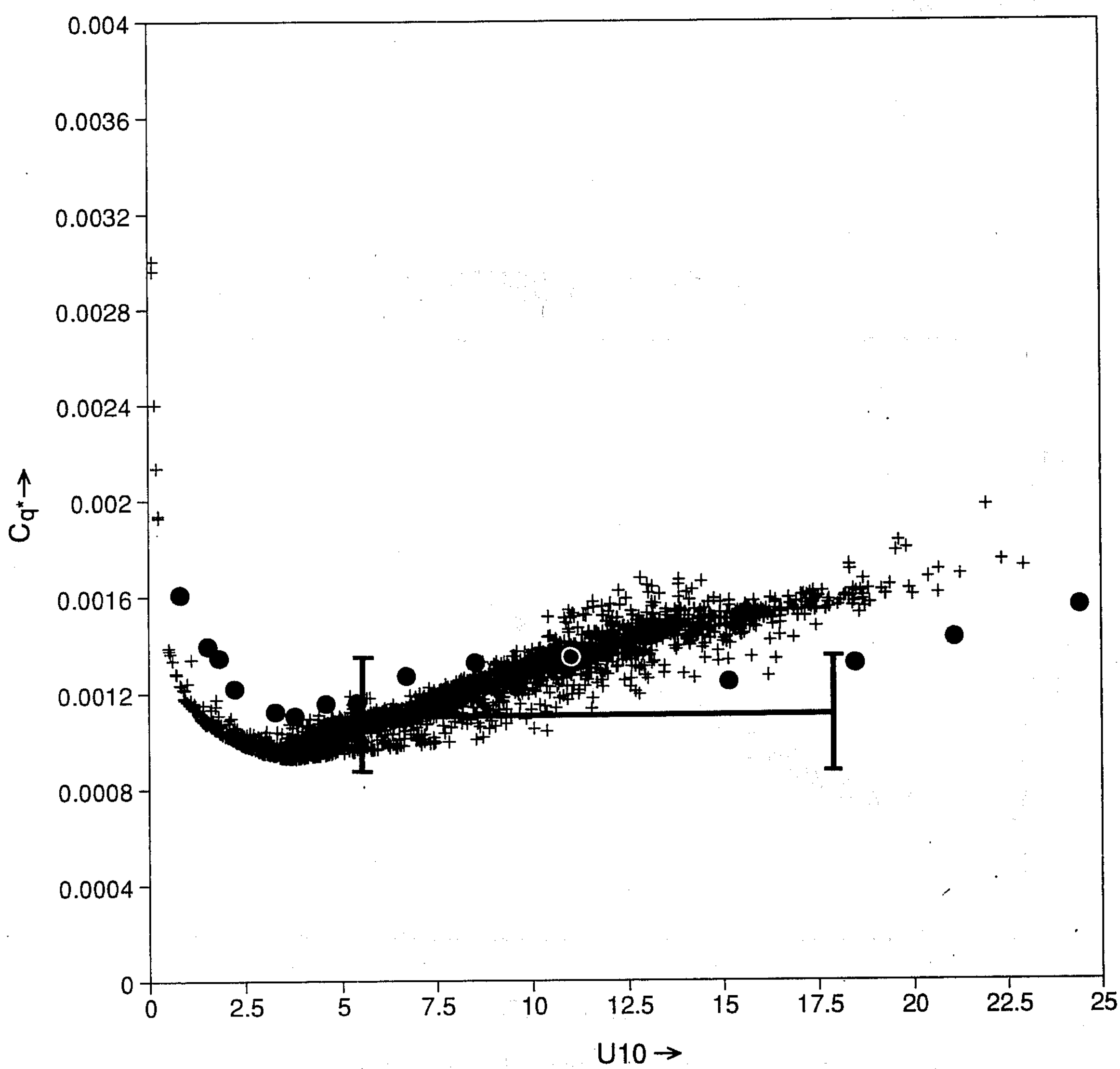

Fig.4 Simulated Stanton number versus wind speed. The horizontal line denotes the mean HEXOS result for moisture flux (De Cosmo et al, 1996) while the dots denote the laboratorty results of Ocampo-Torres et al (1994).

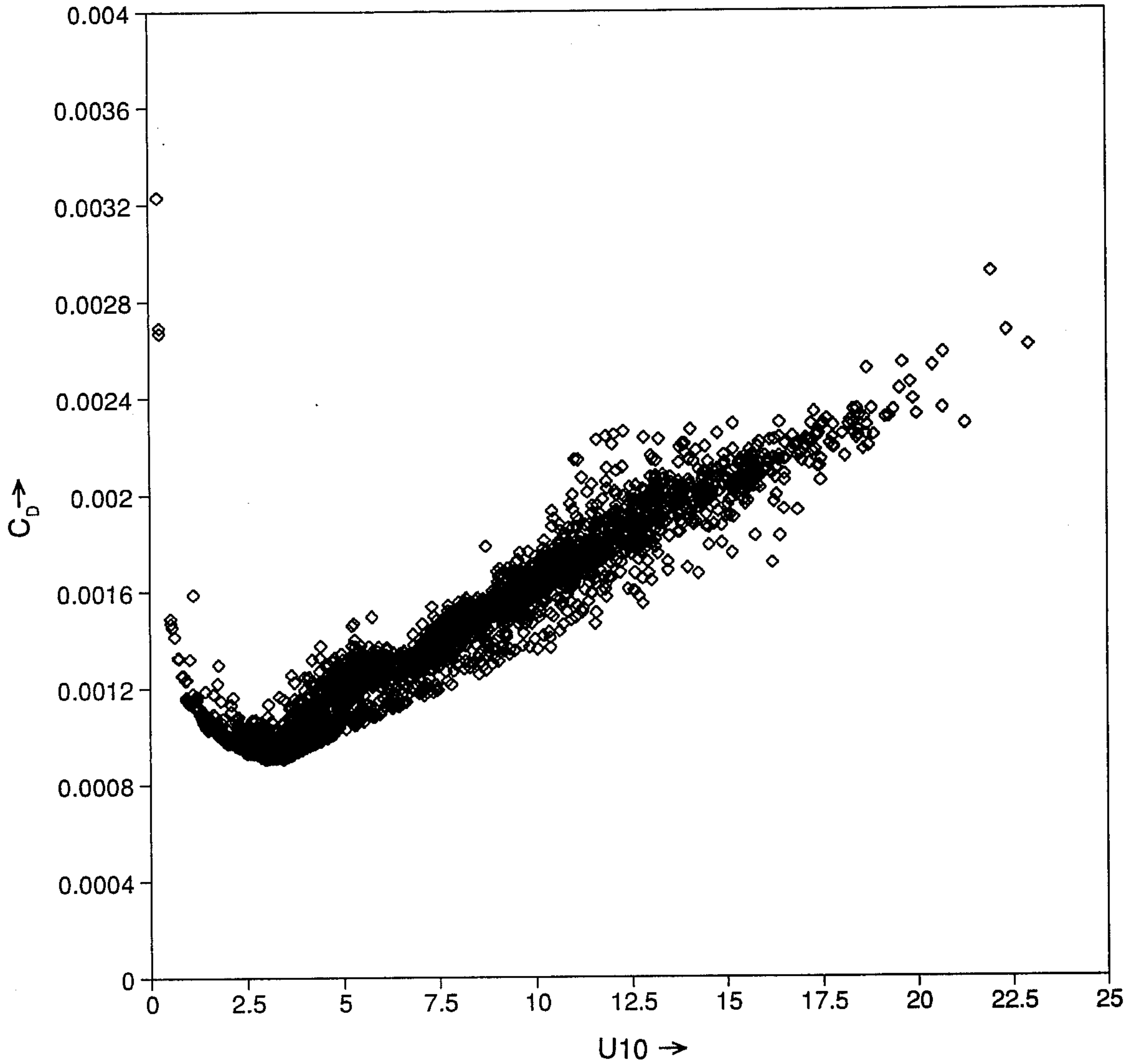

Fig. 5 Simulated Drag coefficient versus wind speed.