ON THE MASS FLUX APPROACH FOR ATMOSPHERIC CONVECTION

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1. INTRODUCTION

Over the last decade the mass flux approach for parameterizing turbulent transport in atmospheric convection has become increasingly popular. Several reasons for this growing interest can be given. First, it has been realised only recently that this concept is not only applicable for cumulus parameterization, for which it was originally developed (Ooyama 1971; Arakawa and Schubert 1974; Betts 1975). Recent studies show that a mass flux concept can also be used for representing transport in the dry boundary layer (Businger and Oncley 1990; Wyngaard and Moeng 1992) and in stratocumulus (Moeng et al. 1992). A second reason is that a mass flux concept is particularly suitable if one wants to include the transport of chemically active tracers in the parameterization (Chatfield and Brost 1987).

In this paper we will review the mass flux approach for both dry and moist convection. In section 2 we discuss the mass flux approximation which lies at the basis of the mass flux approach. Secondly, we discuss entrainment and detrainment rates that describe the lateral mass exchange between up- and downdrafts. Although these are crucial processes in any mass flux parameterization, fundamental definitions of these rates are lacking in the literature. Therefore, in section 3, we propose basic definitions for these exchange rates and discuss the relation with definitions used in operational parameterizations. In section 4, large eddy simulation (LES) results of entrainment and detrainment rates for shallow cumulus convection are discussed. Furthermore the impact of these rates on mass flux parameterizations is explored, using a one-column model. This part is a short review of the work by Siebesma and Cuijpers (1995) and Siebesma and Holtslag (1996). Inspired by the results of section 4, we propose and evaluate in section 5 a practical parameterization for entrainment and detrainment in shallow cumulus convection, much along the lines of Nordeng (1994). Conclusions and future perspectives are discussed in section 6.

2. THE MASS FLUX APPROXIMATION

2.1 Introduction

The basic assumption in the mass flux approach is that vertical turbulent mixing can be well approximated in terms of bulk updrafts and downdrafts. Without making a specific choice for how to define the updraft (+) and downdraft (-) part we can formally rewrite the vertical turbulent flux of an arbitrary field ϕ into three separate terms (see (A6) of the Appendix for a derivation)

$$\left(\overline{w'\phi'}\right) = a_+ \overline{w'\phi'}^{+} + (1-a_+)\overline{w'\phi'}^{-} + a_+ \left(w_+ - \overline{w}\right)\left(\phi_+ - \phi_-\right) \qquad (2\text{-}1)$$

The third term on the rhs of (2-1) describes the contribution of the turbulent transport due to the bulk up- and downdrafts. Formally, the mass flux approximation implies that one can neglect the first two terms of the rhs of (2-1)

$$\begin{aligned}\left(\overline{w'\phi'}\right) &\approx a_+ \left(w_+ - \overline{w}\right)\left(\phi_+ - \phi_-\right) \\ &\equiv M\left(\phi_+ - \phi_-\right)\end{aligned} \qquad (2\text{-}2)$$

where in the last step of (2-2) a mass flux M is introduced. This result, originally introduced for cumulus parameterizations (Ooyama 1971; Arakawa and Schubert 1974; Betts 1975) implies that one only needs to parameterize the mass flux M and bulk fields in the up and downdrafts in order to estimate turbulent fluxes.

2.2 Theory

Concerning the validity of (2-2) some interesting ideas have been put forward by Wyngaard and Moeng (1992). We define the updraft part to be that area where the vertical velocity is positive (*u*) and the downdraft area as the complementary part (*d*). By assuming that w and ϕ have a gaussian joint probability density function (pdf) they show that

$$\overline{w'\phi'} = b\,\sigma_w \left(\phi_u - \phi_d\right) \qquad \text{with } b = \frac{\sqrt{2\pi}}{4} = 0.627 \qquad (2\text{-}3)$$

where σ_w is the rms value of w. In order to make contact with (2-1) we rewrite (2-3) in terms of a mass flux form, still assuming that w has a gaussian distribution so that $M = \sigma_w/\sqrt{2\pi}$. This implies

$$\overline{w'\phi'} = \nu_{ud} M \left(\phi_u - \phi_d\right) \qquad \text{with } \nu_{ud} = \frac{2\pi}{4} \approx 1.57 \qquad (2\text{-}4)$$

This result means that *given* a *gaussian* joint pdf the mass flux term in (2-1) explains roughly 60% ($\sim 1/\nu_{ud}$) of the total flux. Note that (2-4) has to be contrasted with the classical mass flux approximation (2-2) which implicitly assumes $\nu_{ud}=1$. In the next two sections we will evaluate these ideas for different types of boundary layers

2.3 Evaluation of the mass flux approximation for the clear and stratocumulus topped PBL

The value of the enhancement factor ν_{ud} has been tested in the PBL both from observations and utilizing LES outputs for $\phi = \{q_t, \theta\}$ where q_t is the total water specific humidity and θ the potential temperature. For the surface layer Businger and Oncley (1990) have found an almost constant value for b ~ 0.6 (implying $\nu_{ud} \sim 1.5$) over a wide stability range in agreement with (2-3). For the dry convective boundary layer Wyngaard and Moeng (1992) have found, using LES data, for the "bottom-up" case again b~0.6 but for the "top-down" case a slightly lower value of b ~ 0.5 due to the fact that the joint pdf is non-Gaussian in that case. Recently, the value of ν_{ud} has also been deduced experimentally for the stratocumulus topped boundary layer such as observed during ASTEX (De Laat en Duynkerke 1996). By conditional sampling of the flight tracks an enhancement factor $\nu_{ud} \sim 1.6$ has been found both in the stratocumulus layer as well as in the dry convective boundary layer below, in agreement with (2-4).

2.4 Evaluation of the mass flux approximation for shallow cumulus

The validity of (2-4) has also been tested for shallow cumulus convection such as observed during BOMEX using LES outputs (Siebesma and Cuijpers 1995). The results for a updraft-downdraft decomposition is shown in Fig.1a for $\phi=q_t$. The solid line denotes the total moisture flux (lhs of (2.4)) while the dashed line denotes the mass flux approximation (rhs of (2-4)). The ratio between the two curves gives the enhancement factor ν_{ud}. As can be seen from the figure the mass flux approximation works quite well below cloud base with a rather constant enhancement factor $\nu_{ud} \sim 1.2$. Above cloud base however, the mass flux approximation contributes to less than 10% of the total turbulent flux. Apparently a simple updraft-downdraft decomposition is not appropriate for cumulus clouds.

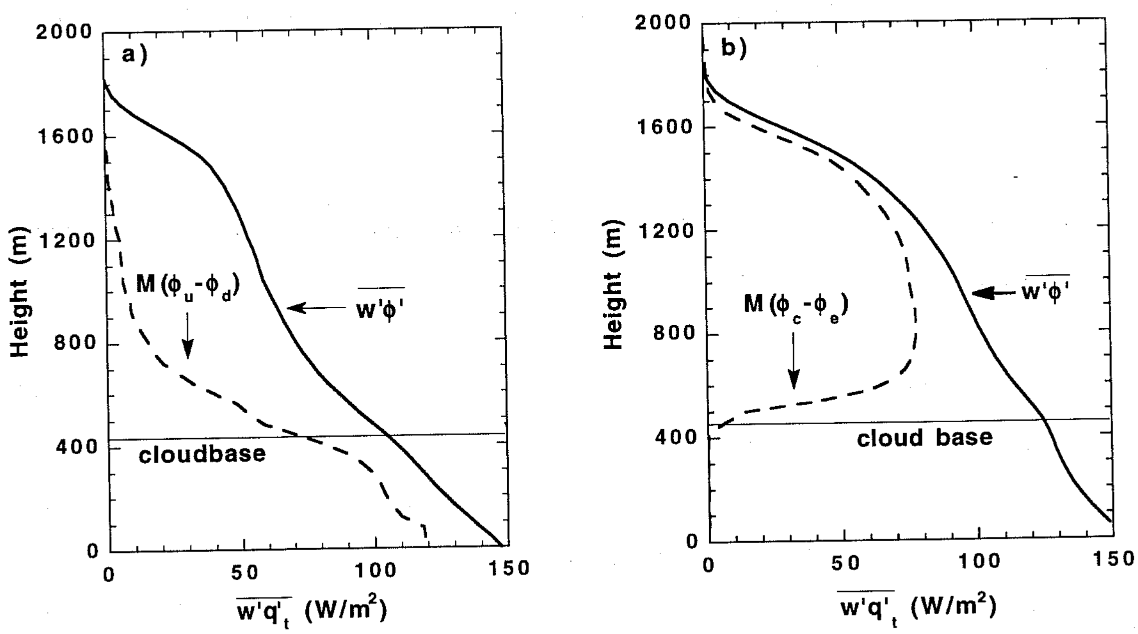

Fig. 1. Comparison of the moisture turbulent flux with the mass flux approximation using an updraft-downdraft decomposition (a) and a cloudcore decomposition (b) for shallow cumulus convection.

Since in cumuli most of the upward transport takes place in "active" cloudy part we explore another decomposition in the cloud layer. If we define the updraft part to be the cloudy part that is positively buoyant (*c*) and the downdraft part as the complementary environmental part (*e*) we expect

$$\overline{w'\phi'} = \nu_{ce} M\left(\phi_c - \phi_e\right) \tag{2-5}$$

with an a priori unknown enhancement factor ν_{ce}. Results for this cloudcore decomposition based on the same BOMEX case are shown in Fig.1b. Note that with this decomposition the mass flux approximation explains 80~90% of the total flux in the cloud layer corresponding to a very modest enhancement factor of $v_{ce} \approx 1.1 \sim 1.2$. Only near cloud base (~500 m) larger deviations are found due to the fact the environmental turbulence term (see (2-1)) gives a significant contribution.

The reason why the cloudcore-decomposition works so well is illustrated in Fig. 2 where we show a scatterplot of values for q_t and w for all the grid points at 1200 m which is in the middle of the cloud layer. A distinction has been made between the environmental grid points (no liquid water), buoyant cloudy grid points and non-buoyant cloudy grid points. From this graph it is immediately clear that the Gaussian joint pdf assumption leading to (2-3) and (2-4) does not hold anymore in the cumulus cloud layer. The distribution has become bimodal with one small cloudy part that is responsible for the fast upward vertical transport and a large environmental part that does the slow compensating downward transport. Apparently the cumulus clouds strongly organize themselves in narrow channels, resulting in a strongly skewed, bimodal and certainly non-gaussian joint pdf. This explains the success of the cloudcore decomposition mass flux approximation for cumulus convection as given by (2-5) with only a very modest enhancement factor close to 1. Similar results have been found for the liquid water potential temperature θ_l.

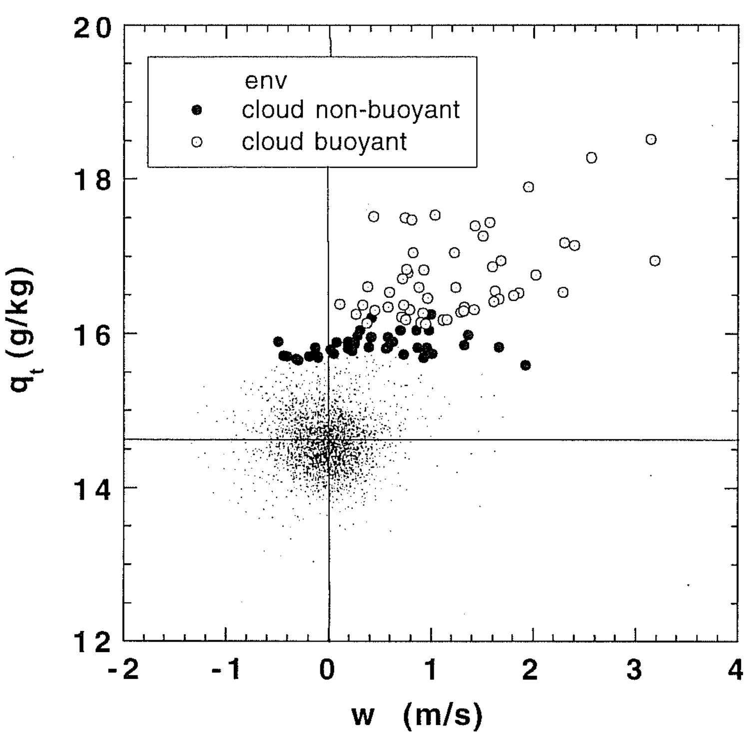

Fig. 2. Scatterplot of the vertical velocity w and the total water specific humidity q_t in the middle of the cumulus cloud layer at 1200 m based on LES output. The thin points represent unsaturated gridpoints, the open circles buoyant oversaturated gridpoints and the solid circles nonbuoyant oversaturated gridpoints.

3. FUNDAMENTALS OF ENTRAINMENT AND DETRAINMENT

3.1 Introduction

Entrainment of dry air into cumulus clouds and detrainment of cloudy air into the environment strongly affects the cloud core field ϕ_c and the environmental field ϕ_e. More general the fields ϕ_+ and ϕ_- (see Fig.3) are strongly affected by lateral mass exchange. Therefore, when using a mass flux approach it is crucial for the calculation of ϕ_+ and ϕ_- to take into account this lateral mass exchange processes.

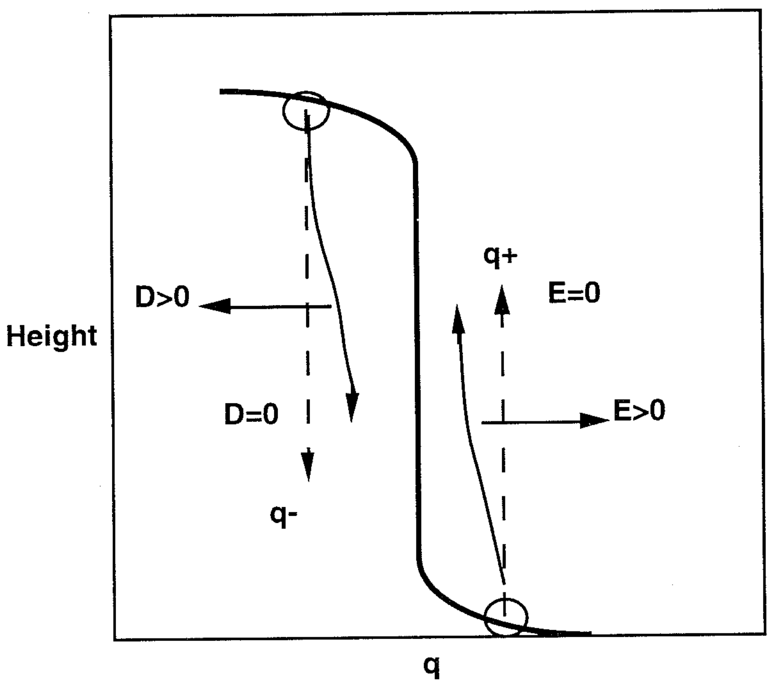

Fig. 3. Schematic humidity profile q in a dry convective boundary layer. A dry sinking parcel with properties q- from the inversion will be moistened during the descent due to detrainment. A moist rising parcel with properties q+ from the surface layer will become dryer during the ascent due to entrainment.

Our objective is to develop a method to estimate entrainment and detrainment rates from a LES model. A direct measurement is even within the LES context quite unpractical since the cloud interface has in general a complicated time varying geometry (see Fig.4). Instead, one can derive budget equations for the decomposed fields and determine the lateral mass exchange between these two regions as residuals of these budget equations (Schumann and Moeng 1991; Siebesma and Cuijpers 1995). In this section we derive some operational definitions of entrainment and detrainment rates based on fundamental budget equations and test their validity with LES. To remain concrete we consider a cloudcore-environment interface in this sequel but it should be understood that all the arguments hold for any arbitrary interface. Just for the sake of simplicity we assume a constant density, but generalization to the quasi-Boussinesq approximation is straightforward.

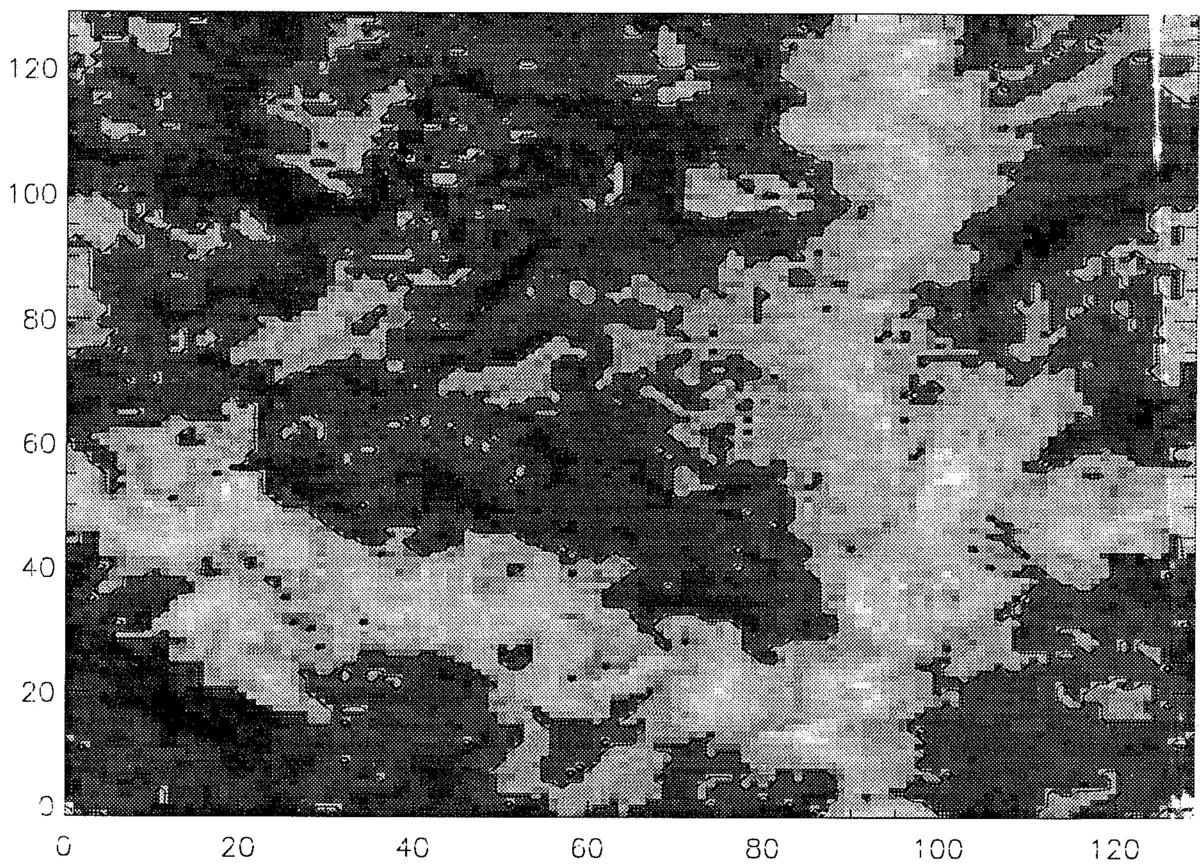

Fig. 4. A LES-snapshot of the vertical velocity on a horizontal slab of 6 by 6 km in the middle of a convective dry boundary layer. The numbers on the x and y axis label the grid points. The dark colors correspond to downward velocities and the light colors to upward velocities. The highly irregular spatial interface between the updrafts and the downdrafts is clearly visible.

3.2 Definitions and approximations

In the Appendix we have derived (following Gregory and Miller 1989) the mass continuity equation at height z of the cloudy part in a domain with area A

$$\frac{\partial a_c}{\partial t} + \frac{1}{A}\oint_{\text{interface}} \hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i)\, dl + \frac{\partial a_c w_c}{\partial z} = 0 \tag{3-1}$$

where we will interpret the subscript c from now on as as cloudcore part. Therefore a_c is the fractional cloudcore cover, $\hat{\mathbf{n}}$ is a unit vector perpendicular to the interface, $\mathbf{u}$ is the full 3d velocity vector of the mass at the interface and $\mathbf{u}_i$ is the velocity of the interface. Note that this equation is slightly different from results previously obtained by Schumann and Moeng (1991) and Gregory and Miller (1989). In most mass flux parameterizations entrainment and detrainment is introduced through the cloudcore continuity equation (for instance Arakawa and Schubert 1974)

$$\frac{\partial a_c}{\partial t} + (D - E) + \frac{\partial a_c w_c}{\partial z} = 0 \tag{3-2}$$

Inspection of (3-1) and (3-2) shows that D-E can be defined as

$$D - E \equiv \frac{1}{A}\oint_{\text{interface}} \hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i)\, dl \tag{3-3}$$

which clearly defines D-E as the *net* mass exchange across the interface. Let us emphasise that it is the mass velocity *relative* to the interface velocity that enters the definition (3-3). This way it is guaranteed that D-E = 0 if the interface is moving with the same velocity as the mass on the interface.

Since entrainment deals with inflow and detrainment with outflow we can proceed by postulating separate definitions for E and D

$$E \equiv -\frac{1}{A}\int_{\hat{\mathbf{n}}\,.\,(\mathbf{u}\,-\,\mathbf{u}_i)\,<\,0} \hat{\mathbf{n}}\,.\,(\mathbf{u} - \mathbf{u}_i)\,dl$$

$$D \equiv \frac{1}{A}\int_{\hat{\mathbf{n}}\,.\,(\mathbf{u}\,-\,\mathbf{u}_i)\,>\,0} \hat{\mathbf{n}}\,.\,(\mathbf{u} - \mathbf{u}_i)\,dl \qquad (3\text{-}4)$$

Therefore (3-4) simply defines entrainment as that part of the contour integral (3-3) where there is a inflow of mass into the cloudy part and detrainment as the complementary part where there is mass outflow into the environment.

Matters complicate if we want to invoke E and D defined by (3-4) into a budget equation for a prognostic field ϕ in the cloudy part (see Appendix for a derivation)

$$\frac{\partial\, a_c\, \phi_c}{\partial t} + \frac{1}{A}\oint_{\text{interface}} \hat{\mathbf{n}}\,.\,(\,\mathbf{u} - \mathbf{u}_i\,)\,\phi\, dl + \frac{\partial a_c\, \overline{w\phi}^{\,c}}{\partial z} = a_c\, F_c \qquad (3\text{-}5)$$

Note that at this point it is not possible to infer E and D as defined in (3-4) into (3-5) since along the contour integral $\hat{\mathbf{n}}.(\mathbf{u} - \mathbf{u}_i)$ is convoluted with the field ϕ. In order to derive equations used for parameterizations in the literature we make the mean field approximation that entrainment transports average properties of the environment into the cloud and detrainment transports average properties of the cloud ensemble into the environment

$$\frac{1}{A}\oint_{\hat{\mathbf{n}}.\,(\mathbf{u}\,-\,\mathbf{u}_i)\,>\,0} \hat{\mathbf{n}}.(\mathbf{u} - \mathbf{u}_i)\,\phi\, dl \approx \frac{\phi_c}{A}\oint_{\hat{\mathbf{n}}.\,(\mathbf{u}\,-\,\mathbf{u}_i)\,>\,0} \hat{\mathbf{n}}.(\mathbf{u} - \mathbf{u}_i)\,dl = D\,\phi_c$$

$$\frac{1}{A}\oint_{\hat{\mathbf{n}}.\,(\mathbf{u}\,-\,\mathbf{u}_i)\,<\,0} \hat{\mathbf{n}}.(\mathbf{u} - \mathbf{u}_i)\,\phi\, dl \approx \frac{\phi_e}{A}\oint_{\hat{\mathbf{n}}.\,(\mathbf{u}\,-\,\mathbf{u}_i)\,<\,0} \hat{\mathbf{n}}.(\mathbf{u} - \mathbf{u}_i)\,dl = -\,E\,\phi_e \qquad (3\text{-}6)$$

Accepting this strong approximation for the moment we can substitute (3-6) in (3-5) and find

$$\frac{\partial\, a_c \phi_c}{\partial t} - E\,\phi_e + D\,\phi_c + \frac{\partial a_c\, \overline{w\phi}^{c}}{\partial z} = a_c F_c \qquad (3\text{-}7)$$

which can be considered as the starting point of many mass flux parameterizations. Likewise, again using the approximation (3-6), we can derive a similar budget equation for the environment

$$\frac{\partial\, (1-a_c)\, \phi_e}{\partial t} + E\,\phi_e - D\,\phi_c + \frac{\partial (1-a_c)\, \overline{w\phi}^{e}}{\partial z} = (1-a_c)\, F_e \qquad (3\text{-}8)$$

In order to obtain separate equations for E and D we substitute the continuity equation (3-2) into (3-7) and (3-8) so that

$$E\,(\phi_c - \phi_e) = \phi_c \frac{\partial M}{\partial z} - \frac{\partial a_c\, \overline{w\phi}^{c}}{\partial z} - a_c \frac{\partial \phi_c}{\partial t} + aF_c \qquad (3\text{-}9a)$$

$$D\,(\phi_c - \phi_e) = \phi_e \frac{\partial M}{\partial z} + \frac{\partial\, (1-a_c)\, \overline{w\phi}^{e}}{\partial z} + \frac{\partial (1-a_c)\, \phi_e}{\partial t} - (1-a)\, F_e \qquad (3\text{-}9b)$$

The result (3-9) has been written in a form slightly different form than Eq. (5-3) used in Siebesma and Cuijpers (1995). However it can be easily written into that form, using the definition

$$\begin{aligned} \overline{w'\phi'}^{c} &= \overline{w\,\phi}^{c} - w_c\,\phi_c \\ \overline{w'\phi'}^{e} &= \overline{w\,\phi}^{e} - w_e\,\phi_e \end{aligned} \qquad (3\text{-}10)$$

so that

$$\begin{aligned} E\,(\phi_c - \phi_e) &= -\,M \frac{\partial \phi_c}{\partial z} - \frac{\partial a_c\, \overline{w'\phi'}^{c}}{\partial z} - a_c \frac{\partial \phi_c}{\partial t} + aF_c \\ D\,(\phi_c - \phi_e) &= -\,M \frac{\partial \phi_e}{\partial z} + \frac{\partial\, (1-a_c)\, \overline{w'\phi'}^{e}}{\partial z} + \frac{\partial (1-a_c)\, \phi_e}{\partial t} - (1-a)\, F_e \end{aligned} \qquad (3\text{-}11)$$

Since all the terms of (3-9), or equivalently (3-11), can be determined in a LES model except E and D, we can use these equations to obtain these rates as a residual. Although (3-11) is mathematically identical to (3-9), it is from the numerical point of view more convenient to use (3-9) in order to estimate E and D.

3.3 Evaluation of the entrainment/detrainment approximation with LES results

The result (3-9) is not incontrovertible. For instance, it has been suggested (Emanuel 1995) that in the case $\phi=q_t$ one should take in (3-9) for ϕ_c the saturation value q_s instead of $q_{t,c}$ as suggested by (3-9). The motivation for this is that total water leaving the cloud should have the saturation value rather than $q_{t,c}$.

We will therefore explore the validity of the approximation (3-6) that leads to (3-9). The problem can be rephrased more clearly if we use (3-6) as an alternative definition of E and D

$$D_\phi = \frac{\oint_{\hat{n}.(\mathbf{u}-\mathbf{u}_l)>0} \hat{n}.(\mathbf{u}-\mathbf{u}_l)\,\phi\,dl}{\phi_c}$$

$$E_\phi = -\frac{\oint_{\hat{n}.(\mathbf{u}-\mathbf{u}_l)<0} \hat{n}.(\mathbf{u}-\mathbf{u}_l)\,\phi\,dl}{\phi_e} \qquad (3\text{-}12)$$

where an "effective medium" E_ϕ and D_ϕ have been indexed to indicate that there might be a ϕ-dependence. If (3-6) is a good approximation then LES-results should support that i) E_ϕ and D_ϕ should be ϕ-independent and ii) the resulting D_ϕ-E_ϕ should coincide with the D-E residual that can be determined as a residual from the continuity equation (3-2).

We have checked this for the BOMEX case (Siebesma and Cuijpers 1995) which is a shallow cumulus case with a well defined steady state. This last fact allows to do long time averaging thereby improving the statistical convergence. First we determine the (time averaged) entrainment rates E for the cloud interface based on two different fields $\phi \in \{\theta_l , q_t\}$ using equation (3-9a). As can be seen in Fig.5a the two fields give exactly the same entrainment rates. The detrainment rate was determined using (3-9b), again for the two fields $\phi \in \{\theta_l , q_t\}$. Moreover we determined D as a residual from the continuity equation (3-2) using the already obtained values of E. The results plotted in Fig.5b again show an amazing data collapse, strongly supporting assumption (3-6). We realise that this check is not a complete proof for the fact that $E=E_\phi$ and $D=D_\phi$ but at least it does show that (3-9) gives consistent results so that indeed we can use these equations as operational definitions for E and D.

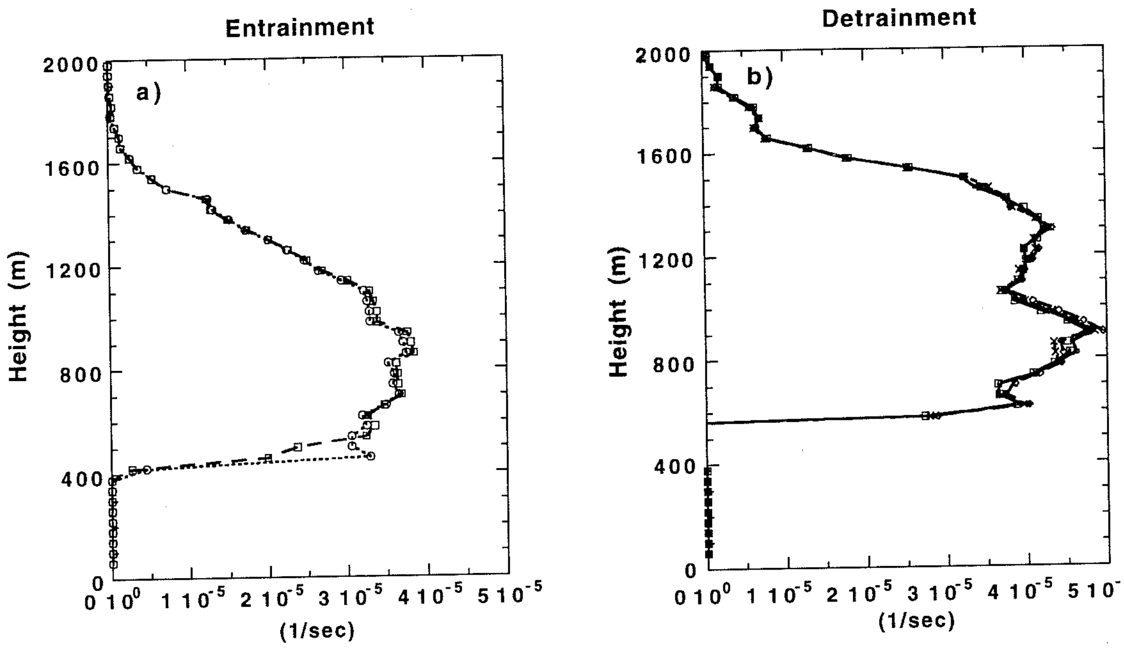

Fig. 5. Entrainment and detrainment rates determined using LES output for based on different fields. In a) entrainment rates are shown obtained as a residual of (3-9) for $\phi=q_t$ (squares) and $\phi=\theta_l$ (circles). In b) as a) but for detrainment. In b) we also plot detrainment results as a residual of the continuity equation (3-2). Note that residuals give an amazing data collapse for all calculations.

4. ENTRAINMENT AND DETRAINMENT IN SHALLOW CUMULUS CONVECTION

4.1 Shallow cumulus convection parameterization

In order to formulate an operational parameterization for non-precipitating shallow cumulus convection, we make the three usual assumptions for the moist conserved variables $\phi = \{q_t, \theta_l\}$:

1) no enhancement factor, $\nu=1$, so that we can use (2-2)
2) cloudcore cover much smaller than 1, $a_c<<1$, so that $\phi_e \approx \overline{\phi}$
3) the cloud ensemble is in a steady state, so that $\frac{\partial \phi_c}{\partial t} = 0$

so that (3-2), (3-7) and (3-8) simply reduce to (Tiedtke 1989; Siebesma and Holtslag 1996)

$$\frac{\partial M_c}{\partial z} = E - D \tag{4-1a}$$

$$\frac{\partial M_c \phi_c}{\partial z} = E\,\overline{\phi} - D\,\phi_c \tag{4-1b}$$

$$\frac{\partial \overline{\phi}}{\partial t} = -\frac{\partial M_c(\phi_c - \overline{\phi})}{\rho \partial z} + F \tag{4-1c}$$

The first term on the rhs of (4-1c) is the turbulent flux divergence within the mass flux approximation that needs to be parameterized. Hence, the parameterization problem is reduced to the determination of M and ϕ_c. These can be obtained from a steady state cloud model such as defined by (4-1a) and (4-1b) *provided* we know E and D plus the boundary conditions of M and ϕ_c at cloud base. This latter, also known as the closure problem, is an interesting and important topic in itself but is outside the scope of this present paper. The entrainment and detrainment rates are usually parameterized in terms of the mass flux:

$$\begin{aligned} E &= \varepsilon M_c \\ D &= \delta M_c, \end{aligned} \tag{4-2}$$

where fractional entrainment and detrainment rates ε and δ are introduced. In Tiedtke (1989) these are assumed to be equal and constant with values based on laboratory experiments with plumes (see Turner (1973) for a review).

$$\varepsilon = \delta = 3 \times 10^{-4}\ \mathrm{m}^{-1}\ . \qquad (4\text{-}3)$$

4.2 LES Results for entrainment and detrainment rates

Using a realistic large-scale forcing and initial profiles, a LES run of 7 hours based on BOMEX has been made during which the cloud ensemble was in a acceptable steady state (Siebesma and Cuijpers 1995). The output of the LES has been used to calculate fractional entrainment and detrainment rates as defined by (4-2) using the method explained in section 3. The following typical values were obtained

$$\begin{aligned} \varepsilon &\approx 1.5 \sim 2.5 \times 10^{-3}\ \mathrm{m}^{-1} \\ \delta &\approx 2.5 \sim 3 \times 10^{-3}\ \mathrm{m}^{-1}\ . \end{aligned} \qquad (4\text{-}4)$$

The spread in the results (4-4) is mainly due to variation of the rates with height. Various sensitivity tests have been made by varying the resolution and domain size of the LES model. In all cases however the fractional entrainment and detrainment rates remained within the range indicated by (4-4). Note that these values appear to be almost one order of magnitude larger than suggested by (4-3) and that the detrainment is systematically larger than the entrainment implying a monotonic decrease of the mass flux with height.

One may wonder how typical the results (4-4) are for shallow convection in general. Recently we have made a LES run of a shallow cumulus convection case during the second Lagrangian of ASTEX (Bretherton and Pincus 1995; Bretherton et al 1995). The same analysis for the cloudcore entrainment and detrainment processes has been applied. We obtained a rather constant value of $1.5\mathrm{x}10^{-3}\ \mathrm{m}^{-1}$ for ε while δ is increasing with height from $3\mathrm{x}10^{-3}$ to $7\mathrm{x}10^{-3}\ \mathrm{m}^{-1}$. These even higher detrainment values are probably due to the fact that the environmental air was quite dry in this case. As a third case we also analysed LES output of the Puerto Rico case (as defined in Cuijpers and Duynkerke 1993) which was a shallow cumulus case with shear. In this case we also found values for ε and δ in the range indicated by (4-4).

4.3 One-column model impacts of entrainment and detrainment

The impact of the entrainment and detrainment rates have been evaluated in Siebesma and Holtslag (1996) using a one-column model derived from the ECHAM3 climate model (Roeckner et al. 1992). In order to focus on the turbulent mixing processes the radiative cooling and the surface fluxes have been prescribed. As a result the only active parameterization schemes are the Tiedtke mass flux scheme (Tiedtke 1989) and a boundary layer scheme for the turbulent mixing in the subcloud layer. For the latter a local diffusion scheme is used as proposed by Louis (1979).

Runs based on BOMEX have been made for two cases: 1) a standard run with the operational values for ε and δ given by (4-3), and 2) a revised run with $\varepsilon=2\times10^{-3}$ m^{-1} and $\delta=2.7\times10^{-3}$ m^{-1} as suggested by the LES results (4-4). The one-column model was initialized with with the same profiles as the LES model and also the prescribed forcings (radiative cooling and surface fluxes) were taken from the LES run. Since we are mainly interested in the thermodynamics we also prescribed the wind fields.

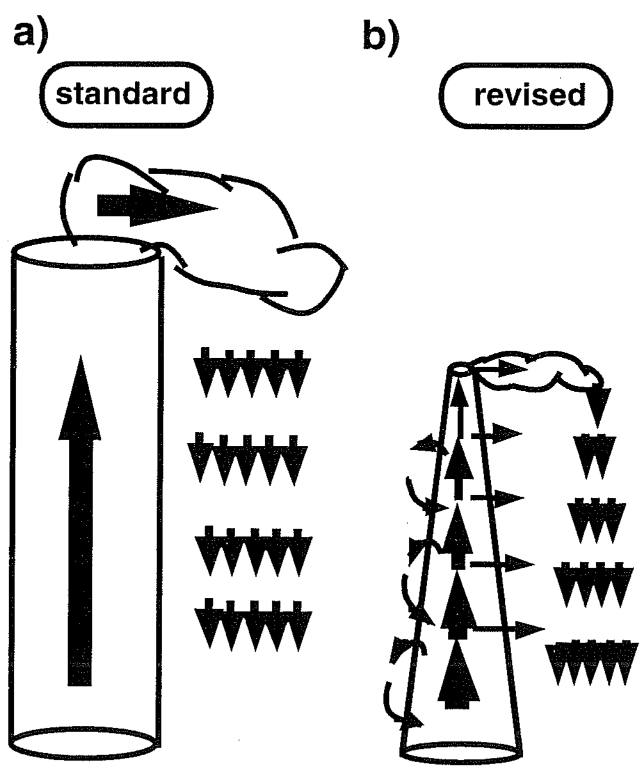

Fig. 6. Schematic picture of the turbulent mixing mechanism of a shallow cloud ensemble. In case of the standard values of ε and δ, the scheme behaves approximately as a non-leaking funnel with massive detrainment at cloud top. When using the enhanced values of ε and δ, as suggested by the LES results, there is more intense lateral mixing and a decreasing mass flux with height due to the fact that d>e and hence little massive detrainment at the top. (Siebesma and Holtslag 1996)

The standard run with the values for ε and δ given by (4-3) strongly overestimates the vertical turbulent mixing of heat and moisture and, as a result, the model is tending to a new equilibrium, reached after 12 hours, where the cloud layer is too dry and too warm and where the inversion has completely disappeared. The revised run however, where values of ε and δ are used as suggested by the LES results (4-4) produces a realistic steady state in agreement with LES and observations. This can also be demonstrated analytically since one can easily solve the set of equations (4-1) assuming linear profiles of the slab averaged fields. We have calculated the cloud fields and mass flux analytically. They compare well with the LES results, *provided* we use the enhanced values for ε and δ.

These findings promote quite a different picture for the dynamics of shallow cumulus convection. With the standard values of ε and δ, the lateral mass exchange is relatively small so that the cloud ensemble acts like a non-leaking funnel (see Fig. 6a). Here moisture and heat are transported almost without losses up to cloud top. The LES results however, suggest a rather different physical mechanism (see Fig. 6b) of a funnel that is heavily leaking and is extensively exchanging mass, heat and moisture all the way from cloud base to cloud top. As a result, there is hardly cloud mass left for massive detrainment at the top so the inversion does not get excessively moistened and cooled by cloud convection. Most of the mixing is done "on the road" to the inversion within the cloud layer. The physics behind this is that there are a lot of shallow clouds that do not reach the inversion at all and already detrain in the cloud layer below the inversion.

In the operational ECMWF model the values for ε and δ have been enhanced for shallow cumuli according to

$$\varepsilon = \delta = 3 \times 10^{-4} \times \max\left(1,\ 1 + \frac{p(z_{lcl}) - p(z)}{150}\right) \qquad (4\text{-}5)$$

where $p(z_{lcl})$ is the pressure (in mb) at cloud base height. One-column tests using (4-5) give an improvement compared with the values (4-3) but still give rise to a rather different steady state solution, mainly due to the fact that the entrainment and detrainment rates in (4-5) still take the same values. As a result the mass flux is constant with height and all the cloud mass is massively detrained in the inversion layer.

In conclusion, we do not claim that the values used in the revised run are universal for ε and δ. They are likely a function of the environmental conditions. On the other hand, the results clearly show that for shallow cumulus convection δ is systematically larger than ε and that both rates are roughly one order of magnitude larger than the standard values (4-3). It is also shown that the mass flux method is a sound approach provided that the correct values for ε and δ are used. What is needed is a dynamical parameterization for this exchange rates based on physical concepts rather than prescribed values like (4-3)-(4-5).

5 NEW ENTRAINMENT AND DETRAINMENT PARAMETERIZATION FOR SHALLOW CUMULUS CONVECTION

5.1 Simple scaling arguments for the fractional entrainment rate

According to (3-9) the tendency of a cloudcore field ϕ_c due to entrainment only is given by

$$a_c \left(\frac{\partial \phi_c}{\partial t} \right)_E = E \left(\phi_c - \phi_e \right) \qquad (5\text{-}1)$$

which can, using $E = a_c\, w_c\, \varepsilon$, be rewritten into

$$\left(\frac{\partial \phi_c}{\partial t} \right)_E = \varepsilon\, w_c \left(\phi_c - \phi_e \right) \qquad (5\text{-}2)$$

On the other hand, if we introduce a turbulent mixing time τ that describes the typical time it takes to homogenize a cumulus field with the environment by turbulent mixing, we have

$$\left(\frac{\partial \phi_c}{\partial t} \right)_E \sim \frac{\left(\phi_c - \phi_e \right)}{\tau} \qquad (5\text{-}3)$$

where the '~' sign should be interpreted as 'of the order of'. If we assume that the turbulent mixing time is of the same order as the eddy turnover time of the most active eddies

$$w_c\, \tau = L \qquad (5\text{-}4)$$

where L is the typical size of these eddies. In the cloud layer we assume that the most active eddies at height z are formed by the clouds that actually do reach that height (see Fig. 7). These clouds have a vertical size of $z - z_{lcl}$, where z_{lcl} is the cloud base height. Inspection of (5-2) and (5-3), combined with $L = z - z_{lcl}$, then learns

$$\boxed{\varepsilon \sim \frac{1}{w_c\, \tau} \sim \frac{1}{L} \sim \frac{1}{z - z_{lcl}}} \qquad (5\text{-}5)$$

Note that these simple arguments already give the correct order of magnitude for deep and shallow convection

$$\varepsilon \sim 10^{-3}\ \mathrm{m}^{-1} \quad \text{for shallow convection}$$
$$\varepsilon \sim 10^{-4}\ \mathrm{m}^{-1} \quad \text{for deep convection} \qquad (5\text{-}6)$$

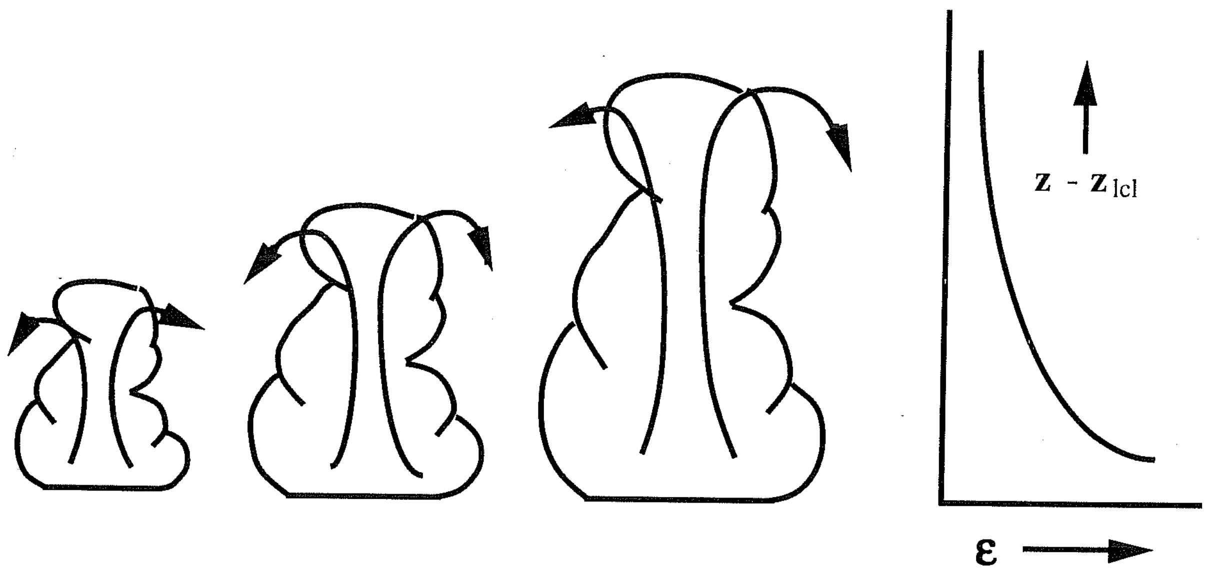

Fig. 7. Schematic illustration of the scaling arguments leading to (5-5). The most active eddies with respect to the turbulent mixing between clouds and environment at height z are the eddies formed by the clouds that actually reach that height.

The result (5-5) has to be contrasted with the usual parameterization of ε based on plume models (Turner 1973)

$$\varepsilon \approx \frac{0.2}{R} \qquad (5\text{-}7)$$

with R being the typical radius of the cloud radius. When taking 500 m for a typical radius of a shallow cumulus cloud, one obtains the traditional underestimated value of ε as given by (4-3).

Since we have used only simple scaling arguments in deriving (5-5), the prefactor needs further determination. As such we will follow an approach proposed recently by Nordeng (1994) derived for deep convection that provides the same general form as (5-5) and moreover, relates the prefactor to the buoyancy.

5.2 Generalized parameterization for entrainment and detrainment

Let us consider the cloudcore equation (3-11) for ϕ_c=w_c and assume that 1) the cloud ensemble is in a steady state and 2) the correlations within the cloudcore can be neglected and 3) that w_e<<w_c. Then (3-11) simplifies to

$$E\, w_c = - M \frac{\partial w_c}{\partial z} + a_c F_c \qquad (5\text{-}8)$$

Furthermore, we make the assumption that the cloudcore source term F_c for w_c is dominated by the buoyancy term

$$a_c\, F_c \approx a_c \frac{g}{T}\left(\theta_{v,c} - \overline{\theta}_v\right) \equiv a_c\, B \qquad (5\text{-}9)$$

On the basis of these of these assumptions (5-8) can be simplified into

$$B - \varepsilon\, w_c^2 - w_c \frac{\partial w_c}{\partial z} = 0 \qquad (5\text{-}10)$$

where we used (4-2). Eq. (5-10) is a well known and useful relation (Simpson and Wiggert 1969) since it relates the vertical cloudcore velocity w_c to the fractional entrainment rate and buoyancy excess of the cloudcore relative to the environment.

On the other hand, we may consider the continuity equation for the cloudcore

$$\frac{1}{M}\frac{\partial M}{\partial z} = \varepsilon - \delta \qquad ((5\text{-}11)$$

or

$$\frac{1}{a_c}\frac{\partial a_c}{\partial z} + \frac{1}{w_c}\frac{\partial w_c}{\partial z} = \varepsilon - \delta \qquad (5\text{-}12)$$

A crucial proposal of Nordeng (1994) is now to link the detrainment to the cloudcore cover divergence since detrainment is closely related with the evaporation of clouds. The remaining entrainment is then coupled to the vertical velocity divergence. This way one obtains two equations out of (5-12)

$$\frac{1}{w_c}\frac{\partial w_c}{\partial z} = \varepsilon \qquad (5\text{-}13a)$$

$$\frac{1}{a_c}\frac{\partial a_c}{\partial z} = -\delta \qquad (5\text{-}13b)$$

Combining (5-13a) and (5-10) and integrating the vertical velocity from cloudbase z_{lcl} to an arbitrary height z provides a parameterization for the fractional entrainment ε only in terms of buoyancy B(z) and the cloud base vertical velocity

$$\varepsilon(z) = \frac{B(z)}{2\left(w_c(z_{lcl})^2 + \int_{z_{lcl}}^{z} B(z)\,dz\right)} \qquad (5\text{-}14)$$

As pointed out by Nordeng (1994) the scaling (5-5) is consistent with (5-14) if B(z) is height independent.

5.3 Evaluation of the generalized parameterization with LES

The parameterizations of ε and δ can be evaluated using the LES results for BOMEX. In Fig. 8a we compare ε as determined directly from (3-9) with the rhs of (5-14). Likewise in Fig. 8b we show the lhs of (5-13b) as determined from LES output and compare it with the detrainment rate δ determined directly from (3-9), again from the LES output. The results are very promising, especially when realising that standard parameterizations of ε and δ, such as (5-7) are easily off by one order of magnitude.

Note that (5-14) offers a consisting framework for a complete parameterization since ε is mainly a function of the cloudcore excess profile $\theta_{v,c} - \overline{\theta}_v$. In Siebesma and Holtslag (1996) it was shown that in general cloudcore excess profiles $\phi_c - \overline{\phi}$ can be calculated correctly when using a mass flux model like (4-1), provided it is feeded with correct entrainment rates. Therefore, (5-14) consistently combined with the mass flux model (4-1) should give correct entrainment rates.

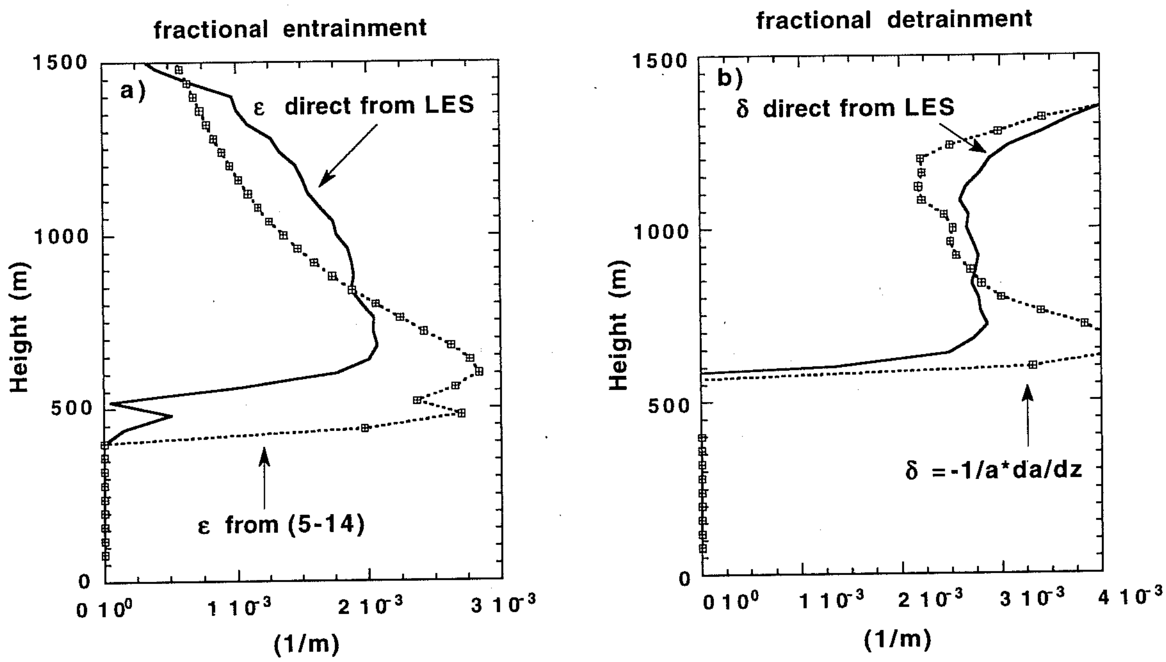

Fig. 8. (a) : Fractional entrainment rate ε directly obtained by LES (full line) and ε obtained by evaluating the rhs of (5-14) using LES output (dotted line). (b): Fractional detrainment rate δ directly obtained by LES (full line) and δ obtained by evaluating the lhs of (5-13b) using LES output (dotted line)

5.4 One-column model impacts of the generalized parameterization

Our objective is to implement the parameterization (5-14) and (5-13b) for ε and δ in the Tiedtke mass flux scheme of the one-column model and perform the same BOMEX run as for the two runs mentioned in section 4.3. The entrainment parameterization (5-14) can be implemented directly. We only have to specify the vertical velocity of the cloudcore at cloudbase which is set to 0.5 m/s. For the fractional detrainment rate δ, the situation is more complicated. The parameterization (5-13b) cannot be implemented directly, since the mass flux scheme does not provide information for the cloudcore cover a_c. Therefore additional assumptions have to be made. In Nordeng (1994) this has been done for deep convection by determining a lowest cloud top height z_e by lifting an entraining parcel with the entrainment rate ε as determined by (5-14) from cloud base to its zero buoyancy level. Furthermore a highest cloud top height z_u by lifting a undiluted parcel with zero entrainment from cloud base to its zero buoyancy level. The cloudcore cover a_c is assumed to be constant with height up to z_e and monotonically decreasing with height to zero between z_e and z_u. When assuming a linear decrease of a_c from z_e to z_u (5-13b) simplifies to

$$\delta = 0, \qquad z \le z_e$$
$$\delta = \frac{1}{z_u - z}, \qquad z_e \le z \le z_u \tag{5-15}$$

which can be easily implemented into the mass flux scheme.

We have performed the same BOMEX one-column run as described in Siebesma and Holtslag 1996) using the parameterization (5-14) and (5-15). Results are displayed in fig. 9a. Let us recall that ideally the one-column model should counteract the large-scale forcing and therefore maintain the initial profiles. However, the implementation of (5-14) and (5-15) leads to erroneous results (see Fig. 9a) with still too intensive mixing. The reason for this resides in (5-15). When launching a undiluted parcel from cloud base it will keep enough buoyancy to break through the inversion. Therefore one diagnoses a erroneous highest cloud top height z_u at 10 km and underestimates the fractional detrainment rate by a factor of 10 (see (5-15)). In general, for shallow convection, an undiluted parcel method will overestimate the highest cloud top height.

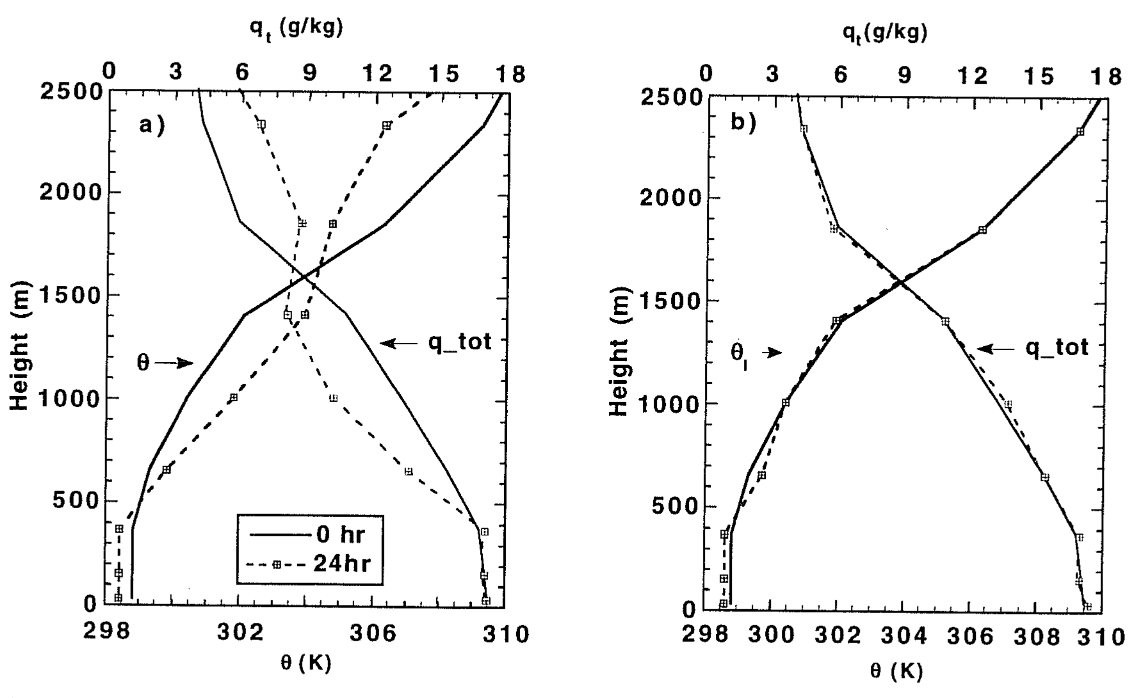

Fig. 9. One-column run results: profiles of potential temperature and total water specific humidity after 0 and 24 h. In (a) parameterization (5-14) for ε and (5-15) for δ is used. In (b) parameterization (5-14) for ε and (5-16) for δ is used.

As an alternative for (5-15) we use the common observed feature that for shallow cumulus convection the mass flux is decreasing monotonically all the way from cloud base to the highest cloud top. For the highest cloud top height we take the height reached by the *entraining* parcel (i.e. z_e). Let us simply assume that the mass flux decreases linear with height from cloud base z_{lcl} to zero at z_e. We then readily find an alternative parameterization for the fractional detrainment rate

$$\delta = \varepsilon + \frac{1}{z_e - z}, \qquad z_{lcl} \leq z \leq z_e \qquad (5\text{-}16)$$

Implementing of (5-16) and repeating the one-column model run now gives excellent results (see Fig. 9b) The mass flux scheme is capable to maintain the steady state using a dynamical parameterization for ε and δ without any tunable parameters!

6. CONCLUSIONS AND PERSPECTIVES

This paper has summarized two important aspects of the mass flux approach for parameterizing turbulent transport: i) the mass flux approximation itself and ii) the lateral turbulent mixing processes.

Concerning the mass flux approximation we have demonstrated that for shallow cumulus convection (2-5) holds, almost without any enhancement factor. For the dry PBL and the stratocumulus topped boundary layer the same parameterization holds, provided that a updraft-downdraft decomposition and a proper enhancement factor ν significantly larger than 1 is used (see scheme below)

Dry PBL Stratocumulus	$\phi_+ \rightarrow \phi_u$ $\phi_- \rightarrow \phi_d$ $\nu_{ud} \rightarrow 1.3 \sim 1.6$	Shallow Cumulus	$\phi_+ \rightarrow \phi_c$ $\phi_- \rightarrow \phi_e$ $\nu_{ce} \rightarrow 1.0 \sim 1.2$

Operational mass flux parameterizations for cumulus convection is common practice by now. (Tiedtke 1989, Gregory and Rowntree (1990), Hack 1994). For the dry PBL the situation is quite different. Also various mass flux parameterizations have been developed for the dry PBL (all with an implicit enhancement factor ν=1) (Randall et al. 1992; Wang and Albrecht 1990). However, in operational large-scale atmospheric models, classical local K-diffusion is still common practice. One reason for this is that all PBL mass flux parameterizations are designed exclusively for the *convective* dry PBL and are not generalized to the *stable* case. In this light it is interesting to investigate how a mass flux concept can be translated into a nonlocal vertical diffusion scheme (Holtslag and Moeng 1991; Deardorff 1972), since both concepts are aiming to go beyond local K-diffusion by including non-local transport. Work in this direction is in progress.

Another challenging future avenue is the possibility of designing one unified mass flux parameterization scheme for the complete cloud-topped PBL, including cumulus and stratocumulus. This way improvement can be expected for present weak points, such as transitions between the various cloud regimes and interactions between the cloud layer and the subcloud layer below. One major complication with such an unified mass flux approach however is the fact that different decompositions will be required (see Fig. 2).

Entrainment and detrainment rates are crucial parameters in determining the updraft and downdraft fields. Fundamental definitions are given in section 4. However, in order to make them operational, simplifications have to be made. LES results show that these simplifications give consistent results. Subsequently, these operational definitions have been used to determine entrainment and detrainment rates for shallow cumulus convection from LES. Substantially higher values for these rates are found than used in present operational cumulus parameterizations. The LES results support the entrainment parameterization (5-14) suggested by Nordeng. We have also implemented this entrainment formula in the Tiedtke scheme and found promising results for the BOMEX when combined with a new detrainment parameterization (5-16).

Concerning the lateral mixing processes, many open problems still remain. A fundamental question is why the entrainment/detrainment approximation (3-6) works so well. This approximation suggests a turbulent mixing process where average environmental field properties are advected into the clouds and where vice versa the average cloud properties are advected into the environment. In reality however, one expects a process where the turbulent mixing is only stretching and folding the interface and where the final mixing across the interface is only taking place by diffusion below the Kolmogorov scale (or at the subgrid scale in a LES model). In this picture the turbulent mixing can be considered as a process that is preparing a extremely long interface and thereby facilitating the final diffusion across this stretched interface. Note that this is a completely different physical picture than suggested by the approximation (3-6). In this respect it is interesting to study the scaling and dynamics of the interface. Work in this direction is in progress.

More practical questions are related to the entrainment parameterization (5-14) and the detrainment parameterization (5-16). Why do these parameterizations work? Do they also hold for other shallow cumulus convection cases?

A final interesting future direction concerns the entrainment and detrainment rates between updrafts and downdrafts in the dry boundary layer. Preliminary results for the convective subcloud layer in BOMEX using LES results suggest $\varepsilon \sim z^{-1}$ in agreement with the general scaling arguments sketched in section 5-1. This opens the way of constructing a mass flux parameterization, including entrainment and detrainment processes, for the dry convective PBL. Results from Petersen et al. (1997) seem to give promising results.

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APPENDIX

Budget equations for conditional sampled fields

a) Definitions and averaging procedures

Consider an arbitrary field ϕ in a domain with an horizontal area A. We denote a horizontal spatial average of this field with an overbar

$$\overline{\phi}(z) \equiv \frac{1}{A}\int_0^{L_x}\int_0^{L_y} \phi(x,y,z)\,dxdy \qquad , A = L_x L_y \qquad \text{(A-1)}$$

Within this domain there exist possibly nonconnected cloudy parts that vary time and space. We define cloud and environment averages as spatial averages over the corresponding parts

$$\overline{\phi}^{c} = \phi_c \equiv \frac{1}{A_c}\iint_{\substack{\text{cloudy}\\ \text{part}}} \phi\,dxdy$$

$$\overline{\phi}^{e} = \phi_e \equiv \frac{1}{A_e}\iint_{\substack{\text{env}\\ \text{part}}} \phi\,dxdy \qquad \text{(A-2)}$$

where A_c and A_e denote the area over the cloudy and the environmental part. Clearly, ϕ_c and ϕ_e are trivially related

$$\overline{\phi} = a_c\,\varphi_c + (1-a_c)\,\phi_e \qquad \text{(A-3)}$$

where a_c=A_c/A is the fractional cloudcover. The vertical turbulent flux of ϕ is defined as

$$\overline{w'\phi'} = \overline{w\,\phi} - \overline{w}\,\overline{\phi} \qquad \text{(A-4)}$$

Likewise we can define turbulent flux within the cloudy and environmental part

$$\overline{w'\phi'}^{c} \equiv \overline{w\,\phi}^{c} - w_c\,\phi_c$$
$$\overline{w'\phi'}^{e} \equiv \overline{w\,\phi}^{e} - w_e\,\phi_e \qquad \text{(A-5)}$$

Again the cloudy and environment fluxes are related

$$\overline{w'\phi'} \equiv a_c\,\overline{w'\phi'}^{c} + (1 - a_c)\,\overline{w'\phi'}^{e} + a_c\,(w_c - \overline{w})\,(\phi_c - \phi_e) \qquad \text{(A-6)}$$

We will also need the average field ϕ_b and the turbulent flux at the cloud boundary

$$\overline{\phi}^{b} = \phi_b \equiv \frac{1}{L_b}\int_{\substack{\text{cloud}\\\text{interface}}} \phi\,dl$$
$$\overline{w'\phi'}^{b} \equiv \overline{w\,\phi}^{b} - w_b\,\phi_b \qquad \text{(A-7)}$$

where L_b denotes the total length of the cloud boundary.

b) Budget Equations

The prognostic equation of ϕ can always be written as

$$\frac{\partial\phi}{\partial t} + \nabla_h.\mathbf{v}\,\phi + \frac{\partial\,w}{\partial z} = F \qquad \text{(A-8)}$$

where **v** denotes the horizontal part of the velocity vector and w the vertical part. Just for the sake of simplicity we assume constant density. The equations can be easily generalized for the quasi-Bousinesq approximation. All possible source and sink terms are collected in F. The objective is now to integrate all the terms of (A-8) over an horizontal cloudy area A(z,t). Applying Leibnitz theorem on the first and third term of the lhs of (A-8)

$$\frac{1}{A}\int_{A_c(z,t)} \frac{\partial \phi}{\partial t}\,dxdy = \frac{\partial\, a\,\phi_c}{\partial t} - \phi_b \frac{\partial a_c}{\partial t}$$
$$\frac{1}{A}\int_{A_c(z,t)} \frac{\partial\, w\,\phi}{\partial z}\,dxdy = \frac{\partial a_c\, \overline{w\,\varphi}^{c}}{\partial z} - \overline{w\,\varphi}^{b}\frac{\partial a_c}{\partial z} \qquad \text{(A-9)}$$

we find

$$\frac{\partial\, a_c\,\phi_c}{\partial t} - \phi_b \frac{\partial a_c}{\partial t} + a_c\overline{\nabla_h.\mathbf{v}\,\phi}^{c} - \overline{w\,\varphi}^{b}\frac{\partial a_c}{\partial z} + \frac{\partial a_c\,\overline{w\,\varphi}^{c}}{\partial z} = a_c\,F_c \qquad \text{(A-10)}$$

If we put ϕ=const we recover the continuity equation

$$\frac{\partial a_c}{\partial t} - \frac{\partial a_c}{\partial t} + a_c\overline{\nabla_h.\mathbf{v}}^{c} - w_b\frac{\partial a_c}{\partial z} + \frac{\partial a_c\, w_c}{\partial z} = 0 \qquad \text{(A-11)}$$

which can be put in a more transparent form by applying the divergence theorem

$$\frac{\partial\, a_c}{\partial t} + \frac{1}{A}\oint_{\text{interface}} \hat{\mathbf{n}}\,.\,(\,\mathbf{u} - \mathbf{u}_i\,)\,dl + \frac{\partial a_c\, w_c}{\partial z} = 0 \qquad \text{(A-12)}$$

where $\hat{\mathbf{n}}$ is a outward pointed unit normal vector at the interface at height z, **u** is the full 3d vector of the (mass) velocity at the interface and $\mathbf{u}_i$ is the velocity of the interface itself. Note that the normal vector does not necessary point into the horizontal direction. Likewise, again using the divergence theorem, the field equation (A-11) can be rewritten into

$$\frac{\partial\, a_c\,\phi_c}{\partial t} + \frac{1}{A}\oint_{\text{interface}} \hat{\mathbf{n}}\,.\,(\,\mathbf{u} - \mathbf{u}_i\,)\,\phi\,dl + \frac{\partial a_c\,\overline{w\phi}^{c}}{\partial z} = a_c\,F_c \qquad \text{(A-13)}$$

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