A TWO-TIME-LEVEL VERSION OF THE IFS SEMI-LAGRANGIAN SPECTRAL MODEL

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1. INTRODUCTION

In September 1991, a three-time-level semi-Lagrangian spectral model was implemented operationally at ECMWF. The increased efficiency of the semi-Lagrangian scheme relative to that of the previous Eulerian version was crucial in making feasible an increase in horizontal resolution from T106 to T213 and in vertical resolution from 19 to 31 layers (*Ritchie et al.,* 1995). At the new resolution, the semi-Lagrangian scheme is stable and accurate with timesteps several times longer than for the corresponding Eulerian scheme, and this is achieved with a computational overhead of only about 20% per timestep.

In principle, it should be possible to obtain a further doubling of efficiency by converting the three-time-level semi-Lagrangian scheme to a two-time-level scheme (*Staniforth and Côté,* 1991). Such two-time-level schemes have been described for a limited-area finite-element shallow-water model (*Temperton and Staniforth,* 1987), a global spectral shallow-water model (*Côté and Staniforth,* 1988), a global finite-element shallow-water model (*Côté et al.,* 1993), global finite-difference shallow-water models (*Bates et al.,* 1990; *Bates et al.,* 1995), limited-area finite-difference multi-level models (*McDonald and Haugen* 1992, 1993), and a global finite-difference multi-level model (*Bates et al.,* 1993).

In this paper, we report on work in progress to apply a similar technique to the ECMWF global spectral forecast model, which is now part of the IFS (Integrated Forecast System) developed in collaboration with Météo-France.

2. RECENT CHANGES TO THE THREE-TIME-LEVEL VERSION

Since it was documented by *Ritchie et al.* (1995), the three-time-level scheme, which forms the basis for the new two-time-level version, has undergone some important changes as listed below.

(1) The semi-Lagrangian scheme reverted to the "fully interpolating in the vertical" form. In the original version, this form gave rise to a growth of eddy kinetic energy during the forecast and a consequent deterioration of the verification scores. This problem has been overcome by two changes to the details of the numerical formulation. In each of the model equations, the interpolations for the "advected variable" and the "right-hand side" terms are now carried out separately. The right-hand-side term is taken as an average between the values at the departure point and the arrival point for all the equations, and the interpolation at the departure point is linear. Simplified cubic interpolation is still used to evaluate the advected variable at the departure point, but in the horizontal a quasi-monotone limiter is applied, based on the scheme proposed by *Bermejo and Staniforth* (1992). The quasi-monotone limiter is also applied in the vertical for the moisture field only. These modifications to the semi-Lagrangian numerics were described by *Hortal* (1994).

(2) The spectral variable is now the virtual temperature T_v instead of the temperature T (this option was described in *Ritchie et al.* 1995), and the moisture field q is treated entirely in gridpoint space.

(3) The Coriolis terms in the momentum equations are incorporated in the semi-Lagrangian advection, as first proposed by *Rochas* (1990); see also *Temperton* (1995).

Meanwhile there have also been important changes to the physical parametrization schemes, notably the introduction of a prognostic cloud scheme (*Tiedtke*, 1993) and a new representation of sub-grid scale orographic effects coupled with a return to "mean" rather than "envelope" orography (*Lott and Miller*, 1996).

With the modifications to the numerics described above, and with the semi-implicit parameter β set to 1 (see Section 2(c) of *Ritchie et al.,* 1995), the three-time-level semi-Lagrangian discretization for each of the model equations can be written compactly in the form

$$\frac{X^{+}-X^{-}}{2\Delta t} = \tfrac{1}{2}\{L^{+}+L^{-}\} + \tfrac{1}{2}\{N^{0}(\underset{\sim}{x}-2\underset{\sim}{\alpha})+N^{0}(\underset{\sim}{x})\} \tag{2.1}$$

where

$$X^{+} = X(\underset{\sim}{x}, t+\Delta t)$$

$$X^{-} = X(\underset{\sim}{x}-2\underset{\sim}{\alpha}, t-\Delta t)$$

$$L^{+} = L(\underset{\sim}{x}, t+\Delta t)$$

$$L^- = L(\underset{\sim}{x}-2\underset{\sim}{\alpha}, t-\Delta t) .$$

Here X is the advected variable and L represents the linear terms; $\underset{\sim}{x}$ is the arrival gridpoint and $\underset{\sim}{x}-2\underset{\sim}{\alpha}$ is the departure point. For the nonlinear terms, $N^0 = N(t)$. The departure point is found by iteratively solving the displacement equation

$$2\underset{\sim}{\alpha} = 2\Delta t\, \underset{\sim}{V}(\underset{\sim}{x}-\underset{\sim}{\alpha}, t) \tag{2.2}$$

where $\underset{\sim}{V}$ is the three-dimensional wind field. Linear interpolation is used to evaluate $\underset{\sim}{V}(\underset{\sim}{x}-\underset{\sim}{\alpha}, t)$ in (2.2), and also to evaluate $\{L^-(\underset{\sim}{x}-2\underset{\sim}{\alpha}, t-\Delta t) + N^0(\underset{\sim}{x}-2\underset{\sim}{\alpha}, t)\}$ in (2.1). The advected variable X^- in (2.1) is evaluated at $(\underset{\sim}{x}-2\underset{\sim}{\alpha})$ using (simplified) cubic interpolation with a quasi-monotone limiter in the horizontal (also in the vertical for q). The advected variables are T, q, $\ln p_s$ (where p_s is the surface pressure), and

$$\underset{\sim}{v}_h + 2\underset{\sim}{\Omega}\times\underset{\sim}{r} \tag{2.3}$$

where $\underset{\sim}{v}_h$ is the horizontal wind field and $\underset{\sim}{\Omega}\times\underset{\sim}{r}$ is the angular velocity at the earth's surface; hence in component form, (2.3) is equivalent to $(u+2\Omega a\cos\theta, v)$ where Ω is the earth's rotation rate, a is the earth's radius and θ is latitude.

3. FORMULATION OF THE TWO-TIME-LEVEL SCHEME

The basic idea behind the two-time-level scheme is to replace the double timestep $(2\Delta t)$ from $(t-\Delta t)$ to $(t+\Delta t)$ in the three-time-level scheme by a single timestep from (t) to $(t+\Delta t)$. Quantities which were formerly evaluated at the central time-level (t) are instead obtained by extrapolation to $(t+½\Delta t)$. Thus, (2.1) becomes

$$\frac{X^+ - X^0}{\Delta t} = ½\{L^+ + L^0\} + ½\{N^*(\underset{\sim}{x}-\underset{\sim}{\alpha}) + N^*(\underset{\sim}{x})\} \tag{3.1}$$

where

$$X^+ = X(\underset{\sim}{x}, t+\Delta t)$$

$$X^0 = X(\underset{\sim}{x}-\underset{\sim}{\alpha}, t)$$

$$L^+ = L(\underset{\sim}{x}, t+\Delta t)$$

$$L^0 = L(\underset{\sim}{x}-\underset{\sim}{\alpha}, t) \ .$$

Again X is the advected variable and L represents the linear terms; $\underset{\sim}{x}$ is the arrival gridpoint and $\underset{\sim}{x}-\underset{\sim}{\alpha}$ is the departure point. The nonlinear terms N^* at time-level $(t+½\Delta t)$ are obtained by extrapolation in time:

$$N^* = \frac{3}{2}N(t) - \frac{1}{2}N(t-\Delta t).$$

The departure point is found by iteratively solving the displacement equation

$$\underset{\sim}{\alpha} = \Delta t \, \underset{\sim}{V}^* (\underset{\sim}{x}-½\underset{\sim}{\alpha}, t+½\Delta t)$$

where the three-dimensional wind field $\underset{\sim}{V}^*$ is also found by extrapolation in time:

$$\underset{\sim}{V}^* = \frac{3}{2}\underset{\sim}{V}(t) - \frac{1}{2}\underset{\sim}{V}(t-\Delta t).$$

The advected variables are the same as for the three-time-level scheme, and the choice of interpolation procedure for each quantity is the same as for its counterpart in the three-time-level scheme.

The physical parametrization routines are called at the arrival gridpoints after the semi-Lagrangian advection, exactly as described for the three-time-level scheme in *Ritchie et al.* (1995). Horizontal diffusion is applied in spectral space after completion of the semi-implicit time-stepping. As the moisture field is no longer transformed to spectral space, there is now no horizontal diffusion of moisture in either the three-time-level scheme or the two-time-level scheme.

4. CHOICE OF REFERENCE TEMPERATURE AND SURFACE PRESSURE

The splitting of the right-hand side of (2.1) or (3.1) into linear and nonlinear terms depends on the choice of reference temperature and surface pressure. This choice turns out to be crucial in determining the stability or otherwise of the two-time-level scheme.

In the three-time-level scheme (*Ritchie et al.,* 1995), for both the Eulerian and semi-Lagrangian versions, the choice is $\bar{T}$ = 300K, $\bar{p}_s$ = 800hPa. This choice was guided by the stability analyses of *Simmons and Burridge* (1981) and *Simmons et al.* (1989). Preliminary experiments with the same choice in the two-time-level scheme clearly showed instabilities in the subtropics where the temperature profile is warm and the surface pressure high. For example, Fig. 1(a) shows a 48-hour forecast of the 500hPa height field using the two-time-level scheme at resolution T213 with Δt = 30 minutes, $\bar{T}$=300K, $\bar{p}_s$=800hPa.

The stability analysis of *Côté and Staniforth* (1988) for the treatment of nonlinear terms in a two-time-level semi-Lagrangian shallow-water model suggested that it might be advantageous to increase the reference temperature and surface pressure. Figure 1(b) shows the same forecast as Fig. 1(a) but with reference values of $\bar{T}$=350K and $\bar{p}_s$=1000hPa; the instabilities have disappeared. A subsequent stability analysis for the two-time-level multi-level model (*Simmons and Temperton*, 1996) confirmed that these reference values were a good choice. Using these values, it has been found possible to integrate the two-time-level scheme stably in the ECMWF model without recourse to any other damping mechanisms such as decentering or time-filtering.

5. EXPERIMENTAL RESULTS

Sets of 10-day forecast experiments (12 at T213, 14 at T106) were run to compare the performance of the two-time-level scheme with that of the corresponding three-time-level scheme. In the model version used for the T106 experiments, the continuity equation was modified following the suggestion of *Ritchie and Tanguay* (1996). The effect of this modification is to improve the simulation of flow over mountains, and as a bonus it considerably improves the global mass conservation of the semi-Lagrangian schemes.

Figures 2(a) and 2(b) show the verification scores for the 500hPa height field in the T213 experiments, for the Northern and Southern Hemisphere respectively. Here the three-time-level model was run in its operational configuration ($\bar{T}$=300K, $\bar{p}_s$=800hPa, Δt = 15 minutes) while the two-time-level scheme was run with $\bar{T}$=350K, $\bar{p}_s$=1000hPa, Δt = 30 minutes. It is evident that the scores are almost equivalent, and that the two-time-level scheme fulfils its promise of almost doubling the efficiency of the model without loss of accuracy.

At the lower resolution of T106, the three-time-level scheme is usually run with a timestep of 30 minutes. Running the two-time-level scheme with Δt = 60 minutes leads to a

Saturday 15 January 1994 12z ECMWF Forecast t+ 48 VT: Monday 17 January 1994 12z
500hPa height Experiment JUO

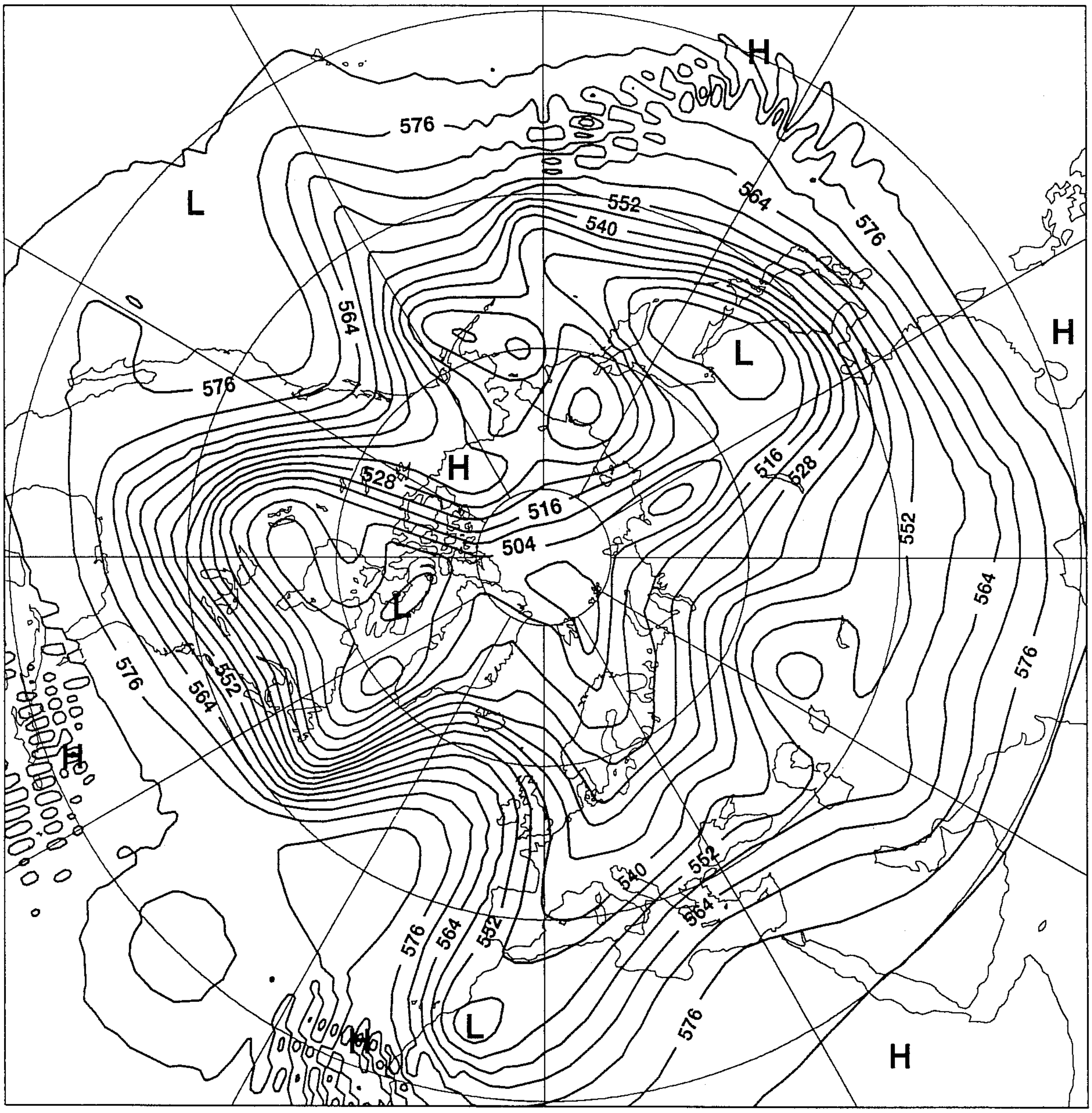


Figure 1(a). 48-hour forecast of the 500 hPa height field (contour interval 6 dam) from 12Z on 15 January 1994, using the two-time-level scheme with Δt = 30 minutes, $\bar{T}$ = 300K, $\bar{p}_s$ = 800 hPa. The model resolution is T213, 31 layers.

Saturday 15 January 1994 12z ECMWF Forecast t+ 48 VT: Monday 17 January 1994 12z
500hPa height Experiment KJF

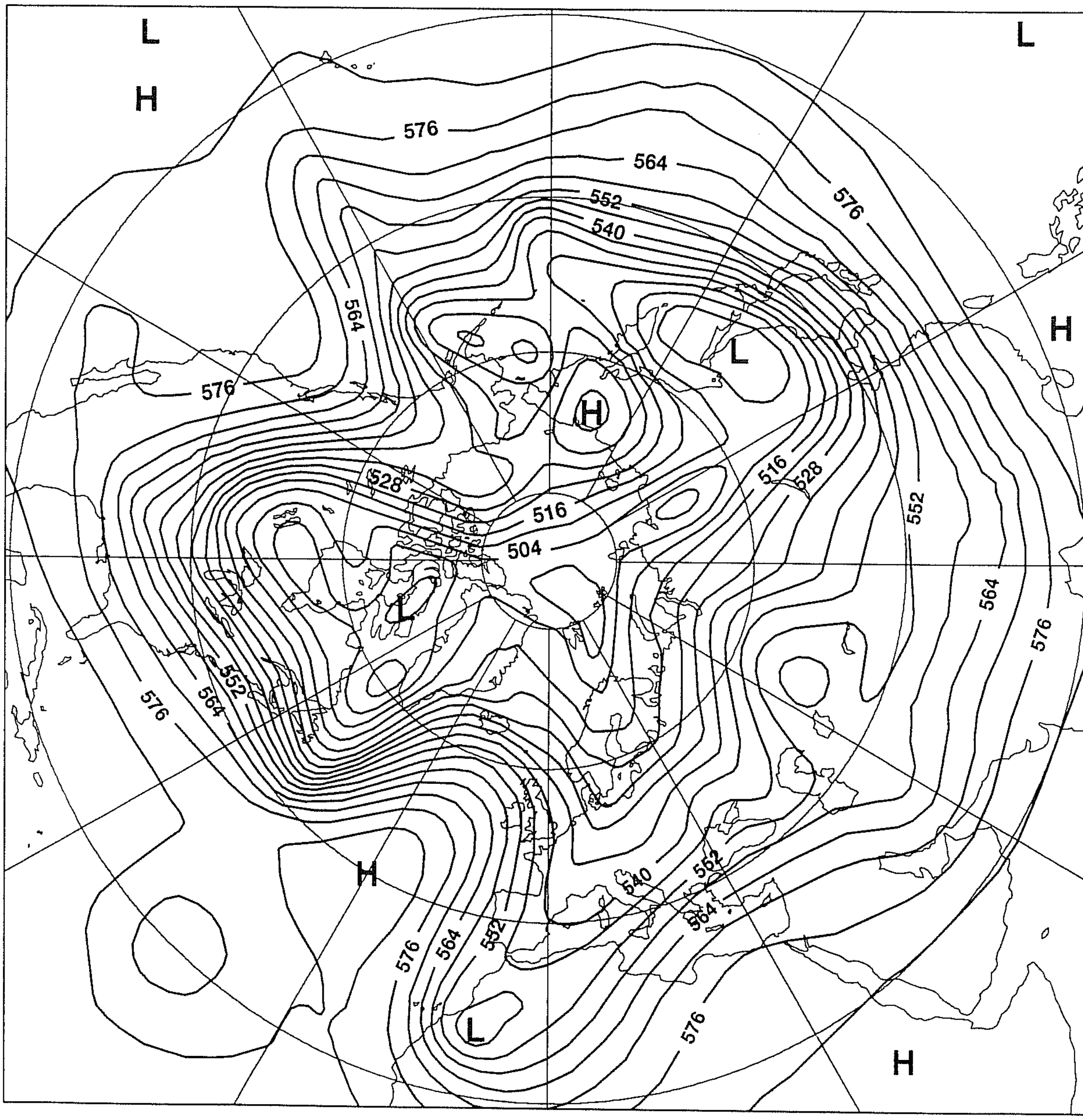
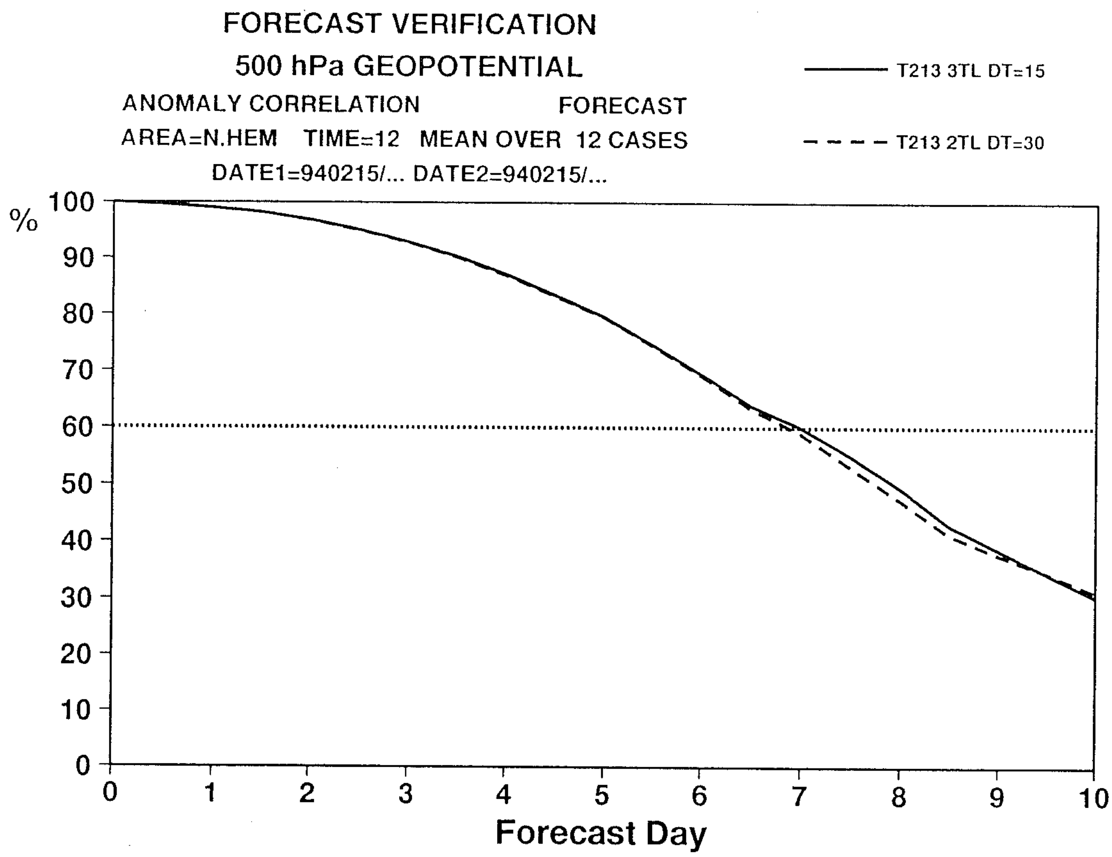


Figure 1(b). As Fig. 1(a), but with $\bar{T}$ = 350K and $\bar{p}_s$ = 1000 hPa.



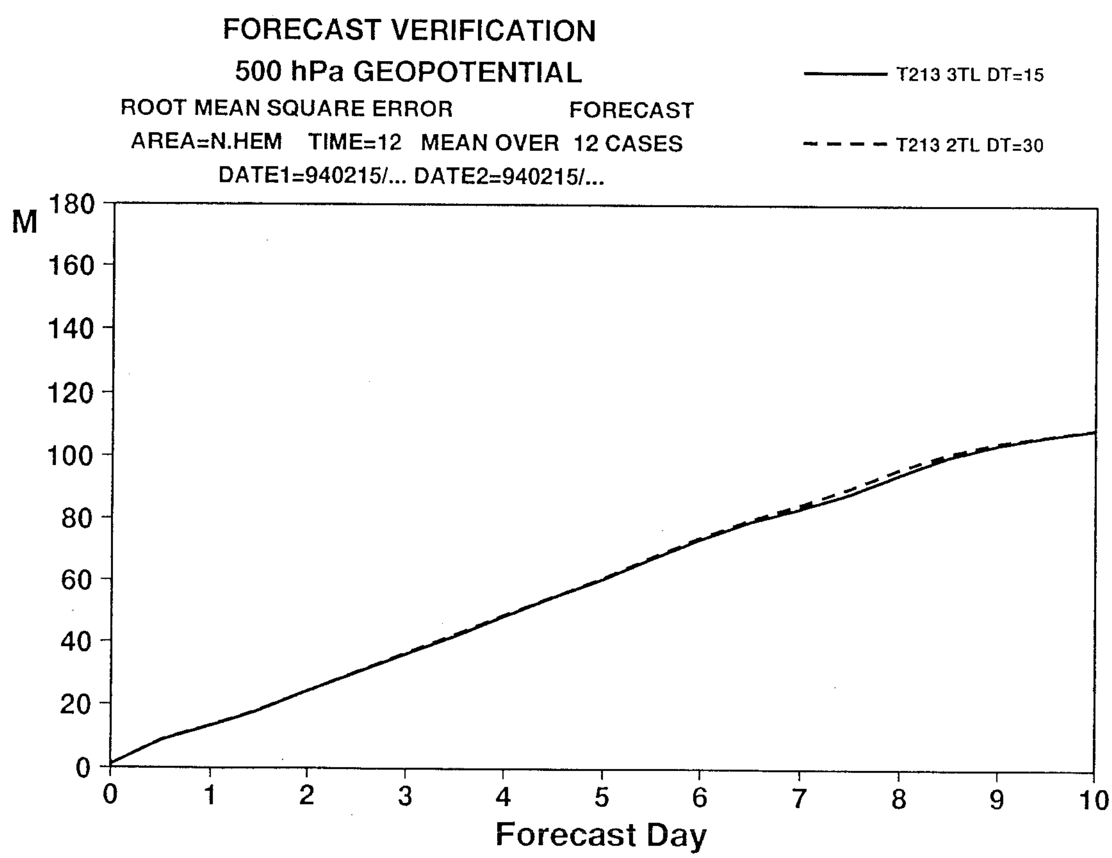
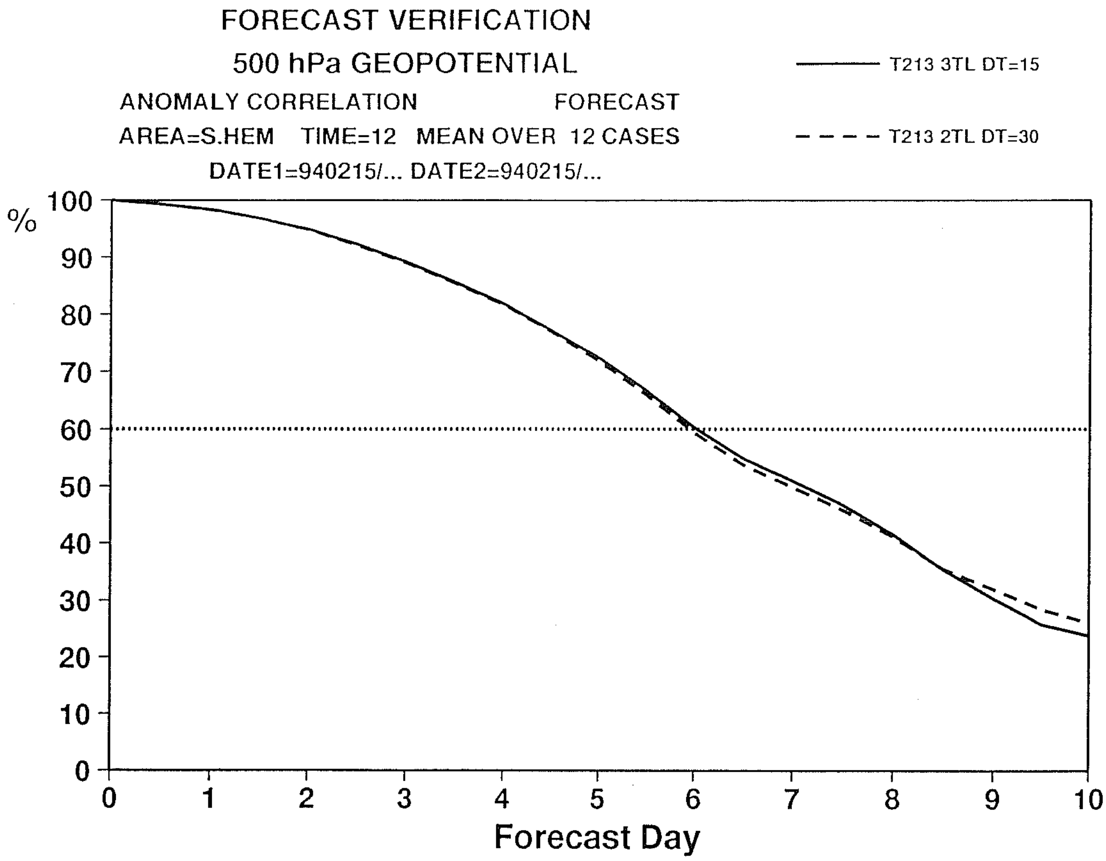


Figure 2(a). Mean verification scores (anomaly correlation coefficient and r.m.s. error) of the 500 hPa height field for the extratropical Northern Hemisphere. Model resolution is T213. Solid line: three-time-level scheme, Δt = 15 minutes. Dashed line: two-time-level scheme, Δt = 30 minutes.



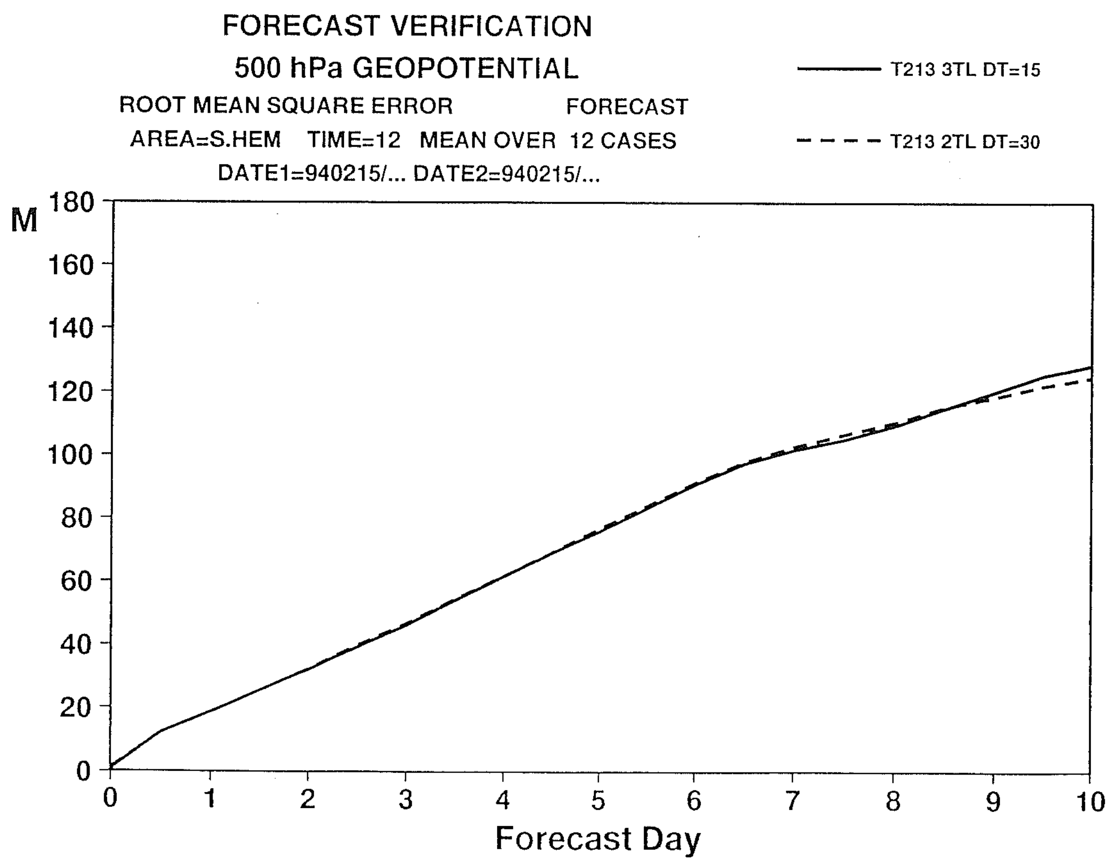
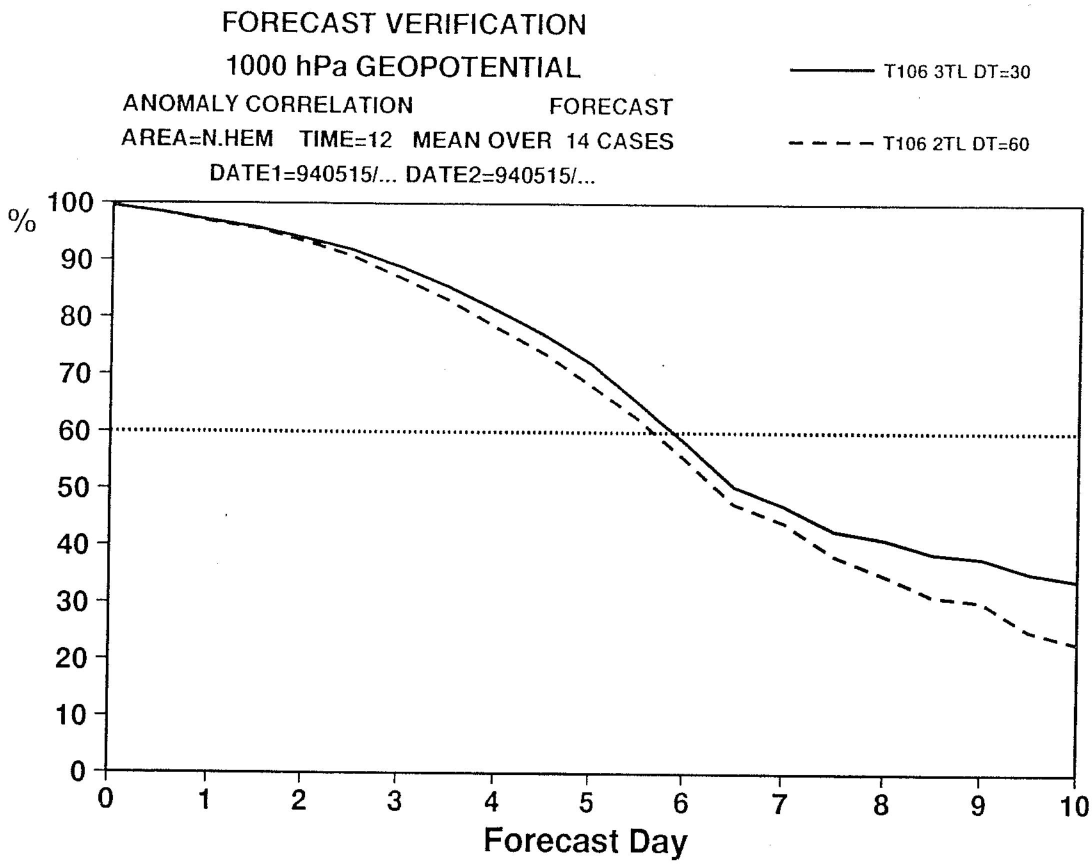


Figure 2(b). As Fig. 2(a), but for the extratropical Southern Hemisphere.



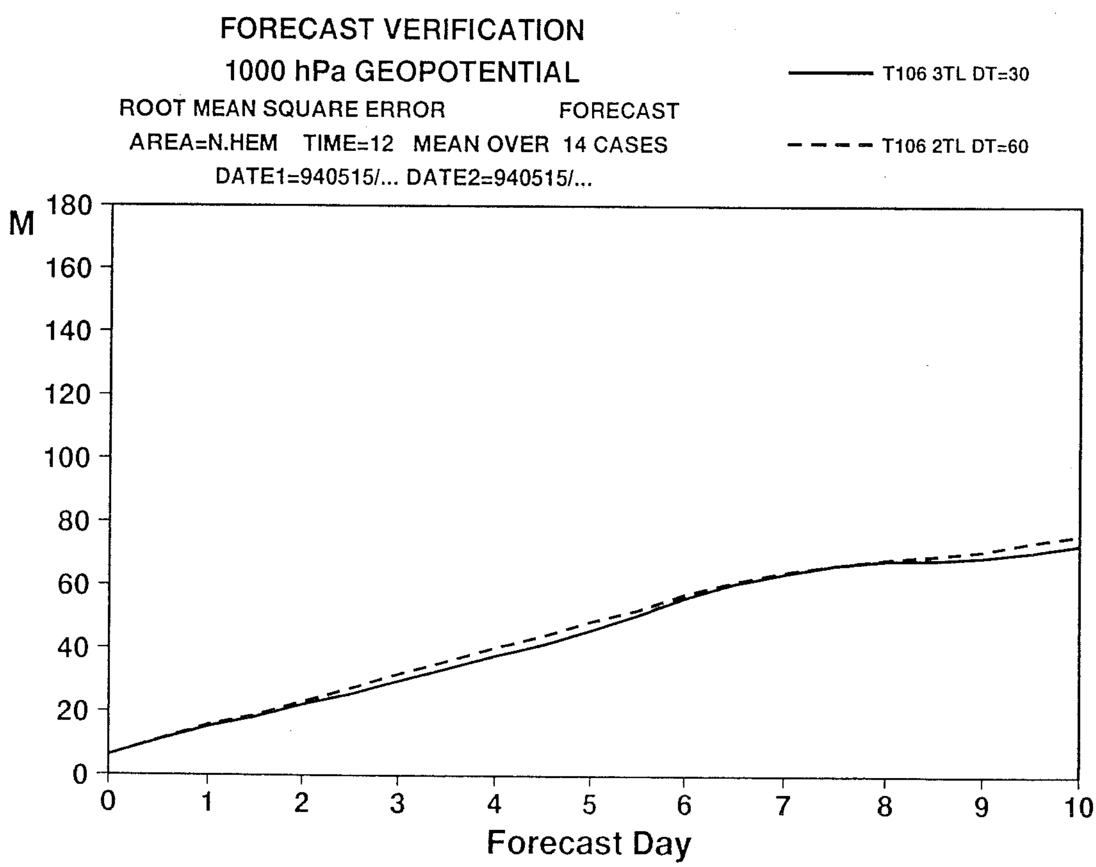


Figure 3. As Fig. 2(a), but for the 1000 hPa height field at model resolution T106. Solid line: three-time-level scheme, $\Delta t = 30$ minutes. Dashed line: two-time-level scheme, $\Delta t = 60$ minutes.

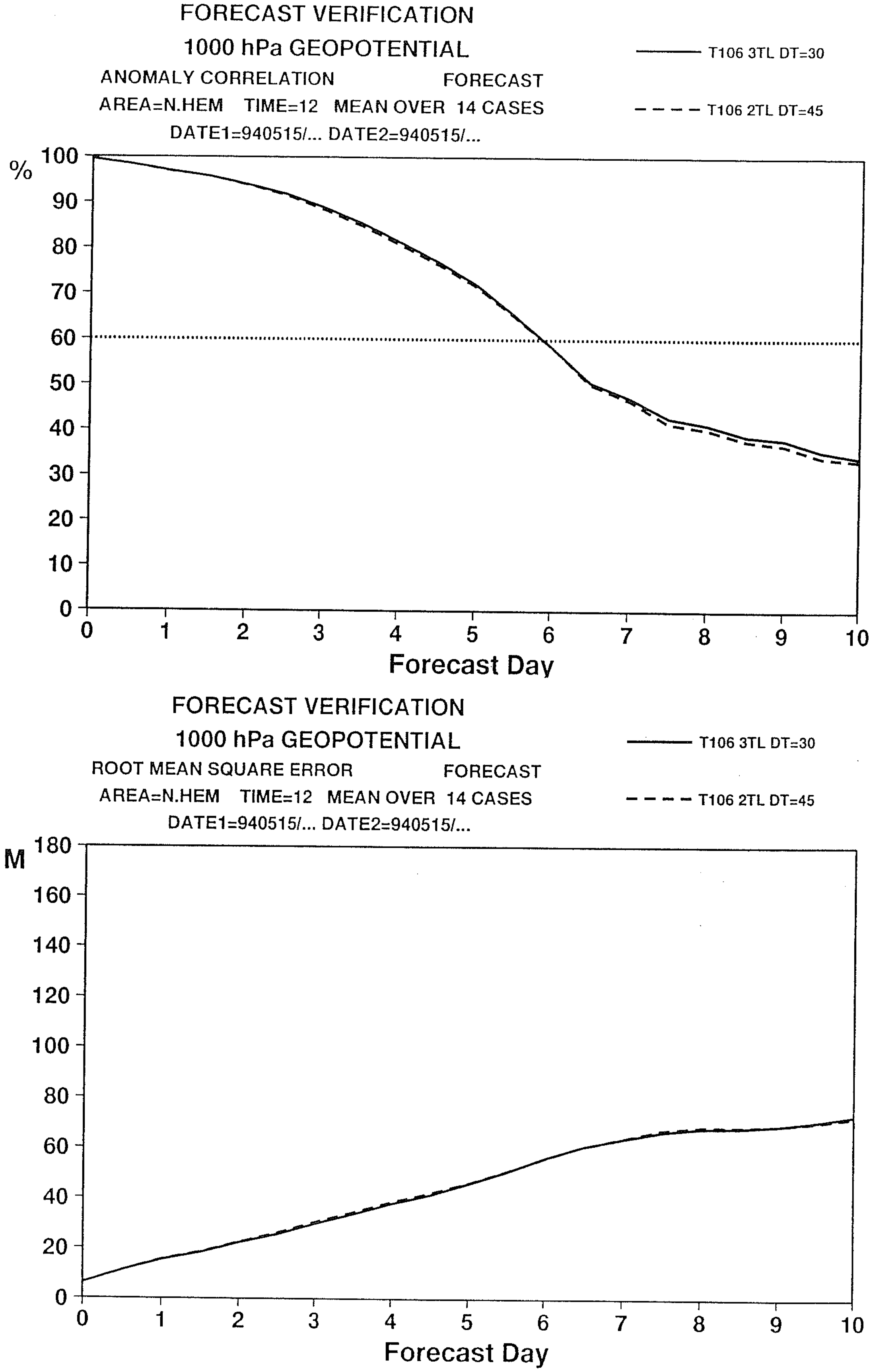


Figure 4. as Fig. 3, but Δt = 45 minutes for the two-time-level scheme.

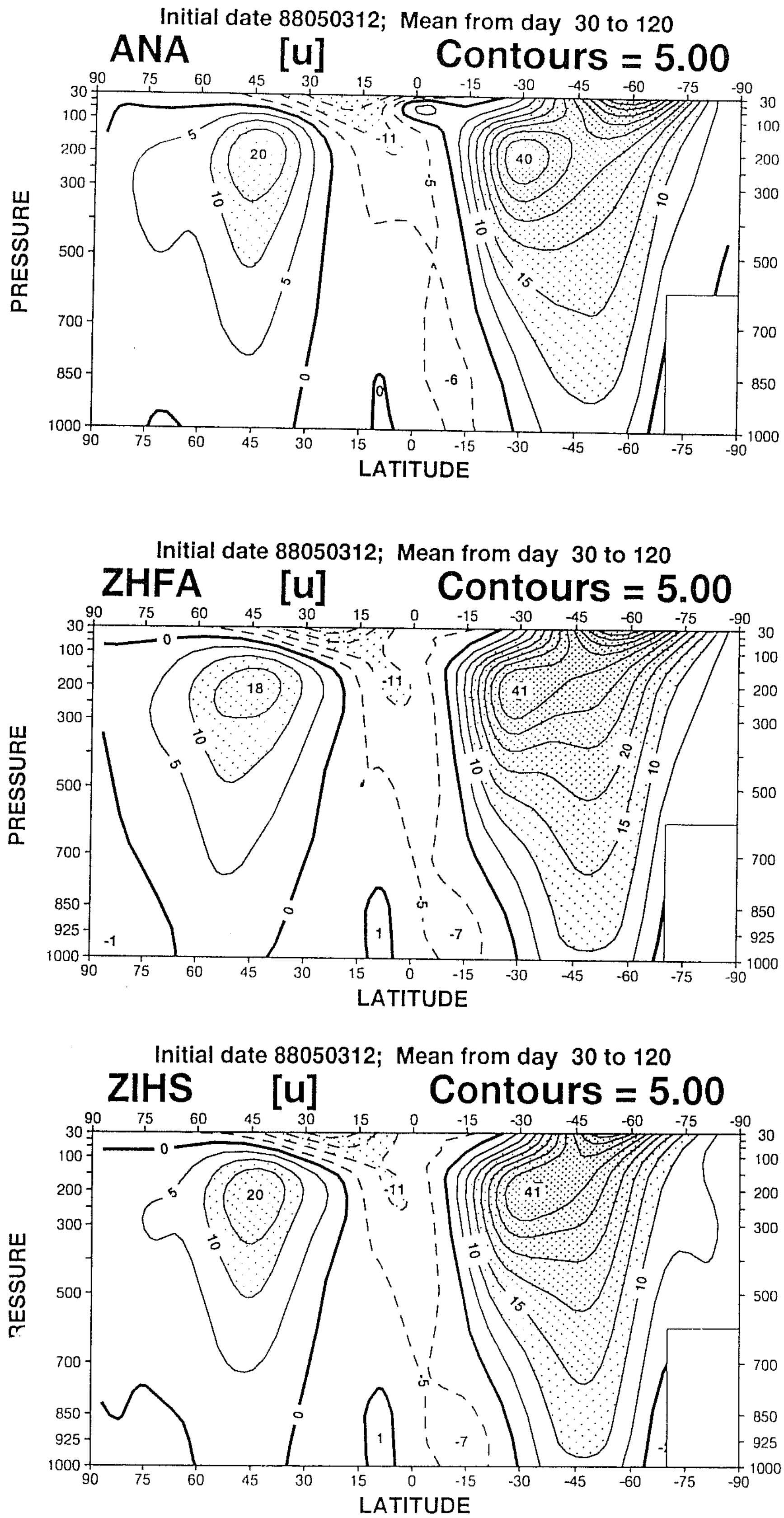


Figure 5. Zonal mean wind from Day 30 to Day 120 of model runs at T63L31. Upper panel, ECMWF analysis. Middle panel, three-time-level scheme with $\Delta t = 30$ minutes. Lower panel, two-time-level scheme with $\Delta t = 45$ minutes.

noticeable deterioration in forecast quality; 1000hPa height field verification scores over the Northern Hemisphere for the two schemes are shown in Fig. 3. Reducing the timestep to Δt = 45 minutes restores the accuracy of the two-time-level scheme (Fig. 4), but of course reduces the computational savings. It is planned to investigate this result further, in the hope of achieving satisfactory accuracy of the two-time-level scheme with Δt = 60 minutes at T106.

A long run (120 days) has also been performed at T63L31 with the two-time-level scheme, using a timestep of 45 minutes. Figure 5 compares the mean zonal wind field from Day 30 to Day 120 with that from a corresponding run with the three-time-level scheme (Δt = 30 minutes) and with the ECMWF analysis. Differences between the two semi-Lagrangian runs are small, and both reproduce the analysed zonal wind field rather accurately. Again, it is hoped that refinements to the two-time-level scheme will enable such runs to be performed with Δt = 60 minutes.

6. CONCLUSIONS AND OUTLOOK

A two-time-level version of the IFS semi-Lagrangian spectral model has been successfully developed. Preliminary experiments revealed instability problems which were resolved by changing the reference temperature and surface pressure for the semi-implicit scheme. At high resolution (T213), the new scheme doubles the efficiency of the three-time-level version without compromising the accuracy. At lower resolution (T106) this goal has not yet been completely achieved, but efforts to do so by refining the two-time-level scheme are continuing.

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