

Hilding Sundqvist  
Dept. of Met., Stockholm University  
S-106 91 Stockholm, Sweden.

## 1. INTRODUCTION

Clouds form as a result of atmospheric condensation, which interacts with the atmospheric mass and wind fields through the liberated heat, and clouds also have a powerful effect on the atmospheric dynamics via the modulation of both shortwave and longwave radiative transfer. With regard to the strong impact that clouds have on the the atmospheric dynamics in general, it is necessary that cloudiness be simulated with greater certainty and accuracy in order to yield useful model results, whether the purpose is for weather prediction or for studies of climate sensitivity (Ramanathan et al., 1983; Slingo, 1990).

The radiative properties of clouds are mainly determined by their microphysical composition, implying that the existence of the entity cloud has to be considered in models that contain radiation calculations. Then, in the case of an ideal model, the condensation process itself as well as the microphysical processes governing the evolution of the hydrometeors, which are formed, should be considered. However, the inclusion of five-six additional dependent variables, and prognostic equations for those, will in general cause unpracticable demands on the computational work in meso-scale and large-scale models. But to allow for the existence of cloud in a model, it is necessary to include at least cloud water content as a dependent (prognostic) variable. This means that the condensate produced by the condensation must be partitioned between cloud water and precipitating water. Consequently, a thorough parameterization of microphysical processes is needed. Not only cloud water is involved in those processes, but also several of the other hydrometeors that form. As these then not are available as individual variables, they have to be diagnostically inferred or deduced from the dependent variables of the model system.

The purpose of the present paper is to discuss the basic elements of parameterization of the microphysical processes in the condensation-cloud-precipitation-evaporation context. We will focus on an approach where cloud water content is the only hydrometeor that is carried as a prognostic variable. The generality of the discussion is not significantly reduced because of this. Rather, as a consequence, we are confronted with both the explicit and indirect parameterization problems, which arise from the fact that some of the needed quantities are not available. Since we will be concerned with only microphysical processes, the discussion will presume in-cloud conditions. A few remarks related to fractional cloud cover will merely be made when appropriate from microphysical points of view. The topic is, in its full extent, too complex to be covered comprehensively here. Elaborate considerations will be left out, but references will be given to special studies, where additional and supplementary papers may be found.

The microphysical processes that operate in clouds are principally the same in both convective and stratiform clouds. But the various processes may be differently emphasized depending on cloud type as a result of differences in intensity. A distinction in this respect should then be taken into account in a parameterization scheme through different magnitudes of the parameters for the different regimes. However, it is instead necessary to

distinguish between liquid and mixed liquid-ice clouds on one hand, and ice crystal clouds on the other. The conditions for formation of the two cloud types, and the way particles grow to precipitation size in the two cases are substantially different. So, in this respect it is necessary to deduce different parameterizations. The discussion in the sequel is therefore divided into one part in which the liquid and mixed liquid-ice regime is elaborately treated, and another where pure ice crystal conditions are considered. We follow Heymsfield and Sabin (1989) and let the division between the two regimes occur at the temperature  $T = -38$  °C.

## 2. PARAMETERIZATION OF CLOUD PROCESSES FOR $T > -38$ °C

Since cloud water content is the only dependent hydrometeor in the prediction system, we consider the tendency equations for temperature,  $T$ , specific humidity (interchangeably called moisture),  $q$ , and cloud water mixing ratio,  $m$ , to highlight the basic questions of this approach. The reason for using the notation "water content" and not "liquid water content" in the present text follows from the fact that this approach does not distinguish between liquid and ice phase cloud particles. Thus, we are considering

$$\frac{\partial T}{\partial t} = A_T + \frac{L}{c_p} Q \quad (1)$$

$$\frac{\partial q}{\partial t} = A_q - Q \quad (2)$$

$$\frac{\partial m}{\partial t} = A_m + Q - G_P \quad (3)$$

where the  $A$ -terms contain all tendency contributions, except those connected with the condensation process. The latent heat of condensation is denoted by  $L$  and the specific heat at constant pressure by  $c_p$ . The rate of latent heat release, or rate of production of condensate, is  $Q$  (dimension  $s^{-1}$ ), and the rate of generation of precipitation is  $G_P$ . As indicated, for the moment we treat the cloud water as if it is liquid; the distinction between liquid and ice phases will be discussed in Sections 3 and 4.

Let us rewrite the moisture equation (2) in terms of relative humidity, which we denote with  $U$ . By definition,  $q = U \cdot q_s$ , where  $q_s$  is the saturation specific humidity. Thus, Equ. (2) turns into

$$q_s \frac{\partial U}{\partial t} = A_q - Q - U \frac{c_p}{L} S_q \frac{\partial T}{\partial t} + \frac{q}{p} \frac{\partial p}{\partial t}; \quad \text{where } S_q = \frac{\epsilon L^2}{R c_p} \frac{q_s}{T^2} \quad (4)$$

The factor including  $S_q$  results from the Clausius-Clapeyron relation;  $p$  is pressure,  $R$  the gas constant for dry air and  $\epsilon = 0.662$  is the ratio of the molecular weights of water and air. Then eliminate the temperature tendency between Eqs. (1) and (4) to obtain

$$Q = \left[ A_q - \frac{c_p}{L} S_q A_T + \frac{q}{p} \frac{\partial p}{\partial t} - q_s \frac{\partial U}{\partial t} \right] (1 + U S_q)^{-1} = \left[ M_q - q_s \frac{\partial U}{\partial t} \right] (1 + U S_q)^{-1} \quad (5)$$

For in-cloud situations, generally  $U \approx 1$  and near steady state; in that case, the rate of release of latent heat is obtained from the humidity, temperature and pressure fields of the model. However, there are situations when the tendency of  $U$  has to be considered. One is when fractional cloudiness is treated, in which case  $U < 1$  and a formulation of the  $U$ -tendency is the basic closure problem (Sundqvist, 1993a). This parameterization ques-

tion is not a part of the present discussion. The other situation, when the tendency of  $U$  has to be considered, is for cirrus (ice crystal cloud) development, which will be discussed in Subsection 3.1.

## 2.1 Condensation

As a result of theoretical studies, laboratory experiments and field studies by cloud physicists, we have a fair understanding of the microphysical processes that take place in condensation and in the subsequent growth of condensate in clouds. It is this insight, by and large concerning individual particles, that we must utilize when we deduce formulations to be valid for ensembles of the scale of a cloud. The principles behind this type of approach are demonstrated in Kessler's (1969) paper.

Empirically we know that condensation in liquid and mixed-phase clouds takes place at very low supersaturations as a result of abundance of cloud condensation nuclei in the air. Hence, at the present state of the art, it seems justifiable to *assume that condensation occurs at 100 % relative humidity, possibly as a weighted mean of liquid-ice mixture*; of course, vapour must be available, either through convergence into the volume in question or through cooling by expansion or radiation. The rate of condensation is given by the rate at which vapour is made available.

We will return to the question regarding the saturation condition for *phase change at low temperatures* (ice clouds).

## 2.2 Release of precipitation

One of the main goals for condensation-cloud parameterization is to describe the partitioning of condensate between cloud water and precipitating water. In his approach, Kessler (1969) — who considered *only liquid water clouds* — included prognostic equations for both cloud water and precipitating water, implying that he could derive explicit formulations not only for "autoconversion" but also for "collection of cloud water by rain". In the first process, the effectiveness of which is a function of the cloud water amount, cloud droplets in their random motion occasionally collide and coalesce. In the second process, raindrops, falling through a cloud, grow by collecting cloud droplets; in that case, the effectiveness is a function of both the cloud water and the precipitating water amount. However, in the present discussion, only cloud water is available, implying that only autoconversion can be described explicitly. The implications of this are discussed in the next three paragraphs.

To describe the autoconversion for release of precipitation, we adopt the form from Sundqvist et al. (1989)

$$G_P = C_0 \cdot m \cdot \left( 1 - \exp \left[ - \left( \frac{m}{b \cdot m_r} \right)^2 \right] \right) \quad (6)$$

where  $C_0$  and  $m_r$  are the basic parameters. The fractional cloud cover  $b$  is included for completeness and for a later comment. Since we are considering in-cloud situations,  $b = 1$  in all the present discussion. The rate of conversion of cloud droplets to precipitation drops is governed by  $C_0$ , while  $m_r$  indicates the amount of cloud

water, around which the conversion becomes effective. The combined effect of those two parameters determines the water content of a cloud for a given forcing. The low generation rate when  $m/m_r < 1$ , accounts for the slow diffusive growth of small droplets. Kessler (1969) simulated this effect by introducing a minimum (threshold) cloud water amount, which has to be exceeded before the autoconversion becomes active.

### 2.2.1 *Diagnostically determined quantities for the coalescence and Bergeron-Findeisen mechanisms*

To obtain a realistic picture of the release of precipitation — and consequently a realistic distribution of cloud water — it is important to account for the effect of coalescence by rain and for the so-called Bergeron-Findeisen mechanism. For the former effect, it is necessary to know the density of the precipitating mass and for the latter effect, the amount of ice/snow in clouds and precipitation must be known. So, the amounts of those quantities have to be determined in the present parameterization approach.

In the case of the coalescence effect, the derivation starts from the growth rate of a raindrop, which is proportional to the cloud water content and to the terminal velocity of the drop. Integrating over the drop size distribution (e.g., Marshall and Palmer, 1948), a growth rate that is a function of precipitating mass is obtained (Kessler, 1969). In our case, the rain water content first has to be deduced. A diagnostic value of this is obtained by dividing the rate of precipitation by a typical terminal velocity. In the case of a Marshall and Palmer distribution, the terminal velocity is proportional to the rain water content raised to the power = 1/8, implying that the velocity varies relatively little even for large variations in water content (Kessler, 1969). Therefore, Sundqvist et al. (1989) accounted for coalescence effects by adopting a prescribed value of the terminal velocity ( $4\text{-}5 \text{ ms}^{-1}$ ) and taking the effect proportional to the square root of the precipitation rate as given by expression (7) below.

The Bergeron-Findeisen mechanism arises in a cloud where ice crystals and water droplets coexist. In this situation, the difference in saturation vapour pressure over water and ice causes a fast growth of the ice particles at the expense of the droplets. A mixed ice-liquid state appears when supercooled droplets freeze and the supply of droplets is replenished through condensation at a rate as fast, or faster than, the freezing process. This heterogeneous freezing is brought about by the freezing nuclei available in the volume in question, or/and by precipitating ice crystals that have formed in layers above. Consequently, the Bergeron-Findeisen effect exists in the temperature interval 235-273 K. The altitude (or temperature), at which the maximum effect appears, is determined by the above mentioned saturation vapour difference, the efficiency of available freezing nuclei and the vertical distribution of ice/snow precipitation. Thus, to take the Bergeron-Findeisen mechanism into account, it becomes necessary to consider the probability for existence or creation of ice crystals. Based on statistics of observational data on ice crystal occurrence in clouds (Matveev, 1984), Sundqvist (1993b) derived an ice probability formulation as a function of temperature.

Then to account for coalescence and Bergeron-Findeisen effects in addition to the autoconversion, we assume  $C_0$  to be given by

$$C_0 = C_{00} \cdot F_{C0}; \quad F_{C0} = 1 + C_1 \cdot \left( \frac{P_{\text{tot}}}{b} \right)^{0.5} + C_2 \cdot f_{B-F} \quad (7)$$

where  $P_{\text{tot}}$  is the rate of precipitation at the level in question,  $f_{B-F}$  is the Bergeron-Findeisen factor, while  $C_1$  and  $C_2$  are additional parameters, the former one including the terminal velocity raised to the power (-0.5). The first term of  $F_{C0}$  in Equ. (7) renders the autoconversion, and the second one yields collection by rain (coalescence) effect; in the third term, the Bergeron-Findeisen factor,  $f_{B-F}$ , is a function of the difference between equilibrium saturation vapour pressure over liquid water and ice, and of the adopted ice crystal probability relation. In Kessler's derivation, the rain water content is raised to the power 7/8 in the term representing collection by rain. In Equ. (7), the power 0.5 was adopted to enhance the effect at small precipitation rates.

An illustration of the Bergeron-Findeisen effect and results from a one-dimensional model are given in Figs. 1 and 2 respectively. The following parameter values are given as a guidance: convective cloud,  $C_{00} = 2 \cdot 10^{-4}$ ;  $m_r = 8 \cdot 10^{-4}$ ; stratiform cloud,  $C_{00} = 1 \cdot 10^{-4}$ ;  $m_r = 4 \cdot 10^{-4}$ ; in both regimes,  $C_1 = 100$  and  $C_2 = 7$ . For further details see Sundqvist (1993b).

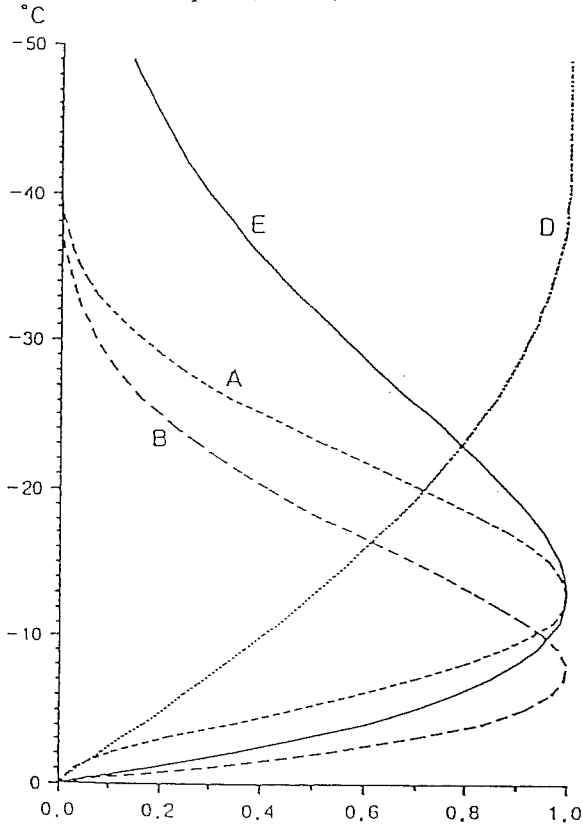


Fig. 1 Curves illustrating the Bergeron-Findeisen effect, based on the ice probability given by curve D; curve E is the difference in saturation vapour pressure over plane water and ice surfaces, normalized by its own maximum; curves A and B show the Bergeron-Findeisen effect for two extreme alternatives; A, if only the local ice probability is regarded; B when it is assumed that the precipitation entering from above exclusively consists of ice/snow; both curves are normalized by their respective maxima.

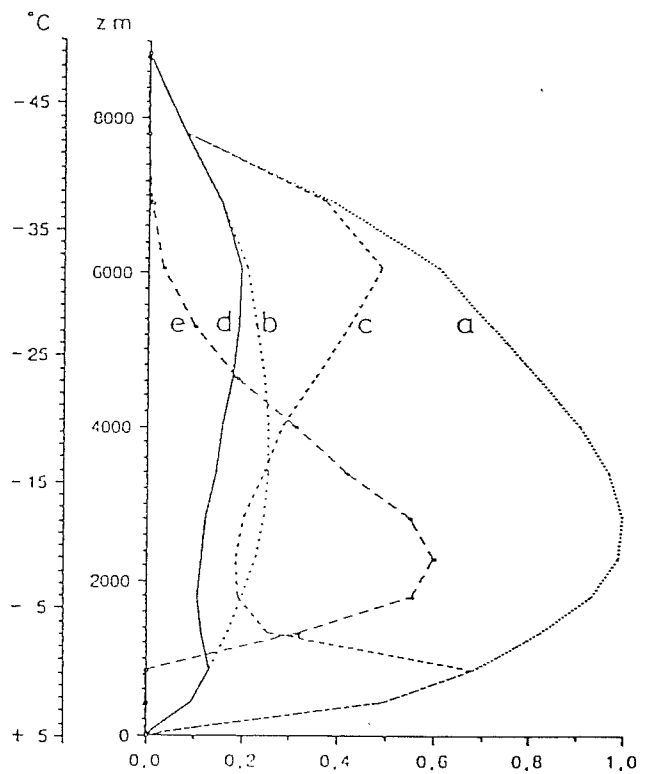


Fig. 2 Results from a study of the coalescence and Bergeron-Findeisen effects in a 1-dimensional model. The curves show the steady-state distribution of cloud water content for a given forcing. Curve a results when only autoconversion is active, i.e.,  $C_1=C_2=0$  in Equ. (7); curve b is the result when autoconversion and coalescence operate together,  $C_1=100$ ; curve c follows when autoconversion and the Bergeron-Findeisen effect operate together,  $C_2=7$ ; curve d shows the result when all three effects operate together. The curves a-d are all normalized by the maximum value of curve a. Curve e shows the actual Bergeron-Findeisen factor  $f_{B-F}$ .

As mentioned, the cloud cover,  $b$ , has been included for completeness in Eqs. (6) and (7). Let us first notice that in the above discussion no specific model resolution is considered. This means that the parameters of the microphysical schemes are merely connected with a particular type of condensation and in-cloud characteristics. Thus, to maintain these features in models that treat fractional cloud cover ( $b < 1$ ), it is necessary to divide the gridpoint value by the cloud cover. Then, ideally these parameters values should be valid for arbitrary grid resolutions.

### 3. PARAMETERIZATION OF CLOUD PROCESSES FOR $T \leq -38$ °C

There are two basic and important differences in the cloud conditions between the liquid or mixed-phase cloud situation and the low temperature (cirrus) regime. First, freezing nuclei appear to be relatively less abundant in the upper troposphere than condensation nuclei are at higher temperatures. Second, as the clouds consist only of ice crystals at the low temperatures, neither accretion nor Bergeron-Findeisen processes are active, implying that the particles grow essentially by vapour deposition.

We consider the same set of equations (1) through (5) as before; the latent heat,  $L$ , now of course includes heat of freezing, and the relative humidity,  $U$ , is taken with respect to saturation over ice.

#### 3.1 Condensation

As a consequence of the sparseness of freezing nuclei at the low temperatures in the upper troposphere, a high supersaturation with respect to ice is required before cirrus can start to form (Heymsfield, 1975; Starr and Cox, 1985). However, after a cirrus cloud has formed, ice crystals act as effective freezing nuclei, and the continued condensation proceeds at a much lower supersaturation with respect to ice. This means that, during the early stages of cirrus development, the relative humidity gradually decreases to a minimum (near steady-state) supersaturation. Hence, it is needed to deduce a relation for the rate of change of  $U$  (in Equ. 5) that will constitute the closure condition for the system (1) - (5). The derivation of a closure condition involves the following principal components.

Starting from the rate of ice crystal mass growth by deposition of vapour for an individual particle, which is proportional to the particle size and the supersaturation, an equation for the growth rate of ice water mixing ratio due to condensation may be obtained assuming that a particle size distribution function is available. This equation has the following principal form

$$\left(\frac{\partial m}{\partial t}\right)_{\text{vd}} = \tilde{N} G f(m) (U - 1) \quad (8)$$

where  $\tilde{N}$  is a function of the size distribution relation and  $G$  (dimension  $\text{m}^2 \text{s}^{-1}$ ) is the quantity Rogers and Yau denote  $\xi_1$ ; the subscript "vd" indicates vapour deposition. The right hand side of Equ. (8) is equivalent to the rate of release of heat due to freezing, that is, equal to  $Q$  of Equ. (5). Combining Eqs. (5) and (8) then gives the  $U$ -tendency.

$$\left[M_q - q_s \frac{\partial U}{\partial t}\right] (1 + US_q)^{-1} = \tilde{N} G f(m) (U - 1) \quad (9)$$

That the  $\partial U/\partial t$ -relation is needed does not become obvious until a description for the release rate of precipitation is considered.

### 3.2 Release of precipitation

As the cloud particles grow essentially by vapour deposition, the growth rate of the cloud water content keeps pace with the rate of condensation. The growth of the water content is composed of a growth, partly of the number density of cloud particles, partly of the mass of the individual particles. The latter effect will eventually lead to release of precipitation, and neglecting the  $A_m$  term in Equ. (3), an approximate steady-state  $m$ -value,  $m_{ss}$ , is obtained from relation (9), with  $\partial U/\partial t = 0$ , and under the condition that we have the steady-state  $U$ -value,  $U_{ss}$ . A typical value of the latter obtained in field experiments is around 5-10 % (Heymsfield, personal communication). However, a relation is needed to describe the rate of release of precipitation during the adjustment stage, that is while  $\partial U/\partial t \neq 0$ . Awaiting a plausible formulation for  $G_P$ , we merely give a principal tentative relation following the above discussion.

$$G_P = f\left(\frac{m}{m_{ss}}, M_q, \frac{U-1}{U_{ss}-1}, \frac{\partial U}{\partial t}\right) \quad (10)$$

The constraint on (10) is that it must yield the steady-state form

$$(G_P)_{ss} = \frac{M_q}{1+U_{ss}S_q} \quad (11)$$

## 4. EVAPORATION AND MELTING

### 4.1 Evaporation

Evaporation will take place when hydrometers are brought into subsaturated air volumes. Liquid and mixed-phase *cloud water* does not survive long in subsaturated air is, due to the small size of cloud particles. Therefore, in view of the typical length of a model time-step, it seems reasonable to assume that cloud water evaporates instantly when it enters a subsaturated volume. A re-evaporation of cloud water due to sub-grid scale circulation (turbulence) in the gridbox where the cloud is created is also conceivable. But that is a situation with a fractional cloud cover, so it is therefore a problem of subgrid-scale parameterization, which is not covered in this discussion.

It seems that evaporation of *ice-phase* cloud water has to be considered separately. When ice water is transported into cloud-free air volumes, those may have a high relative humidity with respect to ice. Therefore, the evaporation may be very slow (or absent if the humidity is higher than the steady-state value,  $U_{ss}$ ). So it may be reasonable to apply a treatment similar to the one for evaporation of precipitation (discussed below). A slow rate of evaporation of cirrus is a plausible explanation to tendency of these clouds to survive the commonly observed long distance transports in the tropics and subtropics.

Evaporation of *precipitating water* proceeds at a slower rate, which means that a more detailed derivation is required for its parameterization. We adopt the Marshall-Palmer (1948) drop size distribution. The rate equation

for evaporation of a single drop is then integrated over the size spectrum to produce the rate of evaporation of the precipitating water mass, which becomes proportional to the square root of the density of the precipitating water (Sundqvist et al, 1989; Sundqvist, 1993a). (Kessler's (1969) derivation yields an exponent of  $13/20$ , because he incorporates ventilation effects.) Since we do not have the precipitating water explicitly available in the present approach, it is necessary to make the same diagnostic interpretation as when we formulated the coalescence effect in Subsection 2.2.1.

For precipitating ice particles, it is important that saturation with respect to ice is applied in order to obtain the commonly observed long fall distances of ice precipitation, which in turn may cause an enhancement of the release of precipitation in underlying liquid and mixed-phase clouds (via the Bergeron-Findeisen effect).

#### 4.2 Melting

When the ice phase is included, it is also necessary that *melting* be taken into account at temperatures higher than 273 K. This may be done with the aid of a melting coefficient in an expression that is similar to the one for evaporation but with the subsaturation replaced by the temperature excess above 273 K (Sundqvist, 1993a). It is sometimes argued that the ventilation is so effective that the melting uses up the local temperature excess implying that relatively strong cooling rates might appear at one single model level and thereby cause pronounced destabilization and even numerical instability. Thus, the matter of melting requires a thorough investigation to resolve this dilemma.

#### 5. PROGNOSTIC TREATMENT OF OTHER HYDROMETEORS?

Let us close the present discussion by returning to the question of including additional hydrometers as dependent variables of the model system. The demand on computational work in model calculations was put forth in the Introduction as one reason for reducing the number of such dependent variables. But this should also be regarded from the physical point of view.

A more complete approach in this respect would mean that we should deal with the four basic variables, namely cloud and precipitating water, in liquid and ice phase. Taking that step, a natural requirement may be that those forms should be divided into bins according to size. An important consequence of this refinement is that relations are needed to describe the processes of transformation between the different states. Lack of adequate insight into such processes may introduce uncertainties in the system. But even with a fair knowledge in this respect, it still seems necessary that we learn more about the characteristic behaviour of such systems as they may possess bifurcation properties because of the general non-linearity in the couplings between the variables (Wacker, 1992). Thus, it appears premature to aim at this level of refinement for application in meso-scale and large-scale models, but it should naturally be pursued in cloud resolving models.

A less ambitious refinement would of course be to consider inclusion of precipitating water as an additional prognostic variable in the approach discussed in this paper. One of the reasons brought up for such a step is that precipitation, which is released in the mid and upper troposphere, does not reach the ground within a



timestep of a high resolution model. The other reason put forth is that diagnostic precipitation stays in the same grid column all the way to the ground (or until it has evaporated), whilst a time-dependent precipitation may be transported into neighbouring columns because of the difference between the horizontal wind speed and the fall speed of the precipitation. Principally this feature would change especially the conditions for evaporation of precipitation and hence render another moisture distribution. Studies of these effects would be interesting, but they are probably justifiable only in models with a horizontal resolution higher than about 50 km.

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