# Boundary-layer parametrization in heterogeneous terrain

P J Mason U.K. Meteorological Office, Bracknell

#### Abstract

Most of the descriptions of the boundary layer and surface transfer processes which are used in numerical weather prediction models are based on the description of level homogeneous terrain. With heterogeneous terrain there remain many difficulties and unsolved problems but recent work now offers some basis to extend current models. From the point of view of the ease of adaptation of existing approaches, it is fortunate that studies of flow over hills, perhaps surprisingly, find that concepts of effective roughness length and displacement heights can be used in a variety of relevant cases. The basis of estimating these quantities will be discussed. When the surface heterogeneity leads to variations in the sign of the heat flux the effective roughness length approach fails and a more explicit treatment of the various surface types is required. Such treatment is required in areas with a mix of land and water surfaces. To provide the required parametrizations of general heterogeneous terrain a combination of effective roughness lengths and an explicit near surface model is recommended.

The suggested modifications are not minor in effect and lead to large increases in momentum roughness length and decreases in temperature and scalar roughness lengths.

## 1 Introduction

Numerical weather prediction models describe motions averaged over areas and require estimates of the corresponding area averages of the surface transfers of heat, moisture and momentum. Apart from the influence of gravity wave generation upon the momentum transfer, these processes are expected to be determined through the interaction of the planetary boundary layer and the surface. Gravity wave generation will depend more on the properties of the free atmosphere and will not be dealt with here. The underlying concept will be to consider the application of some filter operation to the real terrain and

atmosphere. This filter should separate the resolved model variables from the subgrid processes and might have a scale of perhaps  $2\Delta x$  in a grid point model with a horizontal mesh point spacing of  $\Delta x$ .

# 2 Framework for Implementation

To describe the transfers over heterogeneous terrain we will seek to either extend and modify existing descriptions over level terrain or, if this is not possible, propose alternatives. Although not the present concern, we must note that the surface transfers depend on a correct description of the whole boundary layer. For the surface fluxes to be correct it will be a pre-requesite that transfers within the boundary layer and between the boundary layer and the free atmosphere are correctly described. Gradients within the boundary layer are usually small and the estimation of entrainment - effectively the boundary depth is critical. In coarse vertical resolution numerical models this entrainment is poorly described in consequence of the large vertical mesh spacings. This raises issues about how the boundary layer is represented and cautions us against unwarranted detail in the surface flux description. In spite of these difficulties we shall see some large influences which should be significant within existing models.

# 3 Homogeneous Terrain and Flows

It is worth beginning with a few remarks concerning the representation of the boundary layer over homogeneous terrain. Although this description will be based on our empirical knowledge of atmospheric surface layer flow it is desirable to bound this empirical description by some requirements. One requirement is for a rational behaviour in the free convective limit. A second requirement is for the description to be formulated so as to match the description of turbulence within the whole boundary layer. This is needed to ensure that the surface fluxes are not sensitive to changes in vertical resolution. It also provides a good basis for determining the interior description. If in the flow interior we use an eddy viscosity  $\nu$  and an eddy diffusivity for heat  $\nu_H$  then with the mixing length assumption of local equilibrium we can assert

$$\nu = f_M(Ri)\ell^2 \left| \frac{\partial U}{\partial z} \right| \tag{1}$$

and

$$\nu_H = f_H(Ri)\ell^2 \left| \frac{\partial U}{\partial z} \right| \tag{2}$$

where  $f_M$  and  $f_H$  are functions of the gradient Richardson number,  $\ell_M$  and  $\ell_H$  are the mixing length scales and  $|\partial u/\partial z|$  the velocity gradient. In a surface layer  $\ell=\kappa z$  where  $\kappa$  is the von Karman constant.

To satisfy free convection scaling both  $f_M$  and  $f_H$  must be  $\propto Ri^{1/2}$  in the limit  $Ri \to -\infty$ . For unstable flows we can impose this dependence for all Ri < 0 and still obtain

a satisfactory fit to the observations. The different sources of observations do of course show significant variation. The functions

$$f_M = (1 - cRi)^{1/2} (3)$$

$$f_H = a(1 - bRi)^{1/2} (4)$$

with c=16, a=1.4, and b=40 provide a reasonable match with available data (Mason and Brown 1992). For stable flows there are less strigent asymptotic requirements. Theoretical arguments suggest that the length scale of turbulence may decrease roughly linearly towards zero at a critical Richardson number. We can also note a need to modify the Prandtl number  $f_H/f_M$  and the implied stress energy ratio as the Richardson number approach its critical value. The functions

$$f_M = (1 - Ri/Ric)^2 (1 - hRi)$$
 (5)

$$f_H = a(1 - Ri/Ric)^2 (1 - gRi) \tag{6}$$

Where Ric is the critical Richardson number, and 1/Ric is less than either g or h, can provide adequate fits to observations. Values of Ric = 0.25, h = 1.75 and g = 2.4 are typical (Mason and Brown 1992).

To derive transfers between the surface and the first mesh point of the model the integrals of velocity and temperature between the surface and this grid point are required. Monin Obukov similarity indicates that these integrated functions can be written as

$$U = \frac{u_*}{\kappa} \left( \log \left( \frac{z}{z_0} \right) - \psi_M \left( \frac{z}{L_M} \right) \right) \tag{7}$$

and

$$\theta - \theta_0 = \frac{\theta_*}{\kappa} \left( \log \left( \frac{z}{z_{0H}} \right) - \psi_H \left( \frac{z}{L_M} \right) \right) \tag{8}$$

where  $u_*$  is the square root of the surface stress  $\tau$  divided by density,  $\theta_* = H/u_*$  where H is the heat flux, and  $\theta_0$  is the surface temperature.  $\psi_M$  and  $\psi_H$  are universal functions of  $z/L_M$  where  $L_M$  is the Monin Obukov length  $u_*^3/\kappa B$  where B is surface buoyancy flux. To obtain equations 7 and 8 we have assumed that the height scale z is measured above a so-called zero plane displacement. The origin for z is of course arbitrary and except for flow over a smooth level surface a correct origin is unclear and has to be determined. This determination involves selecting a co-ordinate origin so as to give the expected logarithmic variation of velocity with height in a neutral stability flow. This is important for the analysis of observations but not of direct consequence to surface flux parametrization in large scale models.

In principle analytic forms for  $\psi_M$  and  $\psi_H$  follow from equations (3) to (6) but given the choices made here, in particular the use of gradient Richardson number in  $f_H$  and  $f_M$ , such analytic forms are too complex to consider. For a practical application  $\psi_M$  and  $\psi_H$  can be obtained by numerical integration and use of either a look up table or a further approximate functional fit.

The asymptotic assumptions for equations 7 and 8 to hold are essentially for a range of heights z very much greater than the length scale of surface roughness elements and very much less than the boundary layer depth. In this limit  $\psi_M$  and  $\psi_H$  should be universal

and  $z_0$  and  $z_{0H}$  (also  $z_{0Q}$  for moisture) should represent the surface roughness. These roughness lengths are no more than values to constants of integration. The roughness length for momentum  $z_0$  is simply the height at which the velocity profile, obtained for z in the asymptotic range, would extrapolate to equal zero. The roughness lengths for temperature and moisture,  $z_{0H}$  and  $z_{0Q}$  are similarly defined relative to values of surface temperature and moisture. Such surface values are in general impossible to measure properly and it is usual to base  $z_{0H}$  on the surface radiation temperature. The surface radiation temperature is not of course equal to temperature at the top of the soil layer, this is a further difficulty not resolved here.

For all except for the smoothest water surfaces practical values of  $z_0$  are linked to pressure forces on the roughness features of the surface. Except for deformable surfaces such as water and to a slight extent some vegetation values of  $z_0$  are independent of wind speed. In contrast to momentum, heat transfer from a surface always depends on molecular processes and in general values of  $z_{0H}$  will depend on wind speed - a Reynolds number dependence. Observations over uniformly vegetated surfaces show the largest ratios of  $z_{0H}/z_0$  to be  $\sim 0.1$ . For irregular surfaces smaller values are obtained and as discussed below this is inherent in heterogeneous or undulating terrain. To avoid double counting when dealing with the specific influence of surface irregularity we shall need to ensure that observed values of  $z_0$  and  $z_{0H}$  are for truly homogeneous surfaces and do not already include some heterogenity.

In spite of the above comments we have a reasonable data base of observations of  $z_0$  and  $z_{0H}$  over various uniform surfaces (Wieringa 1991). If we were to consider heterogeneity or orography on a short horizontal scale then the asymptotic requirement of equation (5) and (6) would hold. For more typical variations on scales of order kilometres there is no guarantee that (5) and (6) will remain appropriate. In fact numerical simulations, and limited observations suggest that equation (5) and (6) remain useful with  $z_0$  and  $z_{0H}$  replaced by effective values.  $z_0^{eff}$  and  $z_{0H}^{eff}$ . This fortunate result may in part relate to the usual robustness of dimensional arguments but it must also be a matter of some luck. A major objective for small scale numerical models will be to progressively refine our understanding and description of inhomogeneous flows. Some progress has been made and in what follows this is surveyed.

## 4 Flow inhomogeneity

Before considering surface inhomogeneities it is useful to explore the influence of flow inhomogeneities. A variation of wind speed in neutral conditions will, because of the non-linear relation to stress, change the drag coefficients e.g. consider a mean flow  $\langle U \rangle$  with a fluctuation u. The local stress will be given by

$$\tau = C_D(\langle U \rangle + u)^2 \tag{9}$$

and the average by

$$\langle \tau \rangle = C_D (1 + \langle u^2 \rangle / \langle U \rangle^2) \langle U \rangle^2 \tag{10}$$

The effective increase in the drag coefficient by a factor of  $(1 + \langle u^2 \rangle / \langle U \rangle^2)$  is typical of this effect and would be expected to be of order a few percent. Variations of temperature

in the flow area will also influence stability effects and as fluxes do not vary linearly with stability they will also change drag coefficients. The fluxes increase with increasing instability and there will be a bias of transfer coefficients towards unstable values. In particular a positive flux will be produced in conditions which are neutral on average and the transfers will not cease at the usual critical Richardson number. Possible modification to transfer coefficients have been considered by Mahrt (1987). Such modifications require estimates of flow inhomogeneities. Ideally these variations in wind speed and stability should be based on local model gradients. The alternative is to estimate typical subgrid scale variations and derive modified transfer coefficients. In principle equations 3 to 6 above could be subject to this process and revised functions derived. There is also an opportunity to use mesoscale models and HAPEX type field experiments to quantify the changes. We note that flow inhomogeneity will lead to an effective modification in values of the f and  $\psi$  functions above but not in the surface roughness length.

# 5 Surface inhomogeneities

We now consider spatial variations in a surface whose local properties are well known. For example an area of mixed forest and fields or land and water. Following a number of theoretical and numerical studies there is now a firm basis for dealing with statistically-homogeneous heterogeneous surfaces. Here statistical-homogeneity implies cyclic repetition in space. We seek to exclude the case of a single surface change such as a coast line. It is difficult to represent a coast line with greater accuracy than the mesh spacing. Anymore refined statement of local influence would clearly have to be dependent on wind direction and in the case of sea breezes, time of day. We leave such developments for the future and here deal with statistically-homogeneous circumstances.

The boundary layer response to cyclic variations on a scale L provides the basis for understanding the response to surface heterogeneity. The influence of the surface will diffuse upwards into the flow with a slope which is found to be  $\sim (u_*/U(z))^2$ , This slope is a matter of some controversy as linear asymptotic theory suggests values  $\sim u_*/U(z)$ . Wood and Mason (1991) show that the non-asymptotic estimate of  $(u_*/U(z))^2$  does in a practical simulation actually match the simulated height scale of the u flow perturbations. A height scale, termed the blending height follows i.e.

$$\ell_b \sim 2 \left( \frac{u_\star}{U(\ell_b)} \right)^2 L$$
 (11)

Well below  $\ell_b$  the flow locally resembles that of the particular locally homogeneous terrain. Well above  $\ell_b$  the effect of the heterogeneity will have blended together and the flow corresponds to that over an "effective" homogeneous terrain. Wood and Mason (1991) found the u flow perturbation at  $\ell_b$  to be 50% of the surface maximum and found the horizontally averaged flow at  $\ell_b$  and above to be well described by effective values of  $z_0$  and  $z_{0H}$ . Above a height of several times  $\ell_b$  the flow perturbations were small and the flow corresponded to these values of effective roughness length.

The heuristic model proposed by Mason (1988) to describe these changes assumes that at  $\ell_b$  the flow is approximately both, in equilibrium with the surface and also independent of flow position. This approximation leads to values of  $z_0^{eff}$  and  $z_{0H}^{eff}$  in very good

agreement with those deduced from full numerical simulations. The essential physics of the averaging process are captured in this assumption. The average transfer coefficients are obtained by averaging fluxes derived consistent with the heuristic model.

Claussen (1990) notes that this model can be applied directly to estimate fluxes in a large scale model. A typical value of  $\ell_b$  is chosen for each mesh spacing and the model flow at this height is then coupled with separate near surface models for each surface type. i.e.

$$\overline{u_*^2(r)} = U^2(\ell_b)\kappa^2 \overline{\left(\log\left(\frac{\ell_b}{z_0(r)}\right) + \psi\left(\frac{\ell_b}{L_M(r)}\right)\right)^{-2}}$$
(12)

and

$$\overline{\theta(\ell_b) - \theta_0(r)} = \overline{\frac{H(r)\kappa}{u_*(r)} \left( \log\left(\frac{\ell_b}{z_{0H}(r)}\right) + \psi\left(\frac{\ell_b}{L_M(r)}\right) \right)^{-1}}$$
(13)

are solved. Here  $\tau$  is horizontal coordinate and  $z_0$ ,  $u_*$ ,  $\theta_0$ , and H are functions of space, the over bar is an area average. In a practical case there might be several surface types and calculation of the surface fluxes in this way is no more than three times the usual computation. The lowest model levels may be below  $\ell_b$  and this method of variables implies interpolation to  $\ell_b$  and subsequent reassignment of values at the lower levels.

If the surface variations involve distinct changes in heat flux such as is typical of land/water changes then this multi-stream approach is essential. If the surface heat fluxes are fairly similar, such as with variations in vegetation over land surfaces then explicit use of the blending height model is not needed as the main influences can be captured with effective values of  $z_0$  and  $z_{0H}$  defined by

$$\left(\log\left(\frac{\ell_b}{z_0^{eff}}\right) - \psi_m\left(\frac{\ell_b}{L_M^{eff}}\right)\right)^{-2} = \overline{\left(\log\left(\frac{\ell_b}{z_0(r)}\right) - \psi_m\left(\frac{\ell_b}{L_m(r)}\right)\right)^{-2}} \tag{14}$$

and

$$\frac{\overline{H(r)}}{(u_*^2(r))^{0.5}} \left( \log \left( \frac{\ell_b}{z_{0H}^{eff}} \right) - \psi_h \left( \frac{\ell_b}{L_M^{eff}} \right) \right) = \overline{\frac{H(r)}{u_*(r)} \left( \log \left( \frac{\ell_b}{z_{0H}(r)} \right) - \psi_h \left( \frac{\ell_b}{L_M(r)} \right) \right)}$$
(15)

The consequences of equations 14 and 15 are shown in Figures 1 and 2 (from Mason and Wood 1991). Figure 1 shows curves of  $z_0^{eff}$  obtained by averaging different fractions of rough and smooth terrain. The rough value is 1 m and smooth value is 0.01 m. f is the fraction of the terrain which is rough and  $z_{0m}$  is an area weighted logarithmic average of the local values o.e. 0.1 m. Corresponding values of  $z_{0H}^{eff}$  are shown for the case when  $z_{0H}$  is 0.1 of the  $z_0$  values. The various curves are for varying stability and the denoted values of wavelength. For  $\lambda=10^5$  m the results are similar to those obtained by averaging geostrophic drag coefficients, but for smaller values of  $\lambda$  there are larger departures from a simple logarithmic average (a logarithmic average is a straight line from -1 to +1 as f goes from 0 to 1). Typically values of  $z_0^{eff}$  are strongly weighted towards the rougher values even when only a small fraction of the domain is rough. In contrast the values of  $z_{0H}^{eff}$  can decrease by more than two order of magnitudes below the smoothest values. This dramatic change to the values of  $z_{0H}^{eff}$  arises in consequence of the momentum transfer being subject to the strongly non-linear averaging process implied by equation 14. Similar values of  $z_{0H}^{eff}$  are obtained if H(r) and  $z_{0H}(r)$  are assumed constant in equation 15. In effect the mean air/surface temperature difference is not strongly

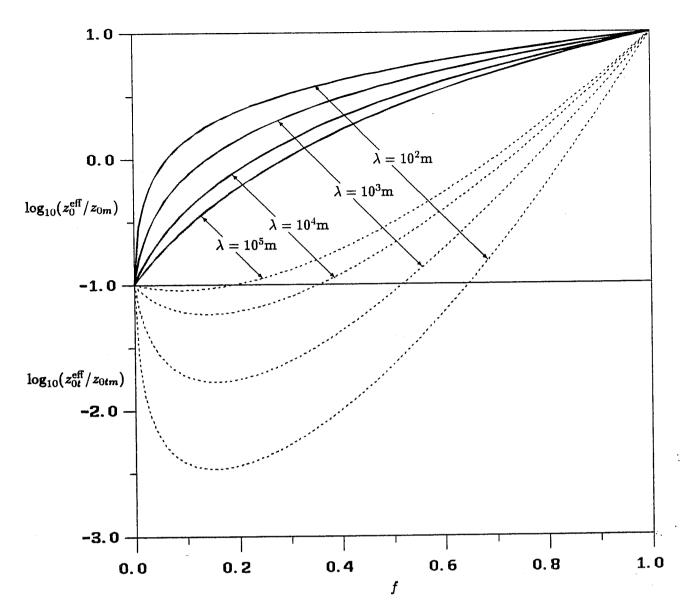


Figure 1: Values of  $\log_{10}(z_0^{\rm eff}/z_{0m})$  (solid lines) and  $\log_{10}(z_{0t}^{\rm eff}/z_{0tm})$  (dotted lines) derived from the heuristic model for step function changes in roughness lengths (from  $z_{0s}$ ,  $z_{0ts}$  to  $z_{0r}$ ,  $z_{0tr}$ ), plotted against  $f_r$ , the fraction of the domain covered by large roughness elements. Results are for  $L_m = -30$ m and for four values of wavelength,  $\lambda$ , as indicated. The form of the step change is as given in Eq. (25). Note:  $\log_{10}(z_{0r}/z_{0m}) = \log_{10}(z_{0tr}/z_{0tm}) = 1$ ,  $\log_{10}(z_{0s}/z_{0m}) = \log_{10}(z_{0ts}/z_{0tm}) = -1$ .

disturbed by the large changes in momentum transfer. Unless these values of  $z_{0H}^{eff}$  are adopted the surface temperature will have errors of order several K for large magnitude positive or negative heat fluxes. Below we shall note an analogous issue when topography enhances momentum transfer.

Figure 2 shows similar curves to those of Figure 1 but with  $\lambda$  fixed at  $10^3$  m and curves for different values of heat flux as implied by the value of Monin Obukov length. Whilst the values of  $z_0^{eff}$  are not strongly influenced by the heat flux, the values of  $z_{0H}^{eff}$  are strongly changed, and become smaller with stable heat fluxes. To capture this behaviour  $z_{0H}^{eff}$  cannot be assumed a constant and rather than make it a function of L direct use of equations 12 and 13 may be more straightforward.

Direct use of equations 12 and 13 also provides a framework to allow for the consequence of heteorogeneity in other surface factors such as soil type and stomatal resistance.

# 6 Flow over orography

As noted above the transfer of momentum between the atmosphere and all surfaces, except for the smoothest sea surface, is dominated by pressure forces acting on the surface roughness elements. Provided these roughness elements are very small compared with the depth of the boundary layer, then, the description provided by equation 7 and 8 should be valid. For larger scale hills and mountains the pressure forces will remain important but the validity of equations 7 and 8 must be questioned. Theoretical arguments to support 7 and 8 are not available, but as noted by Grant and Mason (1990) there are observational and numerical results to support their use with values of effective roughness length.

To obtain such values of effective roughness length it is useful to start with the pressure forces themselves. Two regimes can be recognised, gentle slopes and steep slopes with flow separation. The latter will be more important in consequence of the larger forces but the influence of gentle slopes should not be neglected. In general the pressure force on a hill will be given by

$$F_P = 0.5C_D A U_A^2 (16)$$

where  $C_D$  is a non-dimensional factor, a drag coefficient, A an measure of frontal area and  $U_A$  a velocity scale. For gentle slopes with a wavelength  $\lambda$  a flow perturbation will extend into the flow a distance  $\sim \lambda/2\pi$  and  $u_\star$  can be taken as at least proportional to any velocity scale. With small slopes  $C_D$  is proportional to the slope, as expected from linear theory, and equation 16 could be written as

$$F_P = C_\ell \theta^2 u_*^2 A_s \tag{17}$$

where  $C_\ell$  is a function of  $z_0/\lambda$ ,  $\theta$  is the slope of terrain and  $A_s$  a horizontal surface area. For sinusoidal hills with  $\theta$  defined as the maximum slope and values of  $z_0/\lambda \sim 10^{-4}$  numerical studies find  $C_\ell$  to be  $\sim 6$  (Newley 1985).  $C_\ell$  depends on the velocity profile above the surface and Belcher (1990) provides an analytic form for the dependence of  $C_\ell$  on  $z_0/\lambda$ ; this agrees well with Newley's numerical results. Typically  $C_\ell$  varies nearly linearly with  $(\ell n(z_0/\lambda))^{-1}$  between  $\sim 4$  for  $z_0/\lambda \sim 10^{-6}$  to  $\sim 10$  for  $z_0/\lambda \sim 10^{-3}$ .

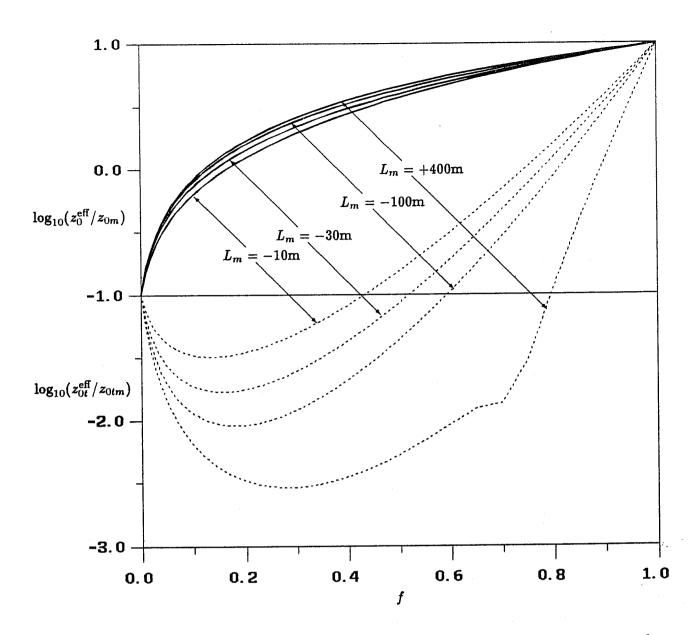


Figure 2: As for Fig. 1 but the wavelength of the variations is fixed at  $\lambda = 1000 \mathrm{m}$  and  $L_m$  is varied as indicated.

To relate this pressure force to a value of  $z_0$  we need to know how to relate the mean surface shear stress  $u_*^2$  to the external flow speed. Given that the influence of the orography only extends a height of order  $\lambda/2\pi$  into the flow a relationship with the velocity at this height would seem appropriate. Theoretical studies (e.g. Belcher 1990) show other somewhat smaller length scales may characterise the drag generating dynamics but the practical consequence of using such smaller scales would be small. We therefore assert that with a flow speed  $U_0$  at  $\lambda/2\pi$  the total surface stress is given by

$$U_0^2 \left(\frac{1}{\kappa} \log \left(\frac{\lambda}{2\pi z_0^{eff}}\right)\right)^{-2} = U_0^2 \left(\frac{1}{\kappa} \log \left(\frac{\lambda}{2\pi z_0}\right)\right)^{-2} + F_P/A_s \tag{18}$$

Here the left hand side of equation 18 is the definition of  $z_0^{eff}$  and the right hand side is the sum of the undisturbed surface stress and the average pressure force per unit surface area. Noting that  $u_*^2 = U_0 \left(\frac{1}{\kappa} \log \left(\frac{\lambda}{2\pi z_0}\right)\right)^{-2}$  we obtain,

$$\left(\log\left(\frac{\lambda}{2\pi z_0^{eff}}\right)\right)^{-2} = \left(\log\left(\frac{\lambda}{2\pi z_0}\right)\right)^{-2} \left(1 + C_{\ell}\theta^2\right) \tag{19}$$

which can be simplified for small values of  $C_\ell\theta^2$ . For slopes of  $\theta\sim 0.2$ ,  $\lambda\sim 10^3$  and  $z_0\sim 0.1$ m we obtain  $z_0^{eff}\sim 0.2$ m. Such an increase in  $z_0$  of a factor of 2 is typical of largest increase which can occur before the alternative steep slope calculation must be considered.

For steep slopes and their associated flow separation the pressure force on the orography is expected to be given by equation 16 with  $C_D$  nearly a constant, A is the frontal or swept area of the obstacle and  $U_A$  a flow speed typical of this local bluff body flow.  $U_A = U(h/2)$  where h is the obstacle height would be a reasonable choice. For smoothly shaped obstacles numerical solutions (Newley 1985) suggest  $C_D \sim 0.3$  but real hills are not so smooth comparisons of derived values of  $z_0^{eff}$  with atmospheric observation suggest that and a slightly higher value of  $C_D$  may be appropriate. To relate this steep slope pressure force to a value of  $z_0^{eff}$  we need to make a nonlinear assertion to derive U(h/2). We assume U(h/2) is not the velocity prevailing in the undisturbed flow but that of the  $z_0^{eff}$  profile. Use of this smaller value of velocity provides good agreement with numerical solutions where  $z_0^{eff}$  can be derived from both the velocity profiles as well as from the the observed forces. To derive  $z_0^{eff}$  we assume

$$U(h/2) = (u_*/\kappa) \ln \left( h/2 z_0^{eff} \right) \tag{20}$$

and 
$$F_p = 0.5C_D (U(h/2))^2 A.$$
 (21)

Where  $u_*$  is related to the total surface stress including the pressure forces and the surface shear stress  $u_{*S}^2$  is assumed to be that due the undisturbed value of  $z_o$  and the value of U in equation 20 i.e.

$$U(h/2) = (u_{*S}/\kappa) \ln (h/2z_o)$$
(22)

The numerical solutions (Wood 1991) suggest that this slightly underestimates the shear stress contribution to the drag but the error is slight. Taking an area average we have

$$Su_{\star}^{2} = \Sigma 0.5C_{d}A \left(U(h/2)\right)^{2} + SC_{N}(U(h/2))^{2}$$
(23)

where S is the surface area considered,  $C_N = \kappa^2/\left(\ln\left(h/2z_o\right)\right)^2$  and the summation includes all obstacles in the area. From these relations we obtain

$$\left(\ln\left(h/2z_o^{eff}\right)\right)^2 = \kappa^2/\left(\Sigma C_d A/S + C_n\right) \tag{24}$$

This expression should be used for terrain with values of peak slope  $\theta$  greater than about 0.2 and in such cases inclusion of the  $C_n$  term is a small correction. Figure 3 shows the curve obtained with this equation compared with values of  $z_0^{eff}$  derived from various field studies and numerical simulations,  $C_n$  has been neglected and  $C_D = 0.4$  has been chosen. The general agreement is encouraging and provides confidence in using the relation. The predicted values of  $z_0^{eff}$  vary between 10m for 300m high hills 2 or 3 kms apart to 100m for 1000m high hills (mountains) 2 or 3 kms apart. There is now good observational support to these values (Grant and Mason 1988, Hopwood 1991). Other more empirical proposals for estimating  $z_0$  (eg Lettau 1969), in consequence of their empirical derivation, give reasonable agreement with equation 24 but do not seem able to fit such wide ranges of data.

A poorly explored aspect of momentum transfer over hills is its stability dependence. Limited observations by Grant and Mason 1990 for moderately unstable and stable buoyancy effect showed a good match with equations 7 and 8 but further work is needed. The effect of orography on heat and moisture variations is also not well explored. Some deduction can however be made for near neutral conditions. Numerical solutions of flow over hills show that the parallel to surface average value of flow speed near the surface only changes slightly even when there is flow separation (Wood 1991). We can thus assert that scalar transfers from the surface will be, on average, unchanged. These transfers depend linearly on the scalar and velocity differences between the surface and the near surface region and this is a clear implication that the scalar transfers will, to the first approximation, be unaffected by the orography. This does not imply no change to  $z_{0H}^{eff}$  but as with an analogous effect in heteorogeneous terrain, a reduction in the 'effective' value. Unless  $z_{0H}$  is reduced the heat transfer efficiency will be increased erroneously by its dependence of  $z_{0H}^{eff}$ . We may write

$$(T - T_S) = \frac{H}{\kappa u_{*o}} \left( \ln \frac{z}{z_{0H}} \right) = \frac{H}{\kappa u_{*e}} \left( \ln \frac{z}{z_{0H}^{eff}} \right) \tag{25}$$

where T and  $T_S$  are horizontal averages,  $z_{0H}$  is the smooth surface value and  $u_{*o}$  is the square root of the true surface shear stress whilst  $u_{*e}$  includes the pressure force. We can also approximately assert that

$$U(z) = u_{\star 0} \left( \ln \frac{z}{z_o} \right) = \frac{u_{\star e}}{\kappa} \left( \ln \frac{z}{z_o^{eff}} \right)$$
 (26)

which leads to the result that

$$\ln\left(\frac{z}{z_{0H}^{eff}}\right) = \ln\left(\frac{z}{z_{0H}}\right) \frac{\ln\left(\frac{z}{z_{0}}\right)}{\ln\left(\frac{z}{z_{0}^{eff}}\right)}$$
(27)

This leads to dramatic reductions in  $z_{oH}^{eff}$  and is strongly recommended as an interim adjustment to ensure, the first approximation result, that heat transfer is unaffected by orography.

One further important aspect of flow over orography needs to be parametrized. Owing in particular to flow separation there is a tendency for the flow to be lifted over a greater height than the mean height of the mountains would imply. This effect is a physical counterpart to the envelope orography used in some large scale numerical weather

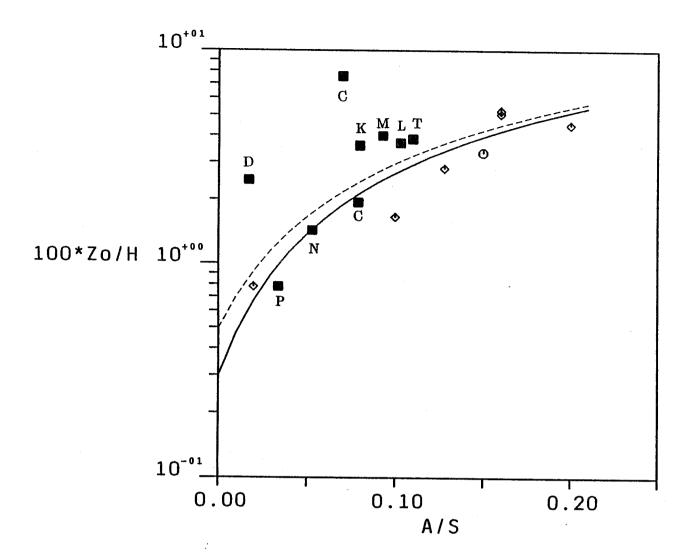


Figure 3: Plots of  $z_0/H$  as a function of A/S. After Grant and Mason (1990) but with extra data points from Hignett and Hopwood (1992). The open symbols are results from a second-order closure model for flow over sinusoidinal orography with  $z_{01} = 0.1$  and 0.3m. The filled symbols are various experimental observations. Details are given in Grant and Mason 1990 but the points P and C are from Hignett and Hopwood 1992. the two points C are for different wind directions.

prediction models. Unfortunately this displacement has proved too difficult to measure in observational studies and has only been estimated approximately in numerical work (Newley 1985). Values of displacement height of about 0.2h where h is the peak - trough height were found for steep sinusoidal orography.

## 7 Suggested procedure

To implement the combined influence of heteorogeneous terrain and orography requires some care. If the averaging over heterogeneous terrain is carried out explicitly by applying equation 12 and 13 to each terrain type then orography must be combined with each uniform surface to give effective values of  $z_0$  and  $z_{0H}$  prior to use of 12 and 13. Similarly if distinct changes in heat flux are ignored the orographic influence should be combined with changing surface type prior to averaging. Note that with the procedures recommended there are only slight differences between dealing with different regions of orography separately and then combining with heterogeneous terrain rules, to simply including all the diverse orography in a single calculation. The former procedure is physically sounder should be most accurate. With these rules it is possible in principle to provide the required parametrizations for heat moisture and momentum. Adequate information on actual surface characteristics and the local roughness lengths remains a problem. Detailed orographic data sets adequate to give reliable estimate of A/S are not available and available parameters such as height variance must be translated to A/S with some assumption of slopes.

## 8 Conclusions

The combination of numerical simulations and some confirming observational studies is providing increasing confidence to parametrizations of boundary layer flow over complex terrain. There is now a firm basis for improvements to current surface parametrizations and future work can be expected to make steady progress in providing wider validity and more accurate specifications.

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