ON THE GROWTH OF ERRORS IN DATA ASSIMILATION SYSTEMS

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1. INTRODUCTION

Considerable progress has taken place in numerical weather prediction over the last decade. It has been possible to extend useful predictive skill in the extra-tropics of the Northern Hemisphere during the winter from less than five days to seven days. Similar improvements, albeit on a lower level, have taken place in the Southern Hemisphere. Another example of improvement in the forecasts is the prediction of intense synoptic phenomena such as cyclogenesis which on the whole is quite successful with the most advanced operational models (Bengtsson (1989), Gadd and Kruze (1988)). A careful examination shows that there are no single causes for the improvements in predictive skill, but instead that they are due to several different factors encompassing the forecasting system as a whole (Bengtsson, 1985).

In this paper we will focus our attention on the rôle of data-assimilation and the effect it may have on reducing the initial error and hence improving the forecast. The first part of the paper contains a theoretical discussion on error growth in simple data assimilation systems, following Leith (1983). In the second part we will apply the result on actual forecast data from ECMWF. The potential for further forecast improvements within the framework of the present observing system in the two hemispheres will be discussed.

2. ERROR GROWTH IN SIMPLE DATA-ASSIMILATION SYSTEMS

The estimation of the rate of growth of forecast errors has been the objective of several recent studies on atmospheric predictability. Some of these studies (Wiin-Nielsen (1986); Dalcher and Kalnay (1987)), have followed the original ideas of Lorenz (1982).

Lorenz used a data base of 100 consecutive ten-day operational forecasts from the European Centre for Medium-Range Weather Forecasts. From this data set of global 500 hPa height data from the winter 1980/81, Lorenz estimated an upper and lower bound of atmospheric predictability. The lower bound is simply the actual average predictive skill for the period; the upper bound, which was obtained from the average growth of the difference between two model forecasts verifying on the same day, shows the predictive skill for a "perfect" model. (Fig. 1). By extrapolating the curve representing the growth of the difference between the two forecasts, Lorenz concluded that, with a perfect model, 8-day forecasts could be as good as the 5-day forecasts in 1980/81 and 14-day forecasts could be as good as the 8-day forecasts, even if the same errors were made during the first forecast day.

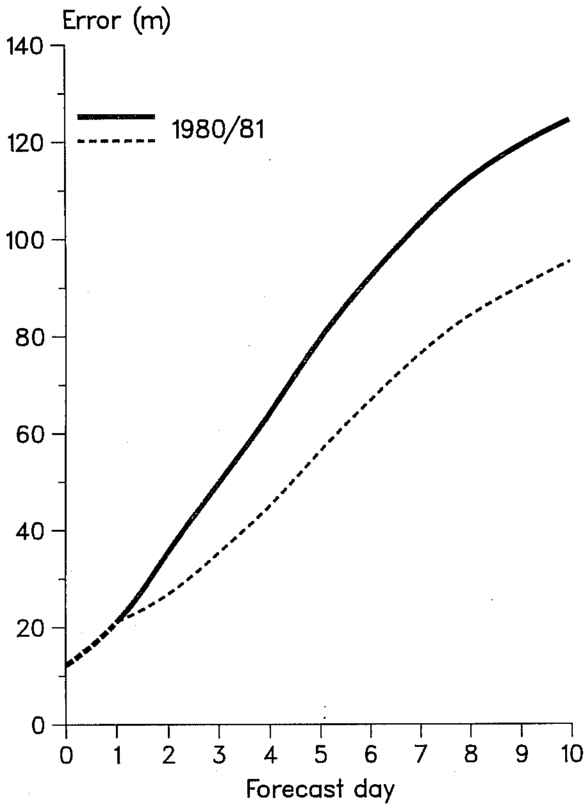


Fig. 1 The full curve shows the Northern Hemisphere RMS error for the 500 hPa height field for the "winter" (DJF) 1980/81 as a function of time. The dashed curve shows the corresponding difference between two consecutive forecasts separated by one day. For further information see text.

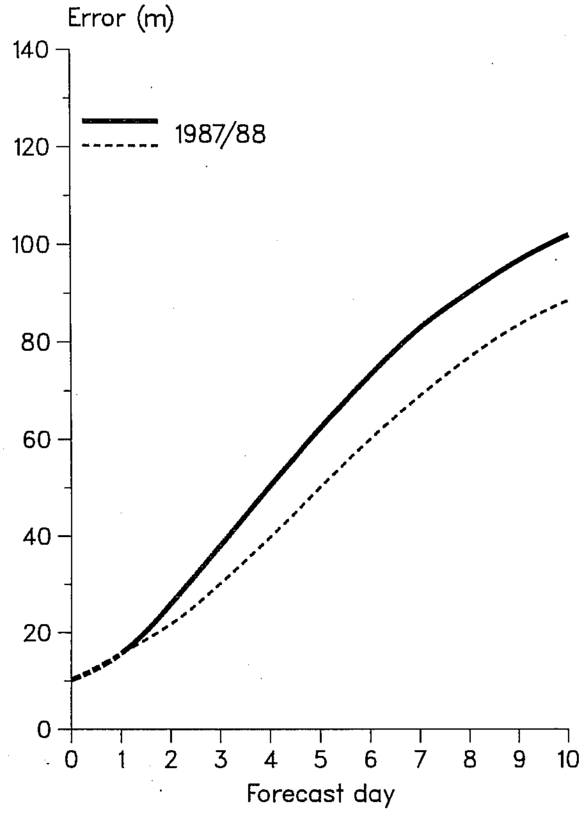


Fig. 2 The same as Figure 1, but for the winter 1987/88. Note the improvement in predictive skill.

ECMWF has repeated Lorenz' predictability estimates. Fig. 2 shows the results for the winter 1987/88. It is easily seen that predictive skill of the ECMWF forecast model has improved between 1980/81 and 1987/88. It can also be noted that the difference between the upper bound (predictability) and lower bound (actual predictive skill) is much smaller. A further examination shows that the improvement in predictive skill in 1987/88 is due to two different factors: the forecast error curve in 1987/88 has **a smaller slope** and the **error at day 1 is significantly lower**. Consequently, the improvement in the forecast is due to model improvements, but also presumably to a more accurate initial state in view of the improvement in the one day forecasts. We will analyse this further below and try to demonstrate that this improvement is essentially due to the impact of a more accurate model used in the data-assimilation process, and improvements in the data-assimilation process itself.

Leith (1978, 1982) has suggested that the deficiencies of a model which make it to differ from the real atmosphere, could be described as an "external" source of error which must be included if we want to represent real forecast error growth. He proposed the following simple growth equation

$$\frac{dV}{dt} = \alpha V + S \qquad (1.1)$$

were V is the model error variance as a function of time. αV describes the inherent tendency for errors to grow due to the unstable nature of the atmospheric dynamics. The term S describes the model error source rate. It is assumed that S is model dependent and can be empirically determined from observed error growth values.

As has been discussed by Dalcher and Kalnay (1987), equation (1.1) is only good for short times and cannot describe the saturation of forecast error which happens at later times. This can be done by replacing equation (1.1) by

$$\frac{dV}{dt} = (\alpha V + S)\left(1 - \frac{V}{V_\infty}\right) \qquad (1.2)$$

where V_∞ is the asymptotic value of the error variance.

In order to examine the effect of data-assimilation on the forecast error, we will follow a procedure proposed by Leith (1983). The initial state is usually determined by two virtually independent assessments: the first guess and the actual observations. If two such independent determinations, z_1 and z_2 of, say, the 500 hPa height with error variances V_1 and V_2 are combined into a better final determination $(V_2 z_1 + V_1 z_2)/(V_1 + V_2)$ then the inverses of the variances, which measure information content, are summed

$$V^{-1} = V_1^{-1} + V_2^{-1} \qquad (1.3)$$

This general statistical principle is used in the objective analysis. By combining equation (1.2) and (1.3) it is now possible to calculate the initial error of a particular forecast as a result of a series of data-assimilation steps.

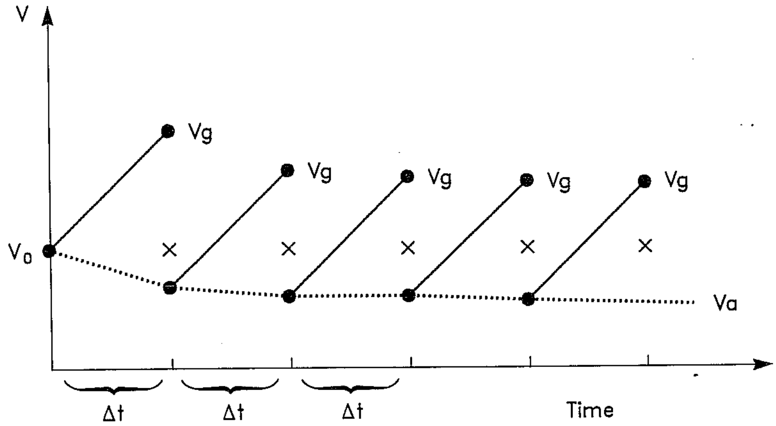


Fig. 3 The successive reduction of the error of the initial state as a function of a data-assimilation process. V_0 demonstrates the observational error variances. V_g' indicates the error variance observational error variance at each data-assimilation step. V_a is the asymptotic error which will develop after a series of data-assimilation steps.

The overall principle is explained in Fig. 3. Let us assume that we start the data-assimilation cycle from a given initial state with initial error variance V_0. This first initial state can consist of an analysis using all available observations at $t = t_0$ using, say, climatology as the first guess.

The initial error variance grows in time during the first data-assimilation step in accordance with equation (1.2). After Δt hours a new initial state is formed by combining the new observations at this time with the first guess obtained from the prediction equation (1.2). The sum of the inverses of the two variances provides the new estimated initial error variance V_n in accordance with equation (1.3).

Following Leith we can easily estimate the **asymptotic value** V_a as it will emerge after a series of data-assimilation steps. In order to simplify the analytic solution, we will replace equation (1.2) by

$$\frac{dV}{dt} = \alpha V + S \tag{1.4}$$

This simplification is justified since we will only use equation (1.2) for short time scales when the ratio of V/V_∞ is very small. We shall now let Δt be a fixed time interval of the data-assimilation cycle. We shall also let V_n be the error variance after the **n**'th data-assimilation cycle. Equation (1.4) when predicted over the time interval Δt leads to a prediction error

$$V_{\Delta t,n} = V_n(1 + \alpha\Delta t) + S\Delta t \tag{1.5}$$

The introduction of new observation with error variance V_0 will lead to a new value V_{n+1} in accordance with the general principle (1.3):

$$V_{n+1}^{-1} = V_0^{-1} + V_{\Delta t,n}^{-1} \tag{1.6}$$

In order to simplify the solution we will introduce

$$v_n = \frac{V_n}{V_0} \;; \qquad \sigma = \frac{S}{\alpha V_0} \text{ (non-dimensional)}$$

Equation (1.5) and (1.6) can then be combined into an iterative expression

$$v_{n+1}^{-1} = 1 + \{(1 + \alpha\Delta t)v_n + \alpha\sigma\Delta t\}^{-1} \tag{1.7}$$

As n increases an equilibrium level

$$v = \lim_{n\to\infty} v_n = \lim_{n\to\infty} v_{n+1} \tag{1.8}$$

is reached which is the factor by which the data-assimilation reduces the observational error variance V_0. It follows that v must satisfy the quadratic equation:

$$v^2 + \eta(\sigma - 1)v - \eta\sigma = 0 \tag{1.9}$$

where $\eta = \frac{\alpha\Delta t}{1 + \alpha\Delta t}$ is a dimensionless parameter which only depends on data-assimilation frequency. The relevant root of equation (1.9), given by:

$$v = [\eta\sigma + \{\eta(\sigma - 1)/2\}^2]^{½} - \{\eta(\sigma - 1)/2\} \tag{1.10}$$

is given in Fig. 4 as a function of $\sqrt{S/V_0}$ for values of η corresponding to time intervals of 3, 6, 12 and 24 hours. α, the inherent error growth, has been put equal to 0.5545 corresponding to an error variance doubling time of 1.25 days. In order to simplify the interpretation, Figure 4 presents the result in terms of relative **error reduction** instead of **error variance reduction.**

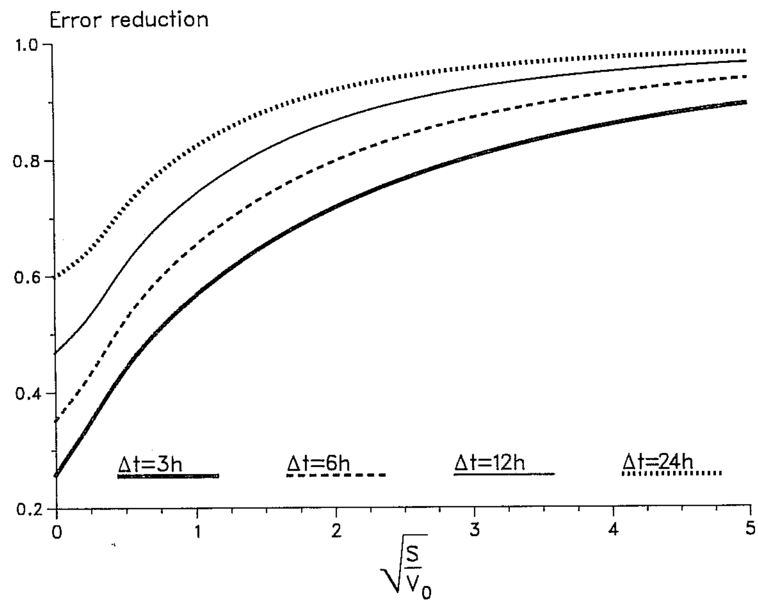


Fig. 4 Assimilation equilibrium error reduction as a function of the ratio between the model error source rate and the observational error. Note the dependence of the data-assimilation frequency on the error reduction. For further information see text.

It follows from Fig. 4 that a data-assimilation system can considerably reduce the initial error and thereby extend predictive skill. As is clearly demonstrated by Fig. 4 the improvement is largest when S/V_0 is small and when the data-assimilation frequency is high (Δt is small).

3. EMPIRICAL APPLICATIONS

In order to provide a realistic estimate of the improvement in predictive skill due to data-assimilation, we need an independent estimate of V_0. This can preferably be obtained by a calculation of the optimum interpolation error (Gandin, 1963) with respect to a given data distribution, using empirical structure functions and climatology as the background field. Observing system simulation studies using the ECMWF multivariate analysis system (Lorenc, 1981) during FGGE, have been reported by Bengtsson (1986). In this way the interpolation error for a given parameter and for a given level depends on the **three-dimensional distribution of mass- height- and wind-observations.** Such calculations show that the 500 hPa height field in the Northern Hemisphere has an initial error of around 15 m during the winter. The major variance reduction is obtained by radiosondes and surface data. The effect of satellite and aircraft data is to reduce the remaining error variance in the central parts of the North Atlantic and North Pacific; the influence in other regions is comparatively small.

The Northern Hemisphere error has by and large been the same over the last ten years. An estimation of corresponding errors for the Southern Hemisphere is more difficult. Satellite observations of wind and temperature are the dominating data source. Due to a reduction in the number of wind observations from geostationary satellites and ships of opportunity, it is presently less good than during FGGE. In recent years a gradual improvement of the surface network has taken place with the establishment of the TOGA buoy network. We have correspondingly estimated the 500 hPa height field error to be 35 m during 1980/81 but improved to 30 m in 1986/87.

By fitting equation (1.2) to the 500 hPa error growth curve (truncated to T40) for the two hemispheres for the winters 1980/81 through to 1986/87 shows a gradual improvement in the model error source rate, S. Figures 5 and 6 respectively show the result for the winters (DJF) 1980/81 and 1986/87 for the Northern Hemisphere and for the Southern Hemisphere respectively, indicating that S has been reduced from $(18m)^2day^{-1}$ to $(12m)^2day^{-1}$. The result varies somewhat from year to year, but a steady trend towards smaller values of S can be noticed. The results for the winter 1987/88 are somewhat better than in 1986/87, indicating a value of S of around $(11m)^2day^{-1}$. A very small improvement in V_0 at the Northern Hemisphere is also indicated.

However, it should be noted, that it is difficult to separate error growth due to α and from due to S; within a certain range an increase in α and decrease in S will not change the overall fit between equation (1.2) and the observed model error growth. Using the same data set for the winter 1980/81 but for global data, Dalcher and Kalnay (ibid) have obtained slightly smaller values for α and slightly larger values for S. However, since it is our principal objective to estimate the predictive skill, we believe it is preferable not to underestimate α, since a small α and a diminishing value for S would provide overly optimistic estimates of achievable predictive skill. Experience also indicates that the inherent error growth in models has a tendency to increase as models successively incorporate more small-scale synoptic features.

In fitting the data for the two hemispheres we have used a 12 hr data-assimilation for the Northern Hemisphere but 6hr for the Southern Hemisphere. We justify this in the following way. In deriving (1.10), which provides an expression for the error reduction due to data-assimilation, we have implicitly assumed that the observational error is the same for all times. This is not strictly the case, in particular not in the Northern Hemisphere where the observing system at 00 UTC and 12 UTC is significantly more accurate than at 06 UTC and 18 UTC. In the Southern Hemisphere the observing system is more homogeneous due to its strong dependence on asynoptic observations.

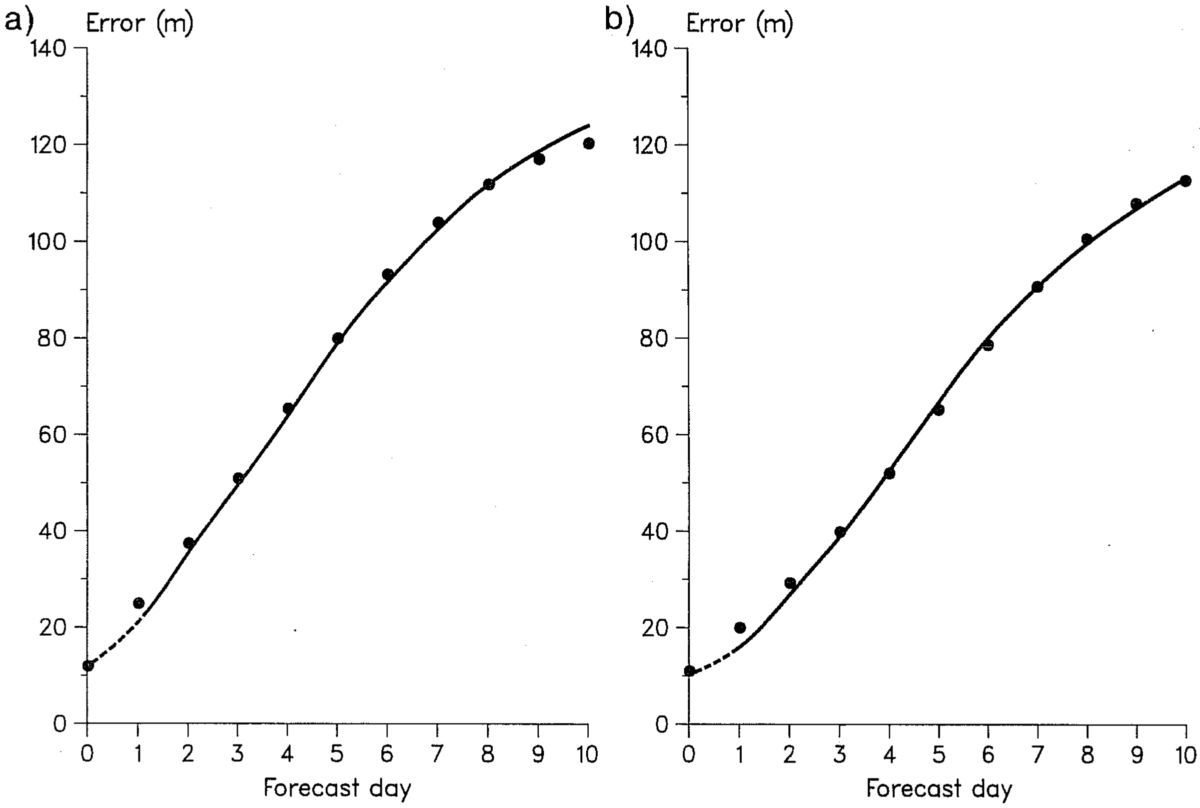


Fig. 5 The full curve shows the Northern Hemisphere RMS error for the 500 hPa height for the winter (DJF) 1980/81 (5a) and 1986/87 (5b). The dots in 5a are equivalent to the $V_0 = (15m)^2$, $S = (17m)^2 day^{-1}$ and $V_\infty = (125m)^2$ according to eq. (1.2). Corresponding values in (5b) are $V_0 = (15m)^2$, $S = (12m)^2 day^{-1}$ and $V_\infty = (120),^2$. For further information see text.

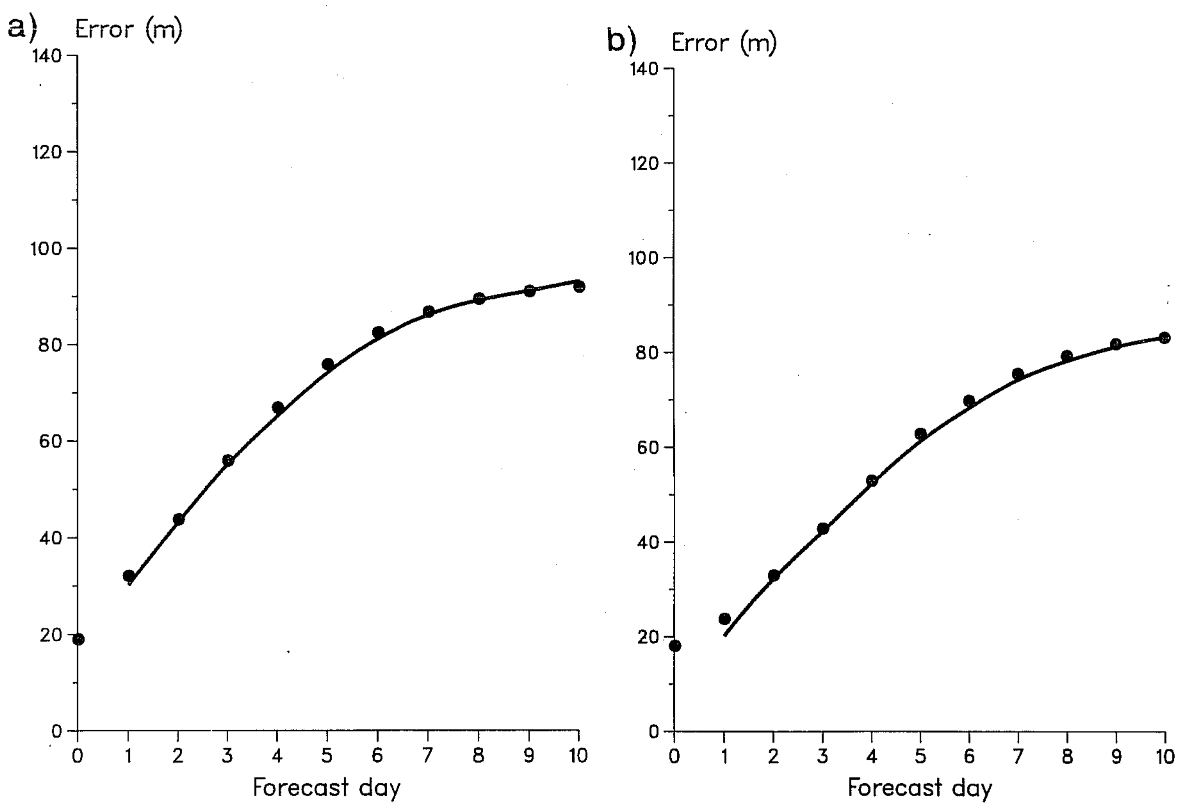


Fig. 6 The same as Figure 5 but for the Southern Hemisphere. (6a) shows the result for the "winter" (DJF) 1980/81, where $V_0 = (35m)^2$, $S = (18m)^2 day^{-1}$ and $V_\infty = (93m)^2$. (6b) shows the result for the "winter" (DJF) 1986/87 where $V_0 = (30m)^2$ $S = (12m)^2 day^{-1}$ and $V_\infty = (85m)^2$.

If we assume that V_0 for the Northern Hemisphere is $(30m)^2$ at 06 UTC and 18 UTC, which, we believe, is a realistic estimate, the effect of 6hr data-assimilation is significantly reduced. As can be seen from Table 1, the relative reduction of the initial error, using a model with an error source rate of $S = (18m)^2 day^{-1}$, for instance, is only 3% compared to 12 hr data-assimilation using data for 00 UTC and 12 UTC only. The relative improvement increases slightly as the forecast model is improved (S is getting smaller).

	Data-assimilation time-step		
Model error S	12 hr	6 hr	6 hr*
$(18m)^2 day^{-1}$	11.7	10.3	11.3
$(12m)^2 day^{-1}$	10.7	9.2	10.2
$(6m)^2 day^{-1}$	9.1	7.4	8.5
0	7.4	5.4	6.7

Table 1
The initial error as obtained through a data-assimilation process for different values of the model error S and data-assimilation frequency. The initial error is 15 m except column 3 (*) where it is assumed that the error is 15 m at 00 UTC and 12 UTC, but 30 m at 06 UTC and 18 UTC.

4. POTENTIAL FOR FURTHER IMPROVEMENT IN FORECAST SKILL

In this final section we will discuss further potential improvements in numerical weather prediction. We do not expect any major improvement in the accuracy of the global observing system at least not before the end of this century, and consequently improvements in numerical weather prediction are mainly expected to come from better models and data-assimilation systems. Figure 7 shows for each hemisphere separately using the same data-assimilation frequency as now, how predictive skill depends on S. In the Northern Hemisphere a 5-day forecast in 1986/87 ($S = (12m)^2 day^{-1}$) is better than a 4-day forecast in 1980/81 ($S = (18m)^2 day^{-1}$)). By reducing the error to $S = (6m)^2 day^{-1}$ a further improvement by another 1.5 days appears feasible. The relative improvement is very similar for the Southern Hemisphere.

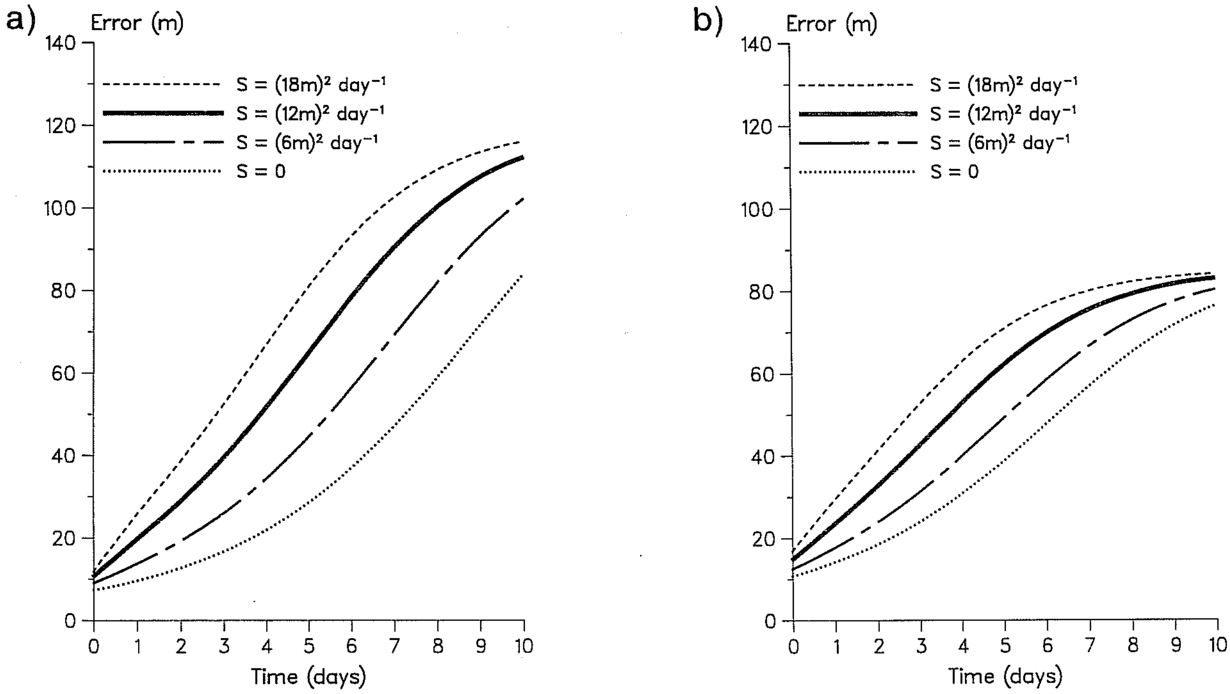
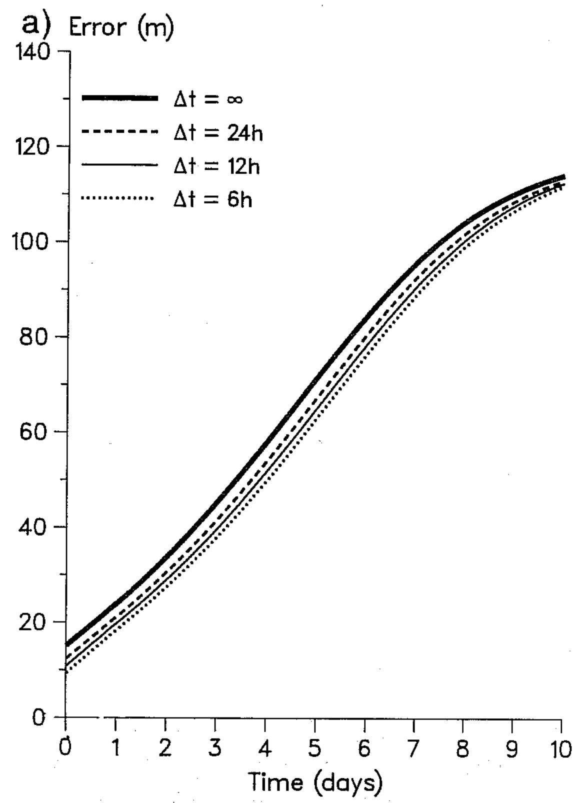


Fig. 7 (a) Improvement in predictive skill for the 500 hPa height field for the Northern Hemisphere as a function of S. The data-assimilation frequency is 12 hrs. The heavy full curve shows the data for the winter 1986/87 corresponding to an error source rate of $(12m)^2 day^{-1}$.
(b) The same result for the Southern Hemisphere but for a data-assimilation frequency of 6 hrs.

As demonstrated in Figure 4, the positive effect of data-assimilation is inversely proportional to the ratio S/V_0. Since this ratio is much smaller is the Southern than the Northern Hemisphere, it follows for present values of S and V_0 that the effect of data-assimilation has so far been much greater is the Southern Hemisphere. This is illustrated in Figure 8, which shows the effect of data-assimilation for the two hemispheres using data for 1986/87. The effect of predictive skill is almost 2 days of the Southern Hemisphere, while only about 12 hr at the Northern Hemisphere. However, as can be seen from Figure 9, the relative importance of data-assimilation for the Northern Hemisphere will increase when the model is becoming more accurate (S smaller).



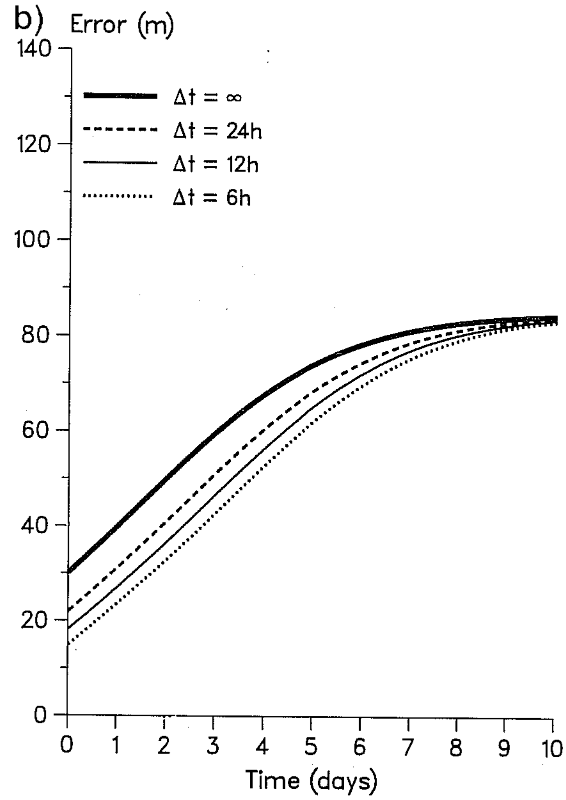


Fig. 8 (a) Improvement in predictive skill for the 500 hPa height field for the Northern Hemisphere as a function of the data-assimilation frequency. $V_0 = (15m)^2$, $S = (12m)^2 day^{-1}$ and $V_\infty = (120m)^2$
(b) The same for the Southern Hemisphere but with $V_0 = (30m)^2$, $S = (12m)^2 day^{-1}$ and $V_\infty = (85m)^2$.

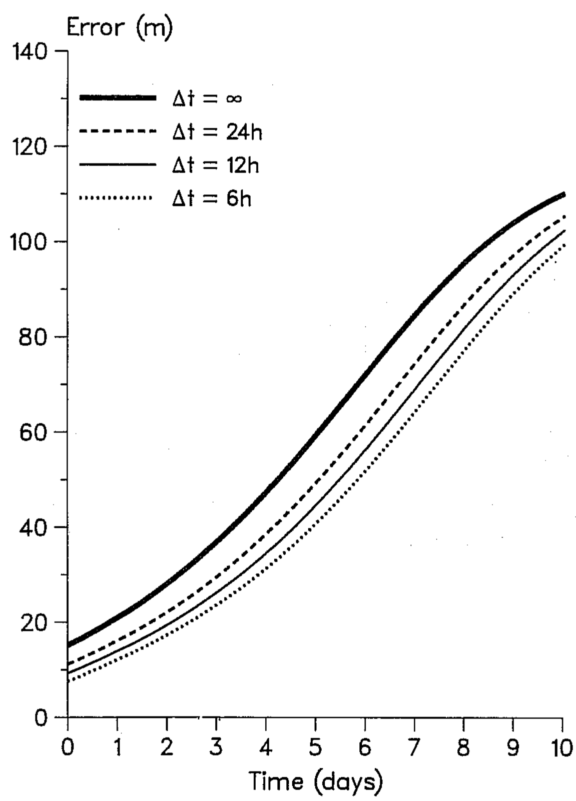


Fig. 9 The same as Figure 8a (Northern Hemisphere) but with $S = (6m)^2 day^{-1}$ instead of $(12m)^2 day^{-1}$. Note the improvement due to the effect of the data-assimilation frequency when S is reduced.

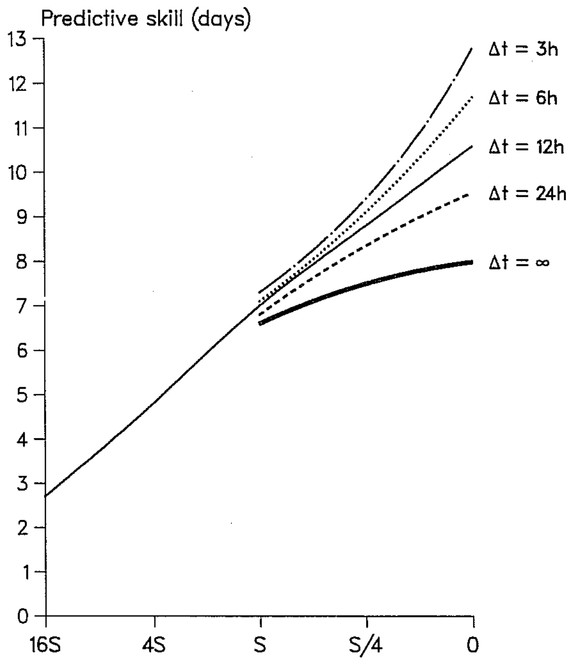


Fig. 10 Predictive skill in days for the Northern Hemisphere as a function of S and Δt. The reference value for $S = (12m)^2 day^{-1}$. Predictive skill is defined as 0.75 of the ratio between forecast error and persistence error. Note the potentially large improvement when models are further improved.

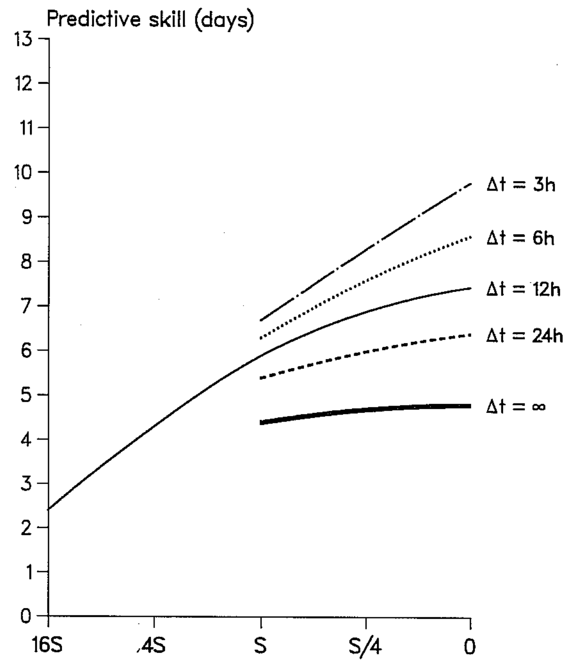


Fig. 11 The same but for the Southern Hemisphere. Note the benefit which has already been achieved through data-assimilation.

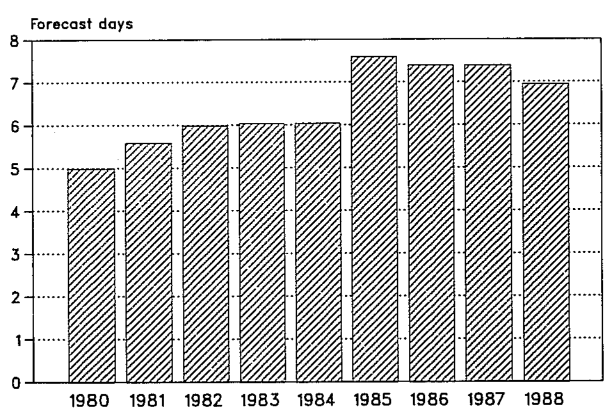


Fig. 12 Predictive skill in days (12 months running mean calculated from monthly values) for the ECMWF model for the 500 hPa height field for the Northern Hemisphere 1980-1988. Predictive skill is given as the time in the forecast when the ratio between the forecast error and persistence error reaches 0.75.

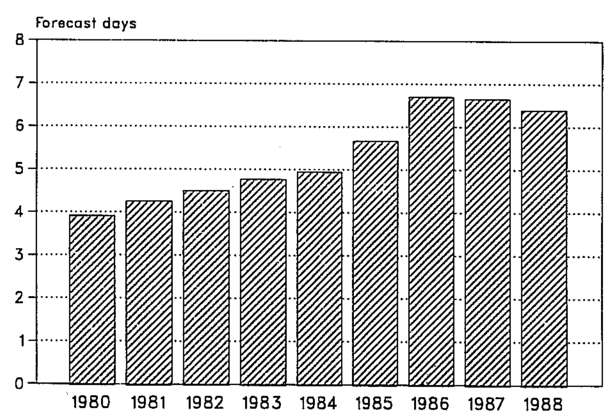


Fig. 13 The same as Figure 12 but for the Southern Hemisphere.

In Figures 10 and 11 an attempt is made to provide an estimate of the upper bound of predictability as a function of S and Δt. As a measure of useful predictive skill we have used 0.75 of the ratio between forecast error and persistence error as suggested by Mason (1986). Figures 12 and 13 show, for comparison, the predictive skill by the ECMWF model from the years 1980-1988, expressed in the time it has taken the forecast model to reach the same error level. The fall in "prediction skill" from 1987 to 1988 and to a smaller degree from 1986 to 1987 is presumably due to an increase in the variance level of the forecast model. This variance has increased due to model changes, in particular a reduction in the vertical diffusion operationally implemented in January 1988. this change has been synoptically beneficial with a better prediction of extreme events such as rapid cyclogenesis. It appears that the recent increase in forecast error variance is more pronounced in this data set than in the truncated T40 data set used elsewhere in this paper.

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