REVIEW OF THE REPRESENTATION OF MOUNTAINS IN NUMERICAL WEATHER PREDICTION MODELS

Fedor Mesinger
Department of Physics and Meteorology, University of Belgrade

William G. Collins
National Meteorological Center, NWS/NOAA, Washington, DC 20233*

Summary: A review is presented of the various approaches taken to arrive at the geometrical shape assumed or considered required to represent mountains in numerical weather prediction and simulation models. Particular attention is paid to the difference between the so-called envelope and "silhouette" mountains. An "edge enhancement" effect of the envelope mountains is briefly analyzed, and results of calculations of $\sqrt{2}$ standard deviation envelope and silhouette mountains on the Gaussian grid of a wavenumber 40 spectral model are presented and compared. A survey is made of the definitions of mountains presently used in a number of leading operational models. Finally, the linear response of the simplest second-order difference schemes to forcing by topography on the Arakawa C and the B/E grid is discussed and compared, following a recent study by Dragosavac and Janjić.

* Lecture presented by F. Mesinger

1. DEFINITION OF MODEL MOUNTAINS - PROBLEMS AND METHODS

Several issues can be identified in choosing an explicit definiton of model orography (e.g., Simmons, 1985). Are the means of terrain elevation over grid squares an appropriate choice, or should some kind of enhancement above these means be undertaken in order to represent the barrier effect of sub-grid scale ridges? If it should, is there an appropriate way of achieving this enhancement? Is there a way to account for the directional dependence of the sub-grid scale ridges? Should an attempt be made to reduce orographic forcing at the smallest resolvable scales? In addition to these questions, specific problems arise in spectral models, related to ripples associated with negative elevations over oceans, and to spreading of continents as a result of spectral fitting.

In the early days of primitive equation modeling grid-square area averaging of the actual terrain elevation was apparently readily accepted as the appropriate method of deriving model mountains. With the limited availability of data sets at the time, the actual averaging was the problem receiving most attention. Thus, the subjectively defined mean values of Berkofsky and Bertoni (1955) were much appreciated and have seen a lot of use. The situation was similar with the later higher resolution values of Gates and Nelson (1975).

A reference to a "severe ... difficulty ... associated with zones of strong topographic gradient" can be found in a paper by Charney (1966), addressed to what he saw as then numerical weather prediction's "islands of resistance" - a term very appropriate for the problems discussed also from today's vantage point. Describing an unrealistic result obtained in calculation of a transport of hypothetical constant volume balloons across the Rocky Mountains, smoothed through the averaging process, Charney invites the reader to consider "what would happen if the Rocky Mountains were an infinitesimally thin knife edge lying along a meridian and extending to the top of the atmosphere". Such a range, points out Charney, "would act as an absolute barrier to the flow, yet the smoothing would reduce it to zero height and zero effect!".

With this idea in mind, a radically new approach in representing mountains was taken by Egger (1972). His mountains consisted of vertical walls, in a sigma system model, placed so as to block the flow in a given sigma layer or layers and thus simulate the barrier effect of steep mountains. The lowest sigma surface, on the other hand, was defined so as to describe the topography of gentle mountain slopes. With this system, Egger has obtained impressive results in simulation of Alpine lee cyclogenesis.

Even though remarkably successful, the method of Egger had a number of obvious imperfections. It could not account for the three-dimensional geometry of steep mountains. In addition, the arbitrariness of the definition of steep mountains and also of the distinction between steep vs. "gentle" mountains were features clearly needing attention in case a more universal application of the method were to be considered.

An attempt to define a method which would in a less arbitrary way correct the problem of area averaging was made by Mesinger (1977). His mountains were obtained by a "valley-filling" procedure, in which sub-grid scale elevation values were compared with the nearest neighboring values. In all cases when it happened that a central elevation value was less than the three greatermost of the four surrounding values, it was, in a block procedure, set equal to the lowest of these three values. These "valley-filled" heights were finally space averaged to obtain model mountains. Justification for the procedure was the idea that the higher elevation values surrounding a valley were those responsible for the mountain blocking effect, and not the elevations of valleys having a neighboring higher elevation value in the directions of both of the two coordinate axes.

Following about the same reasoning, but without resorting to the use of actual sub-grid scale data, Bleck (1977) has simply multiplied his height values by a factor of 2, and subsequently applied a smoothing procedure. He found a mountain enhancement procedure necessary to obtain cyclogenesis in the Genoa lee cyclogenesis cases he was simulating. Indeed, after this rather effective mountain enhancement, Bleck obtained results substantially more successful than those reported earlier (Bleck, 1976).

Less encouraging were the results of Hills (1979). Increasing also rather drastically the height of mountains in the U.K. Meteorological Office 5-level general circulation model, he found some aspects of the mean flow improved, but a reduced transient behavior was an undesireable feature of his experiments. However, it is now known that a much too low level of transient activity observed in the control version of the model was due to reasons other than the representation of mountains, so that the discouraging result would have been avoided had the experiments been done using a model with a higher level of transient activity as indeed has been done later by Wallace et al. (1983).

Following-up on the method of Mesinger (1977) but not satisfied with the effect achieved, Mesinger and Strickler (1982) have in addition to the valley filling added to their terrain heights one half of the standard deviation of the original values. This has raised the maximum elevation of their Alps from 2107 to 2417 m, a much more reasonable value from the point of view of the visual inspection of maps of Alpine topography. They point out that the procedure "would have left the elevation of a flat plateau unchanged, but would raise the elevation of a ragged mountain chain, the more so the more rugged the mountain chain is". This extra enhancement Mesinger and Strickler have performed for one of their two lee cyclogenesis cases in which mountains were found necessary for cyclogenesis. Of these two cases, that with the extra enhancement was clearly one in which they have been more successful.

Independently, and following a suggestion of J.-F. Geleyn, Wallace et al. (1983) have enhanced the grid-square mean terrain values by adding two standard deviations of the sub-grid scale orography. Their "sub-grid scale" orographic values, as well as those of very many authors since, were mean orographic heights for 10' x 10' grid squares contained in a data set made available by the U.S. Navy. Wallace et al. have also added a contribution meant to account for the blocking effect of still smaller scale features, again using the information contained in the U.S. Navy data set. The multiplicative constant of the standard deviation, 2 in their work, would yield a model orography equal to the maximum sub-grid scale values in the case that the sub-grid scale orography has a 2-dimensionally sinusoidal shape. Comprehensive experimentation was performed to assess the effect of the resulting "envelope" orography: a clear improvement in forecast accuracy beyond day 4 was established, as well as a reduction in time-mean error near the end of the forecast range. Forecast improvement was confirmed by further experimentation for another period by Tibaldi (1986), and for that period the rate of growth of systematic error was substantially reduced.

Later experiments done mostly with spectral T63 and subsequently T106 resolution led to the adoption of less enhanced "√2 sigma", and "1 sigma" envelope orographies for these two consecutive versions of ECMWF operational models. For a comprehensive review of various related experimental results the reader is referred to a report by Jarraud et al. (1986b), as well as to papers by ECMWF authors in the present volumes. An interesting result of these experiments is that some detrimental effects identified at lower resolution were found not to be present at the T106 resolution (Simmons, 1985; Jarraud et al., 1986b).

As opposed to the envelope approach which, based on a sinusoidal idealization of the sub-grid scale topography, obviously involves only a very modest effort of taking advantage of the available information on its actual shape, a number of authors did continue looking at ways of arriving at grid-square heights which would to a greater degree and perhaps more appropriately be based on that information. One of such attempts is that of Radinovic (1985). He has used a "valley-filling" approach similar to the method of Mesinger (1977), but had the valleys filled according to independent one-dimensional height comparisons along each of the two axes. Thus, he obtains a more intensive orographic enhancement than that which would have resulted from a two-dimensional comparison.

Another such attempt is that of Pfaendtner et al. (1985). They introduce what they call "significant height" orography, designed "to emphasize the effect on the atmospheric flow of the tallest peaks in regions of rugged terrain". This Pfaendtner et al. do by averaging only the highest one third of their sub-grid scale values. In this way, as they have put it, "the deep valleys and canyons of the Andes Mountains, for example, are not allowed to over exaggerate the effective height of that range as they do (through their substantial contribution to the standard deviation of height) in the Wallace et al. scheme". The concept, Pfaendtner et al. point out, borrows from the approach used by oceanographers to determine "significant wave heights".

The approach of this group following perhaps the most obvious physical concept is that of "silhouette" mountains (Mintz, personal communication; Mesinger, 1985). As silhouette mountains, the average height of the silhouette which sub-grid scale terrain presents to the horizontal airflow is calculated. Average height of the silhouette can of course be calculated independently for the projections of the grid-square terrain onto the yz and onto the xz plane, and the two values obtained can subsequently be averaged; or, if this is felt justified by the difference schemes of the model, a more complex averaging scheme can be used (e.g., Mesinger et al., 1987). Silhouette mountains have been used in an analysis model by Bergporsson (WMO, 1983), and they have been implemented in an operational spectral model at the U.S. National Meteorological Center (NMC; for test results see Caplan, 1985).

As referred here as well as in other papers in these volumes, many tests have been made on the impact of various enhanced orographies as compared to control results obtained using (grid-square) mean orography. In experiments with no gravity wave drag parameterization, these tests have generally shown that a substantial reduction of the systematic model errors can be achieved by the use of enhanced orographies. Results, of course, are resolution dependent, since more and more of the actual orography is resolved as the resolution is increased. On the other hand, tests of the impact of the differences among various enhanced orographies may not have been so numerous, and, because of reduced sensitivity, are more difficult to interpret. The situation is further complicated by the fact that the reduction in systematic errors that can be achieved by the parameterization of gravity wave drag, without an enhancement of orography, is similar to that achieved by various enhanced orographies. Moreover, as noted by Jarraud et al. (1986b), a third physical process, sub-grid scale stress due to mountains, acts also largely in the same direction as it primarily reduces overall westerly flow in middle latitudes. Thus, a danger exists (e.g., Wallace et al., 1983; Simmons, 1985), that by optimizing results for longer forecast ranges and climate simulations, orographic representation may be tuned to compensate for deficiences (or absence) of another of these mechanisms. In addition to sophisticated experimentation, therefore, it seems advisable to seek guidance also from various physical as well as simple model considerations.

Regarding the definition of model mountains, an interesting attempt from that point of view is that of Chouinard et al. (1986). In one set of their experiments, the envelope used for orographic enhancement was based on the orographic variance in those sub-grid scales only for which they have considered the effective Rossby number likely to be less than unity. The remainder of the sub-grid scale variance Chouinard et al. have used to construct gravity wave launching heights needed by the gravity wave drag parameterization scheme.

Of course, adequate treatment of the blocking effect of sub-grid scale orography should also include anisotropic influences due to dominant orientation of mountain ranges in each grid-square (e.g., Sadourny, 1985). While, to the authors' knowledge, no attempt of this kind has as yet been made, it would seem that the construction of an ad-hoc technique accounting for these influences should not cause great problems.

As to the reduction of orographic forcing at the smallest resolvable scales, perhaps most of the authors as a final step did apply to their terrain fields a smoothing procedure of one kind or another (e.g., regarding grid point models, Wallace et al., 1983, p. 698; Pfaendtner et al., 1985, p. 333). It appears generally believed that, because of the inadequate performance of difference schemes at the smallest scales, orographic forcing at these scales is likely to do more harm than good. The understanding of this problem is, however, clearly inadequate; applicability of any experimental results should of course be considered limited to specific schemes used by the model. One may also note that the smoothing practice is not consistent with ample forcing being at the same time applied at the smallest scales through various physical parameterization schemes. In this connection an effort of Mesinger (1985; see also Mesinger et al., 1987) is perhaps worth mentioning: for reasons other than those just given it was felt desirable to have terrain elevations the same for groups of four neighboring grid points. The four-point elevation values have however been derived using a "silhouette" averaging procedure, and thus have perhaps not much affected the barrier effect of the original single-point values. In this way, as a byproduct, orographic forcing has been removed in the "4 to 2 grid intervals" range, while at the same time hopefully minimizing undesirable effects of smoothing. With ever increasing availability of computing power, perhaps one should expect more efforts of this type in the future.

Specific problems related to the representation of mountains and/or possible need for some kind of a smoothing step arise in spectral models, in connection with orographic "ripples" over oceans, and spreading of the orography. The latter is associated with raising of coastal regions in the vicinity of high mountains, which is undesirable in view of increased local discrepancies between model terrain heights and elevations of surface observations (Simmons and Jarraud, 1984). Production of enhanced orographies for spectral models typicaly includes techniques aimed at reducing these problems and/or their effects; Gaussian filtering, repeated setting of negative heights to zero followed by spectral fitting, or adding of the envelope increment over land points only (Simmons and Jarraud, 1984; Jarraud et al., 1986a, 1986b; Chouinard et al., 1986).

Finally, difficulties arise in defining orography for fine mesh models which are to be run using initial/boundary conditions derived from coarser mesh models. Obviously, pragmatic approaches have to be used to avoid/minimize the mismatch between the two orographies. Some will be briefly mentioned in Section 3 of our lecture.

2. ENVELOPE VS. SILHOUETTE MOUNTAINS

With the beneficial impact of an enhanced orography perhaps established, and/or expected on the basis of theoretical and idealized modeling studies (e.g., Pierrehumbert, 1984; Pierrehumbert and Wyman, 1985) the questions of why one should make one or another choice of the enhancement technique and if and/or how much this choice matters come to the forefront. A comparison of mountains resulting from various techniques is therefore of interest. We shall consider from that point of view the difference between envelope and silhouette mountains.

Some insight can be gained from elementary considerations, to which we are led by a remark of Manabe (personal communication) reporting about his envelope Tibet apparently having awkward fence-like additions along its edges. This, of course, could be a result of the large standard deviation at the edges of a plateau-like mountain. In a milder form, the effect has been noted also by Chouinard et al. (1986, p. 98) who in comparing their (2 sigma) envelope against "standard" mountains point out that the envelope enhancement around plateaus generally "follows the edges where the gradient is steepest".

The "minimum" elementary mountain which can be used to demonstrate the problem is a one-dimensional two-grid-interval mountain, as shown in the upper panel of Fig. 1. The case illustrated by the figure is that of the edge of the "actual mountain" being located at a grid point, that is, in the middle of a grid-mesh. In that situation, $\sqrt{2}$ sigma envelope mountain has the shape as shown in the middle panel of the figure; obviously an undesirable result, in view of the fence-like extensions not being justified by any sub-grid scale detail of the actual mountain. In addition, both the envelope and the (most straightforward) silhouette mountain, shown in the lower panel, suffer from a broadening problem. In coastal regions, and in grid point models, this can of course be easily avoided by defining model points to be sea points with zero elevation in case more than a half of the grid-square is occupied by sea (e.g., Mesinger et al., 1987). In spectral models, as mentioned already, adding the enhancement increment over land points only was found helpful (Simmons and Jarraud, 1984).

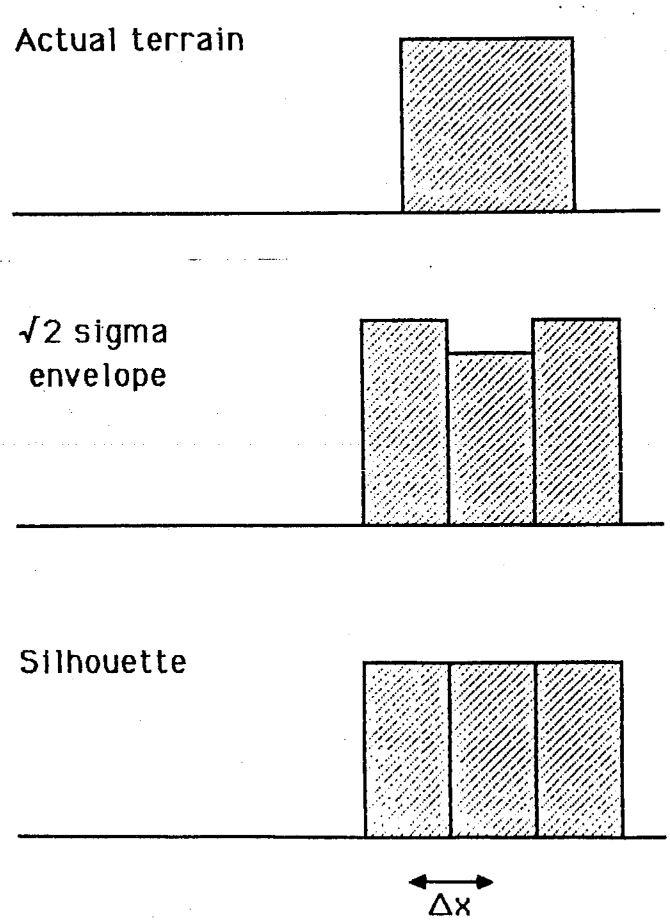

Fig. 1. An example of the "√2 sigma envelope" and the "silhouette" mountain.

The considered "edge enhancement" of a vertical edge of a mountain, of course, depends on the location of the edge within the grid-mesh, and disappears when the edge is located exactly at the boundary of the grid-mesh. Furthermore, it is dependent on the multiplicative constant of the envelope. Thus, one might wonder if it is possible to optimize the multiplicative constant so that for a step-like mountain the envelope does not surpase the elevation of the step for any location of the edge within the grid-mesh. A simple analysis shows that this is, however, not possible, as no matter how small the multiplicative constant is, the envelope edge will surpass the elevation of the step when the edge is located sufficiently close to the boundary of the grid-mesh.

Of more interest of course are differences and possible problems in the representation of existing mountains. Calculations we have recently made of the ($\sqrt{2}$ sigma) envelope and the silhouette mountains, on the same global grid, may serve to illustrate the situation. The mentioned 10' x 10' U.S. Navy data were used. Calculations have been performed in the course of preparations of an enhanced terrain field for the NMC R40 medium-range operational model (Gerrity, 1985). They have been done on the Gaussian longitude-latitude grid of the model, with 128 points and 102 points in the east-west and the north-south directions, respectively. Calculations of the two fields included the rule, mentioned already, of defining grid points to be sea points with zero elevation if more than half of the area of the grid-square was covered by sea.

In order to check on the possible appearance of features reminiscent of the preceding expample, as well as to compare the enhancements resulting from the two techniques, we have had a look at profiles obtained across major mountain ranges. The maximum difference we have noticed between the two terrains occurred at an east-west profile across Andes. This profile, at the latitude of 13.2°S, is shown in the upper panel of Fig. 2. The profile shown, at

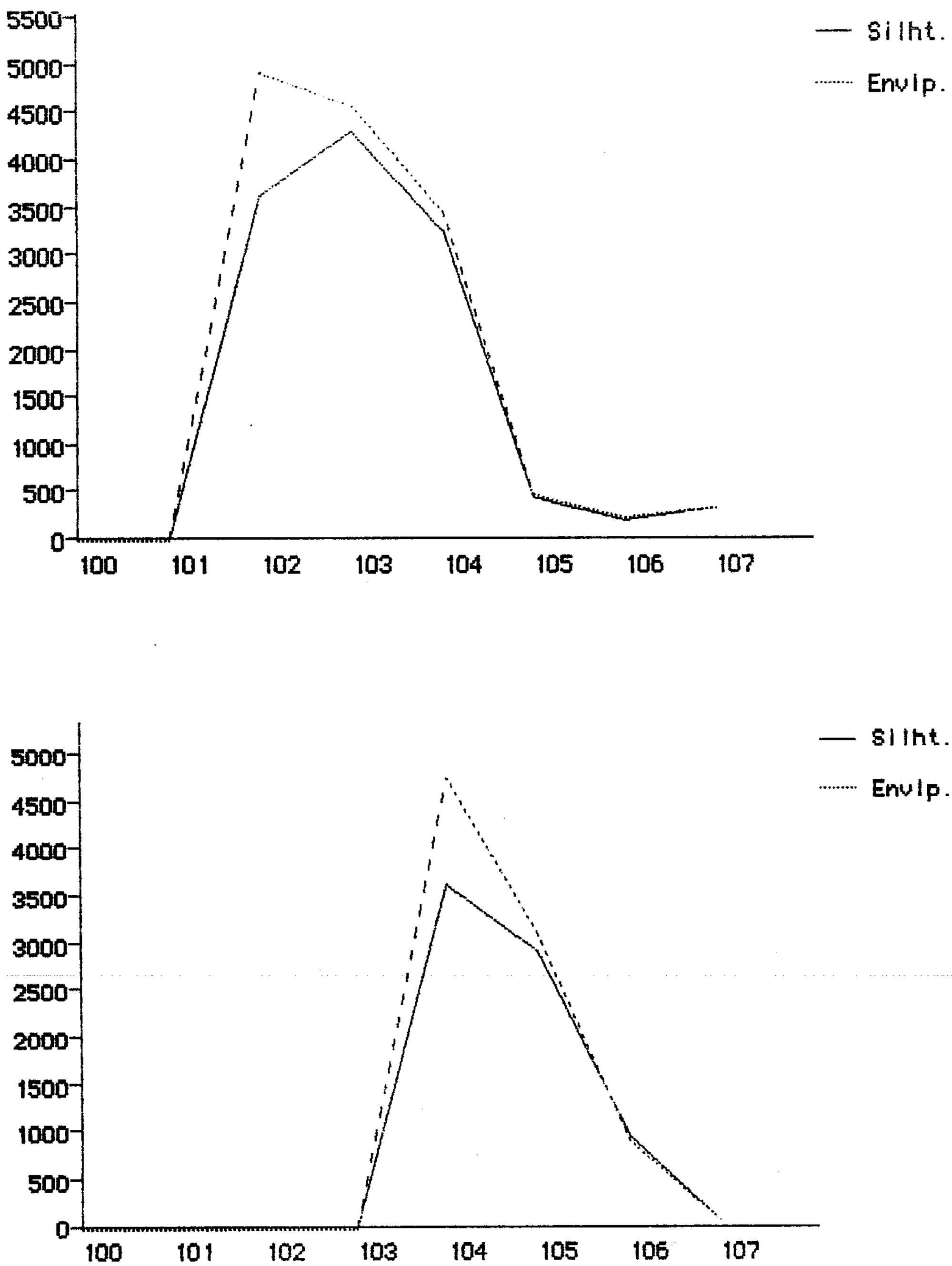

Fig. 2. Profiles of the √2 sigma envelope and the silhouette terrain across the the Andes, at 13.2°S (upper panel) and at 30.0°S (lower panel), calculated on a Gaussian 128 x 102 point grid. The numbers on the abscissa are grid line indices, increasing eastward from Greenwich; the numbers on the ordinate show elevation in meters.

the same time, is the east-west profile of both of the terrains with the greatest elevation value of the continental point situated next to sea level. At that first continental point the envelope terrain is 1287 m higher than the silhouette terrain, a feature clearly reminiscent of the edge enhancement problem. At other points of this profile, the two terrains are seen not to be that much different.

The situation is similar at the profile of the next-to-greatest sea level to first continental point elevation difference across Andes, at 30.0°S. At that profile, shown in the lower panel of Fig. 2, the envelope terrain is at the first continental point 1111 m higher than the silhouette terrain. Since the second continental point is this time substantially lower than the first, this difference is now at the same time the difference between the maximum heights of the two profiles.

Finally, in Fig. 3 we show north-south profiles across Himalayas. The profiles shown are those at the point of maximum elevation, which occurred at the same grid point in the two terrains. Some edge enhancement is again seen, but not to the extent it was present in the profiles of the preceding figure. The difference in the elevation of the two peaks shown amounts to 434 m.

Also at other major mountain ranges peaks of the $\sqrt{2}$ sigma envelope terrain were found to be generally higher than those of the silhouette terrain. Maximum elevations of some of the major mountain ranges are shown in Table 1. Of the ranges entered, the Rocky Mountains are the only range having a silhouette peak higher than the $\sqrt{2}$ sigma envelope peak.

For a two-dimensional view of the two terrains we first show in Fig . 4 our $\sqrt{2}$ sigma envelope and the silhouette Andes. The accuracy of the values shown is

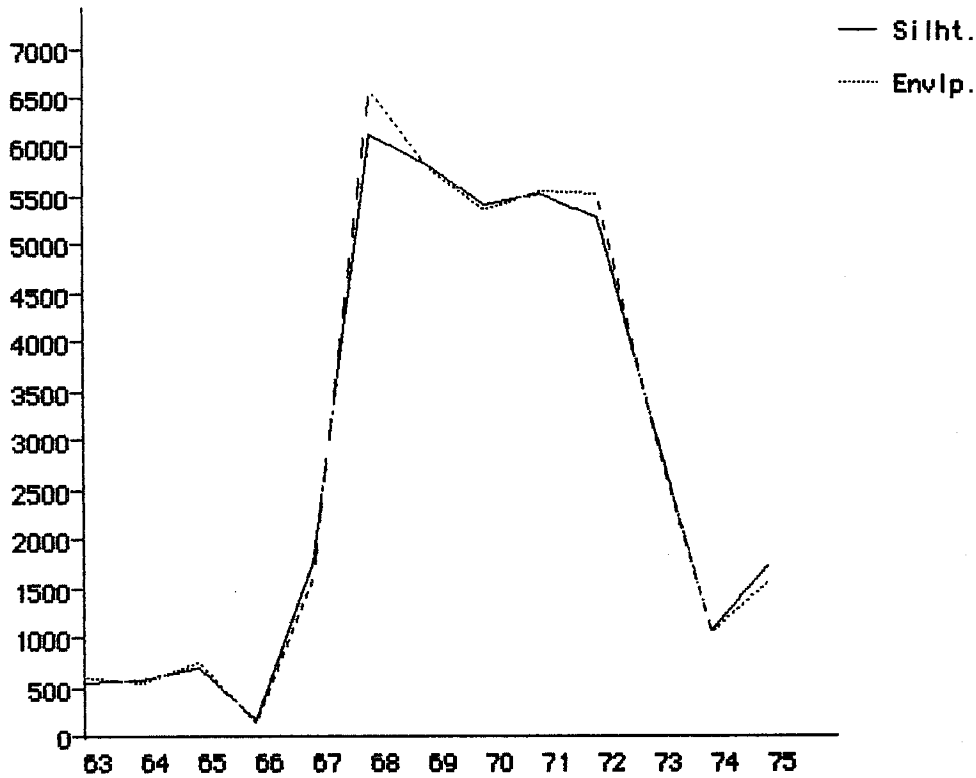

Fig. 3. Profiles of the √2 sigma envelope and the silhouette terrain across the Himalayas, at 84.4°E longitude, calculated on a Gaussian 128 x 102 point grid. The numbers on abscissa are grid line indices, increasing northward from the South Pole; the numbers on the ordinate show elevation in meters.

Table 1. Maximum elevation of some of the major mountain ranges of the √2 sigma envelope and the silhouette terrain, calculated on a Gaussian 128 x 102 point grid.

	√2 sigma envelope	Silhouette
Himalayas	6620 m	6186 m
Andes	5268 m	4847 m
Antarctic	4142 m	4057 m
Rocky Mountains	3409 m	3445 m
Greenland	3209 m	3167 m
Alps	2747 m	2600 m

to some extent degraded by the space interpolation to a 2.5×2.5 deg longitude×latitude grid done within the contouring routine. The associated smoothing effect should of course be expected to affect the smallest scales of the two terrains the most.

Fig. 4. √2 sigma envelope (left hand panel) and the silhouette (right hand panel) Andes, calculated on a Gaussian 128 x 102 point grid. Contour interval is 500 m, and numbers printed show elevation in meters.

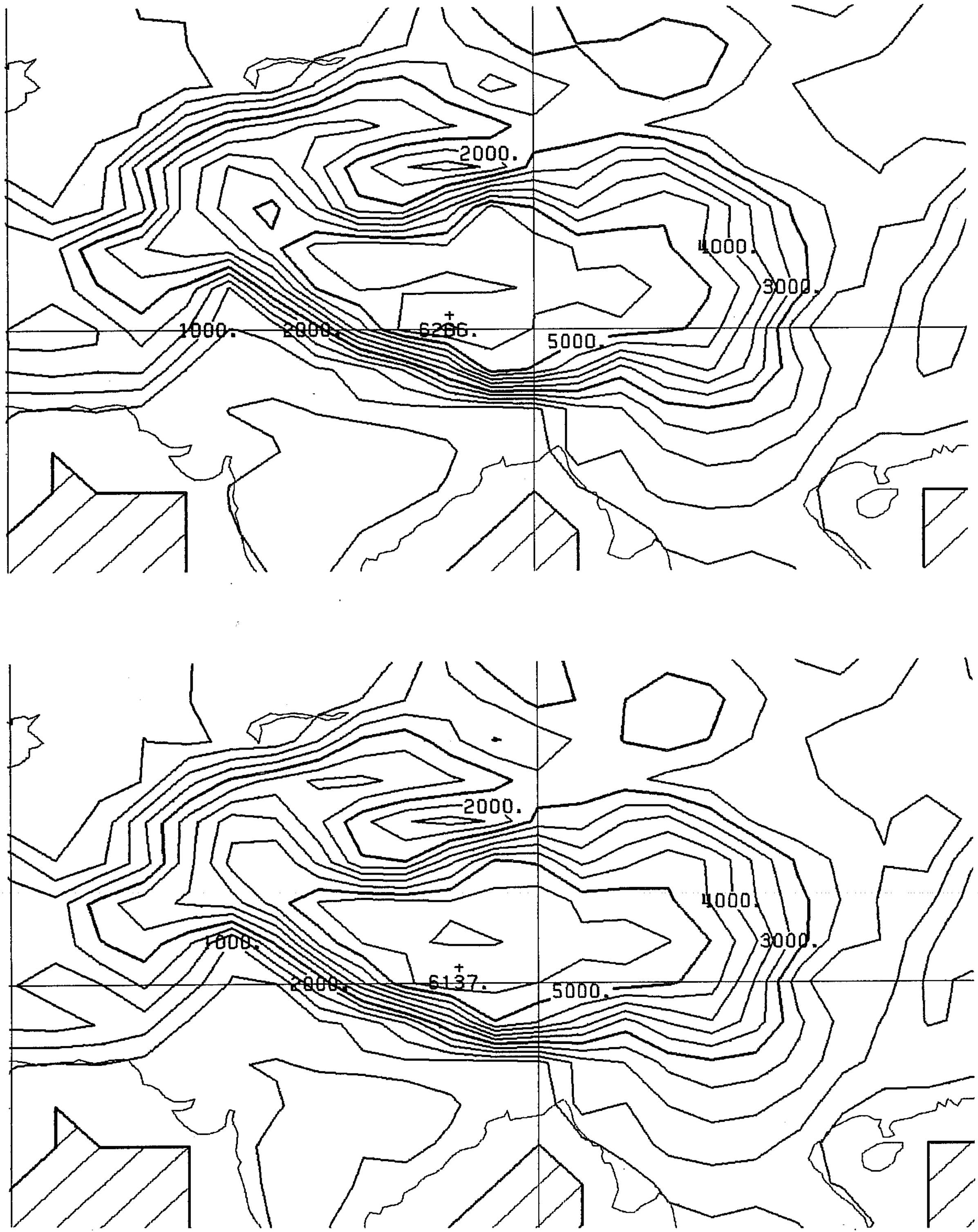

Fig. 5. √2 sigma envelope (upper panel) and the silhouette (lower panel) Himalayas, calculated on a Gaussian 128 x 102 point grid. Contour interval is 500 m, and numbers printed show elevation in meters.

The differences between the two terrains being on a relatively small space scale, one may not be surprised to find the maps rather similar. Nevertheless, a greater steepness of the envelope Andes, shown in the left hand panel, can be noticed. For example, seven 500 m interval contours packed closely together can be seen crossing 30°S latitude on the envelope map, and only five of them in about the same position on the silhouette map. The maximum elevations of the two Andes have not been reduced much by the interpolation within the contouring routine compared to the original values seen in Table 1; less than 100 m in both cases. Accordingly, the fairly large difference between the perhaps effective barrier height of the two Andes, presumably resulting from an unrealistic enhancement of the envelope version, has remained visible in the maps. This tendency for an exaggerated height of the ($\sqrt{2}$ sigma) envelope Andes may have played a role in the mixed results obtained for the Southern Hemisphere in testing the sensitivity to envelope orography (e.g., Simmons, 1985; Jarraud et al., 1986b).

The edge enhancement of the envelope Himalayas, on the other hand, has been very much removed by the interpolation routine. Silhouette Himalayas, predictably, did not change much. As a result, the two Himalayas, Fig. 5, are remarkably similar to each other.

An additional matter of interest is the effect of smoothing that may be performed on terrain fields such as those considered here, since, as mentioned already, frequently a smoothing operation is indeed performed as a final step in preparation of model terrain. Specifically, a smoothing operation in the form of spectal fitting of the Gaussian grid data is being done as the last step in preparations of terrain fields for spectral models (e.g., Jarraud et al., 1986a, 1986b; Chouinard et al., 1986) and has also been done to obtain the R40 terrain using each of the two fields considered here. The fitting results in reduction of the maximum elevations of mountain ranges, and also in the appearance of the already mentioned ocean ripples (e.g., Jarraud and Baede, 1985, p. 23). As reflected on maps of the type shown in Figs. 4 and 5, the R40 fitting has resulted in the reduction of the maximum elevations of our silhouette Andes and Rockies by 122 and by 177 m, respectively; and in ripples with the maximum depth west of Andes, of 351 m.

3. MOUNTAINS OF SOME OPERATIONAL MODELS

For an overview of present preferences, we shall in this section list methods that have been used to produce terrain fields of a number of operational models. In our list we shall include centers participating in the WMO/CAS Data Study and Intercomparison Project (e.g., Lange and Tokkola, 1986).

At the French Meteorological Service, two models are run operationally, "Emeraude", a spectral hemispheric T79 model, and "Peridot", a nested limited area model. Mean orography is used for Emeraude, and a $\sqrt{2}$ sigma envelope for Peridot. The reasons for not using an enhanced orography for Emeraude are (i) that forecasts are done only up to 96 h, and (ii) a desire to minimize the number of Peridot grid points underneath the Emeraude ground surface, for better nesting of the Peridot model (J.-F. Geleyn, personal communication).

At the Weather Service of the Federal Republic of Germany, a system is used allowing for a different height of mountains at velocity points, to be used for blocking of the flow, and at mass points, to be used for surface pressure and other surface variables. Envelope orography (1 sigma) is used at velocity points, and a mean orography at surface pressure points (Edelmann, 1985).

At the Japan Meteorological Agency, a spectral hemispheric T42 model is run operationally, with standard orography, modified to smooth small scale features. However, it was found that (1 sigma) envelope orography "improves the daily forecasts" and that "it remarkably reduces the systematic forecast errors of zonal mean and planetary wave fields" (Iwasaki and Sumi, 1986). In view of correspondence between patterns of errors and improvements achieved, Iwasaki and Sumi conclude, furthermore, that "we may expect that the larger enhancement of orography would result in further improvement".

At the U.K. Meteorological Office, a global and a limited area fine mesh model are run operationally. For the global model, a mean orography is used. Limited area orography used at the beginning of the fine mesh assimilation cycle is derived by a bilinear interpolation of the global model orography. Subsequently, during the first 3 h of the assimilation cycle, the orography at selected grid points is gradually adjusted (grown) to a fine mesh grid-square value (Davies, personal communication).

At the U.S. National Meteorological Center, as already mentioned, in the spectral medium-range forecasting model a version of silhouette terrain is used. The terrain used operationally in the regional (grid point) "Nested Grid Model" (Hoke et al., 1985) is derived from average terrain fields, converted to spectral form and reevaluated on the grids of the model.

Finally, at ECMWF, since the introduction of the T106 model on 1 May 1985, a one standard deviation envelope orography is used (Lange and Tokkola, 1986; papers by ECMWF authors in present volumes). Note that prior to that, a $\sqrt{2}$ standard deviation envelope was used for the T63 model.

4. RESPONSE OF VARIOUS GRIDS TO FORCING BY OROGRAPHY

Very recently (Dragosavac and Janjić, 1986) a possibility has been brought up that same topographic forcing on various horizontal grids may not result in the same response. Accordingly, the choice of a geometrical shape appropriate for a given terrain may not be independent of the choice of the horizontal staggering of the variables of the model.

Dragosavac and Janjić have considered the linearized shallow water equations

$$
\begin{aligned}
&\partial_t u + U\partial_x u + V\partial_y u - fv + g\partial_x h = -g\partial_x h_t, \\
&\partial_t v + U\partial_x v + V\partial_y v + fu + g\partial_y h = -g\partial_y h_t, \\
&\partial_t h + H\partial_x u + H\partial_y v + U\partial_x h + V\partial_y h = 0.
\end{aligned}
\tag{1}
$$

Here h_t is height of the topography, and other symbols have their customary meaning. They have allowed for topography of an arbitrary shape, assuming

$$
h_t = \mathrm{Re}\left[\hat{h}_t e^{i(kx+ly)}\right]. \tag{2}
$$

Dragosavac and Janjić have sought wave solutions of (1) in the usual form

$$
\begin{bmatrix} u \\ v \\ h \end{bmatrix} = \mathrm{Re}\left[\begin{bmatrix} \hat{u}(t) \\ \hat{v}(t) \\ \hat{h}(t) \end{bmatrix} e^{i(kx+ly)}\right].
$$

In addition to three solutions of the homogeneous system, they have obtained a stationary, particular solution, forced by topography,

$$
\begin{aligned}
\hat{u} &= \frac{(kU+lV)kg - ifgl}{-(kU+lV)^2 + f^2 + gH(k^2+l^2)}\,\hat{h}_t, \\
\hat{v} &= \frac{(kU+lV)lg + ifgk}{-(kU+lV)^2 + f^2 + gH(k^2+l^2)}\,\hat{h}_t, \\
\hat{h} &= \frac{-gH(k^2+l^2)}{-(kU+lV)^2 + f^2 + gH(k^2+l^2)}\,\hat{h}_t.
\end{aligned}
\tag{3}
$$

The first two of these equations, $(3)_1$ and $(3)_2$, can be rewritten in terms of the vorticity and divergence amplitudes; one obtains

$$\hat{Z} = \frac{-fg(k^2+l^2)}{-(kU+lV)^2 + f^2 + gH(k^2+l^2)} \hat{h}_t ,$$

$$\hat{D} = \frac{i(kU+lV)g(k^2+l^2)}{-(kU+lV)^2 + f^2 + gH(k^2+l^2)} \hat{h}_t . \qquad (4)$$

As has been done for a number of problems (see, for example, the review paper by Janjić and Mesinger, 1984), one can now repeat this excercise using the simplest second order schemes on each of the four (Arakawa) horizontal grids, A, B/E, C and D. This has been done by Dragosavac and Janjić; they have then compared the amplitudes of the resulting mountain-induced particular solution to those of the solution in the continuous case, $(3)_3$ and (4).

Results which Dragosavac and Janjić have obtained for the ratios of the (total) height amplitude of the two solutions, $(\hat{h}+\hat{h}_t)_A/(\hat{h}+\hat{h}_t)$, $(\hat{h}+\hat{h}_t)_B/(\hat{h}+\hat{h}_t)$, ..., for various grids, are shown in Fig. 6. For the diagrams displayed Dragosavac and Janjić have used the values $f = 10^{-4}\ s^{-1}$, $d = 200$ km, $U = 10\ m\ s^{-1}$, $V = 0$ and $H = 10^4$ m. Here d is the shortest distance between points in which the same variable is carried on each of the grids. Note that results are in each case shown for the largest scales only, those covering wave lengths of 8 grid distances and more. The most striking feature of the figure is perhaps the general magnitude of the error. Since there are convincing reasons against use of the non-staggered (A) grid, and the semi-staggered D grid (e.g., Janjić and Mesinger, 1984), of most interest are the results for grids B and C. For these grids within the considered wavenumber range height amplitude errors of up to and more than 20 % can be seen.

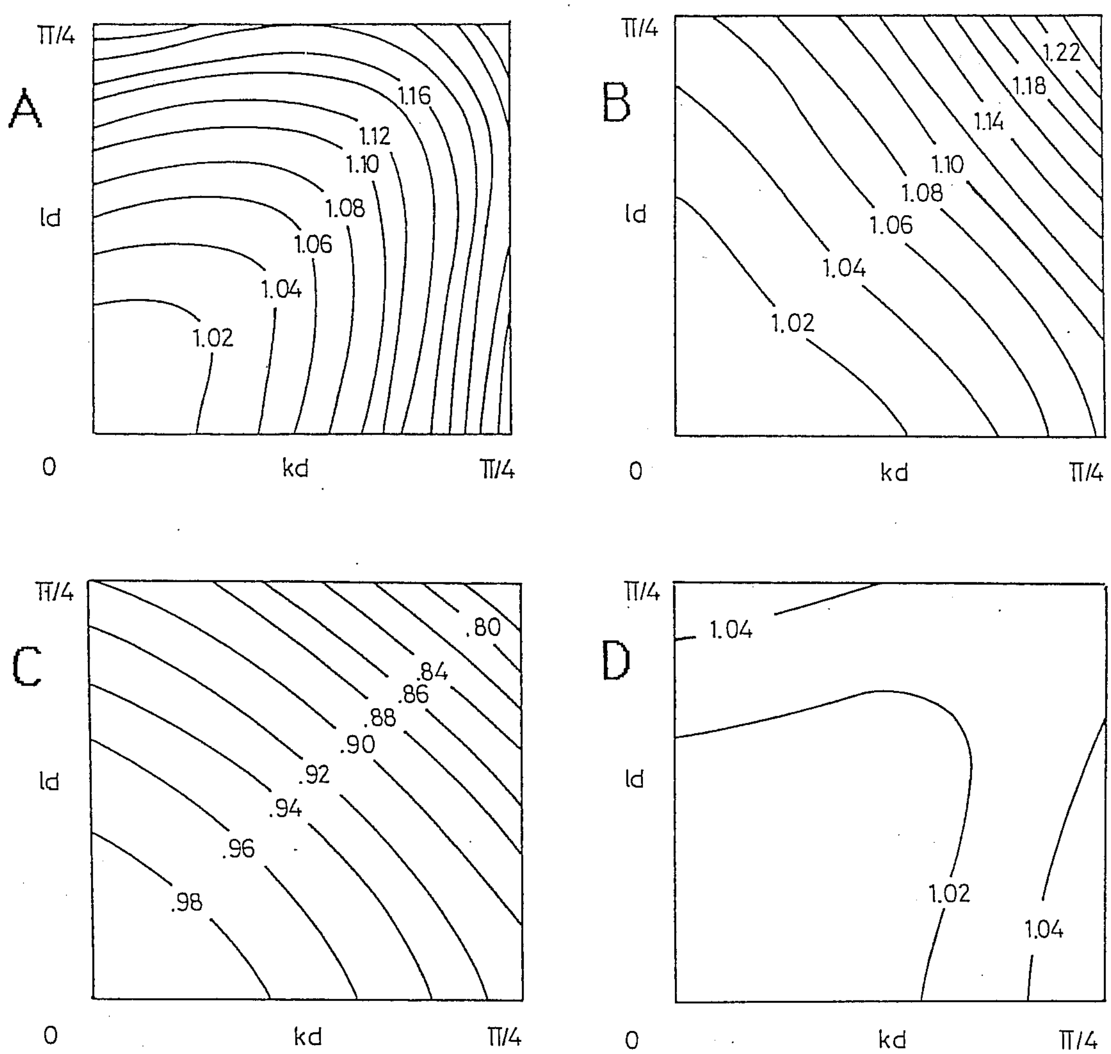

Fig. 6. Ratios of the total (shallow water depth plus terrain) height amplitude of the finite-difference and the continuous solution, for various horizontal grids. See text for further detail.

Ratios of the vorticity amplitudes, for grids C and D, are shown in the upper panel of Fig. 7 Differences between the ratios for these two grids are too small to plot. For grids A and B, the ratios in the considered wavenumber range are very close to unity, and are not shown.

Finally, ratios of the divergence amplitudes are shown in the lower panel of the figure. Differences between the divergence ratios for the four grids are too small to plot.

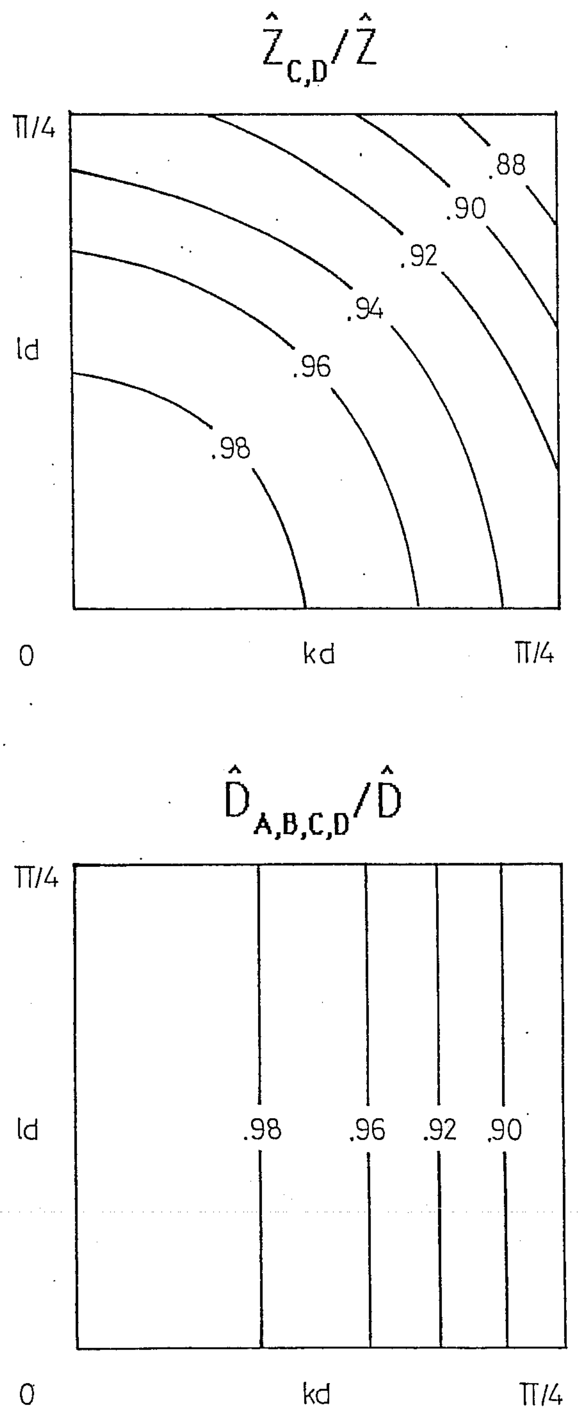

Fig. 7. Ratios of the vorticity amplitudes of the finite-difference and the continuous solution, for grids C and D, upper panel. Ratios of the divergence amplitudes of the finite-difference and the continuous solution, for any of the four grids, lower panel. See text for further detail.

The results summarized, obviously, can be considered simply as information on effects of a chosen horizontal grid on stationary solutions induced by given mountains.

An additional possibility, one that has typically been the main objective of analyses of this type in the case of other problems, is to use these results in forming judgment on the relative advantages of various grids for atmospheric modeling. From this point of view, and considering for reasons already mentioned the B and C grids only, the study of Dragosavac and Janjić would appear to favor the B grid. As pointed out, the divergence amplitude error is about the same on all of the grids. The height amplitude error on B and on the C grid are of similar magnitude, with the wavenumber region of accurate solutions (say, of errors less than 2 or less than 4 %) on B grid being somewhat greater. Finally, a rather large vorticity amplitude error appears on the C grid, while at the same time a very accurate solution for vorticity is obtained on the B grid.

A point of some interest may be that of the B grid amplitudes being each affected in a different way - one being magnified, one affected very little, and one being reduced. On the C grid, on the other hand, all of the amplitudes are seen to be reduced, to a degree which is not too different among the three amplitudes.

5. CONCLUDING COMMENTS

While the perhaps resolution dependent need (Dell'Osso, 1984) for some kind of mountain enhancement compared to grid-square elevation means would appear to have been clearly established, and is also to be expected on the basis of physical reasoning, exactly what kind of enhancement is appropriate and to what extent the choice matters are unsettled questions. In this connection, our discussion and comparison of the $\sqrt{2}$ standard deviation envelope vs. silhouette mountains has emphasized a number of points. One is the undesirable edge enhancement effect of the envelope technique for steep mountain sides. As another, a generally highly similar intensity of enhancement resulting from the

$\sqrt{2}$ sigma envelope and the silhouette technique could be stressed.

However, regarding grid point models, according to the study of Dragosavac and Janjić the effect of given model mountains should not be independent of the choice of horizontal grid. To the extent the linear framework of their model is applicable to atmospheric flows of interest, this may be suggestive of an approach in constructing model mountains which would take into account properties of the grid as well as of the difference schemes of the model. For example, for a given choice of the horizontal grid and schemes of the model, one could try to modify amplitudes of wavenumber components of actual mountains so as to achieve in some sense an optimum response of the finite-difference solution to forcing by model orography.

Acknowledgments. Part of the work reported here has been done while the first of the authors (FM) enjoyed the hospitality of the U.S. National Meteorological Center. It is a pleasure for him to acknowledge the support and encouragement of Dr. William Bonner, Director; and of Dr. John A. Brown, Jr., Head of the Development Division of the Center, in which the work has been performed.

We are both grateful to Dr. Jordan Alpert, of the Development Division of the U.S. National Meteorological Center, who has produced for us plots of the silhouette and envelope mountains used for Figs. 4 and 5 of this manuscript.

References

Berkofsky, L., and E. A. Bertoni, 1955: Mean topographic charts for the entire earth. *Bull. Amer. Meteor. Soc.*, **36**, 350-353.

Bleck, R., 1976: Numerical modeling of terrain-induced mesoscale cyclogenesis. *Ann. Meteor. (Neue Folge)*, No. 11, 147-150.

Bleck, R., 1977: Numerical simulation of lee cyclognesis in the Gulf of Genoa. *Mon. Wea. Rev.*, **105**, 428-445.

Caplan, P., 1985: Recent changes in the new medium-range forecast (MRF) model. Office Note 307, National Meteorological Center, NWS/NOAA, 15 pp.

Charney, J. G., 1966: Some remaining problems in numerical weather prediction. In *Advances in Numerical Weather Prediction*, Travelers Research Center, Inc., Hartford, CN, 61-70.

Chouinard, C.,M. Béland and N. McFarlane, 1986: A simple gravity wave drag parametrization for use in medium-range weather forecast models. *Atmosphere-Ocean*, **24**, 91-110.

Dell'Osso, L., 1984: High-resolution experiments with the ECMWF model: A case study. *Mon. Wea. Rev.*, **112**, 1853-1883.

Dragosavac, M., and Z. I. Janjic, 1986: Stationary solutions of linearized shallow water equations induced by topography on various grids. *Mon. Wea. Rev.* (in press).

Edelmann, W., 1985: The third approach of handling the orography in models using the p-coordinate. *LAM Newsletter*, No. 11, 107-116. [Available from Deutscher Wetterdienst, Postfach 185, 6050 Offenbach am Main].

Egger, J., 1972: Numerical experiments on the cyclogenesis in the Gulf of Genoa. *Beitr. Phys. Atmos.*, **45**, 320-346.

Gates, W. L., and A. B. Nelson, 1975: A new (revised) tabulation of the Scripps topography on a 1° global grid. Part I: Terrain heights. The RAND Corporation, Santa Monica, CA, 132 pp.

Gerrity, J. P., 1985: NMC/GFDL Medium-Range Prediction Model. *Res. Activ. in Atmos. and Oceanic Modelling*, Rep. No. 8, WMO, Geneva, 5.4.

Hills, T., 1979: Sensitivity of numerical models to mountain representation. *Workshop on Mountains and Numerical Weather Prediction*, ECMWF, Reading, U.K., 139-161.

Hoke, J. E., N. A. Phillips, G. J. DiMego and D. G. Deaven, 1985: NMC's regional analysis and forecast system -- results from the first year of daily, real-time forecasting. *Preprints, Seventh Conf. Numerical Weather Prediction*, Montreal, Amer. Meteor. Soc., 444-451.

Iwasaki, T., and A. Sumi, 1986: Impact of envelope orography on JMA's hemispheric NWP forecasts for winter circulation. *J. Meteor. Soc. Japan*, **64**, 245-258.

Janjic, Z. I., and F. Mesinger, 1984: Finite-difference methods for the shallow water equations on various horizontal grids. *Numerical Methods for Weather Prediction*, Seminar 1983, Vol. 1, ECMWF, Reading, U.K., 29-101.

Jarraud, M., and A. P. M. Baede, 1985: The use of spectral techniques in numerical weather prediction. *Large-Scale Computations in Fluid Mechanics*, Part 2, Lect. Appl. Math., Vol. 22, Amer. Math. Soc., 1-41.

Jarraud, M., A. J. Simmons and M. Kanamitsu, 1986a: Impact of an envelope orography in the ECMWF model. *Physical Parameterization for Numerical Models of the Atmosphere*, Seminar 1985, Vol. 2, ECMWF, Reading, U.K., 199-249.

Jarraud, M., A. J. Simmons and M. Kanamitsu, 1986b: Sensitivity of medium-range weather forecasts to the use of an envelope orography. Tech. Rep. No. 56, ECMWF, Reading, U.K., 83 pp.

Lange, A., and E. Tokkola, 1986: Results of the WMO/CAS NWP Data Study and Intercomparison Project for forecasts for the Northern Hemisphere in 1985. World Weather Watch Tech Rep. No. 3, WMO, Geneva, 78 pp.

Mesinger, F., 1977: Forward-backward scheme, and its use in a limited area model. *Contrib. Atmos. Phys.*, **50**, 200-210.

Mesinger, F., 1985: The sigma system problem. *Preprints, Seventh Conf. Numerical Weather Prediction*, Montreal, Amer. Meteor. Soc., 340-347.

Mesinger, F., Z. I. Janjic, S. Nickovic, D. Gavrilov and D. G. Deaven, 1987: The step-mountain coordinate: model description, and performance for cases of Alpine lee cyclogenesis and for a case of an Appalachian redevelopment. Submitted to *Mon. Wea. Rev.*

Mesinger, F., and R. F. Strickler, 1982: Effects of mountains on Genoa cyclogeneses. *J. Meteor. Soc. Japan*, **60**, 326-338.

Pfaendtner, J., R. Balgovind, D. Duffy, H. M. Helfand and E. Kalnay, 1985: Response of the GLA fourth order model to changes in horizontal resolution and terrain heights. *Preprints, Seventh Conf. Numerical Weather Prediction*, Montreal, Amer. Meteor. Soc., 333-335.

Pierrehumbert, R. T., 1984: Linear results on the barrier effects of mesoscale mountains. *J. Atmos. Sci.*, **41**, 1356-1367.

Pierrehumbert, R. T., and B. Wyman, 1985: Upstream effects of mesoscale mountains. *J. Atmos. Sci.*, **42**, 977-1003.

Radinovic, Dj., 1985: A valley filled orographic representation ~~in numerical~~ weather forecast models. Tech. Memo. No. 99, ECMWF, Reading, U.K., 58 pp.

Sadourny, R., 1985: Midlatitude systematic errors in numerical models. GARP Special Rep., No. 43, WMO, Geneva, III, 156-174.

Simmons, A. J., 1985: The treatment of orography in numerical weather prediction models. GARP Special Rep., No. 43, WMO, Geneva, II, 57-77.

Simmons, A. J., and M. Jarraud, 1984: The design and performance of the new ECMWF operational model. *Numerical Methods for Weather Prediction*, Seminar 1983, Vol. 2, ECMWF, Reading, U.K., 113-164.

Tibaldi, S., 1986: Envelope orography and maintenance of quasi-stationary waves in the ECMWF model. *Advances in Geophysics*, (in press).

Wallace, J. M., S. Tibaldi and A. J. Simmons, 1983: Reduction of systematic forecast errors in the ECMWF model through the introduction of an envelope orography. *Quart. J. Roy. Meteor. Soc.*, **109**, 683-717.

WMO, 1983: Numerical Weather Prediction Progress Report for 1982. WMO, Geneva, 61-64.