TECHNICAL MEMORANDUM

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Extensions to AF 82 GRIB (Gridded Binary)

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1. INTRODUCTION

The initial specification for the WMO bit-oriented code GRIB (AF 82 - Gridded Binary) envisaged that extensions would be required and agreed as extended needs are identified. The purpose of this document is to define conventions and standards to be used at ECMWF. These conventions and standards will be used until such time as extensions to GRIB are approved by WMO. They thus form a suggested set of extensions which could provide input to whatever mechanism is chosen by WMO for defining the full GRIB code.

2. REQUIREMENTS

ECMWF wish to use GRIB to represent:

- global fields containing full grid description information;
- fields of data as spherical harmonic coefficients;
- fields of data on polar stereographic grids;
- fields of data on regular or Gausian latitude/longitude grids;
- sub-areas of standard grids;
- fields of mean values;
- fields of the differences between two sets of values.

Experience with coding and decoding software indicates the need to enhance the block length descriptors to indicate the amount of zero fill used (particularly in the binary data block) to achieve an even number of octets. To preserve the generality of decoding software it is necessary to ensure that the starting point and the extent (length) of the packed binary values be available.

Extensions to AF 82 GRIB to meet these requirements are presented below.

3. THE PRODUCT DESCRIPTION BLOCK

Block 1 (Product Description Block) has been sufficiently defined for pilot applications, but requires some enhancement.

Reference is made to code table 0291 for indicator of parameter. This table contains details of units, etc., which are not suitable for use with GRIB.

Reference is made also to code table 4252 (unit of time range), with the comment that this table is to be updated later. It is suggested that code tables suitable for character codes may not necessarily be suitable for use with binary data. Furthermore, the compilation of special code tables for GRIB would enable a clear, concise definition of AF 82 GRIB, without the need to cross refer to other code tables. This process would be completely in keeping with the current inclusion of table 1 and table 2 within the GRIB definition. It is thus suggested that tables be compiled, to provide similar information to that contained in code table 0291 and code table 4252, and that other tables be included as necessary.

The following is a suggested revised specification of octets contents for block 1.

<u>Block 1 (Product definition block)</u>

Octet Number	Contents	
1 - 4 GRIB	(Coded CCITT-ITA No. 5)	
5	Identification of Centre	
6	Model identification	
7	Grid definition	
8	Flag (see code table 1)	
9	Indicator of parameter (see code table 2)	
10	Indicator of type of level (see code table 3)	
11-12	Value of level(s) (see code table 3)	
13	Year of century)	
14	Month)	
15	Day)	Reference time of data
16	Hour)	
17	Minute)	
18	Indicator of unit of time (see code table 4)	
19	Time 1	
20	Time 2	
21	Time range flag (see code table 5)	
22-24	Reserved	

Notes

1) Octet 7 may be set to 255 to indicate a non-standard grid, in which case the grid will be defined in block 2.

2) Where octet 7 defines a standard grid, that grid may be defined in block 2 provided the flag in octet 8 indicates inclusion of block 2.

CODE TABLES RELATIVE TO BLOCK 1

TABLE 1

Octet no 8: Flag indication relative to Blocks 2 and 3

Code	Block 3	Block 2
0	Omitted	Omitted
1	Omitted	Included
2	Included	Omitted
3	Included	Included

TABLE 2

Octet no 9: Indicator of Parameter

(To be developed - based on code table 0291 but:-
1) reference values are not appropriate;
2) units should be MKS;
3) some additions are required.)

TABLE 3

Octet no 10, 11 and 12: Fixed level or layers represented

	Octet No 10	Octet No 11	Octet No 12
0 99	special level (as code table 0491)	0	0
100	isobaric level	pressure in hPa	(2 octets)
101	layer between two isobaric levels	pressure of top (kPa)	pressure of bottom (kPa)
102	mean sea level	0	0
103	fixed height level (above mean sea level)	height (m) above MSL	(2 octets)
104	layer between two fixed hight levels	hight of top (hm above MSL)	height of bottom (hm aove MSL)
105	fixed height level (above ground)	height (m)	(2 octets)
106	layer between two fixed height levels	hight of top (hm above ground)	height of bottom (hm above ground)
107	sigma level	sigma value in 1/10000 (2 octets)	
108	layer between two sigma levels	sigma at top (1/100)	sigma at bottom (1/100)
109	hybrid level	level number	(2 octets)
110	layer between two hybrid levels	level no of top	level no of bottom

TABLE 4

Octet no 18: Unit of Time

Code	Meaning
0	Minute
1	Hour
2	Day
3	Month
4	Year
5	Decade (10 years)
6	Normal (30 years)
7	Century (100 years)
8 to 29	Reserved
30	Minute
31	5 minute
32	10 minute
33	15 minute
34	20 minute
35	30 minute
36 to 39	Reserved
40	Hour
41	2 hour
42	3 hour
44	4 hour
45	6 hour
46	8 hour
46	12 hour
47 to 49	Reserved
50	Day
51	2 day
52	3 day
53	4 day
54	5 day
55	6 day
56	7 day
57	10 day
58	30 day
59	Reserved
60	Month
61	2 month
62	3 month
63	4 month
65	6 month
66	8 month
67	10 month

Code	Meaning
70	Year
71	2 year
72	3 year
73	4 year
74	5 year
75 to 79	Reserved
80	Decade (10 years)
81	20 year
82	30 year
83 to 89	Reserved
90	Century (100 years)

TABLE 5

Octet no 21: Time Range Flag

Code	Meaning
0	Product valid for time T1
1	Average (time T1 to time T2)
2	Accumulation (time T1 to time T2)
3	Difference (time T2 - time T1)
	. . .
10	Time 1 occupies 2 octets (octets 19 and 20) Product valid for time T1.

Notes

1) Time T1 is obtained by adding time 1 to the reference time.

2) Time T2 is obtained by adding time 2 to the reference time.

3) For analysis products time 1 will be zero.

4) For forecast products time 1 will indicate the forecast period; the reference time will be the valid time of the initial data on which the forecast was based.

5) Provision is made to extend time 1 over two octets to assist with extended range forecasts.

4. THE GRID DESCRIPTION BLOCK

Block 2 (Grid Description Block) has not yet been defined. A proposed general definition for octets 1 to 6 follows, in which octet 6 would indicate the data representation type. It is further proposed that the remainder of block 2 should be dependent on the data representation type used.

Block 2 (Grid Description Block)

Octet Number	Contents
1 - 3	Length of grid description block (octets)
4	number of unused bits at end of block 2
5	Reserved
6	Data representation type (see table 6)
7 -32	Grid definition (according to data representation type - octet 6 above)
33-	Vertical co-ordinate parameters

Notes

1) The number of vertical co-ordinate parameters may be obtained by subtracting 32 from the length of the grid description block, and dividing the result by 4.

2) Vertical co-ordinate parameters are used in association with hybrid vertical co-ordinate systems. When used in conjunction with a surface pressure field and an appropriate methematical expression the vertical co-ordinate parameters may be used to interpret the hybrid vertical co-ordinate.

3) Each vertical co-ordinate parameter is represented in 4 octets, using the scheme for representing floating point numbers described in af 82.5.4.

4.1 Grid Definition - Regular Latitude/longitude grid

For a regular latitude/longitude grid the grid definition would take the form:-

Octet Number	Contents
7 - 8	Ni - no. of points along a latitude
9 -10	Nj - no. of points along a meridian
11-13	La - latitude of origin
14-16	Lo - longitude of origin
17	Area flag (see table 7)
18-20	La - latitude of extreme point
21-23	Lo - longitude of extreme point
24-25	Di - i direction increment
26-27	Dj - j direction increment
28	Scanning mode (flags - see table 8)
29-32	Reserved

Notes

1) In the following, bit positions within octets are referred to as bit 1 to bit 8, where bit 1 is the most significant and bit 8 is the least significant bit. Thus, an octet with only bit 8 set would have the integer value 1.

2) Latitude, longitude, and increments are deg x 1000.

3) Latitude values are limited to the range 0 - 90000; bit 1 is set to indicate South latitude.

4) Longitude values are limited to the range 0 - 180000; bit 1 is set to indicate West longitude.

5) Octet 28 (scanning mode) is composed as follows:-
   bit 8 - set to 1 to indicate West to East
   bit 7 - set to 1 to indicate South to North
   bit 6 - set to 1 to indicate the point scan first along meridional lines, then along lines of latitude.
   The full significance of numeric values for the scanning mode is indicated in table 8.

6) The latitude and longitude of the extreme point from the first data point should always be given.

7) Where items are not given the appropriate octet(s) should have all bits set to 1.

4.2 Grid Definition - Gaussian latitude/longitude grid

For a Gaussian latitude/longitude grid the grid definition would take the form:-

Octet Number	Contents
7 - 8	Ni - no. of points along a latitude
9 -10	Nj - no. of points along a meridian
11-13	La - latitude of origin
14-16	Lo - longitude of origin
17	Area flag (see table 7)
18-20	La - latitude of extreme point
21-23	Lo - longitude of extreme point
24-25	Di - i direction increment
26-27	N - number of latitude lines between a pole and the equator.
28	Scanning mode (flags - see table 8)
29-32	Reserved

Notes

1) Latitude, longitude, and increments are deg x 1000.

2) Latitude values are limited to the range 0 - 90000; bit 1 is set to indicate South latitude.

3) Longitude values are limited to the range 0 - 180000; bit 1 is set to indicate West longitude.

4) The number of latitude lines between a pole and the equator is used to establish the variable (Gaussian) spacing of the latitude lines; this value must always be given.

5) Octet 28 (scanning mode) is composed as follows:-
   bit 8 - set to 1 to indicate West to East
   bit 7 - set to 1 to indicate South to North
   bit 6 - set to 1 to indicate the points scan first along meridional lines, then along lines of latitude.

   The full significance of numeric values for the scanning mode is indicated in table 8.

6) The latitude and longitude of the extreme point from the first data point should always be given.

7) Where items are not given the appropriate octet(s) should have all bits set to 1.

4.3 Grid Description - Spherical Harmonic Coefficients

Spherical harmonic coefficients represent fields of data in spectral space (sometimes refered to as wave space) instead of grid point values. The variability of the coefficients can be considerably reduced if the real part of the (0,0) coefficient, representing the mean value, is treated separately. The remaining coefficients, subsequent to such separation, are reduced in range, and can be stored as differences from a minimum value using the same technique as is employed for grid point values in FM 82 GRIB. Since there are several ways of representing spherical harmonic coefficients, the Grid Description should be defined in general terms. The form suggested below, while being defined for one type of representation, is sufficiently general to be capable of extension to other representations.

Octet Number	Contents
7 - 8	J - pentagonal resolution parameter.
9 -10	K - pentagonal resolution parameter
11-12	M - pentagonal resolution parameter
13	Representation type (see table 9)
14	Representation mode (see table 10)
15-32	Reserved

Notes

1) The pentagonal representation of resolution is general. Some common truncations are special cases of the pentagonal one:

Triangular	M = J = K
Rhomboidal	K = J + M
Trapezoidal	K = J , K > M

2) The representation type (octet 13) indicates the method used to define the norm.

3) The representation mode (octet 14) indicates the order of the coefficients, whether global or hemispheric data are depicted, and the nature of the parameter stored (symmetric or antisymmetric).

CODE TABLES RELATIVE TO BLOCK 2

TABLE 6

Octet no 6: Data Representation Type

Code	Meaning
0	latitude/longitude grid
1	Mercator projection
2	Stereographic projection
3	Lambert conformal projection
4	Gaussian latitude/longitude grid
	. . .
50	Spherical harmonic coefficients

TABLE 7

Octet no 17: Area flag

Code	Meaning
1	Direction increments not given
2	Direction increments given

TABLE 8

Octet no 27: Grid Point Scanning Mode (flags)

Code	Meaning
0	Points scan from East to West Points scan from North to South Adjacent points on latitude circles are consecutive.
1	Points scan from West to East Points scan from North to South Adjacent points on latitude circles are consecutive.
2	Points scan from East to West Points scan from South to North Adjacent points on latitude circles are consecutive
3	Points scan from West to East Points scan from South to North Adjacent points on latitude circles are consecutive
4	Points scan from East to West Points scan from North to South Adjacent points on meridional circles are consecutive.
5	Points scan from West to East Points scan from North to South Adjacent points on meridional circles are consecutive.
6	Points scan from East to West Points scan from South to North Adjacent points on meridional circles are consecutive.
7	Points scan from West to East Points scan from South to North Adjacent points on meridional circles are consecutive.

TABLE 9

Octet no 13: Spectral Data Representation Type

Code	Meaning
1	"ECMWF" representation type (to be more fully defined)

TABLE 10

Octet no 14: Spectral Data Representation Mode

Code	Meaning
1	"ECMWF" representation mode (to be more fully defined)

5. THE BINARY DATA BLOCK

Block 4 (Binary Data Block) should enable a decoding programme to unpack the packed data values. To enable this to be done, it is necessary to indicate the type of binary data block, and the number of unused bits. Currently only 1 type of binary data block is defined; a modified form to accommodate spherical harmonic coefficients is suggested. It may be desirable to accommodate a few other alternative representations later. Since it is envisaged that there would be a maximum of 15 such alternatives, these two quantities require 4 bits each, and can be contained in the previously reserved octet 4.

Block 4 (Binary Data Block)

Octet Number	Contents
1 - 3	Length of binary data block (octets)
4	Flag (see table 11) (first 4 bits) Number of unused bits at end of block 4 (last 4 bits)
5 - 6	Scale factor (E)
7 -10	Reference value (minimum of packed values)
11	Number of bits containing each packed value
12-	Variable, depending on the flag value in octet 4
.	
.	
.	Zero fill to even number of octets

5.1 Grid Point Data

Octet Number	Contents
12-	Binary data.

5.2 Spherical Harmonic Coefficients

Octet Number	Contents
12-15	Real part of (0,0) coefficient (stored in the same manner as the reference value (octets 7-10)
16-	Binary data

Notes

1) Removal of the real (0,0) coefficient reduces considerably the variability of the coefficients, and results in better packing.

2) For some spherical harmonic representations the (0,0) coefficient represents the mean value of the parameter represented.

CODE TABLES RELATIVE TO BLOCK 4

TABLE 11

Octet no 4: Flag (first 4 bits only)

Code	Meaning
0	Grid point data - packed binary data begins at octet 12
1	Spherical harmonic coefficients - packed binary data begins at octet 16

Appendix I - AF 82 GRIB

This appendix contains the definition of AF 82 GRIB as issued by WMO and sanctioned by the Chairman of the CBS for experimental use.

B. Manual on codes

1. Global practices

1.1 New codes

Experimental Code

Members operating automated centres are invited to arrange and use the WMO bit-oriented code given below for exchange of processed information between automated centres as appropriate, as soon as it is practicable.

AF 82 GRIB (Gridded Binary) - PROCESSED DATA IN THE FORM OF GRID POINT VALUES EXPRESSED IN BINARY FORM

Code form :

Block 1	GRIB + 20 octets for product definition
Block 2	(Length + octets for grid description)
Block 3	(Length + octets for bit map)
Block 4	Length + octets for data values
Block 5	7777

Notes:

(1) GRIB is the name of the pure binary, computer-to-computer code for the exchange of processed data (in the form of grid point values).

(2) The GRIB coded analysis or forecast consist of a continuous bitstream made of a sequence of octets (1 octet = 8 bits).

(3) The octets of a GRIB message are grouped in blocks;

Block number	Name	Contents
1	Product definition block	Identification of the coded analysis or forecast
2	Grid description block (optional)	Grid geometry as necessary
3	Bit map block (optional)	The bit per grid point, placed in suitable sequence, indicates omission (bit 0) or inclusion (bit 1) of data at respective points
4	Binary data block	Data values
5	End block	Indicators of end of message.

(4) It will be noted that the GRIB code is totally worthless for visual data recognition. When printed out, a GRIB message appears as a garbled mixture of alphanumeric characters, except for the four ones at the beginning (= GRIB) and the four last ones (= 7777).

(5) The representaton of data by means of series of bits is independent of any particular machine representation and presents therefore a universal character.

(6) Block length is expressed in octets. Block 1 has a fixed length of 24 octets. Blocks 2, 3 and 4 have a variable length which is included in the first 3 octets of the block. Block 5 is four octets long.

REGULATIONS

AF 82.1

General

AF 82.1.1

The GRIB code shall be used for the exchange of grid point data expressed in binary form.

AF 82.1.2

The GRIB code shall always contain an even number of octets.

AF 82.1.3

The beginning and the end of the code are identified by 4 octets code according to the International Telegraphic Alphabet No. 5 to represent, respectively, the word "GRIB" and the indicator "7777". All other octets included in the code shall represent data in binary form.

AF 82.1.4

Each block included in the code shall always contain an even number of octets. Whenever the data included in one block can be fitted into an odd number of octets, or whenever the last octet required for that purpose is not completely filled with data, then the above rule shall be applied by, either adding one octet of zeros or by performing both of these operations, as necessary.

AF 82.2

Block 1 (Product definition block)

AF 82.2.1

Block 1 shall have a constant length of 24 octets, including the four character-coded octets used to include the name "GRIB".

AF 82.2.2

The eighth octet of the block shall be used to indicate the inclusion or the omission of Blocks 2 or 3 or of both of them.

AF 82.3

Block 2 (Grid description block)

82.3.1

The length of the block, in units of octets, shall be expressed in binary form over the group of the first three octets of the block, that is over 24 bits.

Note: In respect of Regulation AF 82.1.4 above, the least significant of these 24 bits is always zero.

AF 82.4

Block 3 (Bit map block)

AF 82.4.1

Regulation AF 82.3.1 applies.

AF 82.4.2

Octet numbers 5 and 6 shall be used to indicate that the bit map is either prederternined and not explicitely included or that the bit map follows.

AF 82.5

Block 4 (Data block)

AF 82.5.1

Regulation 82.3.1 applies.

AF 82.5.2

Data shall be coded using a minimum of bits in order to provide for the accuracy required by international agreement.

AF 82.5.3

Data shall be coded in the form of scaled deviations from a reference value.

Notes : (1) The reference value is normally the lowest value of the data set which is represented.

(2) The actual value Y is linked to the coded value X, to the reference value R and to the scale factor E by means of the following formula:

Y = R + X*2**E

AF 82.5.4

The reference value shall be represented over four octets as a single precision floating point number, consisting of a leading sign bit, a 7-bit characteristic and a 24-bit binary representation.

Note: (1) The characteristic is convertible to a power of 16 by subtracting 64 from its 7-bit representation.

(2) The reference value R is linked to the binary numbers s; A; B; representing the sign (1 bit) positive coded as "0", negative coded as "1"; a biased exponent (exponent + 64) (7-bits); and the mantissa (24-bits) by means of the following formula:

R = s*2**(-24)*B*16**(A-64)

AF 82.6

Block 4 (end of message block)

AF 82.6.1

The end of message block shall always be 4 octets long, character-coded as "7777".

Specification of octets contents

Note: Octets are numbered 1, 2, 3.... etc., starting at the beginning of each block.

Block 1 (Product definition block)

Octet Number	Contents
1 - 4 GRIB	(Coded CCITT-ITA No. 5)
5	Identification of Centre (see F_1F_2 (FM 49-VII) See WMO Publication No 386 Volume I, Part II, Attachment II-9, Table A)
6	Model identification (allocated by originating centre)
7	Grid Definition (see NNN.... Catalogue of grid used by centre F_1F_2 ((FM 47-V, FM 49-VII) See Volume B of WMO Publication No. 9)
8	Flag, (see Reg. AF 82.2.2 and code table 1)
9	Indicator of parameter (see a_1a_1, code table 0291)
10	Indicator of type of level (see code table 2)
11 - 12	Height, pressure etc. of levels (see code table 2)
13	Year of century)
14	Month)
15	Day) Reference time of data
16	Hour)
17	Minutes)
18	Indicator of unit of time range (see code table 4252, to be expanded later)
19	Time range
20 - 24	Reserved

Block 2 (Grid description block)

Octet Number	Contents
1 - 3	Length of octets of grid description block
4	Reserved
5 -	Grid description (to be developed later)
.	
.	

Block 3 (Bit map block)

Octet Number	Contents
1 - 3	Length of block
4	Reserved
5 - 6	Table of reference: (a) if the octets contain zero, a bit map follows; (b) if the octets contain a number, it refers to a pre-defined bit map provided by the centre
7	The bit map. Contiguous bits with a bit to data point correspondence ordered as defined in the grid definition.
.	
.	
-	Zero fill as necessary to make an even number of octets

Block 4 (Binary data block)

Octet Number	Contents
1 - 3	Length in octets of binary data block
4	Reserved
5 - 6	Scale factor (E)
7 - 10	Reference value (minimum value)
11	Number of bits containing each data value
12 -	Binary data
.	
.	
-	Zero fill as necessary to make an even number of octets
7777	End of message (Coded CCITT - ITA No. 5)

CODE TABLES RELATIVE TO BLOCK 1

TABLE 1

Octet No. 8: Flag indication relative to Blocks 2 and 3

Code	Block 2	Block 3
0	Omitted	Omitted
1	Included	Omitted
2	Omitted	Included
3	Included	Included

TABLE 2

Octets No. 10, 11 and 12: Fixed levels or layers for which the data are included

Octet Number 10		Octet Number 11	Octet Number 12
Code Fig.	Meaning	Contents	
0 99	special level, see code table 0491	0	0
100	isobaric level	pressure in hPa (2 octets)	
101	layer between 2 isobaric levels	pressure of top kPa	pressure of bottom kPa
102	mean sea level	0	0
103	fixed height level (above MSL)	height above MSL in metres (2 octets)	
104	layer between 2 fixed height levels (above MSL)	height of top hm	height of bottom hm
105	fixed height level (above ground)	height in metres (2 octets)	
106	layer between 2 fixed	height of top	height of bottom
107	sigma level	sigma value in 1/10000 (2 octets)	
108	layer between 2 sigma levels	sigma value of top in 1/100	sigma value of bottom in 1/100

Appendix II - ECMWF CONVENTIONS

This appendix contains the conventions to be adopted by ECMWF for the initial use of GRIB.

1. GRIB ENHANCEMENTS

All of the enhancements suggested in the main body of this document will be used until such time as alternative, WMO approved features are available.

2. ADDITIONAL CONVENTIONS

The following additional conventions will enable some continuity of notation with respect to currently defined ECMWF procedures, or will clarify the use to be made of certain features.

2.1 Product Description Block

The product description block (block 1) is mandatory, and will always be present. The use of specific items will be as follows:-

Octet Number	Contents
1 - 4	GRIB
5	98 (Centre identifier)
6	10-19 (N48 Grid point model), or
	20-29 (T63 Spectral model) or
	30-39 (T106 Spectral model), or
	nn (R.D. reference number), or
	255 (resolution not given)
7	255 (grid definition. The appropriate values from 1 to 12 would be used for the areas currently disseminated via GTS)
8	1 (Flag - denotes block 2 present)
9	128 + PP code (parameter type)
10-21	As defined in main document
22	n1n1 for dissemination products
23	n2n2 for dissemination products
24	n3n3 for dissemination products

2.2 Remaining Blocks

All remaining blocks will conform to the suggested extensions to GRIB contained in the main part of this document.

2.3 Field Representation

Where latitude/longitude fields of grid point values are represented, the ECMWF MARS archiving practice will be to avoid repeated points and latitudinal or longitudinal wrap-around. All data originating at ECMWF represented in grid point space will represent a geographical area with the following conventions:

(i) the origin will be the north west corner of the area;

(ii) the extreme point will be the south east corner of the area;

(iii) the scanning mode will be as defined for code 1 in table 8, i.e. from west to east, from north to south, with adjacent points on latitude circles consecutive.

APPENDIX III

Packing Data to be represented in WMO GRIB Code

1. Introduction

In GRIB code, packed data items are represented as scaled differences from a minimum value. Data are unpacked using the expression

$$U_j = r + P_j \cdot 2^s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where U_j (j=1....n)	are the real unpacked values
r	is a reference value
P_j (j=1....n)	are packed values (positive integers)
s	is a scale factor (signed integer)

To optimise accuracy with respect to packing density, it is necessary to choose a suitable scale factor, s, given a packing density of i bits per packed value.

2. Computation of Scale Factor

Let A_j (j=1....n) be the n real values to be packed

For max (A) = A_m, we have

$$U_m = r + P_m \cdot 2^s$$

But $P_m < 2^i - 1$ where P_m contains i bits

$$\therefore \quad U_m = r + (2^i - 1) \cdot 2^s \quad \text{if ALL bits of } P_m \text{ used.}$$

This represents values of A such that

$$r + (2^i-1)2^s - 0.5(2^s) < A < r + (2^i-1)2^s + 0.5(2^s)$$

Thus, A_m is less than

$$r + (2^i-1)\, 2^s + 2^{s-1}$$

$$= r + 2^{s-1}(2^{i+1}-1)$$

We require s to be the least integer such that

$$r + 2^{s-1}(2^{i+1}-1) \quad > A_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

i.e. $2^{s-1}(2^{i+1}-1) > A_m - r$

But $2^{i+1}-1 > 0$

$$\therefore \quad 2^{s-1} > \frac{A_m - r}{2^{i+1}-1}$$

$$s > \log_2\left(\frac{A_m - r}{2^{i+1}-1}\right) + 1$$

Thus s must be fixed such that

$$s = \text{floor}\left[\log_2\left(\frac{A_m - r}{2^{i+1}-1}\right)\right] + 2$$

where floor [ ] = greatest integer not exceeding [ ].

Examples

1. Let $r = 0$, $A_m = 55.0$, $i = 2$

$$\frac{A_m - r}{2^{i+1}-1} = \frac{55}{7}$$

$\therefore s = \text{floor}\ (2.2974) + 2 = 4$

Check:

$$P_m = \text{int}\left(\frac{55.0}{16} + 0.5\right)$$

$$= \text{int}\left(\frac{63}{16}\right) = 3_{10} = 11_2$$

2. Let $r = 0$, $A_m = 56.0$, $i = 2$

$$\frac{A_m - r}{2^{i+1}-1} = \frac{56}{7} = 8 = 2^3$$

$\therefore s = \text{floor}\ (3) + 2 = 5$

From example 1 above it can be seen that there is appropriate compensation for the increase in range.

3. Let $r = 0$, $A_m = 0.9374995$, $i = 3$

$$\frac{A_m - r}{2^{i+1}-1} = 0.062499966$$

$$\log_2\left(\frac{A_m - r}{2^{i+1}-1}\right) = -4.000000769$$

$\therefore s = \text{floor}\ (-4.000000769) + 2 = -5 + 2 = -3$

4. Let $r = 0$, $A_m = 0.9375$, $i = 3$

$$\frac{A_m - r}{2^{i+1} - 1} = 0.0625 = 2^{-4}$$

$\therefore s = \text{floor}(-4) + 2 = -4 + 2 = -2$

5. Let $r = 0$, $A_m = 0.937501$

$$\frac{A_m - r}{2^{i+1} - 1} = 0.062500066$$

$$\log_2 \frac{A_m - r}{2^{i+1} - 1} = -3.999998461$$

$\therefore s = \text{floor}(-3.999998461) + 2 = -4 + 2 = -2$

NB: From (2) above $A_m < r + 2^{s-1}(2^{i+1} - 1)$

for $s = -3$, $i = 3$, $r = 0$ $\quad A_m < 2^{-4}(2^4 - 1)$

i.e. $\quad A_m < 0.9375$

3. PACKING ERROR

3.1 Reference (minimum) value

The precision of the reference value is 6 hexadecimal digits (i.e. 7.2 decimal digits).

3.2 Representational error

Since $U_j = r+P_j.2^s$ the error due to the number of bits used for packing is $\pm(0.5)2^s$

3.3 Obtaining sufficient accuracy

If s is such that data can be packed into i bits, the effect of packing the same data into packed values each of (i+1) bits would be to reduce the required scale factor to (s-1). Suppose an accuracy of $\pm\varepsilon$ is required, and the representational error is large compared to the error of the reference value

i.e. $|\varepsilon| < (0.5)2^s$

or $|\varepsilon| < 2^{(s-1)}$

thus $s > 1+\log_2(|\varepsilon|)$

Let s_2 be the scale factor required if i_2 bits were to be used, computed as indicated in 2 above. Since the required value, s, can be determined, the number of bits required to pack the values to the necessary precision is given by:

$$i = i_2 + (s_2 - s)$$