SIMULATIONS OF DIRECTLY-FORCED MOTIONS IN THE TROPICS USING SIMPLE ANALYTIC MODELS

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Despite all the complications, there are many features of the tropical atmosphere which can be reproduced in extremely simple analytical models, thus giving a nice illustration of the most important processes and helping understanding of the phenomena involved. In particular, I've looked at the response of an atmosphere which is initially at rest to simple forms of imposed heating. There are various models of the vertical structure which lead to the same mathematics, but I will choose one of them, namely a uniformly stratified incompressible atmosphere with a rigid lid at the tropopause. Then if I take non-dimensional coordinates where the height z goes from zero at the surface to π at the tropopause, the solution is especially simple if the heating rate Q is proportional to sin z, for then the vertical velocity w and temperature perturbation θ are also proportional to sin z, and the horizontal velocity v and the geopotential perturbation ϕ are proportional to cos z (Fig.1).

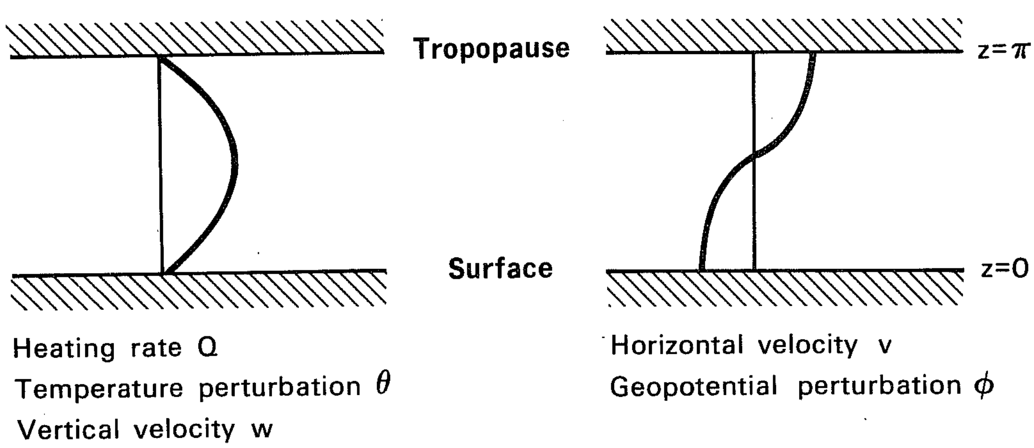

Fig.1. The vertical structure used for deriving the model equations.

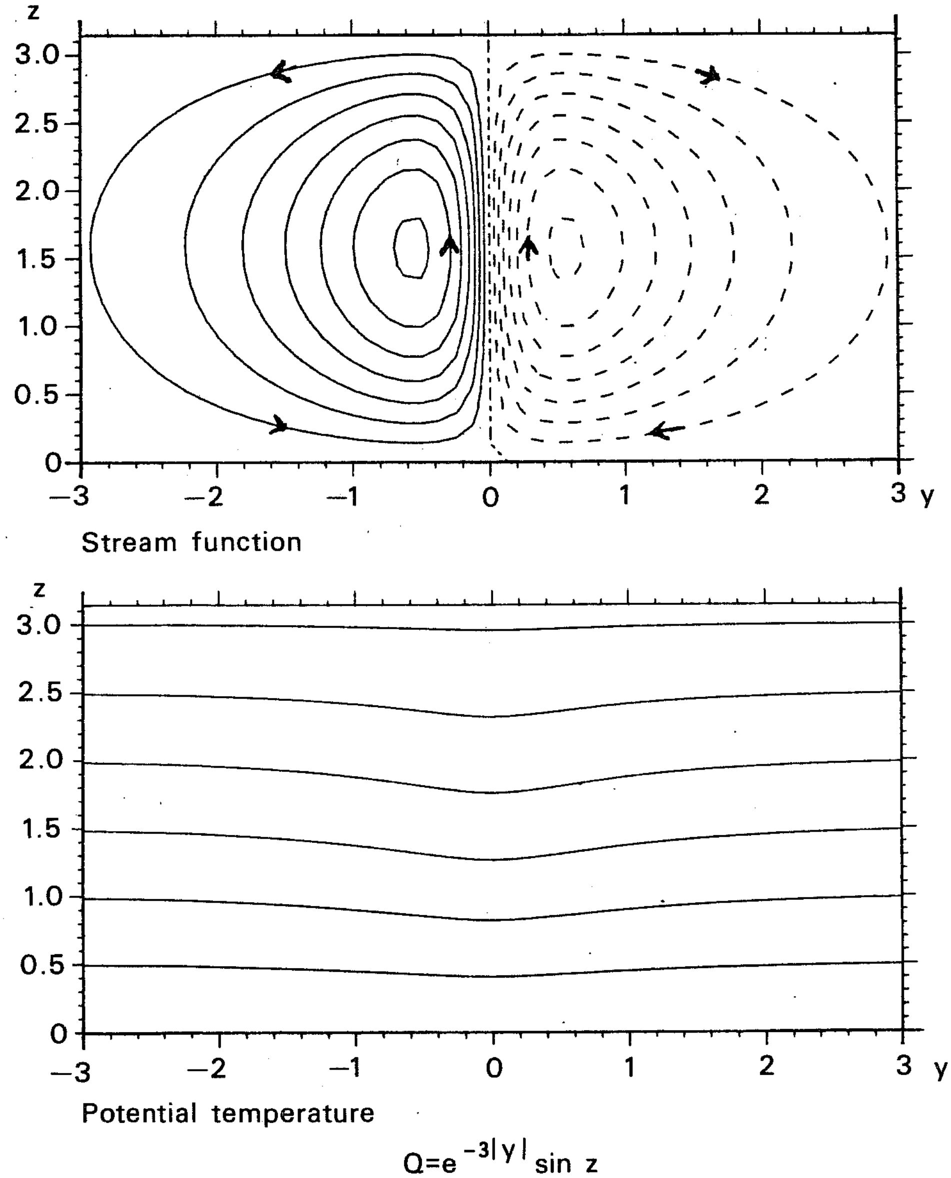

Fig.2. Streamlines and potential temperature distribution for the form of heating shown.

The simplest case is a non-rotating frictionless atmosphere where, if we take out the z-dependence above, we get equations which I hope you will recognise in the non-dimensional form

$$\theta_t + w = Q \qquad (heat)$$

$$w = -v_y \qquad (incompressibility)$$

$$v_t = -\phi_y \qquad (momentum)$$

$$\phi = -\theta \qquad (hydrostatic)$$

where y is the horizontal coordinate.

Some sort of friction has to be added if the system is to reach a steady state, and the simplest form is one where $\partial/\partial t$ is replaced by $\partial/\partial t + \epsilon$ in the equations. This corresponds to Newtonian cooling and Rayleigh friction and the equations become

$$\theta_t + \epsilon\theta = v_y + Q$$

$$v_t + \epsilon v = \theta_y \quad .$$

The steady-state solution is simply one where hot air rises producing the circulation and potential temperature distribution shown in Fig.2.

If we had an axisymmetric situation with a hot-spot in a non-rotating atmosphere, and we plotted contours of w and used arrows to represent low-level flow, we would give a picture like Fig.3 with air rushing in to the hot spot symmetrically on all sides.

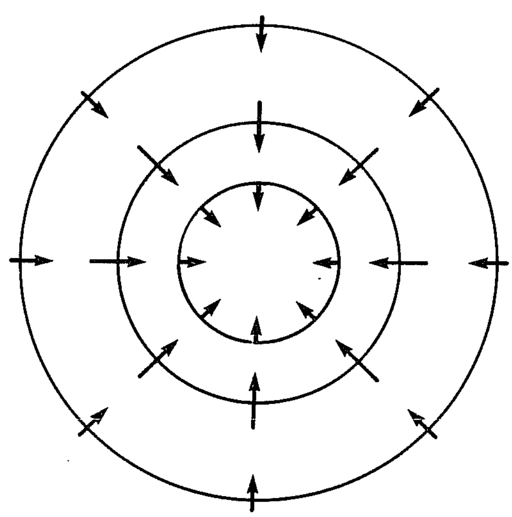

Fig.3. Low-level radial inflow into a circular heat source in the non-rotating case.

This is all very simple and easy to understand, but what happens if we put the hot spot somewhere in the tropics on a rotating earth? Near the equator there are effects which tend to constrain the flow to be predominantly east-west. Off the equator there are effects which induce rotation. These effects are readily investigated by adding the Coriolis term to the equations formulated above. If y is now

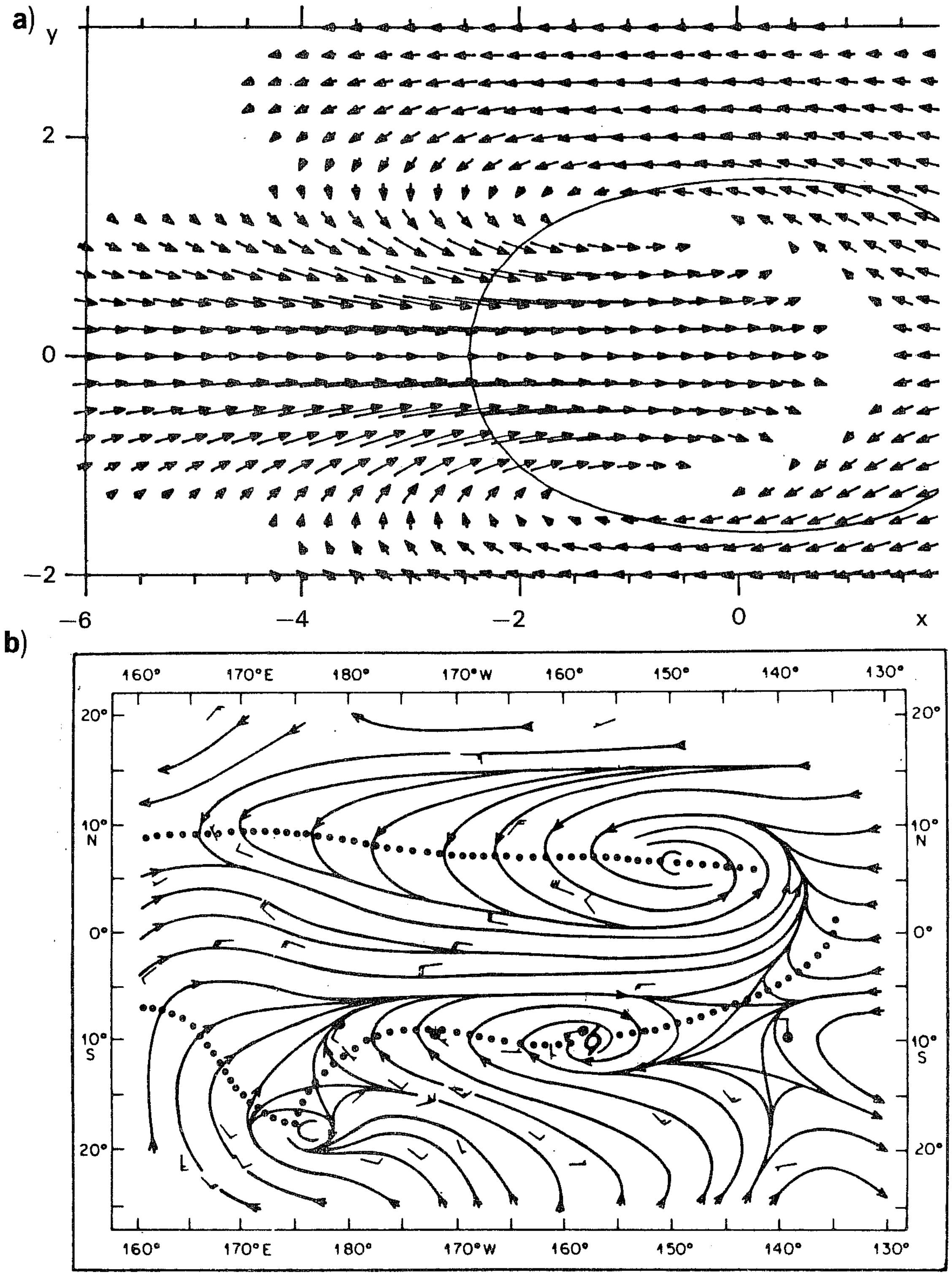

Fig.4 (a) Low-level flow associated with the heating zone shown by the contour. The unit of distance is the equatorial Rossby radius, which is about 10° of latitude (from Heckley and Gill 1984).

(b) Surface wind analysis on 15 November 1982. Trough lines which delineate the envelope of westerly winds are dotted. Each long wind barb is 5 m s^{-1} (from Sadler and Kilonsky 1983).

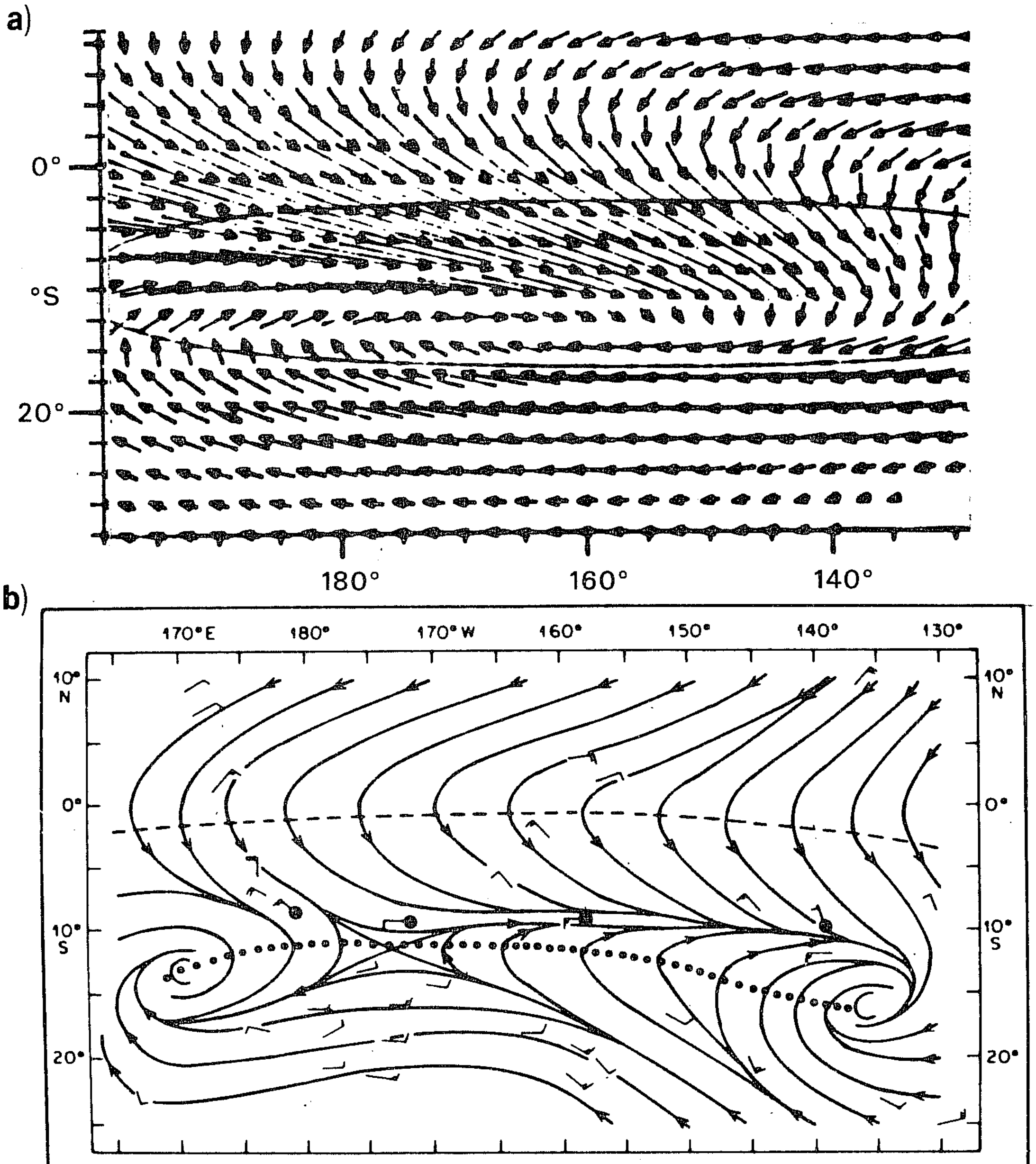

Fig.5 (a) As for Fig.4(a), but for a different shape and location of the heating zone. (*Note*: These diagrams were prepared long before the wind analyses which are used for comparison.)

(b) As for Fig.4(b) but for 21 March 1983.

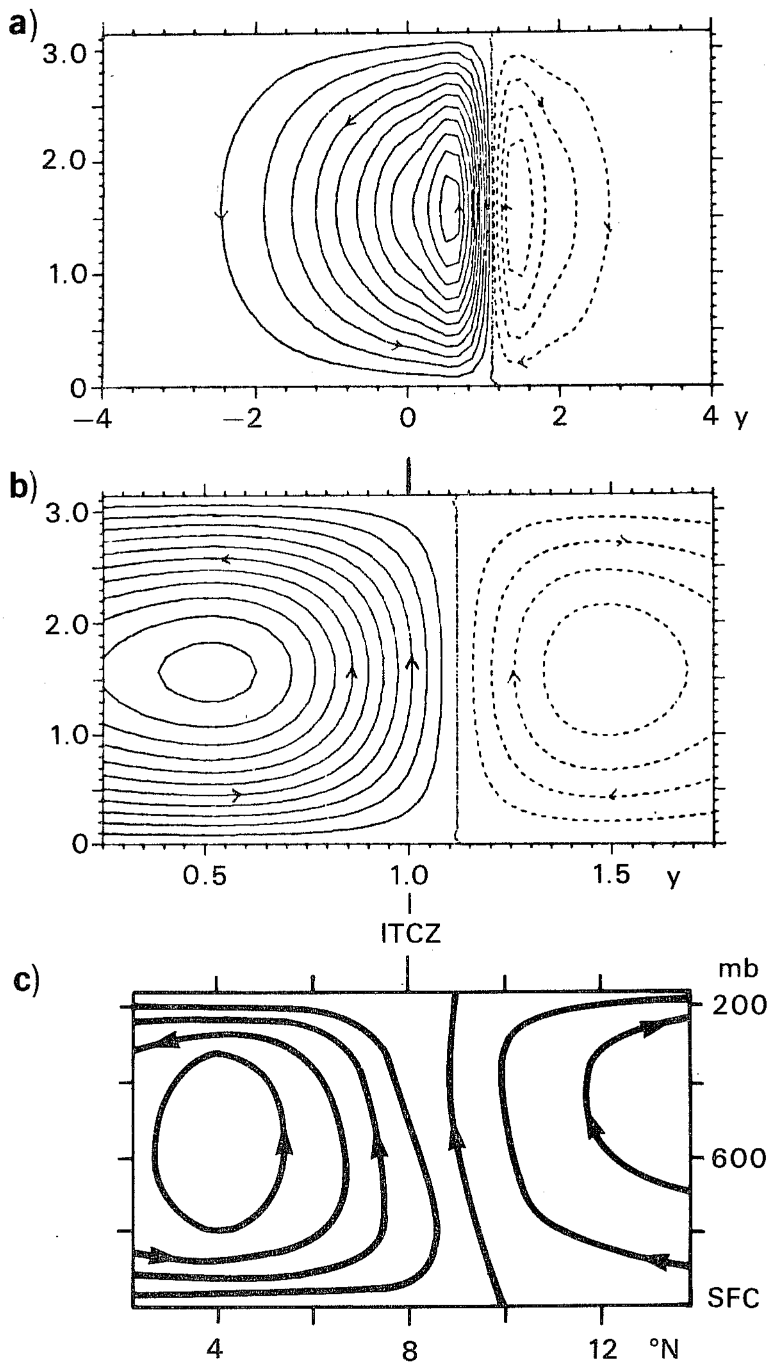

Fig.6. (a) Streamline of meridional flow for heating centred at $y = 1$ and extending from about $y = 5$ to 1.5. (The expansion of a delta function to 60 terms is used - see Heckley and Gill 1984.) (b) An exposed view of the solution in the neighbourhood of the heating zone. The centre of the heating zone is at $y = 1$. (c) Observed meridonal flow relative to the ITCZ in the Atlantic during GATE. The data is taken from the composites given by Frank (1984) with the mean northward flow removed. The centre of the cloud band is at the position marked 8°N (its mean position) and the degree markings serve top give the distance to the centre. The contour interval is 1.8 $ton\ m^{-1}s^{-1}$ ($180\,mb\ m\,s^{-1}$).

taken to be distance northwards from the equator and x distance eastwards, and (u,v) the horizontal velocity, then the non-dimensional equations on an equatorial beta-plane are simply (cf. Gill 1980,1982a)

$$\theta_t + \epsilon\theta = u_x + v_y + Q$$

$$u_t + \epsilon u - \tfrac{1}{2} y v = \theta_x$$

$$\tfrac{1}{2} y u = \theta_y \quad .$$

The above form applies when the east-west scale is large compared with the natural meridional scale which is the equatorial Rossby radius, equal to about 10° of latitude. I will now show you three solutions taken from a paper with Bill Heckley (Heckley and Gill 1984). Fig.4a shows the low-level flow into a heating zone on the equator. See how different it is from the non-rotating case. This pattern is typical of that seen in the equatorial Pacific during the remarkable 1982/3 event which has been discussed in a recent paper with Rasmusson (Gill and Rasmusson 1983), e.g. Fig.4b shows a surface wind analysis for 15 November 1982 prepared by Sadler and Kilonsky (1983). Note (i) the tendency for the flow to be zonal, (ii) the twin cyclones which, in the analytic solution, are on the western flank of the heating region, (iii) the strongest flow is the westerly jet to the west of the forcing, (iv) the poleward part of the meridional flow is mostly in the forcing region, and (v) the equatorward part of the meridional flow is mainly west of the forcing region and feeds the westerly jet. All these features can be explained theoretically (Gill 1980). However, an important feature not in the model is the potential for spawning tropical cyclones. One is marked in Fig.4(b) at 10°S 157° W and a few days later tropical cyclone IWA formed near 8° N 167°W. Note also that the upper-level model flow is the reverse of that shown with twin anticyclones at the western flank of the heating zone. Again, this is a frequently observed pattern.

The low-level flow due to an elongated heating region centred on 10°N ($y = 1$) is shown in Fig.5(a). Again, similar patterns are observed, e.g. Fig.5(b) shows the surface wind analysis for March 21, 1983 – the trough corresponding to the heating zone being in the southern hemisphere in this case. Note (i) cross-equatorial flow changing from easterly to westerly zonal component near the equator, (ii) westerly flow poleward of the heating zone, and (iii) cyclonic flow at the western flank of the heating zone. Note also that there is a much stronger inflow from the equator-wide side than the poleward side. This is a feature of solutions for heating along lines of large zonal extent like the ITCZ show, e.g. in Fig.11.18 of my book (Gill 1982a). Fig.6(a,b) shows a similar solution for heating independent of latitude, but the line of heating is spread over a small but finite width. For comparison, the streamfunction for flow relative to the cloud band during GATE is shown in Fig.6(c), which is based on the data given by Frank (1983).

A third example of the response to a patch of heating is shown in Fig.7, where the heating is centred about 20° latitude. This shows that some asymmetry in the structure of a tropical cyclone can be produced purely from the variation in the

Coriolis parameter.

Transient solutions showing the response of the atmosphere to suddenly applying a heat source have been studied by Heckley and Gill (1984). These solutions have similar features to those found following the initialisation of general circulation models.

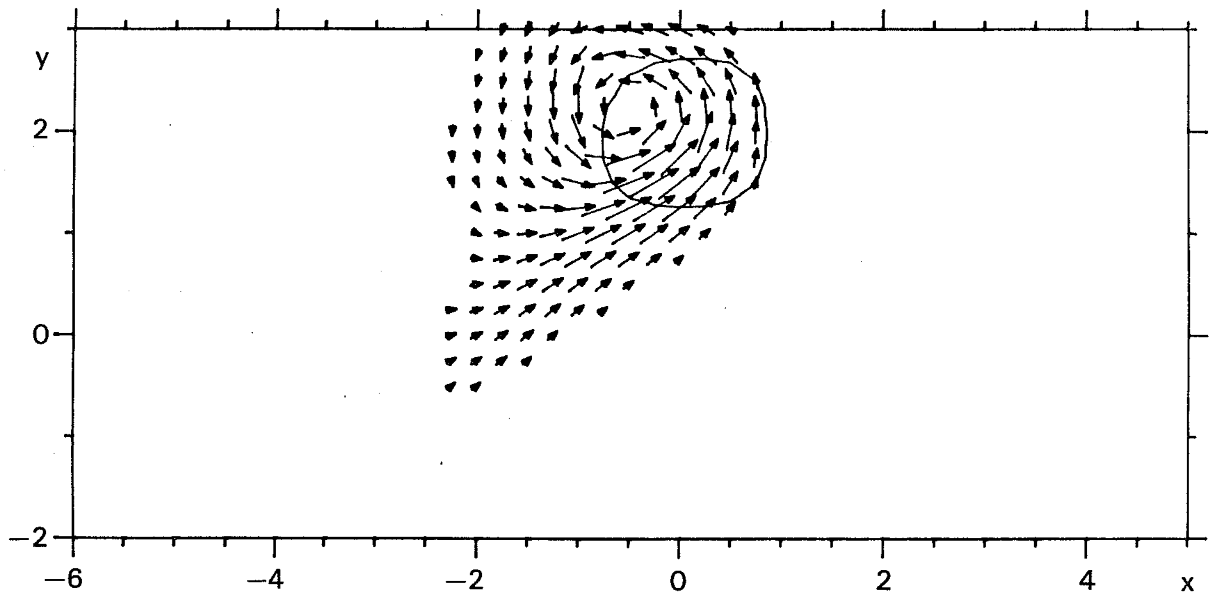

Fig.7. Same as Fig.4(a) but for a different shape and location of the heating zone. The centre is at $y = 2$ (about 20°N).

Another use of this type of solution has been to follow the course of the 1982/3 warm event in the Pacific Ocean. The location of heat sources and sinks for each month was decided purely on the basis of outgoing longwave radiation anomaly patterns, since these indicate regions of high clouds and hence of latent heat release. The model then gives the low-level wind pattern, which agreed quite well with the observed wind anomaly pattern. Fig.8 shows an example after Gill and Rasmusson (1983).

The linear solutions work remarkably well, and one of the reasons may be that specifying the heating rate is close to specifying the divergence field in the active region. The dynamics seem to determine how the flow into or out of the active region distributes itself. Despite the success of the linear solution, there are some aspects where there seem to be systematic differences from observation. For instance during the 1982/3 event there was rather little anomalous low-level inflow into the heating region from the east, whereas the inflow from the west was very strong. The linear solution leads one to expect asymmetry, but not as much as observed.

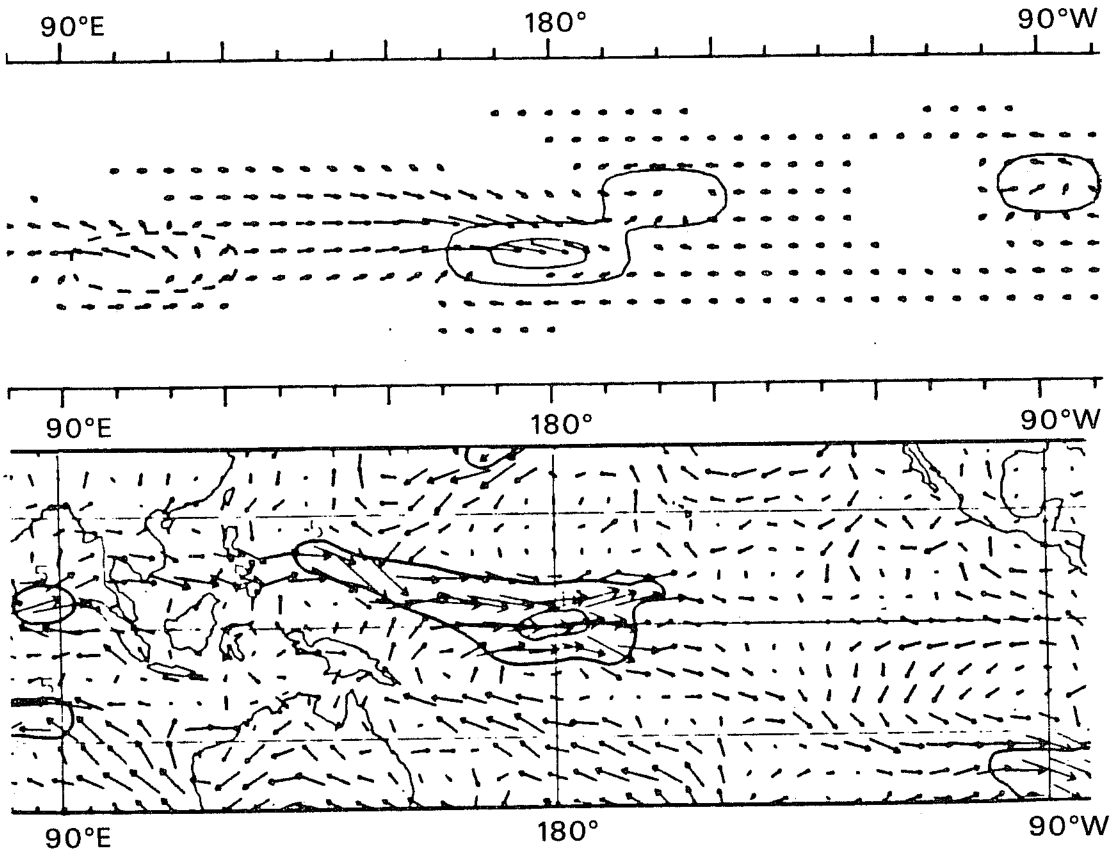

Fig.8 The upper panel shows the low-level flow for the heating distribution shown (broken contours negative). This distribution was chosen solely on the basis of the outgoing longwave radiation anomaly field for September 1982. The lower panel shows the observed low-level flow anomaly for comparison (picture expanded from Gill and Rasmusson 1984).

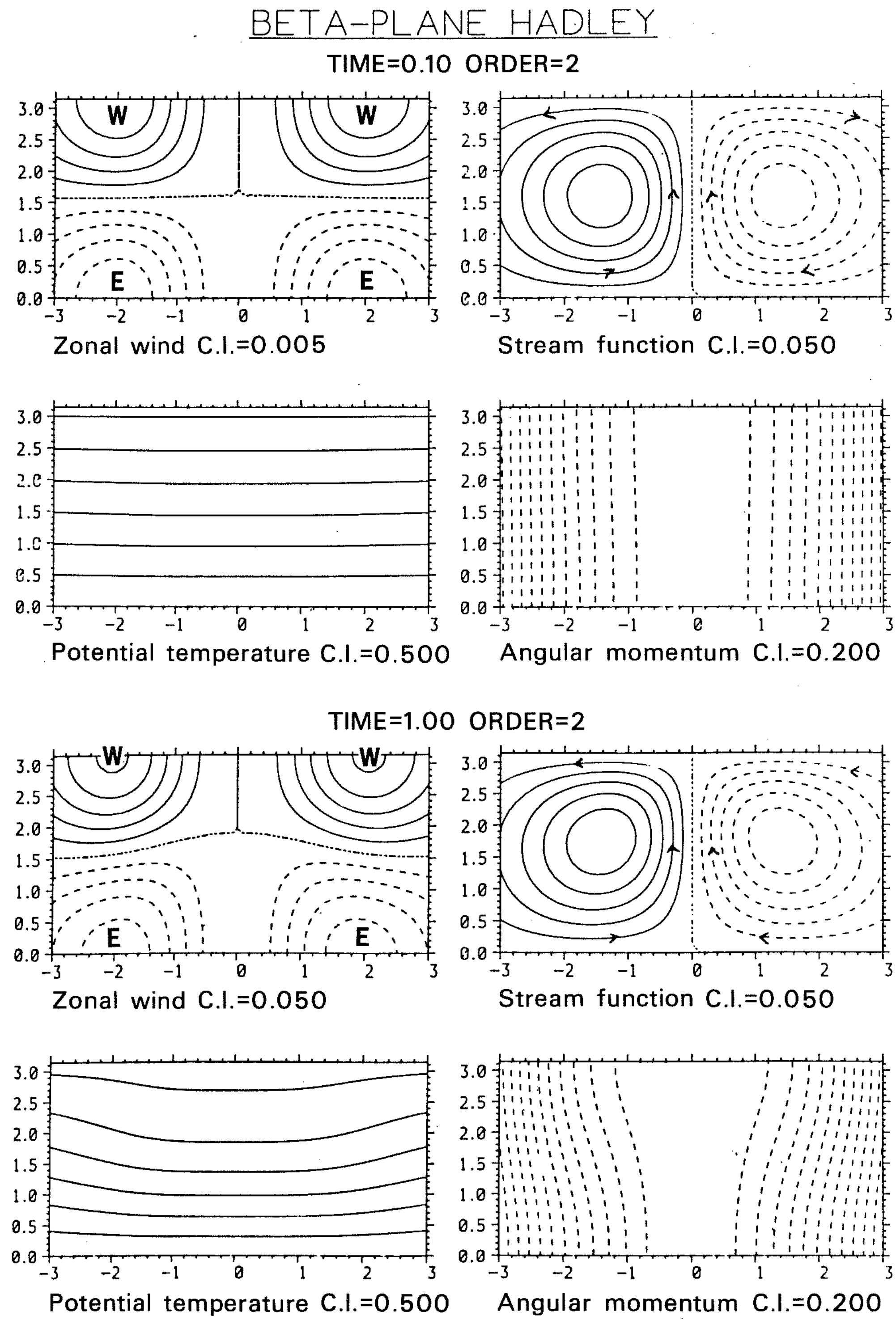

Fig. 9 Meridional sections of zonal wind, streamfunction, potential temperature and angular momentum for the linear case (upper panel) and with a nonlinear correction (lower panel). The upper panel corresponds to the solution 0.1 units of time after switching on a heat source given by sin z $D_0(y)$ for the inviscid case. The lower panel corresponds to one unit of time after the switch on, where the wind is that taken for nonlinear effects to become significant.

The flows induced in the tropics are usually so large that non-linear effects are important, so it is worthwhile examining effects of non-linearities using some of the theoretical solutions. This is easily done in principle by carrying out a small-amplitude expansion. In practice, however, the algebra soon gets out of hand and it is not a trivial matter to calculate the first non-linear correction. Fig.8 shows the simplest case of a Hadley cell with (a) the linear solution as in Gill (1980), and (b) the solution with the non-linear correction. The warming in the heating zone and the advection of angular momentum are the most obvious effects. A student of mine is now calculating analytic solutions when the heating zone is of limited zonal extent. Nobre (1983) has already calculated such solutions numerically for a heat source whose zonal extent is 180°, i.e. half way round the earth. At 700mb, the effect of the non-linearities was not so obvious, but at 300mb the effects were quite substantial. For instance, the easterly upper-level outflow from the heating zone was greatly reduced, whereas strong equatorward and westerly flows developed on the eastern flanks of the heating zone.

Another aspect of the linear model which leaves something to be desired is the *a priori* specification of the heating function. In practice, the heating is largely due to the release of latent heat of condensation in clouds. If the heating is specified, it produces a low-level convergence as observed, but it is this very same convergence which brings in the moisture which fuels the heating. To allow the heating and convergence fields to develop a natural realtionship, it is necessary to include moist processes in some way. I feel it is important to find the simplest possible model so some understanding can be built up from illustrative case studies.

With this in mind I've developed what I call a moist shallow-water model (Gill 1982b,1982c). The vertical structure of the variables other than humidity is assumed to be the same as considered above. The moisture profile, on the other hand, is assumed to be such that nearly all the moisture is below the central level and so is advected in the same direction. Since the same vertical profile is assumed at each location, a single variable q, which may be taken to be the total precipitable moisture, suffices to define the moisure field at any point. This is assumed to have a saturation value $\overline{q}$. If this value is reached and the tendency is for further increase, the excess moisture is converted into precipitation, and latent heat is released (with the same vertical profile as Q). Thus in the one-dimensional case, the moisture equation is

$$q_t + R = -P$$

where

$$R = (qv)_y + r(q-\overline{q}) \quad .$$

The term proportional to r represents evaporation which strives to bring the atmosphere up to the saturation moisture level. If the atmosphere is saturated $(q-\overline{q})$ and $R < 0$, then q remains equal to $\overline{q}$ and the precipitation rate $P = -R$. In all other circumstances, P is zero. In many cases, it is convenient to linearise about the saturated state so the convergence term in R becomes $(\overline{q}\, v)_y$.

As an example, consider the linear problem where the heating rate Q is sinusoidal in y. Then the equations for the dry case without friction are

$$\theta_t + w = Q \equiv \cos \pi y \qquad (1)$$

$$w = -v_y$$

$$v_t = \theta_y \quad .$$

The steady-state solution is simply,

$$w = Q, \quad \theta = 0 \quad .$$

Now let us see what happens in the moist case. The region near $y = 0$ with the strongest rate of convergence is where precipitation persists. Then the moisture equation gives

$$P = -\overline{q}\, v_y = \overline{q}\, w$$

in this region $|y| < \xi$, say. There is now a net heating over the whole region which in practice would need to be balanced by radiative losses which could be represented by Newtonian cooling. In the non-diffusive model, the atmosphere just warms up at a rate $\theta_t = A$ which must be uniform in space so as not to give an acceleration. So in the dry region (1) gives

$$A + w = Q \quad \text{for} \quad \xi < y < 1 \quad .$$

In the moist region

$$A + w = Q + P = Q + \overline{q}\, w$$

$$i.e. \quad A + (1-\overline{q})\, w = Q \quad \text{for} \quad 0 < y < \xi \quad .$$

The conditions are that the average vertical motion is zero

$$\int_0^1 w \, dy = 0 \quad ,$$

which also ensures that the amount of moisture prescribed in the moist region is just balancing the amount which can be supplied by evaporation elsewhere. The other condition is that

$$w = 0 \quad at \quad y = \xi$$

so that the atmosphere is ascending in the region of precipitation and descending in the dry region. This gives as the equation for ξ

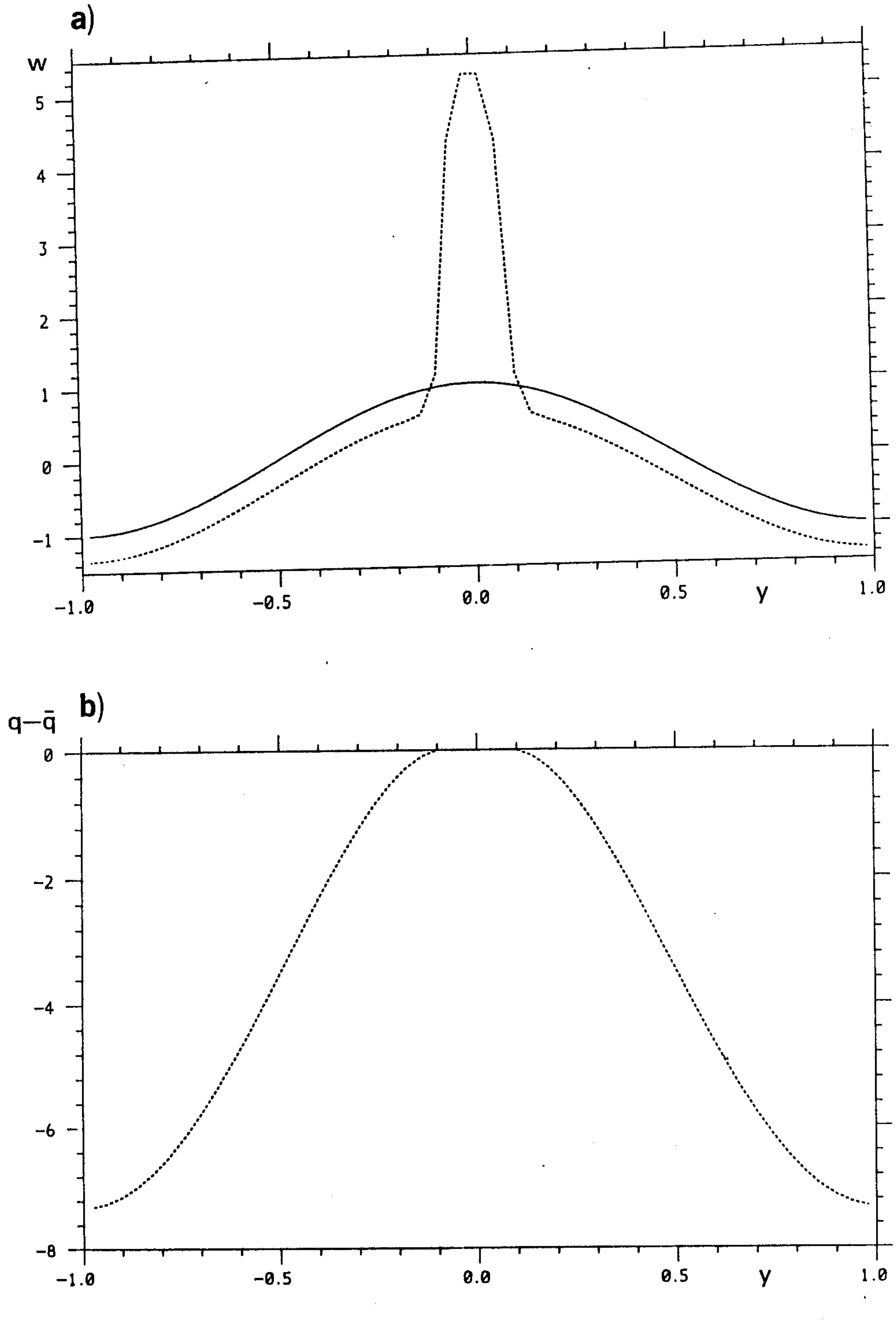

Fig. 10 (a) The vertical velocity distribution for the case with periodic heating proportional to cos πy. The solid line shows the solution in the absence of moisture effects, given approximately by $w = \cos \pi y$. The dotted line shows the solution obtained with moisture effects. Now there is strong vertical motion in a narrow convergence zone of heavy precipitation. This zone intensifies as the saturation moisture level increases toward the value for moist instability. (b) The distribution of the moisture variable $(q - \overline{q})$ for the same case. Zero corresponds to the saturation level.

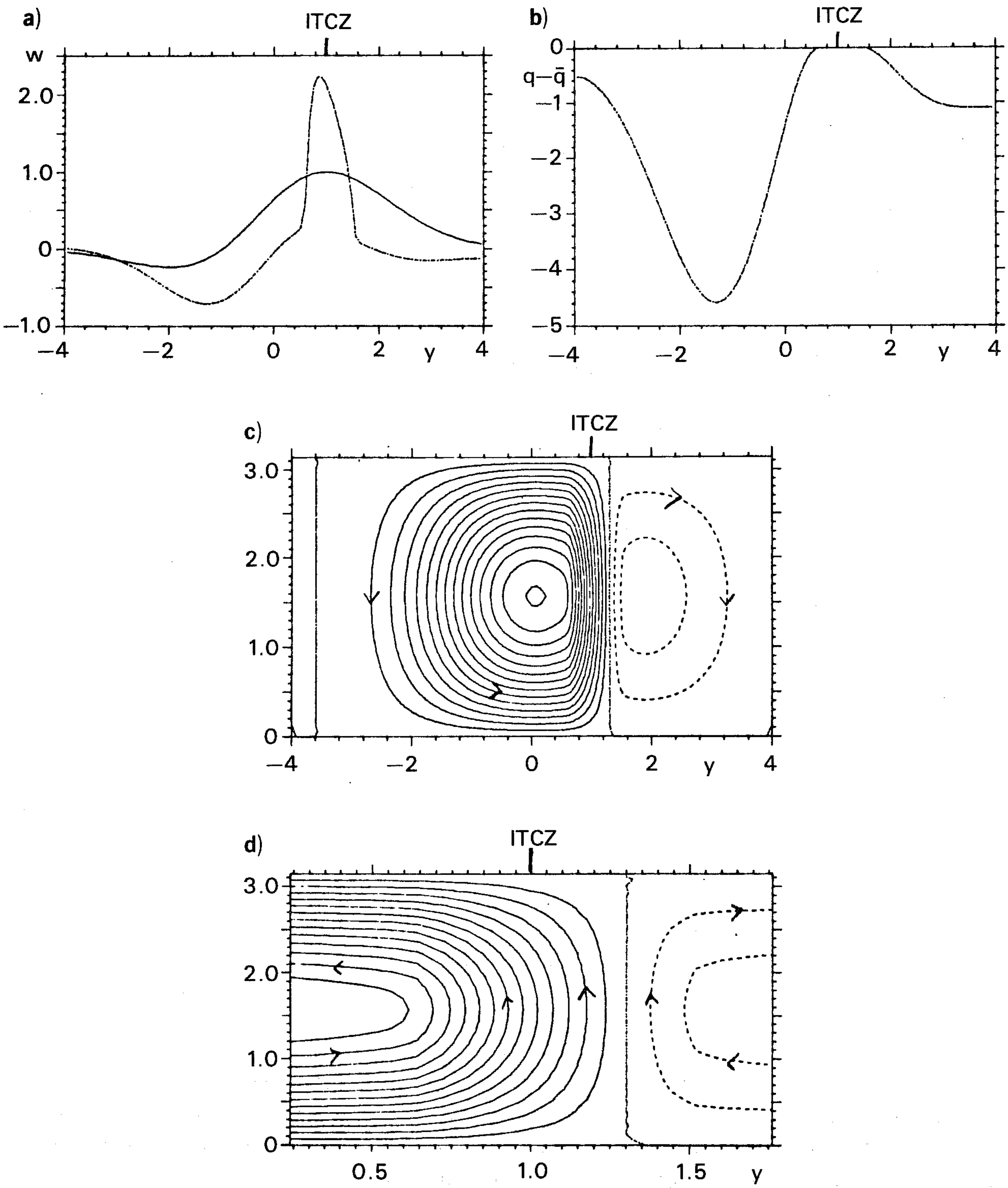

Fig. 11 (a) Distribution of vertical velocity w (dotted line) for the moist case with heating rate (solid line) proportional to sin z ($D_0(y) + D_1(y)$). (b) The moisture variable ($q - \bar{q}$). (c,d) The streamfunction on different y intervals.

$$\tan \pi\xi = \pi(\xi - 1 + 1/\overline{q})$$

If $\overline{q} = 8/9$, the solution is $\xi = 0.293$, the maximum value of w is 3.6 and the minimum value is −1.6. Figure 10 shows a solution obtained numerically but with $\partial/\partial t$ in the equations replaced by $\partial/\partial t - \nu\,\partial^2/\partial y^2$ with $\nu = 0.01$. The value of r was 0.1. Note the concentrated region of upward motion where latent heat is being released, and this region would show as a line of clouds.

We can now do the same sort of problem on an equatorial beta-plane to generate an ITCZ. The heat and momentum equations are now

$$\theta_t + \epsilon\theta = v_y + Q + P$$

$$u_t + \epsilon u - \tfrac{1}{2} y v = 0$$

$$v_t + \epsilon v + \tfrac{1}{2} y u = \theta_y$$

and we can choose

$$Q = (1 + y)\exp(-\tfrac{1}{4} y^2) \quad .$$

The solution for the dry case was obtained in Gill (1980) and gives a broad Hadley cell with rising motion on the side of the equator where the heating is strongest. The moist equations above were solved for this case as well, with $\epsilon = 0.1$ and $\partial/\partial t$ replaced by $\partial/\partial t - \nu\,\partial^2/\partial y^2$ with $\nu = 0.03$. Fig.11(a) shows the vertical velocity distribution obtained, giving a concentrated convergence zone, and (b) shows the moisture distribution. The streamfunction is plotted in Fig.11(c) in the same format as Fig.6(a). The circulation is significantly altered from the dry case with the concentrated upflow and the appearance of a second cell on the poleward side of the convergence zone. Fig.11(d) shows an expanded view for comparison with Fig.6(b).

Thus to summarise, I think these models with highly simplified physics have much to offer in helping us to understand the tropical circulation, and there is a lot more that can be done with them than I've managed to do so far!.

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