T R E A T M E N T O F THE BOUNDARY LAYER IN GENERAL CIRCULATION MODELS - THEORETICAL ASPECTS

B Y

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C O N T E N T S

1. Introductory remarks

For an improvement of the current General Circulation Models (GCMs) the effects in the boundary layer (besides several other effects) have to be incorporated into the models in a much better way as it is done up to now. Of course a successful parameterization of these effects can be achieved only, if the physical processes in the boundary layer are well understood.

For this reason it seems to be useful in these lecture notes first to discuss some important features of the atmospheric boundary layer, mainly its structure, which is studied by the results of a numerical model of the Planetary Boundary Layer (PBL). Most interesting are special similarities in the PBL, which are the basis for so-called resistance- and transfer-laws. These laws are very suitable tools in the parameterization procedure.

2. Generalities on the Planetary Boundary Layer (PBL)

2.1. The definition of the PBL

The PBL is an abstraction; the real atmospheric boundary layer (ABL) would be a PBL if certain conditions (defining the PBL) are fulfilled. However these conditions are never satisfied simultaneously in the ABL. Therefore any results obtained for the PBL can be only applied to the ABL, if the former is a tolerable approximation to the ABL.

These are the conditions defining the PBL:

(a) the boundary layer flow is turbulent
(b) the mean flow as well as the turbulent characteristics are horizontally homogeneous
(c) the mean flow as well as the turbulent characteristics are stationary
(d) the molecular transport of momentum, heat and moisture can be neglected as being small compared with the corresponding

turbulent transports
(e) no sources or sinks of water vapour by evaporation or condensation do exist
(f) there is no divergence of radiative fluxes.

2.2. Remarks on PBL-models, the closure problem

Due to the turbulence one is unable to describe a PBL mathematically as defined by the forementioned conditions (a) - (f), unless one makes some additional assumptions; however that would mean that one constructs mathematical m o d e l s of the PBL, which itself is a physical model of the ABL.
Additional assumptions are necessary because of the c l o s u r e problem.
The PBL-equations are obtained from the momentum equations (averaged for the mean flow) and the thermodynamic energy equation applying the forementioned conditions (a) - (f)

$$d/dz(\tau_x/\bar{\varrho}) = -f\{\bar{v} - \bar{v}_g\} \quad (2.1)$$

$$d/dz(\tau_y/\bar{\varrho}) = f\{\bar{u} - \bar{u}_g\} \quad (2.2)$$

$$d/dz(q/c_p\bar{\varrho}) = 0 \quad (2.3)$$

$$d/dz(j/\bar{\varrho}) = 0 \quad (2.4)$$

and the appropriate boundary conditions. $\bar{\varrho}$ is assumed to be constant; the geostrophic wind $\bar{v}_g$,in a baroclinic PBL varying with height z, is considered to be given. Since for the 6 variables (τ_x , τ_y , $\bar{u}$, $\bar{v}$, q , j) only 4 equations are available one has to hypothesize two further relationships between these variables in order to close the system.
The simplest closure method is to use the flux-gradient relations

$$\tau_x/\bar{\varrho} = k_m\, d\bar{u}/dz \quad (2.5)$$

$$\tau_y/\bar{\varrho} = k_m\, d\bar{v}/dz \quad (2.6)$$

$$q/c_p\bar{\varrho} = k_h\, d\bar{\vartheta}/dz \quad (2.7)$$

$$j/\bar{\varrho} = k_s\, d\bar{s}/dz \quad (2.8)$$

in the eqs. (2.1)-(2.4) giving

$$d/dz\{k_m\, d\bar{u}/dz\} = -f\{\bar{v} - \bar{v}_g\} \quad (2.9)$$

$$d/dz\{k_m\, d\bar{v}/dz\} = f\{\bar{u} - \bar{u}_g\} \quad (2.10)$$

$$d/dz\{k_h\, d\bar{\vartheta}/dz\} = 0 \quad (2.11)$$

$$d/dz\{k_s\, d\bar{s}/dz\} = 0 \quad (2.12)$$

and then to prescribe the $k_m(z)$, $k_h(z)$ and $k_s(z)$ by a hypothesis. The most restrictive hypothesis would be $k_m = const., k_h = k_s = 0$, leading with the appropriate boundary conditions to the well-known Ekman wind spiral.

Another closure method makes use of Prandtl's relation between the eddy diffusion coefficient k_m and the mixing-length ℓ

$$k_m = \ell^2\, |d\bar{v}/dz| \quad (2.13)$$

One inserts (2.13) in (2.9),(2.10) and makes then a hypothesis for the mixing-length profile $\ell(z)$ and for k_h/k_m and k_s/k_m , which both vary with z. Many different hypotheses for $\ell(z)$ have been made, of course most of them regard also the dependence on the stratification.

In recent years an increasing number of authors use the so-called higher order closing. This means that additional equations are derived for the cross-correlations $\tau_x/\bar{\varrho} = -\overline{u'w'}$, $\tau_y/\bar{\varrho} = -\overline{v'w'}$, $q/c_p\bar{\varrho} = \overline{\vartheta'w'}$ and $j/\bar{\varrho} = \overline{s'w'}$, in which of course triple-correlations and other correlations with the pressure fluctuations appear. If one makes hypotheses for all these correlations four additional equations (possessing the additional variables $\bar{\vartheta}$ and $\bar{s}$) are available and the set of equations (2.1) - (2.4) is closed; a second-order-closing is made. In the same way one can go one step further and derive additional equations for the triple-correlations needed in the forementioned equations In these equations then appear quadruple-correlations and some other triple-correlations for which hypotheses have to be made; that would be a third-order-closing.

At the present time for most of the PBL-models the closing is done by some kind of a mixing-length hypothesis. The more sophisticated models with a second-order-closing have not yet shown up to now better results than models with a simpler closing procedure.

For PBL-models with a prescription of $k_m(z)$ or $\ell(z)$ two groups of models can be distinguished: single-layer models or two-layer models. Whereas in the first group $k_m(z)$ and $\ell(z)$ are prescribed for the whole PBL, these prescriptions are made in the second group of models separately for the outer and the inner (or surface-) layer, of course with taking care of the continuity at the interface.

2.3. Coordinate systems used in the PBL

In the PBL Cartesian coordinates are used generally, the vertical direction of which is denoted by z . Two directions are available to orientate the horizontal coordinates, i.e. the direction of the geostrophic wind (in baroclinic cases that one at the surface) and the direction of the surface stress. Both orientations are useful, the choice of them depends on the problem to be treated.

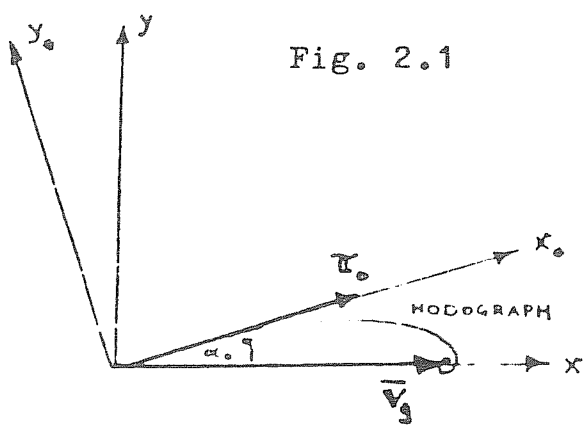


Fig. 2.1

The angle between these two directions is α_0 , the cross-isobar angle.

The coordinate system, the x-axis of which is orientated in the direction of the surface stress, is called a n t i t r i p t i c .

2.4. External and internal parameters

In the PBL we have to distinguish external and internal parameters. The external ones are the large-scale parameters (denoted by a bar) as for instance given by the GCM; these are $\bar{v}_{g0}$ and $(\bar{v}_{gT} - \bar{v}_{g0})/z_T$, $\bar{\vartheta}_T$, $\bar{\vartheta}_0$, $\bar{s}_T$ and $\bar{s}_0$. In addition some local parameters as z_0 and f have to be considered as external parameters. To a given set of external parameters belong the sought turbulent fluxes at the surface $\tau_0/\bar{\varrho}$, $q_0/c_p\bar{\varrho}$ and $j_0/\bar{\varrho}$ or expressed in characteristic fluctuations $-v_*|v_*|$, $-\vartheta_*|v_*|$ and $s_*|v_*|$;

these are the so-called internal parameters.(Occasionally internal parameters are combined with the local parameter f , for which no clear distinction is made in external or internal). The following combination of parameters can be listed

	external	internal
Charact. velocity	$\lvert\bar{\mathbf{v}}_{g0}\rvert$	$\lvert\mathbf{v}_*\rvert$
Charact. temperature	$\bar{\vartheta}_T - \bar{\vartheta}_0$	ϑ_*
Charact. moisture	$\bar{s}_T - \bar{s}_0$	s_*
Charact. dynamic length-scale	$\bar{H} = \lvert\bar{\mathbf{v}}_{g0}\rvert/f$	$H_* = \varkappa\lvert\mathbf{v}_*\rvert/f$
Charact. thermal length-scale	$\bar{L} = \mathbf{v}_{g0}^2/\{\beta[\bar{\vartheta}_T-\bar{\vartheta}_0]\}$	$L_* = \mathbf{v}_*^2/\{\varkappa\beta\vartheta_*\}$ OBUKHOV STABILITY LENGTH
Stability parameter	$\sigma = \bar{H}/\bar{L}$ $= \beta[\bar{\vartheta}_T-\bar{\vartheta}_0]/(f\lvert\bar{\mathbf{v}}_{g0}\rvert)$ $= Le\,[\bar{\vartheta}_T-\bar{\vartheta}_0]/\bar{\vartheta}$	$\mu = H_*/L_*$ $= \varkappa^2\beta\vartheta_*/(f\lvert\mathbf{v}_*\rvert)$
Parameter for baroclinicity	$\bar{\lambda}_x = \bar{\mathbf{v}}_{g0}^{\circ}\{\bar{\mathbf{v}}_{gT}-\bar{\mathbf{v}}_{g0}\}/z_T$ $\bar{\lambda}_y = [\mathbb{k}\times\bar{\mathbf{v}}_{g0}^{\circ}]\{\bar{\mathbf{v}}_{gT}-\bar{\mathbf{v}}_{g0}\}/z_T$	$\lambda_{*x} = \mathbf{v}_*^{\circ}\{\bar{\mathbf{v}}_{gT}-\bar{\mathbf{v}}_{g0}\}/z_T$ $\lambda_{*y} = [\mathbb{k}\times\mathbf{v}_*^{\circ}]\{\bar{\mathbf{v}}_{gT}-\bar{\mathbf{v}}_{g0}\}/z_T$

The ratios of internal to external velocities, temperatures and moistures are denoted by

$$C_g = \lvert\mathbf{v}_*\rvert/\lvert\bar{\mathbf{v}}_{g0}\rvert$$ Geostrophic drag coefficient (2.13)

$$C_h = \vartheta_*/\{\bar{\vartheta}_T-\bar{\vartheta}_0\}$$ Transfer coefficient for heat, a Stanton number (2.14)

$$C_s = s_*/\{\bar{s}_T - \bar{s}_0\}$$ Transfer coefficient for moisture, a Dalton number (2.15)

3. The structure of the PBL

In order to get an impression what does the PBL look like some remarks will be made in this chapter about the structure of the PBL; most of the pictures presented are results of a numerical (single-layer) PBL-model (section 3.3.).

3.1. The PBL as boundary layer in a rotating system

The main difference between the ABL (or PBL) and a boundary layer on a flat plate in a windtunnel is the much larger length - -scale for the former and by this the necessity to consider the PBL as a boundary layer in a rotating system. Whereas the boundary layer thickness at the flat plate is $\delta \propto (\nu x/U)^{1/2}$, the thickness of the PBL is $\delta \propto (\tilde{k}_m/f)^{1/2}$;i.e. at the flat plate the thickness of a steady state boundary layer increases with increasing distance x from the edge, but the thickness of the PBL is independent of any distance.

Another consequence of the rotation of the earth is the overshooting in the upper part of the APL, a phenomenon the physics of which are difficult to understand. This is very much different to boundary layers in non-rotating systems, where this overshooting does not exist.

The amount of overshooting depends remarkably on the thermal stratification; it is most pronounced in very stable cases, in which it occurs at lower levels than under neutral or unstable conditions. This is connected with the phenomenon of the low level jet. However a pronounced low level jet occurs, when the overshooting is strengthened by an appropriate baroclinicity. Also the non - stationarity has an effect on the low level jet.

Of course the phenomenon of low level jet can not be simulated by a GCM since its vertical resolution is not sufficient.

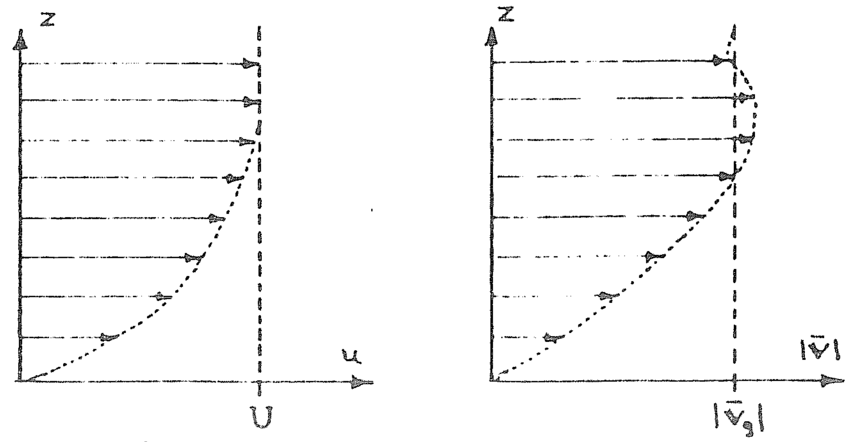


Fig. 3.1

The displacement-thickness for a boundary layer is defined as that distance, up to which the surface must be elevated in order to have the same mass transport by the flow, which is completely uneffected by the surface.

With that definition for the displacement thickness it can be written

$$\delta^* = \int_0^\delta (1 - u/U) \, dz \qquad (3.1).$$

However in the ABL (or PBL) one has due to the rotation of the earth a turning of the wind with height, the hodograph is like a spiral, see fig. 2.1 . Therefore we have two velocity components, by which two different displacement heights can be formed

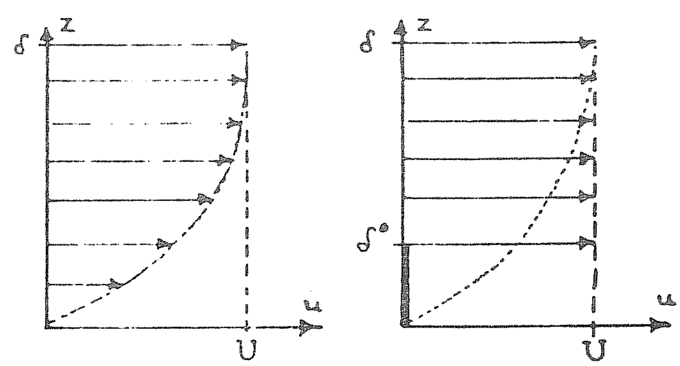


Fig. 3.2

$$\delta^*_u = \int_0^\delta (\bar{u}_g - \bar{u}) \, dz/ \bar{u}_g \quad , \quad \delta^*_v = \int_0^\delta (\bar{v}_g - \bar{v}) \, dz/ \bar{v}_g \qquad (3.2)$$

Inserting herein the velocity defects of eqs.(2.2) and (2.1) one obtains

$$\delta^*_u = \{\tau_y(\delta) - \tau_{yo}\}/(f \bar{\varrho} \bar{u}_g) \quad , \quad \delta^*_v = \{\tau_{xo} - \tau_x(\delta)\}/(f \bar{\varrho} \bar{v}_g)$$

It can be seen that one of these two displacement-thicknesses vanishes, if one choses the antitriptic coordinate system; then one has the boundary conditions

$$z = o: \quad \tau_x(\delta) = \tau_y(\delta) = 0 \quad , \quad z = 0: \quad \tau_{xo} = \bar{\varrho} \, u_*^2 \quad \tau_{yo} = 0$$

Using these conditions in the foregoing formulae one gets for the displacement-thickness of the PBL

$$\delta^* = u_*^2/(f \, \bar{v}_g) \qquad (3.3)$$

Since $\bar{v}_g = |\bar{v}_g| \sin(|\alpha_o|)$ the eq. (3.3) can be written

$$\delta^* = C_g u_* / \{f \sin(|\alpha_o|)\} = H_* C_g / \{\varkappa \sin(|\alpha_o|)\}$$

or with the abbreviation

$$\varkappa \sin(|\alpha_o|)/C_g = N \tag{3.4}$$

$$\delta^* = H_* / N \tag{3.5}$$

In a later chapter it will be shown, that N is a universal function, i.e. it is independent of any external parameter. This N appears in the resistance law, see section 5.2.1., which will be used in the parameterization procedure. With (3.5) one has found out, that N is nothing else than the displacement thickness of the PBL nondimensionalized with the internal scale height H_*.

3.2. <u>Similarity in the PBL</u>

We speak about similarity in the PBL, if the vertical profiles of the considered variables (e.g. the Reynolds' stress, the velocity defect, the eddy diffusion coefficient or others) nondimensionalized in the appropriate way will be independent on any e x t e r n a l parameter.

The proof of an existing similarity, i.e. of an appropriate nondimensionalization, can be undertaken by considering the equations and the boundary conditions governing the processes in the boundary layer.

The PBL-equations (2.1)-(2.3) read in a coordinate system, the orientation of which is not yet fixed, and for a baroclinic PBL

$$d/dz(\tau_x/\bar{\varrho}) = - f\{\bar{v} - \bar{v}_{go}\} + f^2 \lambda_y z \tag{3.6a}$$

$$d/dz(\tau_y/\bar{\varrho}) = f\{\bar{u} - \bar{u}_{go}\} - f^2 \lambda_x z \tag{3.6b}$$

$$d/dz(q/c_p \varrho) = 0 \tag{3.6c}$$

The flux-gradient relations (2.5)-(2.7) have the form

$$\tau_x/\varrho = k_m \, d/dz(\bar{u} - \bar{u}_{go}) \tag{3.7a}$$

$$\tau_y/\varrho = k_m \, d/dz(v - v_{go}) \tag{3.7b}$$

$$q/c_p\bar{\varrho} \quad k_h \, d/dz(\bar{\vartheta} - \bar{\vartheta}_T) \tag{3.7c}$$

The closure hypothesis should be

$$k_m(z) = \text{prescribed}, \quad k_h(z) = \text{prescribed} \qquad (3.8a),(3.8b)$$

Since the 6 equations (3.6a)-(3.7c) are coupled, 12 boundary conditions are needed, 6 at the top ($z = z_T$) and 6 at the bottom z_o

$$z = z_T: \quad \tau_{xT}/\bar{\varrho} = k_{mT} f \lambda_x \quad , \quad \tau_{yT}/\bar{\varrho} = k_{mT} f \lambda_y \qquad (3.9\ a,b)$$

$$(\bar{u} - \bar{u}_{go})_T = f \lambda_x z_T \ , \quad (\bar{v} - \bar{v}_{go})_T = f \lambda_y z_T \qquad (3.9\ c,d)$$

$$q_T/c_p\bar{\varrho} = q_o/c_p\bar{\varrho} \quad , \quad (\bar{\vartheta} - \bar{\vartheta}_T)_T = 0 \qquad (3.9\ e,f)$$

$$z = z_o: \quad \tau_{xo}/\bar{\varrho} = u_*^2 \quad , \quad \tau_{yo}/\bar{\varrho} = v_*^2 \qquad (3.10\ a,b)$$

$$(\bar{u} - \bar{u}_{go})_o = - \bar{u}_{go} \quad , \quad (\bar{v} - \bar{v}_{go})_o = - \bar{v}_{go} \qquad (3.10\ c,d)$$

$$q_o/c_p\bar{\varrho} = |v_*|\vartheta_* \quad , \quad (\bar{\vartheta} - \bar{\vartheta}_T)_o = \bar{\vartheta}_o - \bar{\vartheta}_T \qquad (3.10\ e,f)$$

The prescription (3.8 a,b) of $k_m(z)$ and $k_h(z)$ cannot be completely arbitrary, it must regard the well-known behaviour in the surface layer

$$k_m(z) = \varkappa |v_*| z \, \Phi_m^{-1}(z/L_*) \qquad z_o \leq z < z_p \qquad (3.11a)$$

$$k_h(z) = \varkappa |v_*| z \, \Phi_h^{-1}(z/L_*) \qquad z_o \leq z < z_p \qquad (3.11b)$$

The eqs. (3.6 a-c),(3.7 a-c) with the boundary conditions (3.9 a-f),(3.10 a-f) and the closure hypotheses (3.8 a,b), which have to regard (3.11 a,b), form a closed system; it is suitable to try if a nondimensionalization could be found, which makes the whole system free of external parameters. In that context it is irrelevant which kind of closure is chosen; the prescription of $k_m(z)$ and $k_h(z)$ has been chosen for the sake of simplicity. The neglection of the moisture is likewise unimportant in this context.

For the nondimensionalization a length-scale Λ , a velocity-scale w and a temperature-scale ϑ_c is used:

$$\tau_x/\bar{\varrho} = w^2 F_u \qquad \tau_y/\bar{\varrho} = w^2 F_v$$

$$q/c_p\bar{\varrho} = w \, \vartheta_c \, F_\vartheta \qquad (3.12\ a-c)$$

$$\bar{u} - \bar{u}_{go} = w \, D_u \qquad \bar{v} - \bar{v}_{go} = w \, D_v$$

$$\bar{\vartheta} - \bar{\vartheta}_T = \vartheta_c \, D_\vartheta \qquad (3.13\ a-c)$$

$$k_m = w \Lambda K_m \qquad k_h = w \Lambda K_h \qquad (3.14\ a,b)$$

$$z = \Lambda Z \qquad (3.15)$$

With this scaling the eqs.(3.6 a-c),(3.7 a-c) and the boundary conditions (3.9 a-f),(3.10 a-f) read

$$dF_u/dZ = - I D_v + I^2 \lambda_x Z \quad , \quad dD_u/dZ = F_u/K_m \qquad (3.16\ a,b)$$

$$dF_v/dZ = I D_u - I^2 \lambda_y Z \quad , \quad dD_v/dZ = F_v/K_m \qquad (3.16\ c,d)$$

$$dF_\vartheta/dZ = 0 \quad , \quad dD_\vartheta/dZ = F \; /K_h \qquad (3.16\ e,f)$$

$$Z = Z_T: \quad F_{uT} = I K_{mT} \lambda_x \quad , \quad D_{uT} = I Z_T \lambda_x \qquad (3.17\ a,b)$$

$$F_{vT} = I K_{mT} \lambda_y \quad , \quad D_{vT} = I Z_T \lambda_y \qquad (3.17\ c,d)$$

$$F_{\vartheta T} = F_{\vartheta o} \quad , \quad D_{\vartheta T} = 0 \qquad (3.17\ e,f)$$

$$Z = Z_o: \quad F_{uo} = u_*^2/w^2 \quad , \quad D_{uo} = - \bar{u}_{go}/w \qquad (3.17\ g,h)$$

$$F_{vo} = v_*^2/w^2 \quad , \quad D_{vo} = - \bar{v}_{go}/w \qquad (3.17\ i,j)$$

$$F_{\vartheta o} = |v_*| \vartheta_*/(w \, \vartheta_c) \quad , \quad D \;_o = (\bar{\vartheta}_o - \bar{\vartheta}_T)/\vartheta_c \qquad (3.17\ k,l)$$

Prescribed: $K_m(Z)$ with $K_m = \varkappa\,|v_*|\,Z\,\Phi_m^{-1}(\mu Z)/w$ (3.18a)

$K_h(Z)$ with $K_h = \varkappa\,|v_*|\,Z\,\Phi_h^{-1}(\mu Z)/w$ (3.18b)

both for $Z_o \leq Z < Z_p$

I is an abbreviation for

$$I = f\Lambda / w \quad (3.19)$$

Similarity exists, if Λ and w can be chosen in such a way that $I = 1$ and also the boundary conditions will be free of any parameter. We will try to reach that by scaling (a) with internal and (b) with external parameters:

(a) with $w = |v_*|$, where $|v_*| = u_*$, i.e. in an antitriptic coordinate system ($v_* = 0$, $\lambda_x = \lambda_{*x}$, $\lambda_y = \lambda_{*y}$), the scale-height becomes according to $I = 1$ in (3.19) $\Lambda = u_*/f$; furthermore with $\vartheta_c = \vartheta_*$ every thing becomes free of any parameter, except the boundary conditions (3.17 h and j), which will read $D_{uo} = -\sin(|\alpha_o|)/C_g$ and $D_{vo} = \cos(|\alpha_o|)/C_g$.

(b) with $w = |\bar{v}_{go}|$, where $|\bar{v}_{go}| = \bar{u}_{go}$, i.e. in a coordinate system orientated with the x-axis along the geostrophic wind ($\bar{v}_{go} = 0$, $\lambda_x = \bar{\lambda}_x$, $\lambda_y = \bar{\lambda}_y$), the scale-height becomes according to $I = 1$ in (3.19) $\Lambda = \bar{u}_{go}/f$; furthermore with $\vartheta_c = (\bar{\vartheta}_* - \bar{\vartheta}_T)$ every thing becomes free of any parameter except the boundary conditions (3.17 g,i,k), which will read $F_{uo} = C_g^2 \sin^2(|\alpha_o|)$, $F_{vo} = C_g^2 \cos^2(|\alpha_o|)$ and $F_{\vartheta o} = C_g\,C_h$.

This means that on no account similarity can exist for the fluxes F_u, F_v, F_ϑ and for the defects D_u, D_v, D_ϑ together. It remains to examine if similarity can be found for the fluxes alone or the defects alone.
For this purpose the defects, respectively the fluxes are eliminated from the eqs. (3.16 a-f); one obtains two sets of equations with the appropriate boundary conditions, one for the fluxes

$$d^2F_u/dZ^2 + I\,F_v/K_m - I^2\lambda_x = 0 \quad (3.20a)$$

$$d^2F_v/dZ^2 - I\,F_u/K_m + I^2\lambda_y = 0 \quad (3.20b)$$

$$dF_\vartheta/dZ = 0 \quad (3.20c)$$

$$Z = Z_T:\quad F_{uT} = I\,K_{mT}\lambda_x\,,\quad F_{vT} = I\,K_{mT}\lambda_y \quad (3.20\ d,e)$$

$$Z = Z_o:\quad F_{uo} = u_*^2/w^2\,,\quad F_{vo} = v_*^2/w^2\,,$$

$$F_{\vartheta o} = |v_*|\,\vartheta_*/(w\,\vartheta_c) \quad (3.20\ f,g,h)$$

and the other for the defects

$$d/dZ(K_m\,dD_u/dZ) + I\,D_v - I^2\lambda_x Z = 0 \quad (3.21a)$$

$$d/dZ(K_m\,dD_v/dZ) - I\,D_u + I^2\lambda_y Z = 0 \quad (3.21b)$$

$$d/dZ(K_h\,dD\,/dZ) = 0 \quad (3.21c)$$

$$Z = Z_T:\quad D_{uT} = I\,\lambda_x Z_T\,,\quad D_{vT} = I\,\lambda_y Z_T \quad (3.21\ de)$$

$$D_{\vartheta T} = 0 \quad (3.21f)$$

$$Z = Z_o: \quad D_{uo} = -\bar{u}_{go}/w \quad , \quad D_{vo} = -\bar{v}_{go}/w$$

$$D_{\vartheta o} = (\bar{\vartheta}_o - \bar{\vartheta}_T)/\vartheta_c \qquad (3.21\ g\text{-}i)$$

Moreover to both sets (3.20),(3.21) belongs the closure hypothesis in prescribing

$$K_m(Z) \quad \text{with} \quad K_m = \varkappa\, |v_*|\, Z\, \Phi_m^{-1}(\mu Z) \qquad (3.22a)$$

$$K_h(Z) \quad \text{with} \quad K_h = \varkappa\, |v_*|\, Z\, \Phi_h^{-1}(\mu Z) \qquad (3.22b)$$

$$\text{both for} \quad Z_o \leq Z < Z_p$$

The application of internal scaling parameters ($w = |v_*|$, $\Lambda = |v_*|/f$, antitriptic coordinate system, $\vartheta_c = \vartheta_*$) allows to make the system (3.20 a-h),(3.22 a,b) completely free of any parameter; however in the system (3.21 a-i),(3.22 a,b) parameters would remain in the boundary conditions (3.21 g-i).
On the other hand applying external scaling parameters ($w = |\bar{v}_{go}|$, $\Lambda = |v_{go}|/f$, geostrophic coordinate system, $\vartheta_c = \bar{\vartheta}_o - \bar{\vartheta}_T$) one is unable to make any of the two systems free of parameters; in the system (3.20 a-h),(3.22 a,b) they will appear in the boundary conditions (3.20 f,g,h) and in the prescription (3.22 a,b); in the second system parameters will appear in the prescriptions (3.22 a,b).

It can be concluded, that similarity in the PBL exist only for the fluxes (not for the defects) and only in the case, in which nondimensionalization is done by i n t e r n a l parameters.

Since on may prescribe the profiles $K_m(Z)$ and $K_h(Z)$ as obeying likewise the similarity, it can be concluded from the flux-gradient relations (3.16 b,d,f) that the profiles of the vertical gradients of the defects must be universal, or - what is the same - the profiles dU/dZ, dV/dZ, $d\Theta/dZ$ are universal. If these profiles are integrated over Z starting at $Z = Z_T$ proceeding downwards one obtains $U_{gT} - U(Z)$, $V_{gT} - V(Z)$, $\Theta_T - \Theta(Z)$; these are again three defect-profiles, which must be universal too (the nondimensionalization is done by internal parameters !). The universality, of course, is restricted to heights Z, which are not to close to the lower boundary at $Z = Z_o$.

It can be shown, that such a similarity exists for other variables too, e.g. the nondimensional mixing-length $L_M = \ell/H_*$, the nondimensional dissipation rate of turbulent kinetic energy $\mathcal{E} = \varkappa^2 \varepsilon/(f\, u_*^2)$ or some spectral properties.

One should realize, that the lower boundary is located at $z = z_o$, the nondimensional form of which is

$$Z_o = z_o/H_* = (\varkappa\, C_g\, Ro_o)^{-1} \qquad (3.23)$$

z_o itself (or Ro_o) is an external parameter. It is true, the lower boundary conditions are free of any parameter, however the height Z_o, at which the lower boundary is located, depends on the external parameter Ro_o. Fortunately in the atmosphere Ro_o is very large, it ranges from $1 \cdot 10^5$ to $1 \cdot 10^9$; therefore Z_o in (3.23) is very small. The similarity exists only for $Z > Z_R$, where roughly $Z_R = 10 \cdot Z_o$ (see WIPPERMANN and YORDANOV, 1972). For $Z_o \to 0$ (i.e. $Ro_o \to \infty$) similarity would exist at any height Z in the PBL; since Ro_o is very large in the atmosphere and therefore close to the condition $Ro_o \to \infty$ we call this kind of a similarity a Rossby-number similarity.

To summarize: In the PBL exist a similarity (called Rossby number similarity) o n l y for variables, which are nondimensionalized by i n t e r n a l parameters. Similarity means, that the vertical profiles of these variables are i n d e p e n d e n t o f a n y e x t e r n a l parameter, they depend only on an internal parameter for the stratification (μ) and on two internal parameters for the baroclinicity (λ_{*x}, λ_{*y}). The variables for which similarity exist are listed in the following table (the nondimensionalization is slightly changed by including also the v.Karman's constant $\varkappa$)

$F_u = \tau_x/(\bar{\varrho}u_*^2)$ $F_v = \tau_y/(\bar{\varrho}u_*^2)$ $F_\vartheta = q/(c_p\bar{\varrho}u_*\vartheta_*)$

$K_m = k_m f/(\varkappa^2 u_*^2)$ $K_h = k_h f/(\varkappa^2 u_*^2)$

$dU/dZ = d(\varkappa\bar{u}/u_*)/d(zf/\varkappa u_*)$ or $D_u = \varkappa(\bar{u} - \bar{u}_{gT})/u_*$

$dV/dZ = d(\varkappa\bar{v}/u_*)/d(zf/\varkappa u_*)$ or $D_v = \varkappa(\bar{v} - \bar{v}_{gT})/u_*$

$d\Theta/dZ = d(\bar{\vartheta}/\vartheta_*)/d(zf/\varkappa u_*)$ or $D_\vartheta = (\bar{\vartheta} - \bar{\vartheta}_T)/\vartheta_*$

Similarity exists for $Z \gg Z_0$ and only by using the a n t i t r i p t i c coordinates.

3.3. Universal profiles in the PBL as obtained by a single-layer model

Universal profiles of the several variables are very suitable to study the structure of the PBL. In this section some of such profiles are shown; they are obtained by a numerical PBL - model (single-layer model, closure by a mixing-length hypothesis), see WIPPERMANN (1971,1973 p. 95-110).

3.3.1. Universal profiles of the velocity-defect and of the Reynolds' stress

Fig. 3.3 shows the profiles for the two defect components $D_u(Z)$ and $D_v(Z)$ in the neutral ($\mu = 0$) and barotropic ($\lambda_x = \lambda_y = 0$) case. The dots are observations (Leipzig wind profile) for a comparison. For $Z < 0.001$ (the lowest level in the figure) D_v remains constant (= 4.6 , it is again the value of N in eq.3.4), but D_u decreases linearly (because of the logarithmic ordinate).

In fig. 3.4 only the $D_u(Z)$ profile is displayed, however for three different stratifications. There are also the heights indicated, where the profiles deviate for 10 % from the logarithmic profiles (broken lines). Note the overshooting, which is increased with increasing stability.

Fig. 3.5 shows universal wind spirals in a barotropic PBL for different stratifications. The numbers give the nondimensional height Z. For $Z < 1\cdot10^{-5}$ the hodographs can be extended by a straight line to the left with D_v being constant.

In order to get an impression how the baroclinicity effects such wind spirals a figure by WIIN-NIELSEN (1974) is presented, fig. 3.6 . It shows spirals for the special case $|\bar{v}_{g0}| = 10$ m/sec, $|\bar{v}_{gT} - \bar{v}_{g0}|/z_T = 4\cdot10^{-3}$ sec^{-1}, k_m = const = 3.6 m^2/sec; these hodographs are not universal as those given in fig. 3.5; they are

Fig. 3.3

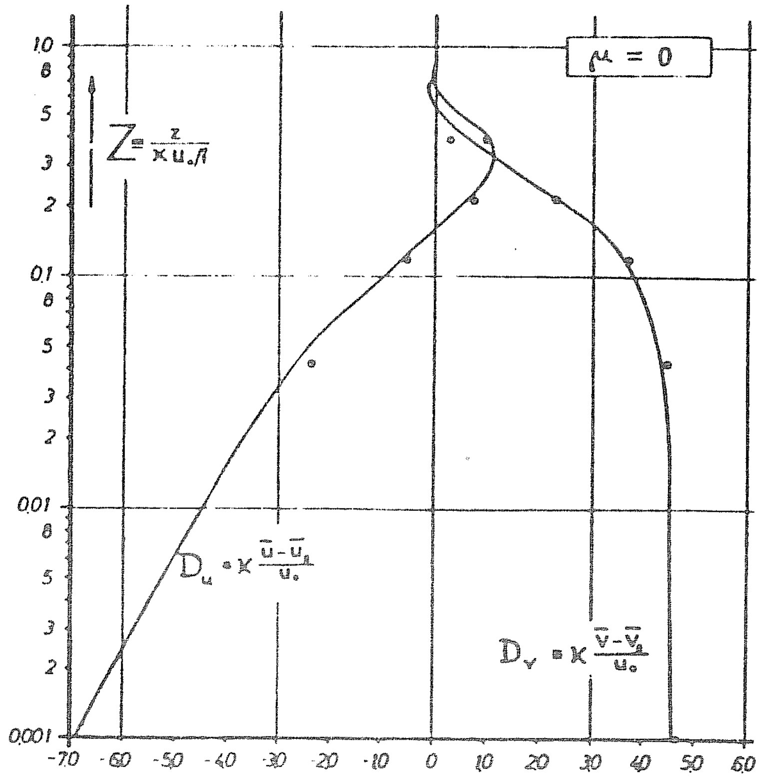
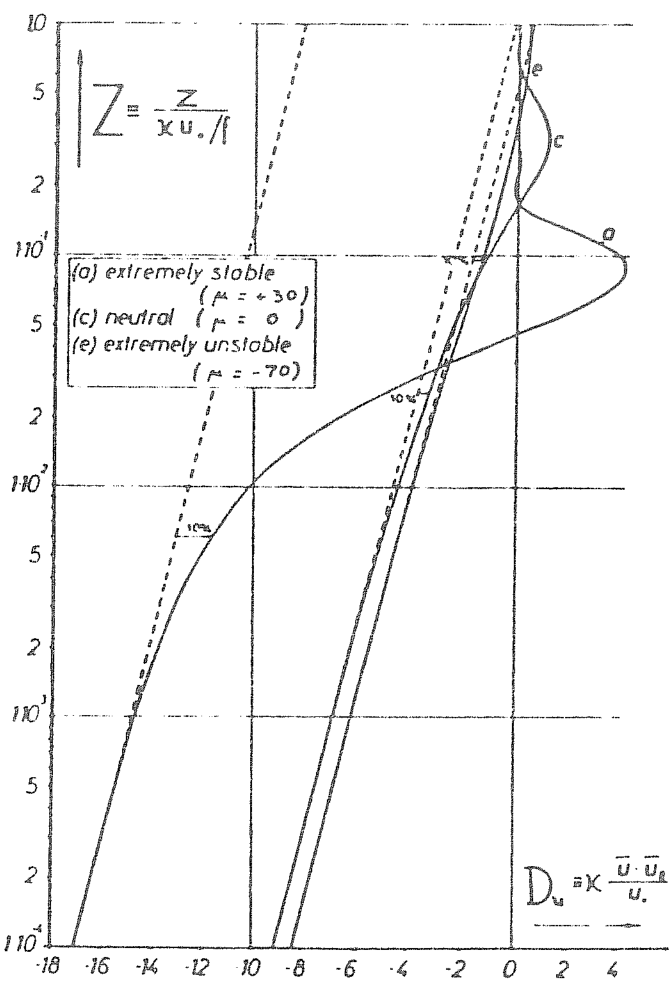


Fig. 3.4



presented for different surface wind directions but for the same thermal wind; the degree - numbers give the veering between 150 m and 1000m.

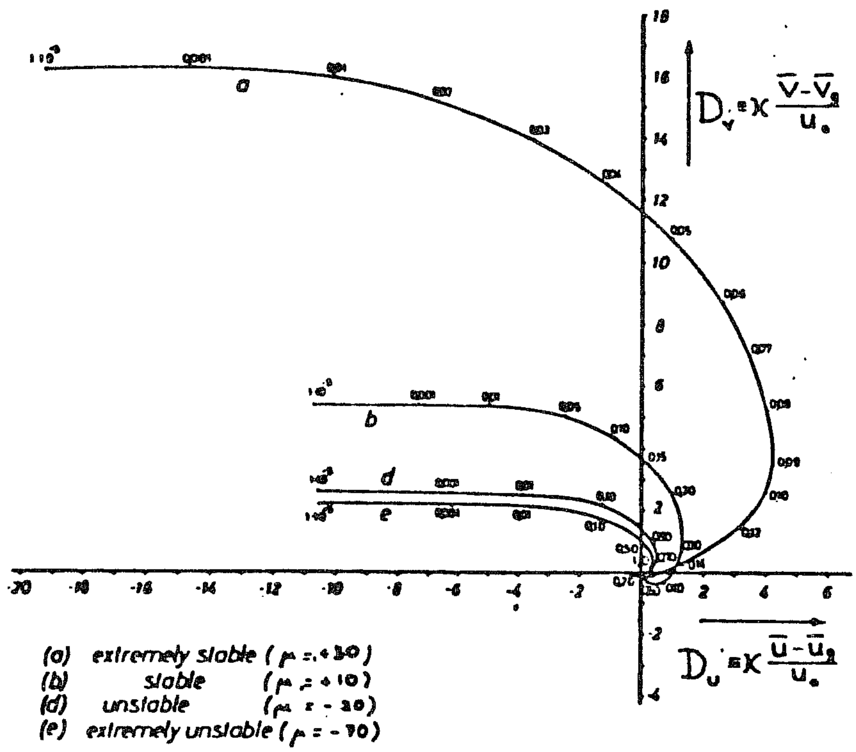
The stress-spirals corresponding to the wind-spirals in fig. 3.5 are shown in fig. 3.7; the numbers give the nondimensional height Z.

In the case that a constant flux layer would exist (let say up to height $Z \sim 0.02$) such a number indicating the height should appear at $F_u = 1$, $F_v = 0$. A figure giving $(F_u^2 + F_v^2)^{1/2}$ as depending on Z would show more clearly that no constant flux layer exist in the lower part of the PBL. On the contrary $d(F_u^2 + F_v^2)^{1/2}/dZ$ is largest at the surface. In this context it is interesting to know, that

$$(dF_u/dZ)_o = - N \qquad (3.23)$$

where N is known from the eq. (3.5) as being the reciprocal of the nondimensional displacement thickness. N appears also as a universal function in the resistance law of the PBL, which will be considered in a later section.

Fig. 3.5



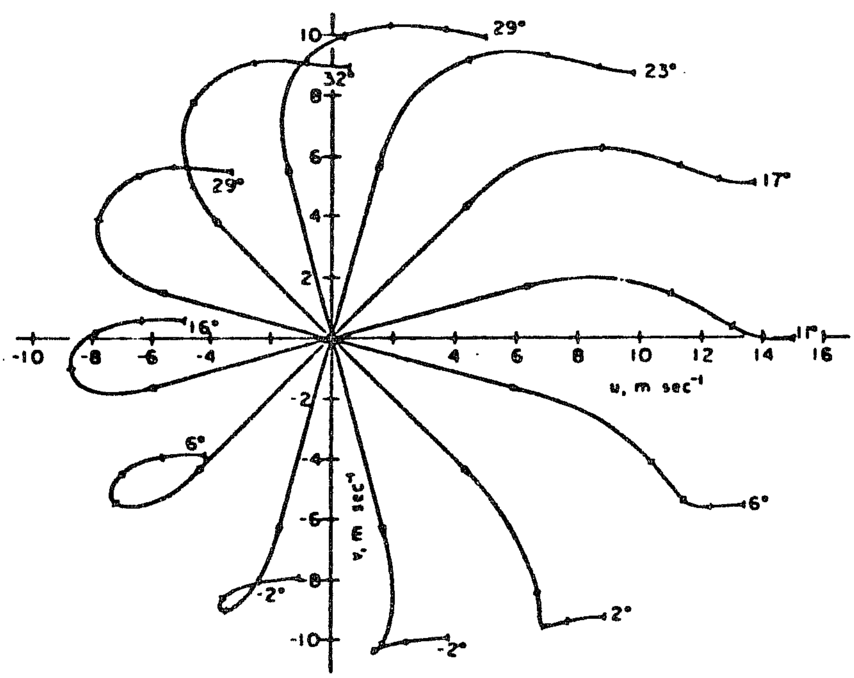
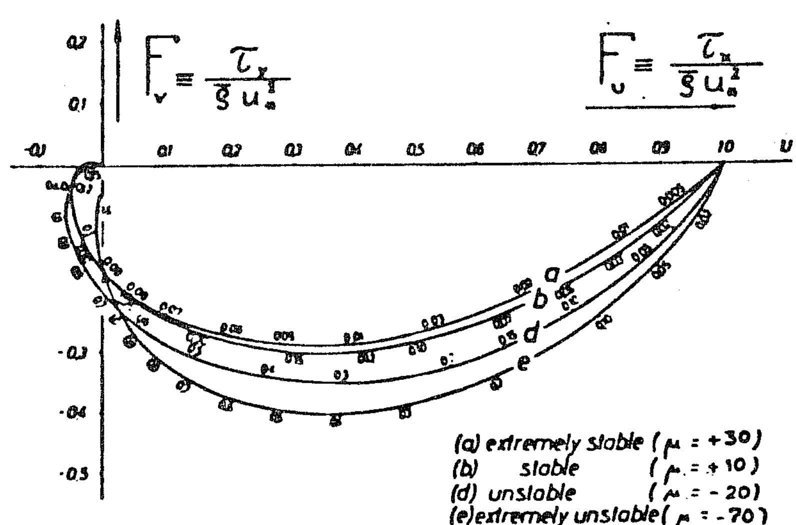


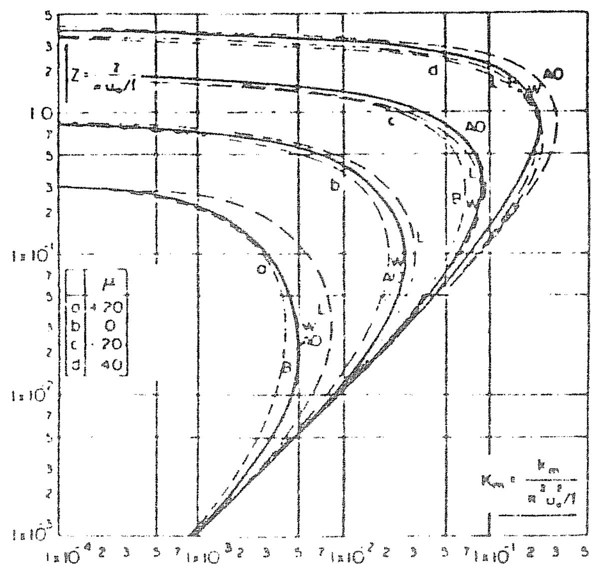
Fig. 3.6

Fig. 3.7



3.3.2. Universal profiles of the eddy diffusion coefficient

In figure 3.8 the profiles $K_m(Z)$ are shown for 4 different thermal stratifications, all in a barotropic PBL ($\lambda_x = \lambda_y = 0$) The broken lines give the results bei other authors (AO = Appleby - Ohmstede, B = Blackadar, L = Lettau) using other hypotheses for the mixing-length.
Baroclinicity effects the profile $K_m(Z)$ too; in the figures 3.9a and 3.9b two examples are given for such profiles in baroclinic conditions, they are compared with the evaluations of observations.



These examples show, that a certain baroclinicity (cold air advection) may reduce the eddy viscosity considerably within rather thin layers; that occurs independently of the thermal stratification, i.e it is not caused by a temperature inversion.
It should be mentioned, that also these profiles in special baroclinic conditions are still universal ones; they depend only on the internal stratification parameter μ and on two internal parameters for the baroclinicity λ_x and λ_y.

Fig. 3.8

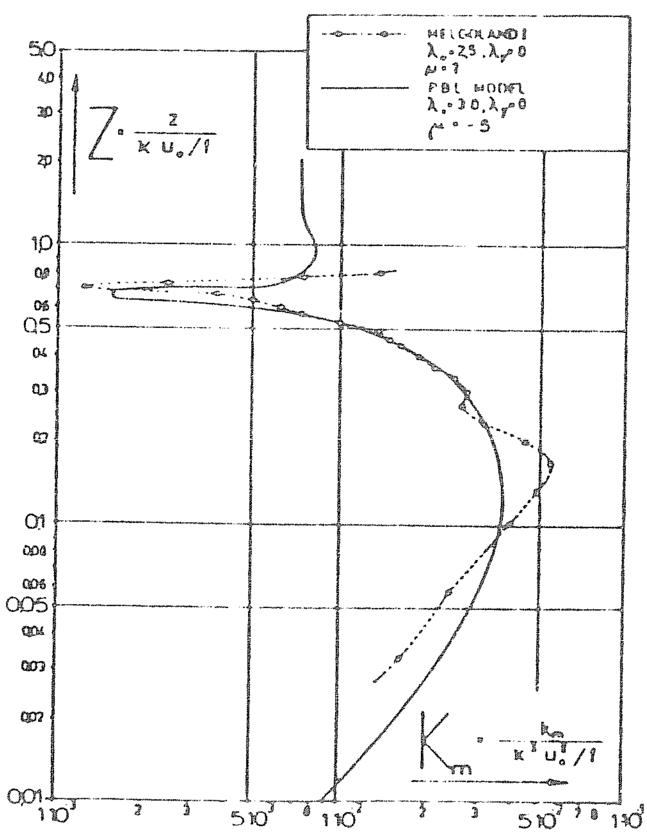


Fig. 3.9a

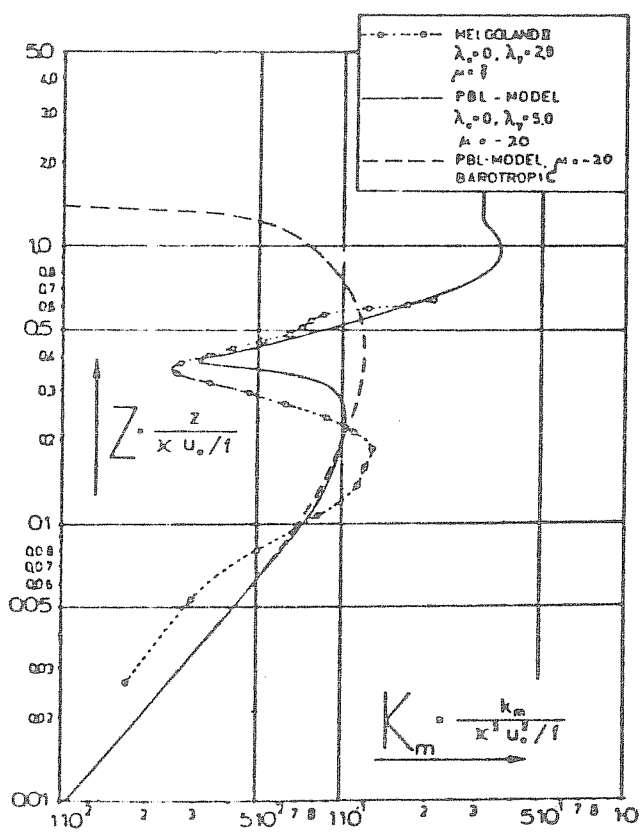


Fig. 3.9b

3.3.3. Universal profiles of the temperature defect

In figure 3.10 the universal profiles $D_\vartheta(Z)$ are shown for different thermal stratification parameters μ in a barotropic PBL according to WIPPERMANN (1975). Since D_ϑ is the temperature defect scaled by ϑ_*, in the neutral case D_ϑ equals 0 over 0, the determination of which gives the curve (μ = 0) in the fig. 3.10 .
One notices that for increasing instability the profiles $D_\vartheta(Z)$ converge to a limiting profile.
The broken lines show where Z_0 must be located for a given Ro_0.

There is no doubt, that the profiles of the temperature defect will be effected by baroclinicity. These effects seem to be very sensitiv to the closure hypothesis, they will not be discussed here.

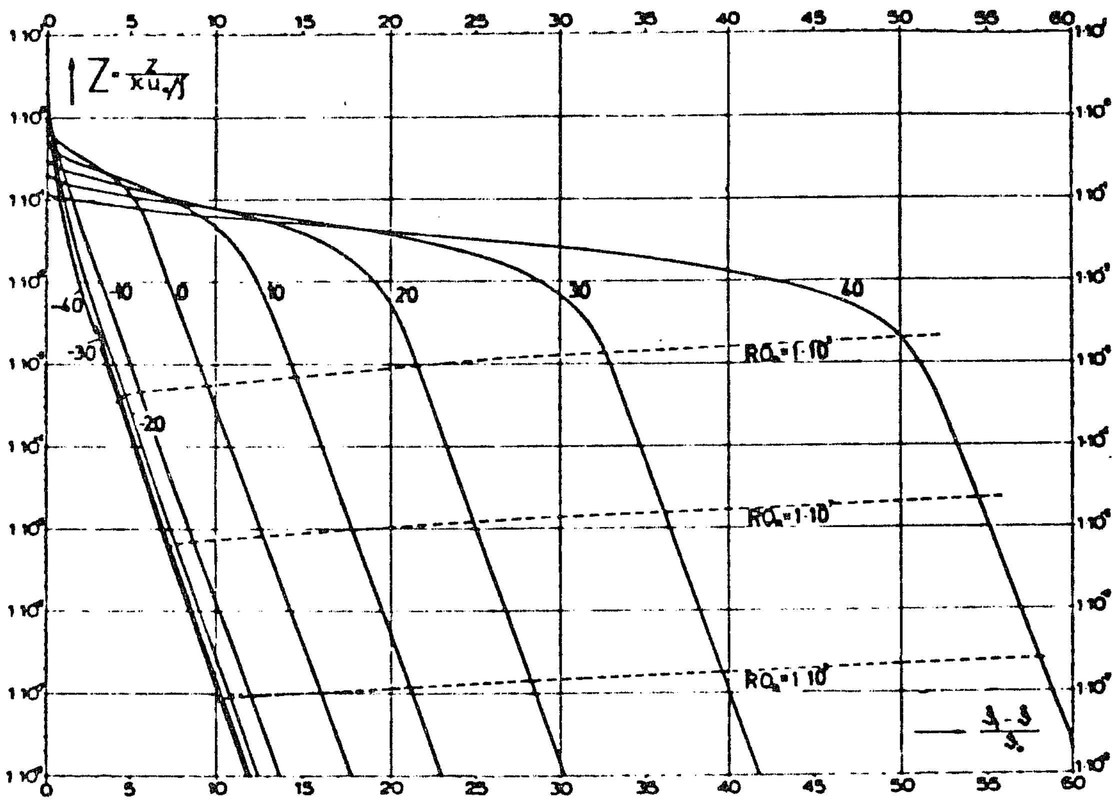
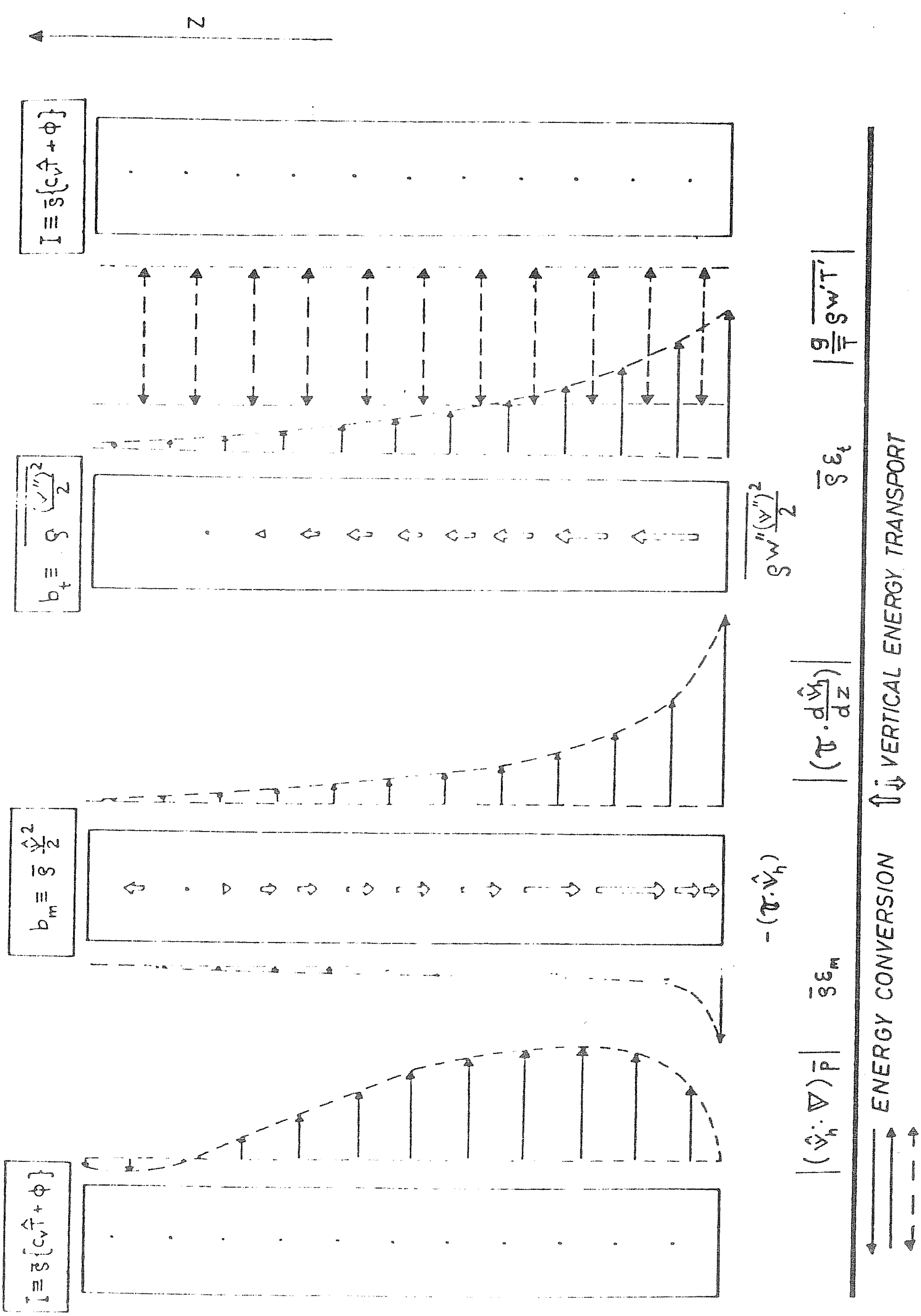


Figure 3.10

3.4. Remarks on the energy budget of the PBL

Figure 3.11 is a schematic picture showing the vertical fluxes (double arrows) of three kinds of energy, the internal and potential energy I , the kinetic energy of the mean flow b_m and the turbulent kinetic energy b_t ; the flux of I is not indicated, it is going upward for unstable stratification ($\mu < 0$) and downward for a stable one ($\mu > 0$); it is constant with height according to eq. (2.3). The transport $- \tau \cdot \bar{v}$ has a maximum in the lower part of the PBL, one of the definitions of the height z_p of the surface layer (KRAUS 1972). Notice that the turbulent flux of turbulent kinetic energy b_t is going upwards whereas the turbulent flux of momentum is goin downward.
The usual arrows indicate energy transformations. That one between I and b_t depends on the stratification, for $\mu < 0$ the transformation is $I \rightarrow b_t$ and the opposite for $\mu > 0$. The mean flow gains energy from the internal and potential one at a rate given by $- \bar{v} \cdot \nabla \bar{p}$ or $- f \, \bar{\rho} \, k \cdot [\bar{v} \times \bar{v}_g]$; the reservoir of I

Figure 3.11



is assumed to have an infinite capacity, it is always able to supply energy to b_m and b_t or to take in energy from other kinds. The assumption of an infinite capacity of I is necessary, since the PBL is steady state but looses permanently momentum.

4. The problem of the treatment of the boundary layer in GCMs

4.1. The importance of the boundary layer effects for large scale motions

The atmosphere interacts with the underlying surface; this interaction occurs through the boundary layer by vertical turbulent transports of momentum, heat and moisture. The turbulent heat flux coming from the surface or going to the surface represents a considerable part of the energy sources, resp. sinks; the same is true for the moisture flux, whereas the turbulent momentum flux (in the boundary layer always directed downward) is responsible for the rate, at which kinetic energy of the large scale motion is transformed into turbuelnt kinetic energy and then dissipated.
The reaction-time of the free atmosphere to these fluxes is 3-5 days, depending on the definition. That makes it necessary to take into account the boundary layer effects in any numerical model, by which large scale motions are simulated for a period longer than this reaction time.

It is known, that under certain conditions an upward turbulent flux of sensible and/or latent heat increases its characteristic (horizontal) length-scale while passing through the boundary layer; this is connected with the transition to more "organized" convective transports.
For a parameterization of the boundary layer effects one considers two separate scales, the small-scale and the somewhat larger convective-scale. The processes on both scales are treated differently, since only the small-scale transports are restricted to the boundary layer, whereas on the convective scale heat and/or moisture is sometimes transported upward far into the free atmosphere. That is the reason why only the small-scale turbulent transports (and their divergences) are understood as the so called boundary layer effects.

4.2. Boundary layer parameterization - what does it mean ?

The horizontal grid-size of the current GCMs is about 300 km. Since the characteristic (horizontal) length-scale of the small scale turbulent transports in the boundary layer is of the order of 10 meters and that one of the convective transports is of the order of 100 meters, these transports cannot be resolved by the GCM. In order to take into account such sub-grid effects like the turbulent transports one has to parameterize them, that means one has to establish relations for the sought sub-grid parameters as depending on the large-scale variables of the GCM. These large-scale variables of the GCM are called sometimes p a r a m e t e r s too, that is why we speak about parameterization (an expression very unprofitably chosen), if we relate the sought sub-grid parameters to the large-scale parameters (i.e. the variables of the GCM).

Since the the sought turbulent fluxes (at the surface) can be written

$$\tau_{xo}/\bar{\varrho} = u_*^2 , \quad \tau_{yo}/\bar{\varrho} = v_*^2 , \quad q_o/c_p\bar{\varrho} = |v_*| \vartheta_* , \quad j_o/\bar{\varrho} = |v_*| s_*$$

parameterization does mean relating the sought internal parameters to the given external ones.

4.3. The parameterization with respect to the vertical resolution of the GCM

Most of the current GCMs have between 5 and 8 levels in the vertical; of course, there are some having more than a dozen, but others have only two.
That means the vertical resolution of most of the current GCMs allows only for one, at most for two levels within the boundary layer. In such cases the turbulent fluxes entering or leaving the boundary layer at the surface must be determined only by the variables given in the first (or in the two first) interior grid level(s). By that one is compelled to use for the parameterization "integrated" relationships; such relationships are valid for the entire boundary layer, they are known as resistance law and transfer laws for heat and moisture.
Models with a higher resolution in the vertical may be able to place the lowest interior grid level in the surface layer. Then the turbulent fluxes at the surface can be determined by the large-scale variables in that grid level with the aid of relationships resulting from the Monin-Obukhov similarity theory. Such models have some other grid levels within the boundary layer; for those grid levels the fluxes and their divergences must be determined too whereby additional relationships are needed for a parameterization.
For the very simple GCMs with only two or three grid levels a parameterization on a physical basis is not possible. The lowest interior grid level is located outside the boundary layer, its variables are such of the free atmosphere. Therefore on can apply for a parameterization only an empirical bulk transfer formula, in which the transfer coefficient has to be prescribed.
These three kinds of parameterization will be described in chapter 5 .

4.4. The necessary assumptions of stationarity and horizontal homogeneity

As to be seen in chapter 5 the parameterization of the boundary layer effects is based on results of the similarity theory either for the whole boundary layer or for the surface layer only. However in both cases the similarity requires stationarity and horizontal homogeneity. That implies the necessity to approximate the real boundary layer by a fictive one, which is steady state and horizontally homogeneous.
It means the variation with time of the external parameters (i.e. of the large-scale variables of the GCM) is understood as a sequence of stationary states with discontinuous changes at times. WIPPERMANN (1973, p.222-243) et. al. have investigated the effect of non-stationarity on the drag coefficient and the cross-isobar angle during a clear day in summer. The deviation between the values in the non-stationary case and those in the corresponding

steady state case is considerable in the morning transition time but tolerable in any other time of the day. Since most of the current GCMs are not able to simulate the diurnal cycle (the radiative energy input is distributed equally over the whole day) no remarkable deviation will be caused by the assumption of stationarity in these models.
On the other hand models being able to simulate the diurnal cycle too need anyway a better resolution in the vertical and have probably a grid level located in the surface layer; such models have been found to be less sensitive to the assumption of stationarity.

The assumption of horizontal homogeneity does not cause an additional error, since the large-scale variables at the grid points of the GCM are anyway averages over an area of about 10^5 km^2.

5. Possible methods for the parameterization of the turbulent fluxes

As explained in section 4.3. the kind of the parameterization procedure depends strongly on the vertical resolution of the GCM, for which the parameterization has to be made. In this chapter three different parameterization procedures will be described.

5.1. Parameterization by using empirical bulk transfer formulas

At present most of the GCMs use for the purpose of parameterization of the turbulent fluxes at the surface bulk transfer formulas with prescribed transfer coefficients, although the vertical resolution of several of these models would allow a more physically based parameterization.
The bulk transfer formulae read

$$\tau_o/\bar{\varrho} = - c_{dM} \bar{v}_M^2 \qquad (5.1)$$

$$q_o/(c_p\bar{\varrho}) = - c_{hM} |\bar{v}_M| \{ \bar{\vartheta}_M - \bar{\vartheta}_o \} \qquad (5.2)$$

$$j_o/\bar{\varrho} = - c_{sM} |\bar{v}_M| \{ \bar{s}_M - \bar{s}_o \} \qquad (5.3)$$

Since the veering of the wind is not considered in such models, a relation for the cross-isobar angle (or any other parameter for the direction of the surface stress) is not needed.
The subscript "o" denotes the values at the surface whereas the subscript "M" indicates values at the first interior grid level of the GCM. There are some GCMs for which the $\bar{\vartheta}$ (resp. $\bar{s}$) and $\bar{u}$, $\bar{v}$ are not computed in the same grid level; in such cases the subscript "M" refers to different grid levels and $|\bar{v}_M|$ in (5.2), (5.3) must be obtained by interpolation.

The bulk transfer coefficients c_{dM} , c_{hM} , c_{sM} are prescribed, they are often assumed equal having a fixed numerical value. Mostly they are obtained by running the model with different values and then chosing those which give the most reasonable results. In some GCMs different transfer coefficients are assigned for sea and for land surfaces.

5.2. The use of the resistance law and the transfer laws for heat and moisture

5.2.1. The resistance law and the transfer laws for heat and moisture

If the real boundary layer can be approximated by a PBL, i.e. by a boundary layer with Rossby-number similarity (see sect.3.2.), in that boundary layer a set of "laws" can be applied, which relate the sought internal parameters α_o , C_g , C_h , C_s to the given external ones $\bar{v}_{gT}$, $\bar{v}_{go}$, $\bar{\vartheta}_T$, $\bar{\vartheta}_o$, $\bar{s}_T$, $\bar{s}_o$

$$\varkappa \sin(|\alpha_o|)/C_g = N(\mu , \lambda_x , \lambda_y) \tag{5.4}$$

$$\varkappa \cos(\alpha_o)/ C_g = \ln(C_g Ro_o) - M_m(\mu , \lambda_x , \lambda_y) \tag{5.5}$$

$$a_{ho}/ C_h = \ln(C_g Ro_o) - M_h(\mu , \lambda_x , \lambda_y) \tag{5.6}$$

$$a_{so}/ C_s = \ln(C_g Ro_o) - M_s(\mu , \lambda_x , \lambda_y) \tag{5.7}$$

Herein is

$$Ro_o = |\bar{v}_{go}|/(f z_o) \tag{5.8}$$

the surface Rossby number and

$$a_{ho} = (k_h/k_m)_o \qquad a_{so} = (k_s/k_m)_o \tag{5.9 a,b}$$

The N , M_m , M_h , M_s in (5.4)-(5.7) are universal functions, i. e. they are independent of any external parameter; they depend only on the internal stratification parameter μ and on the two internal (in antitriptic coordinates !) parameters λ_x , λ_y for the baroclinicity. Mostly N , M_m , M_h , M_s are called B , A , C , D , however in Russian and also in some French papers $N = A$ and $M_m = B$.

The resistance law (5.4),(5.5) was first derived by KAZANSKII and MONIN (1961) for the neutral and barotropic case; it has been extended to diabatic cases by BLACKADAR (1967) and simultaneously by MONIN and ZILITINKEVICH (1967) and later on also to baroclinic cases by YORDANOV and WIPPERMANN (1972).
The eqs. (5.4)-(5.7) can be derived in differnt ways; these are summarized by WIPPERMANN (1973, p. 157-163). The simplest method of derivation is based on the matching of inner and outer layer similarity theories.

5.2.2. The determination of the universal functions N and M

For an application of the four equations (5.4) - (5.7) the four universal functions N , M_m , M_h , M_s have to be known. In principle it is impossible to derive these functions (varying with the stratification and the baroclinicity) theoretically.One has to rely on the evaluation of measurements in the boundary layer, whereby for N the equation (3.5) may be very useful, which relates it to the nondimensional displacement thickness. Unfortunately only two sets of data exist (Great Plains and Wangara), which allow such an evaluation. Several authors ZILITINKEVICH and CHALIKOV (1968), CLARKE (1970), CLARKE and HESS (1974), ARYA (1975) and others have published such evaluations. In the fig. 5.1 - 5.3 the results by ARYA (1975) are shown for $N(\mu)$, $M_m(\mu)$, $M_h(\mu)$; since no suffient data are available for an evaluation of

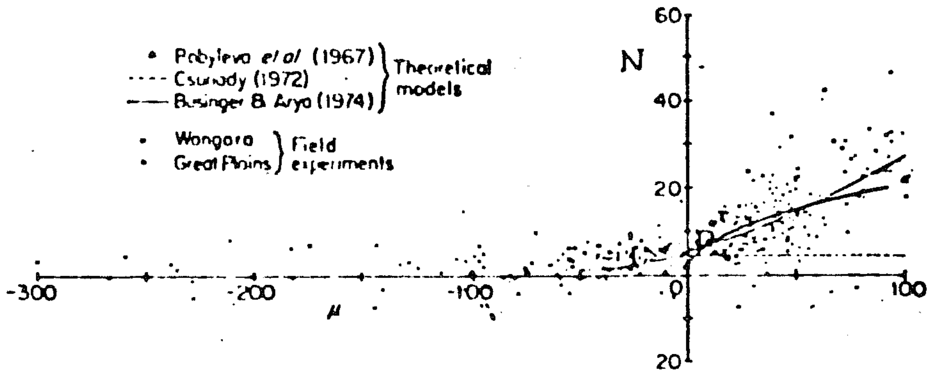


Figure 5.1

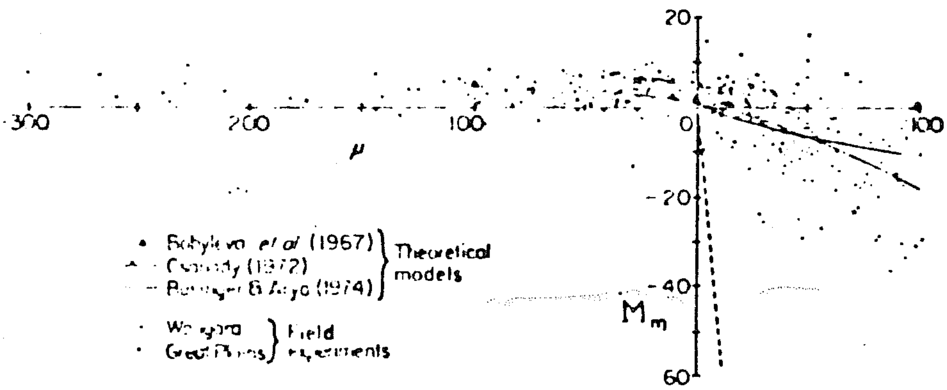


Figure 5.2

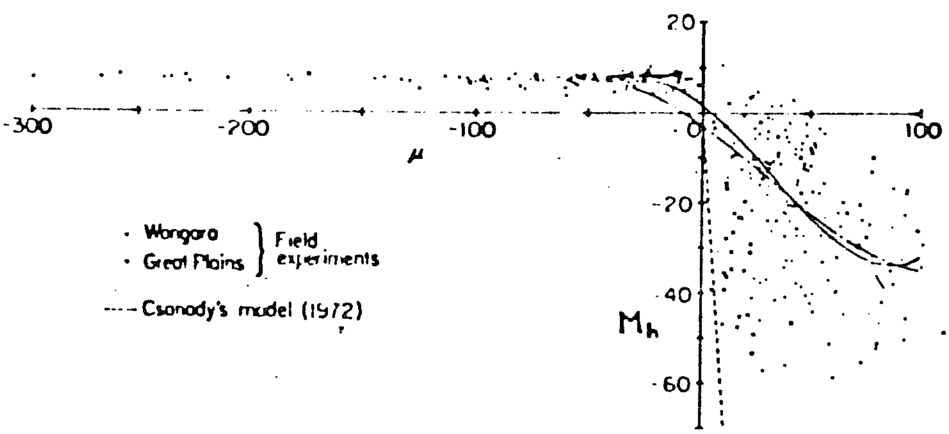


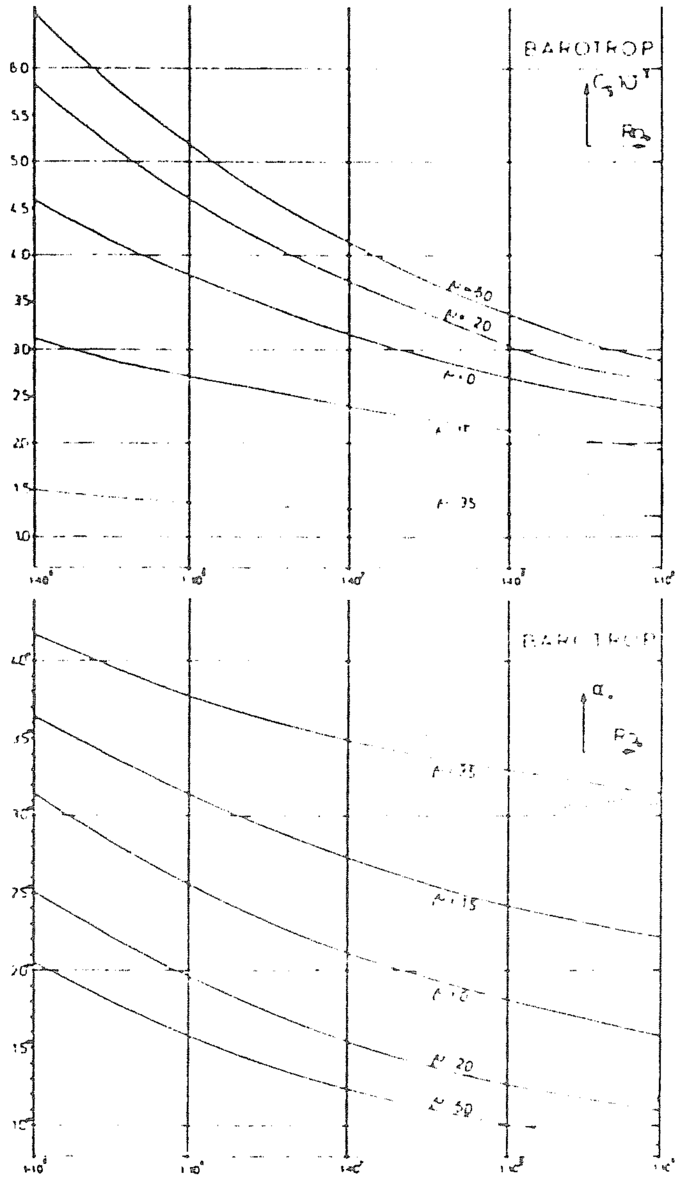
Figure 5.3

M_s , usually on puts $M_s = M_h$.

There is a very large scatter of the data, especially for the stable stratification ($\mu > 0$). ARYA (1975) gives by the thin solid lines in the figures 5.1 - 5.3 the best fitted polynomials (for the Wangara data only):

$$N(\mu,0,0) = 5.14 + 0.142\mu + 0.00117\mu^2 - 0.0000033\mu^3$$
$$M_m(\mu,0,0) = 1.01 - 0.105\mu - 0.00099\mu^2 + 0.0000008\mu^3$$
$$M_h(\mu,0,0) = -2.95 - 0.346\mu - 0.00187\mu^2 + 0.0000211\mu^3$$
$$M_s(\mu,0,0) = M_h(\mu,0,0) \qquad (5.10\ a\text{-}d)$$

With the knowledge of N , M_m and M_h one is able to construct with the aid of eqs. (5.4)-(5.6) diagrams for the sought parameters α_o , C_g , C_h as depending on Ro_o and μ . The figures 5.4 and 5.5 are examples for such diagrams based on the resistance law. They are obtained by WIPPERMANN (1973, p.180) using functions N ,



Figures 5.4a , 5.4b

M_m gained from a numerical PBL-model; they are slightly different from those given by (5.10 a,b). There exist similar diagrams for the heat transfer coefficient C_h .

There are several reasons for the large scattering of the data in the figures 5.1 - 5.3; one reason and probably the strongest is the fact of disregarding the baroclinicity in evaluating the universal functions. All observations (except the morning transition runs) have been used in the evaluation whatever the baroclinicity has been. Therefore the resulting functions N , M_m and M_h have been assigned to the barotropic case.

In the universal functions the baroclinicity may be taken into account with the aid of the formulae derived by YORDANOV (1975) for the differences of these functions in the baroclinic and the barotropic case

$$N(\mu, \lambda_x, \lambda_y) - N(\mu,0,0) = c_1(\lambda_x) + c_2(\lambda_y)$$
$$M_m(\mu, \lambda_x, \lambda_y) - M_m(\mu,0,0) = - c_2(\lambda_x) + c_1(\lambda_y) \qquad (5.11\ a,b)$$

The values of the constants are not yet well determined, they are in the neutral case roughly

$$c_1(0) = 0.1 \quad , \quad c_2(0) = 0.2 \qquad (5.11c)$$

Since the observational data are to poor for an evaluation of the constants c_1 and c_2 ,one has to rely on numerical PBL-models. Their results show a fast decrease of c_1 and c_2 with increasing stratification; this means that the effect of baroclinicity may be neglected in rather stable conditions. However for an increasing instability also the two constants will increase; see for instance WIPPERMANN (1973, p.199).

5.2.3. The determination of the effective roughness-length

A practical application of the formulae (5.4)-(5.7) requires also the knowledge of the surface Rossby number (5.8). While $|\vec{v}_{g0}|$ is computed at each grid point by the GCM and f is known, the roughness-length z_0 must be prescribed for each horizontal grid point. For this purpose it has to be determined as a value representing the roughness averaged over an area of about 300 km x 300 km. If sufficient data of the roughness are available in such an area one can obtain the needed value of z_o by forming an average of the local values z_o'

$$z_o = \exp\left\{\sum_i A_i \left[\ln(z_o')\right]_i\right\} \quad , \quad \sum_i A_i = 1 \tag{5.12}$$

where the subscript "i" stands for a special vegetation type and A_i is an area-weight of that vegetation type. The logarithmic averaging of z_o' is necessary since $C_g \propto \ln(z_o)$, see eq.(5.5). Some determinations of this kind for z_o have been published, e. g. KUNG and LETTAU (1961).
The z_o' ranges over five orders of magnitude, the averaged values z_o (over an area corresponding to the square of the grid - size of the current GCMs) still range over three orders of magnitude. At the present time it seems to be hopeless to get sufficient informations to form z_o according to (5.12) for each grid-point over land; for most of the grid-points one has to guess the values, which of course will vary with the seasons. A procedure of that kind may be still better than the use of a constant value z_o at all grid-points over land as made in most current GCMs.

It should be mentioned, that the use of an overall roughness length z_o (5.12) requires to assume a completely plain area , over which the z_o' are averaged (the square of the horizontal grid-size of the GCM); any small-scale or meso-scale topography has to be neglected. Of course, there is no doubt, that a considerable part of the total drag is caused by the topographic irregularities, but one ist still unable to determine it. Since the boundary layer flow over an irregular topography is horizontally inhomogeneous almost all of the boundary layer theories are invalid. Here is a remarkable gap in our understanding of the hydrodynamical processes in the baundary layer, it is necessary to fill it by an enhanced research work in the future.

Over the sea the situation is much different, since there z_o characterizes the stage of the sea surface, which is effected by the surface stress itself. CHARNOCK (1955) has given a relationship by dimensional arguments, it relates z_o to the surface stress

$$z_o = v_*^{\,2}/(c_3\; g) \tag{5.13}$$

For the nondimensional constant c_3 a value between 20 and 30 is appropriate; it can be shown, that a slight change in c_3 has only negligible effects on the further results.
Eq. (5.13) allows to eliminate z_o in eq. (5.5), in which now the (geostrophic) Lettau-number

$$Le = g/(f\,|\bar{v}_{g0}|) \tag{5.14}$$

appears instead of the surface Rossby-number Ro_o; the same elimination of z_o can be made in the heat transfer law (5.6), resp. in the moisture transfer law (5.7). The laws (5.4)-(5.7) have over sea the form

$$\varkappa \sin(|\alpha_*|)/C_g = N(\mu, \lambda_x, \lambda_y) \tag{5.15}$$

$$\varkappa \cos(\alpha_*)/C_g = \ln(Le/C_g) - M_m^*(\mu, \lambda_x, \lambda_y) \tag{5.16}$$

$$a_{ho}/C_h = \ln(Le/C_g) - M_h^*(\mu, \lambda_x, \lambda_y) \tag{5.17}$$

$$a_{so}/C_s = \ln(Le/C_g) - M_s^*(\mu, \lambda_x, \lambda_y) \tag{5.18}$$

where

$$M_n^* = M_n - \ln(c_3) \quad , \quad n = m, h, s \tag{5.19}$$

Instead of the diagrams in the figures 5.4a and 5.4b one can now construct diagrams, which show C_g and α_o depending on Le and μ. The figures 5.5a and 5.5b are examples for such diagrams valid over sea, corresponding to the diagrams 5.4a and 5.4b (valid over land).

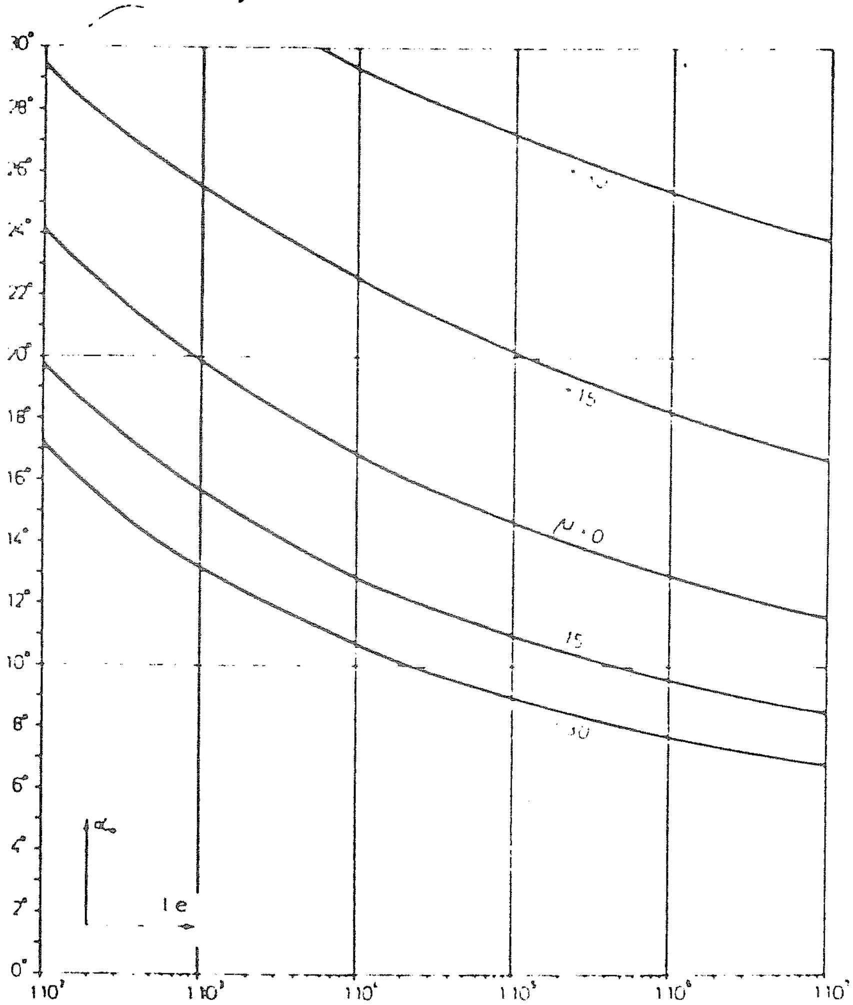


Figure 5.5a

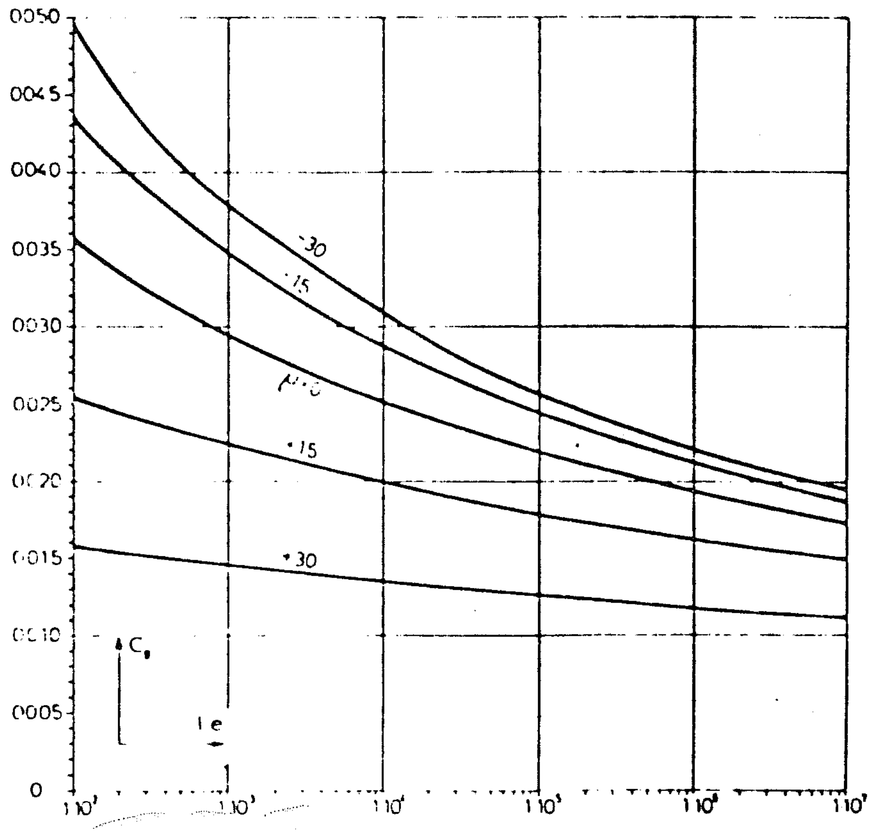
While over land the surface Rossby-number has to be used, for which the roughness-length z_o must be given, over sea the more simple Lettau-number has to be used, which is directly provided by the GCM.

5.2.4. Conversion of the given external parameters in the needed internal ones

(a) the stratification parameter

For an application of the relations (5.4)-(5.7) over land, resp. (5.15)-(5.18) over sea one has to know the internal stratification parameter $\mu = H_*/L_*$; however given is only the external stratification parameter $\sigma = Le\{\bar{S}_1 - \bar{S}_o\}/\bar{\sigma}$. The conversion of σ into μ requires an iterative procedure. CHALIKOV (1968) was the

Figure
5.5b



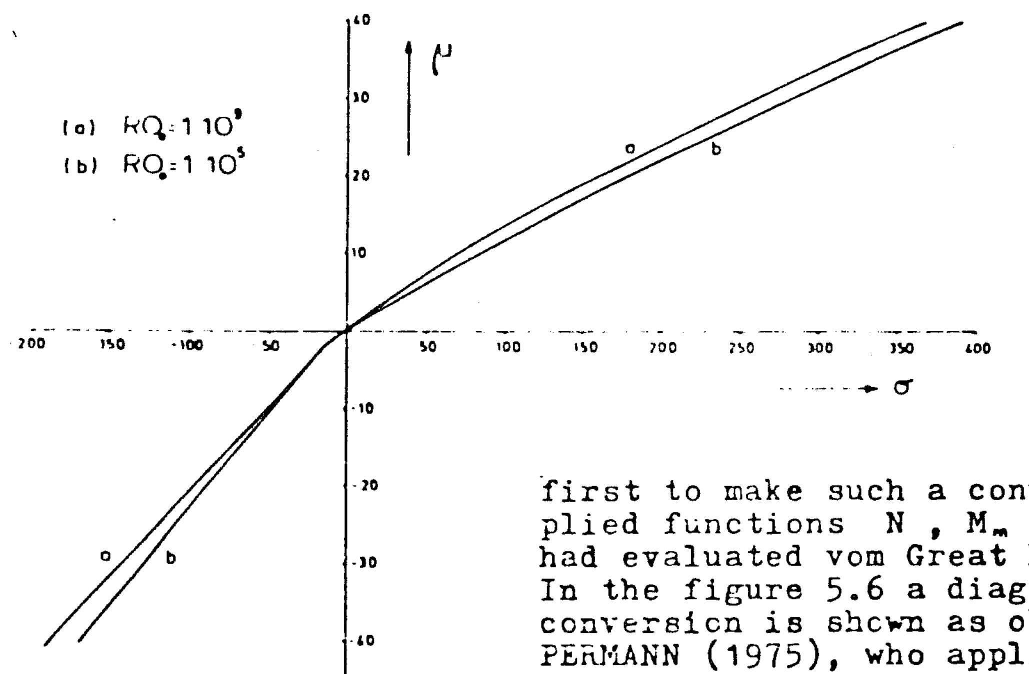


Figure
5.6

first to make such a conversion; he applied functions N , M_m , M_h ,which he had evaluated vom Great Plains data set. In the figure 5.6 a diagram for such a conversion is shown as obtained by WIPPERMANN (1975), who applied functions N , M_m , M_h obtained by a numerical PBL-model. One notices only a very small dependence on the surface Rossby-number suggesting to neglect that for practical purposes.

(b) the parameters for the baroclinicity
Just as for the stratification also the parameters for the baroclinicity have to be converted from the external (provided by the GCM) in the internal ones, which are needed. The GCM provides the thermal wind $\bar{v}_{gT} - \bar{v}_{g0}$ probably in two components according to the orientation of the grid. There is no difficulty to transform these in a component directed along the geostrophic wind at the surface $\bar{v}_{g0}$ and perpendicular to it. The thermal wind, of course, is needed in antitriptic coordinates, but the cross-isobar angle is

not yet known and depends itself on the baroclinicity. Therefore an iterative procedure would be necessary by applying eqs. (5.4), (5.5). In order to save such work it may be sufficient to rotate the coordinate system into the antitriptic one by the cross-isobar angle $\alpha_o(Ro, \mu, 0, 0)$ valid in the barotropic case (as shown for instance in figure 5.4b).

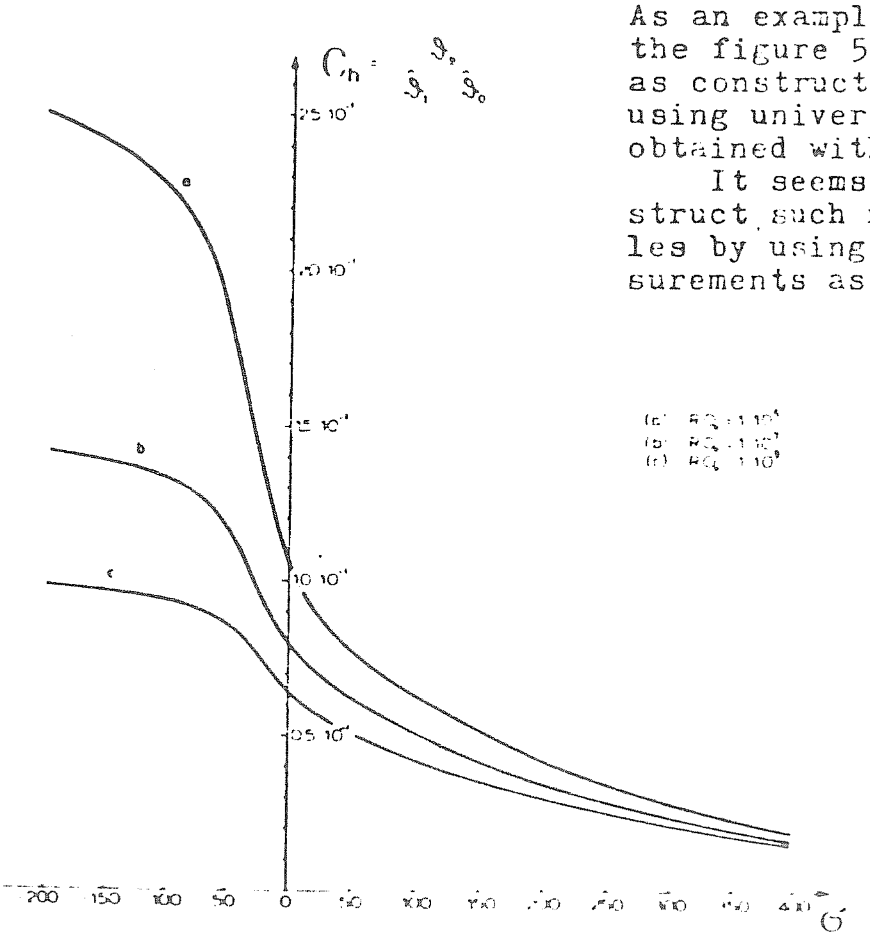
5.2.5. Formulation for a practical application

Being able to convert (with the aid of a diagram like that one in figure 5.6 or a corresponding table) the external stratification parameter σ given by the GCM into the needed internal stratification parameter μ one can compute the sought C_g, α_o, C_h and C_s by eqs. (5.4)-(5.7), resp. (5.15)-(5.18) over sea. Such computations have to be made only once. One obtains

over land:	$\alpha_o(\sigma, Ro_o)$	over sea:	$\alpha_o(\sigma, Le)$	(5.20)
	$C_g(\sigma, Ro_o)$		$C_g(\sigma, Le)$	
	$C_h(\sigma, Ro_o)$		$C_h(\sigma, Le)$	
	$C_s(\sigma, Ro_o)$		$C_s(\sigma, Le)$	

These functions can be nomographed or tabulated. Nomograms of this kind have been presented first by CHALIKOV (1968), who used the Great Plain data set for the evaluation of the universal functions N and M needed in the computation. CLARKE (1970) evaluated the functions N and M from the Wangara data set and constructed such diagrams too. Some other authors have given similar nomograms As an example for such nomograms in the figure 5.7 $C_h(\sigma, Ro_o)$ is shown as constructed by WIPPERMANN (1975) using universal functions N and M obtained with a numerical PBL-model.

It seems to be the time to construct such nomograms (5.20) or tables by using all available data, measurements as well as results by the different theoretical PBL-models, every thing reasonably weighted.



One should keep in mind that all the nomograms mentioned above are valid for a barotropic PBL only, since the universal functions N and M, which are needed for the construction have been evaluated from data disregarding the baroclinicity. For the time being one may consider the real atmospheric boundary layer as being approximated by a barotropic one, since the observational data are not yet sufficient to allow an evaluation of correct values of the coefficients $c_1(\mu)$ and $c_2(\mu)$ in the eqs. (5.11 a,b). At a future stage one probably will have additional nomograms for corrections with respect to baroclinicity

$$\delta c_g(0, \lambda_x, \lambda_y)\ ,\quad \delta\alpha_o(0, \lambda_x, \lambda_y)\ ,\quad \delta c_h(0, \lambda_x, \lambda_y)\quad \text{and}$$
$$\delta c_s(0, \lambda_x, \lambda_y) \tag{5.21}$$

which of course give - as a first step - these corrections for neutral stability only.

5.2.6. Difficulties for a practical application

(a) The proximity to the equator

Approaching the equator $f \to 0$ and therefore $Ro_o \to \infty$, which makes the formulae (5.4)-(5.7), resp. (5.15)-(5.18) inapplicable; also the internal scale height $H_* = \varkappa u_*/f \to \infty$.

In comparison to the boundary layers in middle and high latitudes in the tropics a strong thermal convection is more predominant, the assumptions for a PBL are less satisfied there (e.g. cloud induced meso-scale circulations violate the condition of horizontal homogeneity).

At the present time a boundary layer theory for the tropic atmosphere does not yet exist, which is suitable for the purpose of parameterization of the surface fluxes. One has to rely on empirical bulk transfer formulae with prescribed transfer coefficients (see section 5.1.). An application of such bulk formulae will be necessary within a latitude belt roughly $5°N > \varphi > 5°S$; $\{f(\varphi=5°) = 1.27\cdot 10^{-5}\ sec^{-1}\}$.

(b) The determination of the temperature $\bar{\vartheta}_o$ and moisture $\bar{s}_o$

In order to form the external stratification parameter $\sigma = Le\cdot\{\bar{\vartheta}_T - \bar{\vartheta}_o\}/\tilde{\vartheta}$ by the variables of the GCM one has to know how $\bar{\vartheta}_o$, the temperature in the level z_o, depends on the temperature $\bar{\vartheta}_s$ in the level $z = 0$; only the latter one is provided by the GCM, either computed by an equation for the heat balance at the surface or (in other GCMs not having grid levels in the underlying ground or sea) given as the observed sea surface temperature and assumed temperatures over land.

The difference $\bar{\vartheta}_o - \bar{\vartheta}_s$ cannot be simply neglected although the vertical distance (z_o) between the two levels is very small; under certain conditions this temperature difference can reach a remarkable amount.

MONIN and ZILITINKEVICH (1968) have proposed an empirical formula

$$\bar{\vartheta}_o - \bar{\vartheta}_s = \vartheta_*\, 0.13\, (Re_*)^{0.45}\ ,\quad Re_* = u_* z_o/\nu \tag{5.22}$$

where ν is the viscosity of the air. Unfortunately an analog relation for $\bar{s}_o - \bar{s}_s$ is still missing.

(c) The determination of the pick-up height Z_T

The GCM, for which the parameterization is made, has to provide the external parameters, namely $\bar{v}_{gT}$, $\bar{v}_{go}$, $\bar{\vartheta}_T$, $\bar{\vartheta}_o$, $\bar{s}_T$, $\bar{s}_o$. In order to pick-up the values $\bar{\vartheta}_T$ and $\bar{s}_T$ one has to know the height Z_T of the top of the thermal boundary layer. This height will generally not coincide with the lowest interior grid level, at which the GCM computes the $\bar{\vartheta}$ and $\bar{s}$; an interpolation will

be necessary.
What is the value of Z_T ? The similarity theory requires, that z_T is proportional to the internal scale height H_* ,where the factor of proportionality depends only on the internal stratification parameter and on the two internal parameters for the baroclinicity

$$z_T = c_4 H_* \quad , \quad Z_T = c_4(\mu, \lambda_x, \lambda_y) \qquad (5.23)$$

There are more than a dozen proposels for the definition of z_T by different authors, but unfortunately most of them contain $|\vec{v}_g|$ instead of $|v_0|$, also many are restricted to a special stability category. ZILITINKEVICH (1972) shows, that $c_4 \propto \mu^{-1/2}$ in stable conditions and $c_4 \propto (-\mu)^{1/2}$ in unstable ones, where the latter result is doubtful.
For the neutral case ($\mu = 0$) the following $Z_T = z_T/H_*$ have been proposed

	Z_T =
HANNA (1969)	0.50
BLACKADAR and TENNEKES (1968)	0.62
CLARKE (1970)	0.75
ZILITINKEVICH et al. (1967)	1.00
MONIN (1970)	1.00
DEARDORFF (1970)	1.12
SHIR (1973)	1.23

ETLING and WIPPERMANN (1975) published a table of Z_T-values in a barotropic PBL with different thermal stratifications and for the following four definitions of Z_T

$$(Z_T)_a = Z(\tau / \tau_o = 0.02)$$
$$(Z_T)_b = Z(\tau / \tau_o = 0.05)$$
$$(Z_T)_c = Z(D_{|v|} = 0.10) \quad , \quad D_{|v|} = (D_u^2 + D_v^2)^{1/2}$$
$$(Z_T)_d = Z(D_{|v|} = 0.50)$$

The values given in table 1 have been obtained with a numerical PBL-model

Table 5.1

μ	-40	-30	-20	-10	0	10	20	30	40
$(Z_T)_a$	2.40	1.93	1.49	0.98	0.57	0.38	0.23	0.15	0.095
$(Z_T)_b$	1.95	1.64	1.22	0.84	0.48	0.32	0.19	0.12	0.080
$(Z_T)_c$	2.00	1.85	1.50	1.08	0.66	0.47	0.31	0.19	0.115
$(Z_T)_d$	0.92	0.91	0.87	0.74	0.51	0.38	0.25	0.17	0.105

One should keep in mind that all the above given Z_T-values are valid for the dynamical boundary layer; the values for the thermal boundary layer are generally slightly different.
The error caused by an incorrectly fixed pick-up height Z_T may be rather large: the similarity theory requires that for $Z \geq Z_T$: $\bar{\vartheta} = \bar{\vartheta}_T$ and $\bar{s} = \bar{s}_T$, but the GCM will in most cases (due to the poor vertical resolution) provide a $d\bar{\vartheta}/dz \neq 0$ and $d\bar{s}/dz \neq 0$ in the range $Z \geq Z_T$; this renders the difficulties in fixing the fictive height Z_T .

At the present time most of the GCMs content themselves with a heat input by radiation equally distributed over the whole day; in such models the transfer laws for heat (5.6), resp.(5.17) and for moisture (5.7),resp.(5.18) seem to be applicable for parameterization - of course, some research work has still to be done for the determination of Z_T . However in GCMs, which include the

diurnal cycle of radiative heating - and there will be more in the future - these two transfer laws (and to some extent also the resistance law) are inapplicable at least in the daytime over land.
That is one of the reasons why some authors, e.g. DEARDORFF (1972), ZILITINKEVICH and DEARDORFF (1974), MELGAREJO and DEARDORFF (1974), ZILITINKEVICH and MONIN (1974), ZILITINKEVICH (1975) and YAMADA (1976) replace the internal scale height H_* by the boundary layer height z_B ; this height z_B is prescribed in some way, for instance as the height z_i which gives the height of the lower border of a temperature inversion. z_B could be also defined as the fixed height of the first interior grid intervall of the GCM, which is even more interesting with respect to parameterization. Of course such a substitution implies that the similarity is lost. It can be seen immediately that the similarity condition $I = 1$ with I according to (3.19) requires with $\Lambda = z_B$ a scaling velocity $w = f z_B$, which introduces via the boundary conditions a new parameter $Z_B = z_B/H_*$. This additional parameter Z_B (an external one) appears now in the resistance law and in the two laws for heat- and moisture-transfer; they are derived in the same way as eqs.(5.4)-(5.7)

$$\varkappa \sin(|\alpha_o|)/C_g = N'(Z_B, \mu, \lambda_x, \lambda_y) \qquad (5.24)\text{-}(5.27)$$

$$\varkappa \cos(\alpha_o)/C_g = \ln(Z_B) + \ln(C_g\, Ro_o) - M'_m(Z_B, \mu, \lambda_x, \lambda_y)$$

$$a_{ho}/C_h = \ln(Z_B) + \ln(C_g\, Ro_o) - M'_h(Z_B, \mu, \lambda_x, \lambda_y)$$

$$a_{so}/C_s = \ln(Z_B) + \ln(C_g\, Ro_o) - M'_s(Z_B, \mu, \lambda_x, \lambda_y)$$

Also the set of equations (5.15)-(5.18), which is valid over sea, can be modified in the same way in order to take into account the parameter z_B .
In (5.24)-(5.27) the functions N' and M' appear instead of N and M in the eqs. (5.4)-(5.7). One notices that the functions N' and M' are no longer universal ones, since they depend on the external parameter z_B .
By using the equations (5.4)-(5.7),resp.(5.15)-(5.18) the height Z_T is fixed, according to (5.23) it depends only on internal parameters. If one uses the equations (5.24)-(5.27), or a corresponding set valid over sea, one is free to chose the height z_B in a reasonable way, for instance $z_B = z_i$ .However one has to pay for it by sacrifying the similarity. The consequence for the practical application would be that the sought parameters will not depend only on σ and Ro_o (see 5.20) but also on Z_B .

The functions N' and M' in (5.24)-(5.27) depend in the barotropic case on z_B , H_* and L_* . These three parameters can either be combined to $Z_B = z_B/H_*$ and $\mu = H_*/L_*$ as done in (5.24)-(5.27) or to $\zeta_B = z_B/L_*$ and $\mu = H_*/L_*$ as done in several other papers, e.g. ZILITINKEVICH (1975). Some authors, e.g. MELGAREJO and DEARDORFF (1974), postulate that the functions N' and M' depend only on ζ_B , which corresponds to the assumption for Z_B being a function of μ only. There is some doubt, that the height z_B , at which in actual cases the variables are picked up, satisfies that assumption; if it really does satisfy the assumption the functions N' and M' are universal again.

At the present time there are only first attempts made to evaluate these stability functions from observational data; a practical application of eqs.(5.24)-(5.27) or of a similar set of equations with ζ_B instead of Z_B seems not yet possible. However in future days with improved GCMs it may become necessary to apply such relations for the purpose of parameterization; then possibly also a prognostic relation for ζ_B or Z_B will be needed, a research area which is presently more important for air pollution problems.

5.3. The application of the Monin-Obukhov similarity theory

5.3.1. The permissible maximum height of the first interior grid level

If the GCM's first interior grid level, at which $\bar{u}$ and $\bar{v}$ are computed, can be placed within the surface layer or so-called constant flux layer, empirical profiles according to the Monin - Obukhov similarity theory can be used for the parameterization. The question arises up to which height this first interior grid level can be placed in order to be sure that the similarity still exists. The height z_P of the surface layer is mostly defined as that height, at which the Reynolds' stress has decreased to 90 % of the value at the surface. Another definition would be that height, where the downward energy flux $\mathbf{\tau} \cdot \mathbf{\bar{v}}$ has its maximum. ETLING and WIPPERMANN (1975) have computed these two heights $(z_P)_1 = z(\tau/\tau_0 = 0.9)$, $(z_P)_2 = z(\mathbf{\tau} \cdot \mathbf{\bar{v}} = max)$ for different stratifications in a barotropic PBL with the aid of a numerical PBL-model. In table 5.2 these heights are listed in nondimensional form ($Z_P = z_P/H_*$)

μ	-40	-20	0	+20	+40
$(Z_p)_1$	0.053	0.045	0.023	0.010	0.004
$(Z_p)_2$	0.056	0.050	0.032	0.027	0.018

Table 5.2

One notices the strong decrease of z_P with increasing static stability.

The conclusion is, that a GCM, which should be able to treat also very stable conditions (let say $\mu > +30$) must have its first interior grid level (for $\bar{u}$ and $\bar{v}$) not higher than $Z = 0.007$ according to the first definition and $Z = 0.022$ according to the second definition. Since the internal scale height H_* is about 600 m for $\mu = +30$ and $Ro = 1 \cdot 10^7$ and a usual roughness-length in middle latitudes, the first interior grid level should not be placed higher than roughly 10 m.

5.3.2. The wind-, temperature and moisture profiles in the surface layer

According to Monin-Obukhov the following relations are valid in the surface layer

$$d\bar{u}/dz = \Phi_m(z/L_*)\, u_*/(\varkappa z) \qquad (5.28)$$

$$d\bar{v}/dz = 0 \qquad (5.29)$$

$$d\bar{S}/dz = \Phi_h(z/L_*)\, S_*/(a_{ho}\, z) \qquad (5.30)$$

$$d\bar{s}/dz = \Phi_s(z/L_*)s_*/(a_{so}\ z) \qquad (5.31)$$

Φ_m, Φ_h, Φ_s are the so-called profile-functions, they depend only on $\zeta = z/L_*$ (= μZ in the notation of the foregoing section). Similar to the functions N and M in section 5.2.2. also these profile functions cannot be derived theoretically, they have to be evaluated from observations. Numerous authors have presented such evaluations, here those by BUSINGER et al. should be used

$z/L_* < 0$	$z/L_* > 0$	
$\Phi_m = (1 - 15\ z/L_*)^{-1/4}$	$\Phi_m = 1 + 4.7\ z/L_*$	(5.32 a,b)
$\Phi_h = 0.74(1 - 9\ z/L_*)^{-1/2}$	$\Phi_h = 0.74 + 4.7\ z/L_*$	(5.33 a,b)

$$\Phi_s = \Phi_h \qquad (5.34)$$

After replacing the profil-functions in (5.28)-(5.31) by the empirical relationships (5.32)-(5.34) the former equations can be integrated over z from z_o up to an arbitrary height z ($< z_p$). One obtains

$$U(z) = \varkappa\bar{u}/u_* = \ln(z/z_o) - \psi_1(z/L_*) \quad , \quad z/L_* < 0 \qquad (5.35a)$$

$$\psi_1(z/L_*) = \ln\{[1+\Phi_m^{-1}]^2[1+\Phi_m^{-2}]/8\} + 2\ \mathrm{tang}(\Phi_m^{-1})$$

$$U(z) = \varkappa\bar{u}/u_* = \ln(z/z_o) + 4.7\ z/L_* \quad , \quad z/L_* > 0 \qquad (5.35b)$$

$$\Theta(z) = (\bar{\vartheta} - \bar{\vartheta}_o)/\vartheta_* = 0.74\{\ln(z/z_o) - \psi_2(z/L_*)\} \quad , \quad z/L_* < 0 \qquad (5.36a)$$

$$\psi_2(z/L_*) = \ln\{[1 + 0.74\ \Phi_h^{-1}]/2\}$$

$$\Theta(z) = (\bar{\vartheta} - \bar{\vartheta}_o)/\vartheta_* = 0.74\ \ln(z/z_o) + 4.7\ z/L_* \quad , \quad z/L_* > 0 \qquad (5.36b)$$

5.3.3. Formulation for a practical application

Unfortunately the sought parameters u_* and ϑ_* appear not only on the left-hand side of the eqs. (5.35),(5.36), they are also contained in the Obukhov stability-length $L_* = u_*^2\bar{\vartheta}/(\varkappa g\vartheta_*)$ on the right-hand side. This stability length has to be obtained by a conversion from an external stability parameter, which is provided by the GCM, e.g. the temperature difference $\bar{\vartheta}_{GL} - \bar{\vartheta}_o$ between the grid level at z_{GL} and the surface. More convenient is to use a bulk-Richardson-number as external stratification parameter

$$Ri_{GL} = g\ z_{GL}\{\bar{\vartheta}_{GL} - \bar{\vartheta}_o\}/(\bar{\vartheta}\ \bar{u}_{GL}^2) \qquad (5.37)$$

Dividing (5.37) by L_* one gets the relation between z_{GL}/L_* and the bulk-Richardson-number Ri_{GL}:

$$z_{GL}/L_* = \varkappa\ Ri_{GL}\ C_{h,GL}/C_{d,GL}^{3/2} \qquad (5.38)$$

The conversion of the external stability parameter Ri_{GL} provided

Notice that $C_{d,GL} = u_*^2/\bar{u}_{GL}^2$

$C_{h,GL} = C_{d,GL}^{1/2}\ \vartheta_*/(\bar{\vartheta}_{GL} - \bar{\vartheta}_o)$

by the GCM into the needed internal stratification parameter $\zeta_{GL} = z_{GL}/L_*$ is attained by an iterative method using the eqs. (5.32) - (5.36); it yields $z_{GL}/L_* = f(Ri_{GL}, z_{GL}/z_o)$. This allows to determine from (5.32)-(5.36) the parameters

$$u_*(\bar{u}_{GL}, Ri_{GL}, z_{GL}/z_o)$$
$$\vartheta_*(\bar{\vartheta}_{GL} - \bar{\vartheta}_o, Ri_{GL}; z_{GL}/z_o)$$
$$s_*(\bar{s}_{GL} - \bar{s}_o, Ri_{GL}, z_{GL}/z_o)$$

Herewith nomograms can be constructed giving directly the transfer coefficients needed in the formulae for the fluxes at the surface

$$\tau_o/\bar{\varrho} = C_{d,GL}(Ri_{GL}, z_{GL}/z_o)\, \bar{u}^2_{GL} \qquad (5.39a)$$
$$q_o/(c_p\bar{\varrho}) = C_{h,GL}(Ri_{GL}, z_{GL}/z_o)\, \bar{u}_{GL} \{\bar{\vartheta}_{GL} - \bar{\vartheta}_o\} \qquad (5.39b)$$
$$j_o/\bar{\varrho} = C_{s,GL}(Ri_{GL}, z_{GL}/z_o)\, \bar{u}_{GL} \{\bar{s}_{GL} - \bar{s}_o\} \qquad (5.39c)$$

Usually one assumes $C_{s,GL} = C_{h,GL}$; the transfer coefficients depend on the two external parameters Ri_{GL} and z_{GL}/z_o. Nomograms for the transfer coefficients depending on these two external parameters have been constructed for example by ARYA (1976). Similar nomograms for $u_*/\bar{u}_{GL}$ and $\vartheta_*/(\bar{\vartheta}_{GL} - \bar{\vartheta}_o)$, both depending on Ri_{GL} and z_{GL}/z_o are given by ZILITINKEVICH (1970) and in a slightly different form by CLARKE (1970).

5.3.4. Difficulties for a practical application

Similar to the heat and moisture transfer laws the temperature ϑ_o and moisture $\bar{s}_o$ is needed in the eqs. (5.36a),(5.36b) (and similar equations for the moisture), whereas the GCM provides only ϑ_s and $\bar{s}_s$, the values directly on the surface. For the determination of the needed values at $z = z_o$ see the section 5.2.6.(b).

5.3.5. The determination of turbulent fluxes at grid levels in the outer layer

In most cases, in which the first interior grid level is located within the surface layer, one or more further grid levels will fall inside the boundary layer. Also at this levels the turbulent fluxes and their divergences must be determined.
For this purpose the use of the flux-gradient relationship is recommended here whereby the profiles $k_m(z)$ and $k_h(z)$ should be prescribed. For such a prescription WIPPERMANN (1974) proposes an empirical formula, obtained by a numerical PBL-model:

$$k_m(z) = \varkappa u_* z \exp\{- c_5(\mu)[z/H_*]^{0.764}\} \qquad (5.40)$$

with values of c_5 as given in table 5.3

μ	-40	-30	-20	-10	0	+10	+20
c_5	1.6	2.1	3.0	4.8	7.8	14.8	27.8

Table 5.3

u_* and therefore H_* as well as $\mu = H_*/L_*$ are known from the values at the surface, where

$$\mu = \varkappa^2 \, Ro_{GL} \, Ri_{GL} \, C_{h,GL} \, / \, C_{d,GL} \qquad (5.41)$$

with $Ro_{GL} = \bar{u}_{G}/(f\ z_{GL})$.
For $k_h(z)$ one assumes that the ratio $k_h(z)/k_m(z)$ observed in the surface layer (e.g. BUSINGER et al. 1971) should be valid in the whole PBL:

$$a_h = k_h/k_m = \Phi_m/\Phi_h = (1 + 4.7\mu Z)/(0.74 + 4.7\mu Z) \qquad \mu > 0$$
$$= (1 - 9\mu Z)^{1/2}/(1-15\mu Z)^{1/4} \qquad \mu < 0 \qquad (5.42\ a,b)$$

Because of the shape of the k_m-profiles, see fig.3.8, it may be useful to form layer-averaged values (according to the vertical grid size) to be applied in the determination of the turbulent fluxes.
Of course, the formula (5.40) is applicable only in a PBL, i.e. in a boundary layer in which the temperature profiles increase or decrease monotonously with height. If a high vertical resolution enables the GCM to simulate also temperature inversions, the formula (5.40) becomes inapplicable. In such cases one has to make additional computations for the $k_m(z)$ and $k_h(z)$ possibly by using the equation for the turbulent kinetic energy (with a closure hypothesis for the length-scale).

6. Acknowledgement

Preparing these lecture notes the author took pattern from earlier papers on this subject as there are the papers by CHALIKOV (1968), CLARKE (1970) and DEARDORFF (1972). He is also very much indebted to Prof. ARYA, University of Washington, Seattle, USA, who kindly permitted him to read the draft of a manuscript for the chapter IX "Parameterizations for Atmospheric Circulation Models" of a book on the atmospheric boundary layer to be published 1977.

7. References

ARYA, S.P.S. 1975 Geostrophic drag and heat transfer relations for the atmospheric boundary layer Quart.J.Roy.Meteor.Soc. 101, 147-161

ARYA, S.P.S. 1976 Parameterization for atmospheric circulation models Manuscript for Chapter IX of a book on the atmospheric boundary layer, to be published 1977

BLACKADAR, A.K. 1967 External parameters of the wind flow in the barotropic boundary layer of the atmosphere Proc.GARP Study Conf.Stockholm, App.IV, 11 pp

BLACKADAR, A.K. TENNEKES, H. 1968 Asymptotic similarity in neutral planetary boundary layers J.Atm.Sci. 25, 1015-1020

BUSINGER, J. et al. 1971 Flux-profile relationships in the atmospheric surface layer J.Atm.Sci. 28, 181-189

CHALIKOV, D.V. 1968 Calculation of eddy fluxes near the ground from synoptic informations
Meteor.Gidrol. No. 8, 10-19 (in Russ.)

CHARNOCK, H. 1955 Wind stress on a water surface
Quart.J.Roy.Meteor.Soc. 81, 639-642

CLARKE, R.H. 1970 Observational studies in the atmospheric boundary layer
Quart.J.Roy.Meteor.Soc. 96, 91-114

CLARKE, R.H. 1970 Recommended methods for the treatment of the boundary layer in numerical models
Austral. Meteor. Mag. 18 (2), 51-73

CLARKE, R.H.
HESS, G.D. 1974 Geostrophic departure and the functions A and B of Rossby-number similarity theory
Bound.Layer Meteor. 7, 267-287

DEARDORFF, J.W. 1970 A threedimensional numerical investigation of a idealized planetary boundary layer
Geophys.Fluid Dyn. 1, 377-410

DEARDORFF, J.W. 1972 Numerical investigation of neutral and unstable planetary boundary layers
J.Atm.Sci. 29, 91-115

DEARDORFF, J.W. 1972 Parameterization of the planetary boundary layer for use in general circulation models
Monthly Weather Rev. 100 (2), 93-106

ETLING, D.
WIPPERMANN, F. 1975 The height of the planetary boundary layer and of the surface layer
Beitr.Phys.Atm. 48, 250-254

HANNA, S.R. 1969 The thickness of the planetary boundary layer
Atm.Env. 3, 515-536

KAZANSKII, A.B.
MONIN, A.S. 1961 On the dynamic interaction between the atmosphere and the earth surface
Bull.Acad.Sci.USSR, Geophys.Ser.5, 786-788

KRAUS, H. 1972 A note on the definition of the surface boundary layer
Beitr.Phys.Atm. 45, 185-195

MELGAREJO, J.W.
DEARDORFF, J.W. 1974 Stability functions for the boundary layer resistance-laws based upon observed boundary layer heights
J.Atm.Sci. 31, 1324-1333

MONIN, A.S. 1970 The atmospheric boundary layer
Annual Rev. Fluid Mech. 2, 225-250

MONIN, A.S.
ZILITINKEVICH, S.S. 1967 Planetary boundary layer and large scale atmospheric dynamics
Proc. GARP-Study Conf.Stockholm, App.V, 37pp.

MONIN, A.S.
ZILITINKEVICH, S.S. 1968 Taking account of micro- and meso-scale phenomena in numerical models of the atmosphere
Proc.WMO-IUGG Symposium on Numerical Weather Forecasting, Tokyo

SHIR, C.C. 1973 A preliminary study of atmospheric turbulent flows in the idealized boundary layer
J.Atm.Sci. 30, 1327-1339

WIIN-NIELSEN, A. 1974 Vorticity, divergence and vertical velocity in a baroclinic boundary layer with a linear variation of the geostrophic wind
Bound.Layer Meteor. 6, 459-476

WIPPERMANN, F. 1973 The planetary boundary layer of the atmosphere, Vol. 7 of Annalen der Meteorologie - Neue Folge, pp.346
Deutscher Wetterdienst,Offenbach/M. FRG

WIPPERMANN, F. 1974 Eddy diffusion coefficients in the planetary boundary layer
Adv. in Geophys. 18A, 407-418

WIPPERMANN, F. 1975 Properties of the thermal boundary layer of the atmosphere obtained with a PBL-model
Beitr.Phys.Atm. 48, 30-45

WIPPERMANN, F.
YORDANOV, D. 1972 A note on the Rossby similarity for flows of barotropic planetary boundary layers
Beitr.Phys.Atm. 45, 66-71

ZILITINKEVICH,S.S. 1970 The dynamics of the boundary layer of the atmosphere
pp 291, Leningrad, Gidromet. Isd. (Russ.)

ZILITINKEVICH,S.S. 1975 Resistance laws and prediction equations for the depth of the planetary boundary layer
J.Atm.Sci. 32, 741-752

ZILITINKEVICH,S.S.
CHALIKOV, D.V. 1968 The laws of resistance and of heat and moisture exchange in the interaction between the atmosphere and an underlying surface
Izv.,Atm.Ocean.Phys. 4, 438-441 (Engl.ed.)

ZILITINKEVICH,S.S.
MONIN, A.S. 1974 Similarity theory for the planetary boundary layer of the atmosphere
Izv.,Atm.Ocean.Phys. 10,353-359 (Engl.ed.)

ZILITINKEVICH,S.S.
DEARDORFF,J.W. 1974 Similarity theory for a planetary boundary layer of time-dependent height
J.Atm.Sci. 31, 1449-1452

YAMADA, T. 1976 On the similarity functions A , B and C of the planetary boundary layer
J.Atm.Sci. 33, 781-793

YORDANOV, D. 1974 Parameterization of the influence of baroclinicity in the planetary boundary layer
Izv.,Atm.Ocean.Phys. 10,480-482 (Engl.ed.)

YORDANOV, D.
WIPPERMANN, F. 1972 The parameterization of the turbulent fluxes of momentum, heat and moisture at the ground in a baroclinic planetary boundary layer
Beitr.Phys.Atm. 45, 58-65

8. List of symbols

A, B, C, D = Similarity functions in eqs.(5.4)-(5.7); here denotes as M_m, N, M_h, M_s

ABL = Atmospheric Boundary Layer

A_i = Weighting factor in the determination of z_o

$a_h = k_h/k_m$ (turb.Prandtl number)

$a_s = k_s/k_m$ (turb.Schmidt number)

b_m = kinetic energy of mean flow

b_t = turbulent kinetic energy

c_p, c_v = specific heats

$C_g = u_*/|\bar{v}_{go}|$ geostroph.drag coeff.

$C_h = \vartheta_*/(\bar{\vartheta}_T - \bar{\vartheta}_o)$ heat transfer coeff. (Stanton number)

$C_s = s_*/(\bar{s}_T - \bar{s}_o)$ moisture transfer coefficient (Dalton number)

$D_u = \varkappa(\bar{u} - \bar{u}_g)/u_*$ non-dimens. velocity defect (x-component)

$D_v = \varkappa(\bar{v} - \bar{v}_g)/u_*$ non-dimens. velocity defect (y-component)

$D = (\bar{\vartheta} - \bar{\vartheta}_T)/\vartheta_*$ nondimensional temperature defect

$F_u = \tau_x/(\bar{\varrho}u_*^2)$ nondimens. eddy flux of momentum (x-component)

$F_v = \tau_y/(\bar{\varrho}u_*^2)$ nondimens. eddy flux of momentum (y-component)

$F_\vartheta = q/(c_p\bar{\varrho}u_*\vartheta_*)$ nondimens. eddy flux of heat

$F_s = j/(\varrho u_* s_*)$ nondimens. eddy flux of moisture

$f = 2\omega\sin(\varphi)$ Coriolis parameter

g = gravitational acceleration

$H_* = \varkappa u_*/f$ internal scale height

$\bar{H} = |\bar{v}_{go}|/f$ external scale height

$I = c_v\bar{\varrho}T$ internal energy per vol.

$I = f\wedge/w$ abbreviation

$\mathbb{k}$ = vertical unit vector

$j = \bar{\varrho}\,\overline{s'w'}$ turbulent flux of moisture

k_m, k_h, k_s = turbulent diffusion coefficients for momentum, heat and moisture

K_m, K_h, K_s = nondimensional turbulent diffusion coeff. $K = kf/(\varkappa^2 u_*^2)$

$Le = g/(f|\bar{v}_{go}|)$ geostrophic Lettau number

$L_* = -c_p\bar{\varrho}u_*^3/(\varkappa\beta q_o)$ or $-c_p\bar{\varrho}u_*^3/\{\varkappa\beta(q_o + h\,j_o)\}$ Obukhov stability length for a dry or moist boundary layer

$\bar{L} = \bar{v}_{go}^2/\{\beta(\bar{\vartheta}_T - \bar{\vartheta}_o)\}$ external stability length

ℓ = mixing-length

$L_m = \ell/H_*$ nondimens. mixing length

M_m, M_h, M_s = Similarity functions in eqs.(5.5)-(5.7) usually called A,C,D

M_m', M_h', M_s' = Similarity functions in eqs.(5.25)-(5.27) usually called a,c,d

N = Similarity function in eq.(5.4), usually called B

N' = Similarity function in eq.(5.24), usually called b

p = pressure

PBL = Planetary Boundary Layer

$q = c_p\bar{\varrho}\,\overline{\vartheta'w'}$ = turbulent flux of sensible heat

$Ro_o = |\bar{v}_{go}|/(f\,z_o)$ surface Rossby number

$Ro_{GL} = \bar{u}_{GL}/(f\,z_{GL})$ grid level Rossby number

$Re_* = u_* z_o/\nu$ surface Reynolds number

s = specific humidity

$s_* = -j_o/(\varkappa\bar{\varrho}u_*)$ characteristic moisture fluctuation

T = temperature

U = velocity outside the boundary layer

$U = \varkappa \bar{u}/u_*$ nondimensional velocity (x-component)

$\bar{u}$ = velocity component (x-direc.)

u_* = friction velocity, x-comp.

$V = \varkappa \bar{v}/u_*$ nondimensional velocity (y-component)

$\bar{v}$ = velocity component (y-direc.)

v_* = friction velocity, y-comp.

$\bar{\mathbb{v}}$ = horizontal velocity

$|\mathbb{v}_*| = (|\tau_o|/\bar{\varrho})^{1/2}$ friction veloc.

x = distance from the edge

$Z = z/H_*$ nondimensional vertical coordinate

z = vertical coordinate

z_o = roughness-length

$Z_o = (\varkappa C_g Ro_o)^{-1}$ nondimensional roughness-l.

α_o = cross-isobar angle

$\beta = g/\tilde{\vartheta}$ buoyancy factor

δ = boundary layer thickness

δ^* = displacement thickness

ε = rate of energy dissipation

$\Theta = \bar{\vartheta}/\vartheta_*$ nondimensional potential (virtual) temperature

$\bar{\vartheta}$ = potential (virtual) temperature

$\tilde{\vartheta}$ = reference temperature

ϑ_c = scaling temperature

$\vartheta_* = -q_o/(\varkappa c_p \bar{\varrho} u_*)$ or $-(q_o + h j_o)/(\varkappa c_p \bar{\varrho} u_*)$ characteristic temperature fluctuation for a dry or moist boundary layer

$\varkappa$ = v.Karman's constant

Λ = scaling length

$\bar{\lambda}_x, \bar{\lambda}_y$ = external parameters for the baroclinicity (geostrophic coordin.)

$\lambda_{*x}, \lambda_{*y}$ = internal parameters for the baroclinicity (antitriptic coordinates)
For definition see p.5

$\mu = H/L$ = internal stratification parameter

ν = kinematic viscosity

Φ_m, Φ_h, Φ_s = nondimensional (universal) profile functions for momentum, heat and moisture

ϱ = density

$\sigma = \bar{H}/\bar{L}$ = external stratification parameter

$\tau_x = -\bar{\varrho}\overline{u'w'}$ eddy momentum flux (x-compon.)

$\tau_y = -\bar{\varrho}\overline{v'w'}$ eddy momentum flux (y-compon.)

Subscripts

g = geostrophic

h = for heat

GL = Grid level (= M , page 19)

m = for momentum or for mean motion

p = denoting the upper border of the surface layer

s = for moisture or denoting the surface at z = 0

T = denoting the top of the boundary layer

t = for turbulent

u,v = for one of the velocity components

o = denoting the height $z = z_o$

ϑ = for the temperature (or heat)

$*$ = denoting an internal parameter

Other symbols

$\overline{(\ )}$ = denoting a large-scale variable provided by the GCM

$\widetilde{(\ )}$ = denoting a layer-averaged value