Coupling the continuity equation with the physics in the IFS
(Work in slow progress)

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- new continuity equation for conservation of dry mass instead of total mass
- net mass transport in mass flux scheme
Mass conservation

Lagrangian/Eulerian mass conservation

\[ \frac{Dm}{Dt} = 0 \quad \text{or} \quad \frac{\partial m}{\partial t} = -\vec{u} \cdot \vec{\nabla}(m) \]

Equation for density

\[ m = \rho V \implies \frac{D\rho}{Dt} = -\rho \frac{1}{V} \frac{DV}{Dt} = -\rho D_3 \]

What if an "unresolved" source of mass?

\[ \frac{Dm}{Dt} = S \implies \frac{D\rho}{Dt} = -\rho \frac{1}{V} \frac{DV}{Dt} + \frac{1}{V} S \]

- in an hydrostatic model, \( S \) need to be in hydrostatic balance,
- in a compressible model, \( S \) has to be understood by the model as a source of mass and not as a source of volume.
Total mass conservation versus dry mass conservation

**Current IFS**

\[
\frac{D\rho_t}{Dt} = -\rho_t D_3 \\
\frac{D\rho_w}{Dt} = -\rho_w D_3 - S_\varphi \\
\Rightarrow \frac{D\rho_d}{Dt} = -\rho_d D_3 + S_\varphi
\]

**New continuity**

\[
\frac{D\rho_t}{Dt} = -\rho_t D_3 - S_\varphi \\
\frac{D\rho_w}{Dt} = -\rho_w D_3 - S_\varphi \\
\Rightarrow \frac{D\rho_d}{Dt} = -\rho_d D_3
\]

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Total mass conservation versus dry mass conservation

In practice:

Current IFS

In the hydrostatic IFS, thanks to the hybrid pressure levels, the continuity equation is reduced to a 2D equation for the surface hydrostatic pressure:

\[
\frac{\partial \pi_s}{\partial t} = \left[ - \int_{\text{surf}}^{\text{top}} \vec{\nabla} \cdot (\vec{v} \frac{\partial p}{\partial \eta}) \, d\eta \right]
\]

New continuity

Add physics tendency to the equation for the surface pressure, but also in the diagnostic equations for \(\omega\) and \(\dot{\eta}\) (the vertical velocity used for vertical advection).

Should the physics mass tendency affect all specific variables?

\[
\frac{\partial (\rho_t \psi)}{\partial t} = - \vec{\nabla} (\rho_t \psi \vec{u}) + S_\psi
\]

\[
\frac{\partial \psi}{\partial t} = - \vec{u} \cdot \vec{\nabla} \psi - \frac{\psi}{\rho_t} \left( \frac{\partial \rho_t}{\partial t} \right) + \vec{u} \cdot \vec{\nabla} \rho_t + \rho_t \vec{\nabla} (\vec{u}) + \frac{S_\psi}{\rho_t}
\]
Error (in %) in the computation of "dry" mixing ratios of tracers (for example CO$_2$) from specific ratios.

Tracer in an explicit cumulonimbus: before/after
Generalisation to a net sub-grid transport of total mass in deep convection parametrisations

HYMACS (Kuell, Gassmann and Bott, 2007), also Grell 3D in WRF

In the grey zone ⇒ replace the parametrised compensating subsidence in the convective column by an ”explicit” 3D subsidence computed by the dynamics.

Net mass advection inside the physics

\[
\frac{\partial (\rho \psi)}{\partial t}_{\text{conv}} = - \frac{\partial (M_u (\psi_u - \overline{\psi}))}{\partial z} = - \left[ \frac{\partial (M_u \psi_u)}{\partial z} + \frac{\partial (-M_u \overline{\psi})}{\partial z} \right]
\]

⇓

\[
\frac{\partial (\rho \psi)}{\partial t}_{\text{conv}} = - \frac{\partial (M_u \psi_u)}{\partial z}
\]

If the compensating subsidence is not parametrised by the convection scheme ⇒ the dynamics is expected to produce the mass adjustment as a consequence of the net mass transport by the physics;
MCs case: 3 June 2015, 00UTC, wind at 200hPa

IFS increment computed by 4DVAR data assimilation.

200hPa Wind difference between simulations without and with deep convection scheme
How does the dynamics react to a parametrised net mass transport?

Academic simulation of a tracer transport by a single column updraft

- subgrid mixing of a passive tracer (not part of the total mass, not part of the buoyancy, unlike moisture)
- passive tracer with same initial profile as moisture
- no wind, temperature and moisture from Klemp and Wilhelmsom, 78
- only the concentration of traceur + total mass in the new scheme are transported by the mass flux (temperature and moisture of parcels adjust instantaneously to the environment, not energetically correct, but only for illustration purpose)
- the vertical integral of the mass tendencies in the column is zero.
How does the dynamics react to a parametrised net mass transport?

- Hydrostatic model,
- Small planet, cubic grid, $dx = 5$ km, $dt = 1$ min

New mass flux scheme

$\Rightarrow$ change total mass with mass flux tendencies using same modifications written for total water tendencies (cf moist/dry mass conservation)
How does the dynamics react to a parametrised net mass transport?

6h. accumulated tendencies for the specific ratio of a passive tracer

Conventional mass flux scheme

New mass flux scheme

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How does the dynamics react to a parametrised net mass transport?

- Non hydrostatic model,
- Small planet, cubic, $dx = 5$ km, $dt = 1$ min

**NH-spectral IFS**

- Hybrid mass vertical coordinate ($\pi_s$ is the hydrostatic surface pressure)
- Thermo equation for internal energy instead of enthalpy
- Two more equations: $\hat{q} = \log\left(\frac{\hat{p} + \pi}{\pi}\right)$ and $d_4$ related to the "vertical divergence".

**New mass flux scheme**

$\Rightarrow$ mass flux tendencies projected on NH pressure departure

$d\hat{\rho} = d(p - \pi)$ and temperature $dT$ such as $d\theta = 0$ (Kuell et al, 2007)
How does the dynamics react to a parametrised net mass transport?

Wind after 1 k

Acc. tracer tend. after 6 hours

\[ \frac{1}{\rho} \frac{\partial \rho}{\partial t} \rightarrow d\hat{\rho}, dT(d\theta = 0) \]
How does the dynamics react to a parametrised net mass transport?

- Non-hydrostatic model
- Small planet, cubic, $dx = 5$ km, $dt = 1$ min

**New mass flux scheme**

$\Rightarrow$ change total mass with mass flux tendencies as in hydrostatic model (physics tendency for hydrostatic surface pressure, $\omega$ and $\dot{\eta}$)
How does the dynamics react to a parametrised net mass transport?

Wind after 1 k

Acc. tracer tend. after 6 hours

NH, $\frac{1}{\rho} \frac{\partial \rho}{\partial t} \rightarrow d\pi$
How does the dynamics react to a parametrised net mass transport?

- Non hydrostatic model,
- Small planet, cubic, $dx = 500$ m, $dt = 5$ s

New mass flux scheme

⇒ mass flux tendencies projected as in hydrostatic model (physics tendency for hydrostatic surface pressure, $\omega$ and $\dot{\eta}$)
How does the dynamics react to a parametrised net mass transport?

Wind after 1 k

Hydrostatic

NH

Acc. tracer tend. after 3 hours

Hydrostatic

NH
How does the dynamics react to a parametrised net mass transport?

- Hydrostatic model
- Small planet, cubic, \( dx = 5 \text{ km} \), \( dt = 1 \text{ min} \)

New mass flux scheme

\( \Rightarrow \) change total mass with mass flux tendencies as in hydrostatic model (physics tendency for hydrostatic surface pressure, \( \omega \) and \( \eta \))

\[
\frac{\partial \psi}{\partial t} = -\vec{u} \cdot \vec{\nabla} \psi - \frac{\psi}{\rho_t} \left( \frac{\partial \rho_t}{\partial t} + \vec{u} \cdot \vec{\nabla} \rho_t + \rho_t \vec{\nabla} (\vec{u}) \right) + \frac{S_\psi}{\rho_t}
\]
How does the dynamics react to a parametrised net mass transport?

Wind after 1 k

Acc. tracer tend. after 6 hours

H, Ref

H, extra terms

H, Ref

H, extra terms
It is possible to conserve dry mass instead of total mass in the hydrostatic IFS. Neutral in term of scores, improve ”dry” mixing ratios for tracers.

It is also possible to parametrise a net sub-grid mass transport (e.g. mass flux scheme) and let the dynamics do the compensating subsidence. The same solution works for both the hydrostatic and NH IFS.

But it is dangerous to play with mass!