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1. Introduction

Multisensor validation of precipitation involves comparison of precipitation estimates at different scales, e.g., comparison of 20-30 km averages from a satellite to 2-km averages from radars to point observations from raingauges, and often to a numerical model output at another scale. A major problem arising when spatial averages of precipitation at one scale are compared to averages at another scale is the fact that precipitation variability is scale-dependent (e.g., see Fig. 1). How this variability changes with scale is a function of the inherent characteristics of the storm and varies with storm type. Moreover, even the uncertainty of the estimates depends on scale, and it differs from sensor to sensor.

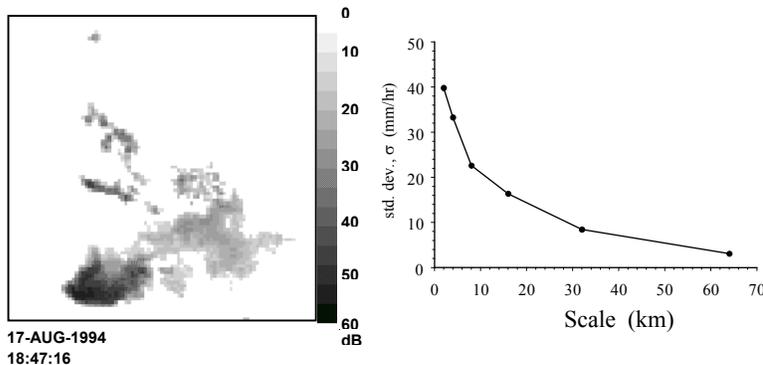


Figure 1. Radar-observed precipitation (left) from the WSR-88D (NEXRAD) radar KICT in Wichita, Kansas on 17 August 1994, along with the standard deviation of non-zero precipitation as a function of grid scale.

In a recent study, Tustison et al. (2001) demonstrated the importance of accounting for the multiscale variability of precipitation in QPF verification studies and showed that typical methods used to change the scale of observations to the scale of the model estimates impose a “representativeness error” which is nonzero even in the case of “perfect” model estimates. We propose the

development of a rigorous methodology, which can explicitly account for the scale-dependent variability and uncertainty of precipitation estimates in any study involving comparison or merging of multisensor observations.

2. Background on the proposed methodology: Scale-Recursive Estimation

The proposed methodology largely utilizes a stochastic scale-recursive estimation (SRE) technique introduced by Chou et al. (1994). This technique can optimally merge observations of a process at different scales while explicitly accounting for their uncertainty and variability at all scales. It requires the specification of a model (called multiscale model) describing how the process variability changes with scale. This multiscale model is defined on an inverted tree structure (see Figure 2) and the estimation algorithm is developed along this tree in two steps. One step involves the multiscale model in its coarse-to-fine scale form:

$$X(\lambda) = A(\lambda)X(\gamma\lambda) + B(\lambda)W(\lambda) \tag{1}$$

where λ is an index to specify the nodes on the tree, $\gamma\lambda$ represents the node on the tree directly above the node specified by λ , $X(\lambda)$ is the state of the system at scale λ , $A(\lambda)$ and $B(\lambda)$ control the scale-to-scale variability of the process, and $W(\lambda)$ is a normally distributed driving noise. This model may be used to describe any process whose coarse-to-fine scale dependence can be

expressed in a linear form. The second step of SRE involves the fine-to-coarse scale form of the model:

$$X(\gamma\lambda) = F(\lambda)X(\lambda) + W^*(\lambda). \quad (2)$$

The parameters $F(\lambda)$ and $W^*(\lambda)$ are defined by $A(\lambda)$ and $B(\lambda)$ by manipulation of (1) and (2). The incorporation of the measurements into the multiscale model is done via the measurement equation

$$Y(\lambda) = C(\lambda)X(\lambda) + V(\lambda) \quad (3)$$

where $V(\lambda)$ is a normally distributed measurement noise which characterizes the error variance of the measuring instrument and $C(\lambda)$ provides the relation between the observed quantity and the state.

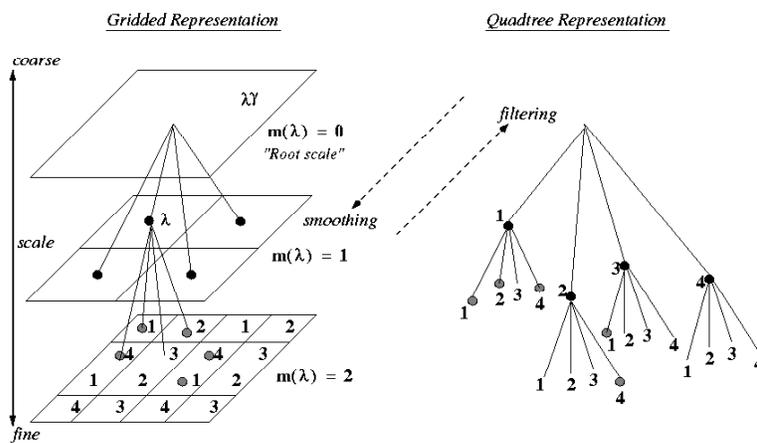


Figure 2. Illustration of the scale-recursive estimation (SRE) technique applied to precipitation measurements. Sparsely-distributed measurements at one scale (gray dots), and measurements at a coarser scale (solid dots), are placed on an inverted quad-tree and merged via filtering and smoothing to obtain estimates at any desired scale together with the uncertainty of these estimates.

Equations (1-3) form the basic equations for the scale-to-scale evolution of the process and the inclusion of its measurements. Figure 2 illustrates the inverted tree, the scale-to-scale composition of the process, and how measurements are placed on the tree for inclusion in the multiscale estimation.

The multiscale recursive estimation algorithm consists of two steps: filtering and smoothing. The filtering step uses

Kalman filtering, incorporating the measurements of the process with (3) and propagating the estimates from fine-to-coarse scales with (2). The second step consists of smoothing in which the estimates from step one are merged with those predicted from (1) along the coarse-to-fine path on the tree. This algorithm has the advantage that it is extremely computationally efficient due to its recursive nature. This is especially important with large data sets available at many scales, such as those available for rainfall (e.g. raingauges, radars, satellites) and for real-time data assimilation applications.

3. Illustrative Example

Suppose one has available radar observations at 2-km resolution and satellite observations at 16-km resolution, or only a sparse network of raingauge observations but complete coverage of the area by a satellite overpass. How is one to combine these observations for obtaining an optimal (in some sense) merged product at one or more desired scales? This optimally merged product can be used to verify a numerical weather model or a precipitation retrieval at the desired scale. To demonstrate the use of the multiscale framework, a simple numerical experiment was conducted.

In all cases, a combination of observations and their uncertainties at 2 km and 16 km were provided and optimal estimates at 8km were computed using SRE (see Fig. 3). These estimates were then compared to the true 8-km averages (known to us in this constructed example) in terms

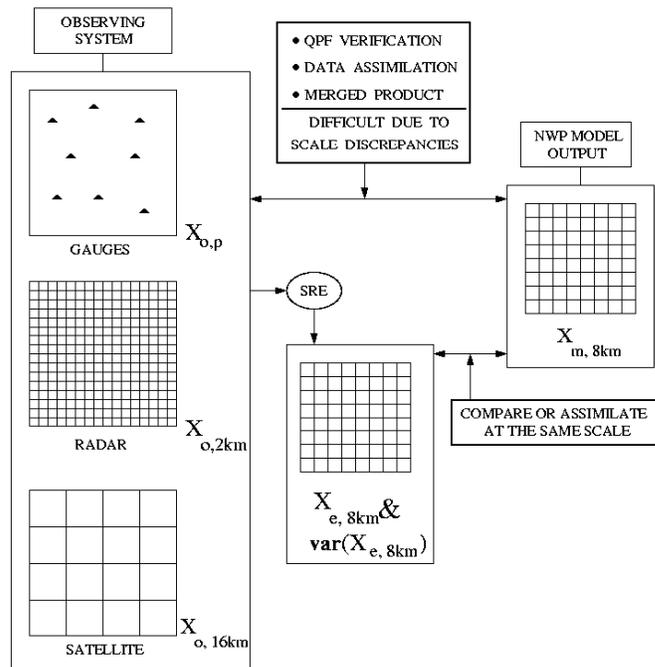


Figure 3. Illustration of the proposed framework for QPF Verification.

of bias, RMSE, standard deviation of the whole field, and mean uncertainty of the estimates. Different scenarios of available observations were tested. A bounded lognormal cascade fitted to the 2-km field was used as the prescribed multiscale model. Table 1 summarizes the results and quantifies the potential of the method for multisensor validation studies. As was expected, increasing the density and resolution of available observations decreases the RMSE and bias in validation (i.e., produces more accurate merged fields at 8km) and increases the accuracy of the merged estimates (smaller uncertainty). More details can be found in Tustison et al. (2003).

Several issues must be carefully studied before SRE can be used with confidence for multisensor validation or data merging. One of those is the sensitivity of SRE to multiscale model selection, observational error, and presence of zeros.

Table 1. Case studies illustrating the application of the SRE methodology for merging observations at different scales. The mean of the KEAX WSR-88D hourly accumulation field at 8 km (which was considered the “true” field in this example) was 1.99 mm, and the standard deviation, 3.01mm. All the values are given in mm.

Case	Observations	SRE estimation at 8 km			
		Bias	RMSE	σ_{est} field	Mean uncertainty of estimates
1	10% sampling at 2 km	0.40	1.13	2.47	0.32
2	50% sampling at 2 km	0.15	0.48	2.84	0.21
3	10% sampling at 2 km; 100 % at 16 km	0.17	0.84	2.80	0.30
4	50% sampling at 2 km; 100 % at 16 km	0.10	0.45	2.88	0.21

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