SPECIAL PROJECT PROGRESS REPORT

All the following mandatory information needs to be provided. The length should *reflect the complexity and duration* of the project.

Reporting year	**2020**……………………………………………………
Project Title:	**Direct numerical simulation of long-term evolution of wind waves: dynamics vs kinetics, with applications to freak waves prediction**
Computer Project Account:	**SPGBSHRI** …………………………………………
Principal Investigator(s):	**Prof V.I. Shrira** …………………………………
	………………………………………………………
Affiliation:	**School of Computing and Mathematics, Keele University, Keele ST5 5BG UK**
Name of ECMWF scientist(s) collaborating to the project (if applicable)	……………………………………………………… ………………………………………………………
Start date of the project:	**2019**……………………………………………………
Expected end date:	**2021**……………………………………………………

Computer resources allocated/used for the current year and the previous one
(if applicable)
Please answer for all project resources

		Previous year		Current year	
		Allocated	Used	Allocated	Used
High Performance Computing Facility	(units)	**500000**	**497089**	**500000**	**103000**
Data storage capacity	(Gbytes)	**100**	**100**	**100**	**100**

Summary of project objectives (10 lines max)

The target of the project is to perform long term DNS simulations and to compare them with high quality observations. As shown by Annenkov & Shrira (2014), higher-order moments of a random wave field, and hence the probability of freak waves, are dependent on spectral shape, not just the on integral characteristics of a wave field. Preliminary estimates show that the found discrepancies in spectral shape can strongly affect higher-order moments, in particular kurtosis, and the difference is expected to be substantial (of the order of 100%). This has huge potential implications for the prediction of extreme wave events. Specific objectives include exploring the discrepancy between the shape of the DNS (verified by observations) and the Hasselmann equation predictions, examining implications for probability of freak waves, mixing via the vortex force and other processes sensitive to the shape of spectra, getting new insights into the input and dissipation functions.

Summary of problems encountered (10 lines max)

No particular problems encountered

Summary of plans for the continuation of the project (10 lines max)

In the next part of the project, we will continue simulations of spectral evolution based on the set of data collected in the Mediterranean Sea with the airborne radar system KUROS. We will improve the performance of the generalised kinetic equation algorithm, which currently has stability issues for low wind forcing, and the DNS algorithm. We will compare the results of simulations with the data. We will study the role of intermittency in the evolution of wind wave spectra. Such intermittency is absent in the kinetic equations models, but evident in the DNS simulations, and in the data.

List of publications/reports from the project with complete references

Annenkov S.Y., Shrira V.I. When is the dynamic non-Gaussianity essential for water wave fields? WISE 2019, Jozankei, Hokkaido, Japan, 12--16 May 2019

Annenkov S.Y., Shrira V.I. Evolution of random wave fields and the role of the statistical closure. 8th International Symposium on Bifurcations and Instabilities in Fluid Dynamics, University of Limerick, Ireland

Annenkov S.Y., Shrira V.I. Evolution of weakly nonlinear random wave fields: kinetic equations vs the Zakharov equation. IX-th International Conference “SOLITONS, COLLAPSES AND TURBULENCE: Achievements, Developments and Perspectives” (SCT-19) Yaroslavl, Russia 5 - 9 Aug 2019.

Summary of results

If submitted **during the first project year**, please summarise the results achieved during the period from the project start to June of the current year. A few paragraphs might be sufficient. If submitted **during the second project year**, this summary should be more detailed and cover the period from the project start. The length, at most 8 pages, should reflect the complexity of the project. Alternatively, it could be replaced by a short summary plus an existing scientific report on the project attached to this document. If submitted **during the third project year**, please summarise the results achieved during the period from July of the previous year to June of the current year. A few paragraphs might be sufficient.

In this project, we study numerically the long-term evolution of water wave spectra, using the standard model based on the Hasselmann kinetic equation (KE), widely used in operational wind wave forecasting, as well as two other models developed by us and based upon different sets of assumptions. The second model is the generalised kinetic equation (gKE), using the same statistical closure as the KE, but derived without the assumption of quasi-stationarity. The gKE does not employ the quasi-stationarity assumption and is valid when a wave spectrum is changing rapidly (e.g. at the initial stage of evolution of a narrow spectrum, or after a rapid change of wind forcing). The third model is based on the Zakharov integrodifferential equation for water waves and does not depend on any statistical assumptions. Since the Zakharov equation plays the role of the primitive equation of the theory of wave turbulence, we refer to this model as direct numerical simulation of spectral evolution (DNS-ZE). The DNS-ZE method allows to study long-term spectral evolution (up to $O(10^4)$ periods), which was previously possible only with the KE. Thus, we are able to perform a direct comparison of spectral evolution with and without the statistical closure.

During the previous Special Project, which ended in 2018, we traced the evolution of initially narrow (both in frequency and angle) spectra subjected only to dissipation at high frequencies, and considered the evolution of higher statistical moments. In the current Special project, the target is to study the evolution of waves generated by wind blowing in the offshore direction. This numerical setup corresponds to conditions during experiments with the KuROS airborne radar during field campaigns in the Lion Gulf in the Mediterranean Sea in 2013, in an area exposed to intense offshore winds (Le Merle et al 2019). In this report, we present numerical results, obtained with three different models, on spectral evolution of wind waves generated by a moderate or strong wind in an initially quiescent sea. We compare the evolution of various spectral parameters, including spectral peak parameters, angular width, and higher moments, emphasizing differences between the models.

I. Wave action spectra and spectral peak

Figure 1 shows the development of wind wave spectra generated by constant wind with speed U equal to $1.2c_0$ and $2.0c_0$, where c_0 is the characteristic phase speed, approximately corresponding the the phase speed at the end of evolution. These values of wind speed are chosen since they are close to minimal and maximal wind speeds in the KuROS experiment. Wind forcing is calculated according to Hsiao & Shemdin (1983), dissipation is applied to small scales.

Spectral evolution demonstrated by all three models in both cases shown in figure 1 is clearly self-similar, with spectral slope corresponding to the theoretical $k^{-23/6}$ angle. Both kinetic equations give nearly identical results, although the gKE develops a certain instability in smaller scales at large time, especially manifested for weaker wind, where the simulation with the gKE even has to be abandoned prematurely. The downshift due to kinetic models is slightly faster for stronger wind, and the spectral peak is more pronounced. The most striking difference between the kinetic models and the DNS is in the shape of the spectral peak, which is narrower with the kinetic models, and the front appears to be much steeper.

Evolution of wave steepness for four different wind speeds is shown in figure 2. After the

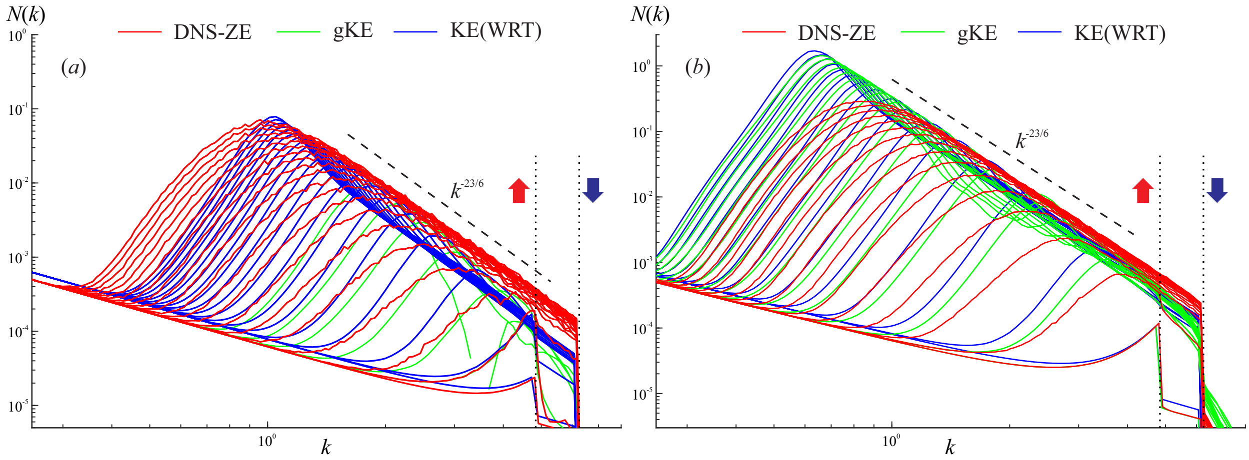

Figure 1: Long-term evolution of wave action spectrum generated by constant wind over initially quiescent sea, with a direct comparison of three different numerical methods. Evolution is simulated by the DNS based on the Zakharov equation (DNS-ZE), standard kinetic equation, WRT algorithm, KE(WRT), and generalized kinetic equation (gKE). (a) Wind speed $U/c_0 = 1.2$, where c_0 is the characteristic phase speed, spectra are plotted every 1000 characteristic periods (b) Wind speed $U/c_0 = 2.0$, where c_0 is the characteristic phase speed, spectra are plotted every 320 characteristic periods

initial rise to a maximum, wave steepness slowly decreases in all models, although the decrease is faster for the kinetic equations, especially for stronger winds.

In figure 3, evolution of spectral peak parameters (amplitude and position of the peak) is shown. In all cases, for large times the values approach theoretical asymptotes known from the analysis of the Hasselmann kinetic equation (Badulin et al 2005). At the same time, for stronger wind there is a large discrepancy in peak amplitude between the kinetic equations and the DNS, which also appears to be growing with time. For weaker wind, this effect almost disappears.

However, for all wind speeds there is a considerable difference is spectral shape: DNS spectra appear to be wider, although a more detailed analysis shows that the width parameters are in fact close, the difference in spectral shape is being manifested by lower *peakedndess* of DNS spectra, rather than different width. This difference in peakedness is best represented by the peakedness parameter

$$\Phi = \frac{\displaystyle\int_0^\infty E(\omega)^2 \,\mathrm{d}\omega}{\left[\displaystyle\int_0^\infty E(\omega)\,\mathrm{d}\omega\right]^2},$$

where $E(\omega)$ is the energy–frequency spectrum (Goda 1970).

The evolution of the peakedness parameter Φ is shown in figure 4. Peakedness for all DNS simulations is much lower than for the corresponding simulations with the kinetic equations, demonstrating the difference in spectral shapes.

II. Angular width

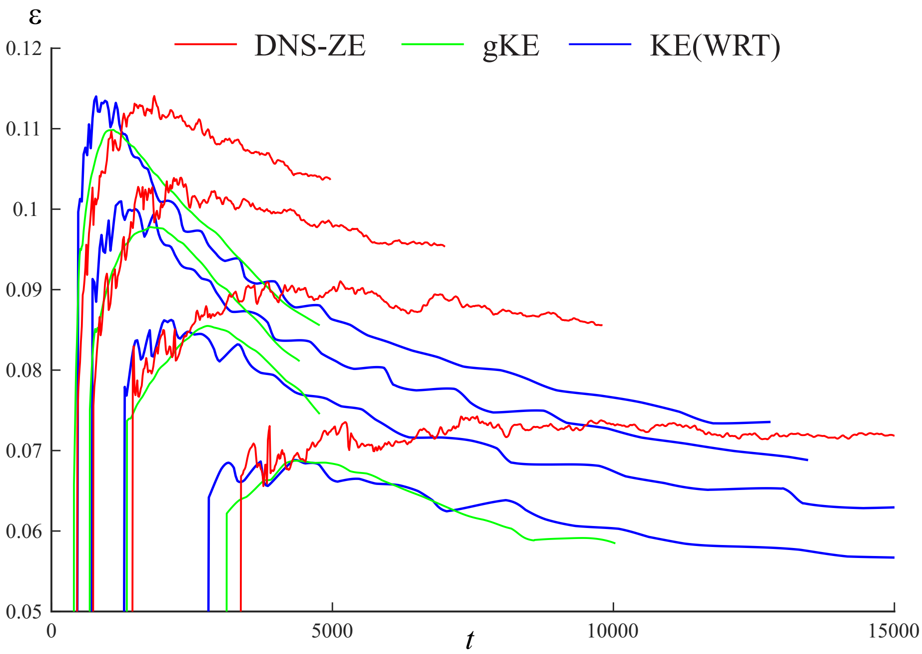

Figure 2: Evolution of wave steepness with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind $U/c_0 = 1.2$, 1.5, 1.8 and 2.1, simulated with three numerical methods

Here we consider the evolution of mean directional spread $\theta_m = \overline{\theta_2(k)}$ defined as the average of the second moment of $D(\theta)$

$$\theta_2(k) = \left(\int_0^{\pi/2} \theta^2 D(k,\theta)\,\mathrm{d}\theta \right)^{1/2} \left(\int_0^{\pi/2} D(k,\theta)\,\mathrm{d}\theta \right)^{-1/2},$$

where $D(k,\theta)$ is the angular distribution function (Hwang et al 2000). Unlike in the absence of wind forcing (cf the final report of the previous Special Project, and also Annenkov & Shrira 2018), the evolution of mean directional spread for wind waves, shown in figure 5, is close for the DNS and the kinetic equations. However, it is clear that due to the integral nature of the averaged directional spread, it is mostly defined by the spectral tail, while for practical applications the angular width of the spectral peak is important (Badulin & Zakharov 2017).

In figure 6, evolution of function the second moment of the directional distribution $\theta_2(k)$ is shown for $k = k_p$ and $k = 0.5k_p$. At these low wavenumbers, the evolution of angular width is markedly different between the kinetic equations and the DNS. At the spectral peak, while the kinetic equations show that the spectral peak is narrow in angle and nearly constant in time, the DNS demonstrates an increase of the angular width to a much higher value. At low frequencies, both the gKE and the DNS are quite different from the Hasselmann kinetic equation, which gives much larger angular width.

III. Higher statistical moments

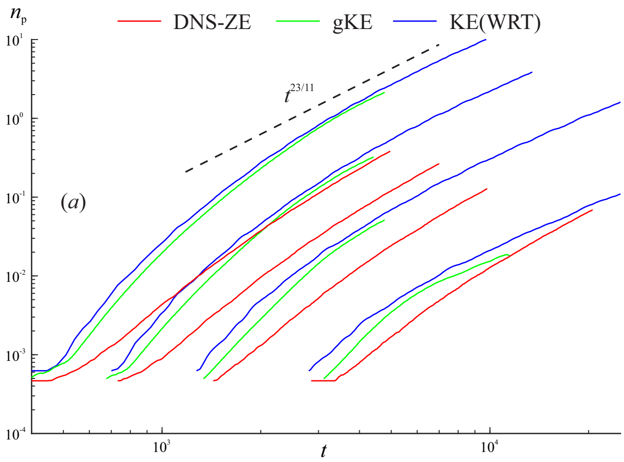

Figure 3: Evolution of spectral peak with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind $U/c_0 = 1.2$, 1.5, 1.8 and 2.1, simulated with three numerical methods. (a) spectral peak amplitude (b) peak wavenumber

In the context of gravity water waves, the kurtosis is commonly used as the main characteristics of the field departure from gaussianity. The component of the kurtosis due to wave interactions (the dynamical kurtosis) is usually assumed to be small for a quasi-stationary wind wave field, which is confirmed by the current simulations. The other component, the bound harmonic kurtosis, can be calculated from the spectrum as

$$C_4^{(b)} = \frac{m_4^{(b)}}{m_2^2} - 3,$$

where

$$m_4^{(b)} = 3 \int \omega_0 \omega_1 n_0 n_1 \, \mathrm{d}\mathbf{k}_{01} + 12 \int \mathcal{J}_{012}^{(4)} \omega_0 \omega_1 \omega_2 n_0 n_1 n_2 \, \mathrm{d}\mathbf{k}_{012},$$

$m_2 = \int \omega_0 n_0 \, \mathrm{d}\mathbf{k}_0$, $n_j = n(\mathbf{k}_j)$, $\mathrm{d}\mathbf{k}_{01} = \mathrm{d}\mathbf{k}_0 \mathrm{d}\mathbf{k}_1$, etc., and the coefficient $\mathcal{J}_{012}^{(4)}$ was derived by Janssen(2009) (see also Annenkov & Shrira 2013).

Evolution of $C_4^{(b)}$ for two wind speeds, obtained with different models, is shown in figure 7. The difference in spectral shapes manifests itself in much higher values of the bound harmonic kurtosis in the DNS simulations, corresponding to increased probability of extreme wave events.

References

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Badulin, S.I., Pushkarev, A.N., Resio, D. & Zakharov, V.E. 2005 Self-similarity of wind-driven seas. *Nonlin. Proc. Geophys.* **12**, 891–945.

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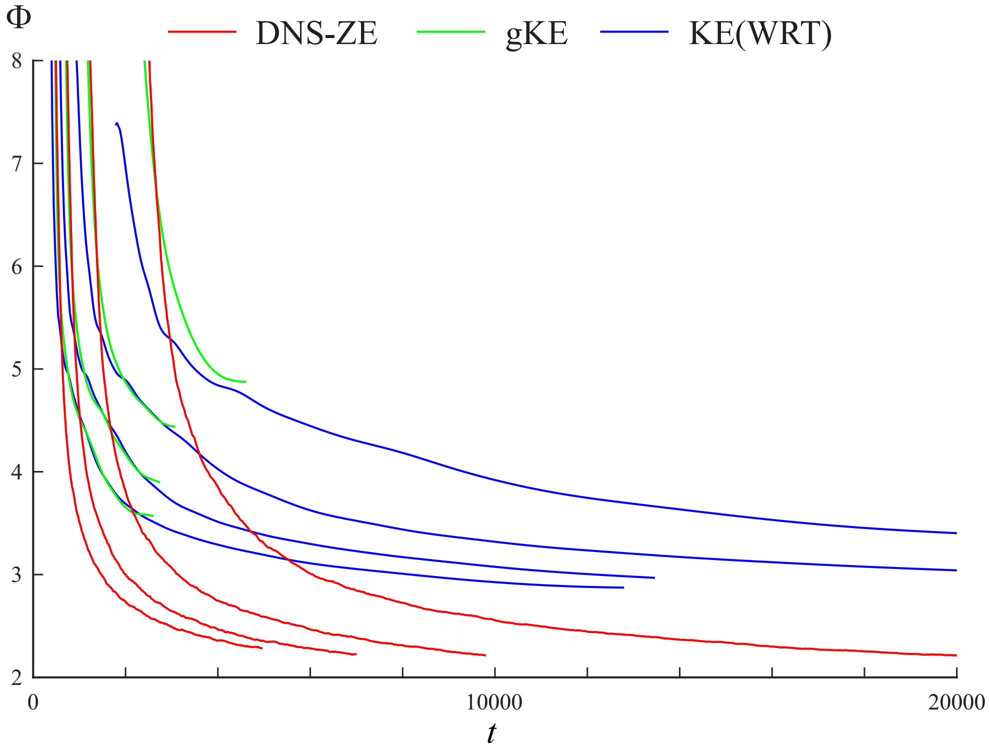

Figure 4: Evolution of spectral peakedness Φ with time (in characteristic periods)

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Hsiao, S.V. & Shemdin, O.H. 1983 Measurements of wind velocity and pressure with a wave follower during MARSEN. *J. Geophys. Res.* **88**, 9841–9849.

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le Merle E., Hauser D. & Tison C. 2019 Directional wave spectra at the regional scale with the KuROS airborne radar: comparisons with models. *Ocean Dynamics* **69**, 679–699.

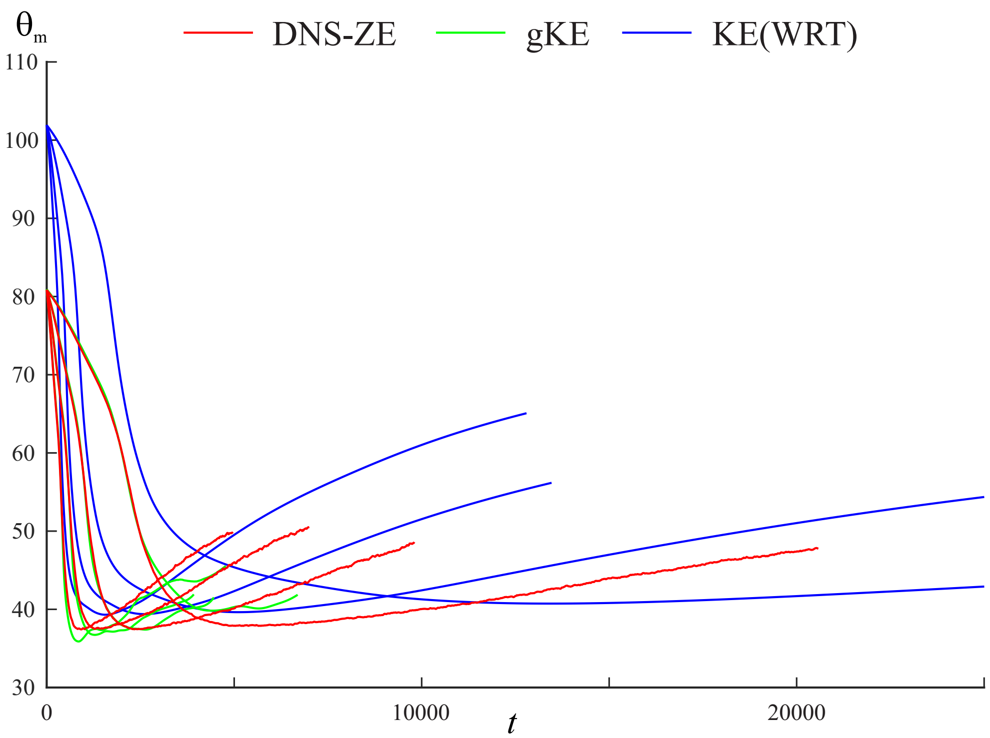

Figure 5: Evolution of average angular width $theta_m$ with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind $U/c_0 = 1.2$, 1.5, 1.8 and 2.1, simulated with three numerical methods

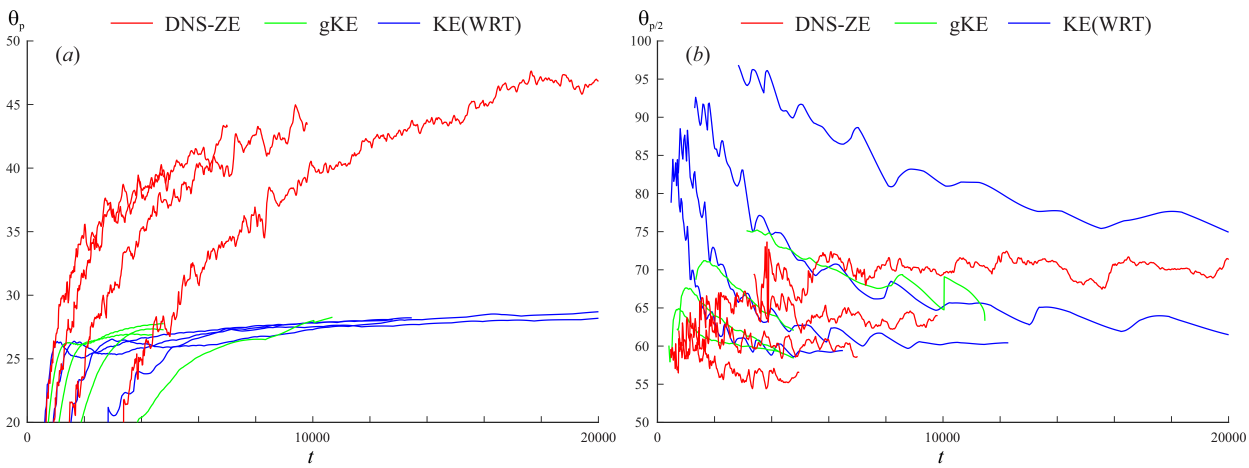

Figure 6: Evolution of angular width θ at (a) wavenumber of the spectral peak and (b) half of wavenumber of the spectral peak with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind $U/c_0 = 1.2$, 1.5, 1.8 and 2.1, simulated with three numerical methods

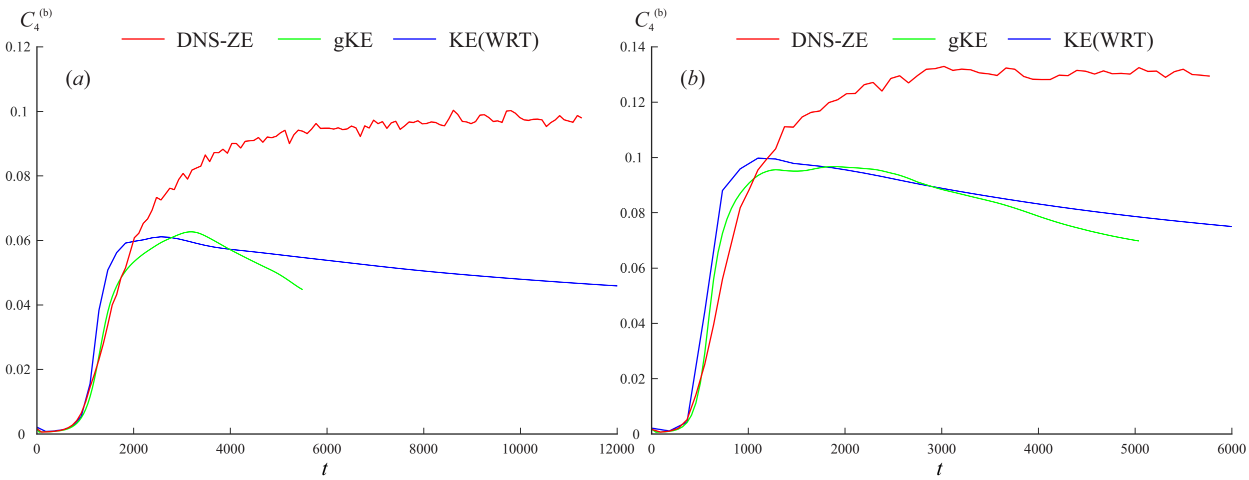

Figure 7: Evolution of bound harmonic kurtosis $C_4^{(b)}$ with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind (a) $U/c_0 = 1.5$ (b) $U/c_0 = 2.0$