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**IFS DOCUMENTATION – Cy41r1**  
**Operational implementation 12 May 2015**

**PART IV: PHYSICAL PROCESSES**

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# Chapter 1

## Overview

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- 1.1 Introduction
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### 1.1 INTRODUCTION

The physical processes associated with radiative transfer, turbulent mixing, convection, clouds, surface exchange, subgrid-scale orographic drag and non-orographic gravity wave drag have a strong impact on the large scale flow of the atmosphere. However, these mechanisms are often active at scales smaller than the resolved scales of the model grid. Parametrization schemes are then necessary in order to properly describe the impact of these subgrid-scale mechanisms on the large scale flow of the atmosphere. In other words the ensemble effect of the subgrid-scale processes has to be formulated in terms of the resolved grid-scale variables. Furthermore, forecast weather parameters, such as two-metre temperature, precipitation and cloud cover, are computed by the physical parametrization part of the model. This part (Part IV ‘Physical processes’) of the IFS documentation describes the physical parametrization package.

The radiation scheme (Chapter 2) performs computations of the short-wave and long-wave radiative fluxes using the predicted values of temperature, humidity, cloud, and monthly-mean climatologies for aerosols and the main trace gases ( $\text{CO}_2$ ,  $\text{O}_3$ ,  $\text{CH}_4$ ,  $\text{N}_2\text{O}$ ,  $\text{CFCl}_3$  and  $\text{CF}_2\text{Cl}_2$ ). The radiation code is based on the Rapid Radiation Transfer Model (RRTM, [Mlawer \*et al.\*, 1997](#); [Iacono \*et al.\*, 2008](#)). Cloud-radiation interactions are taken into account in detail by using the values of cloud fraction and liquid, ice and snow water contents from the cloud scheme using the McICA (Monte Carlo Independent Column Approximation) method (McRad, [Morcrette \*et al.\*, 2008a](#)). The solution of the radiative transfer equations

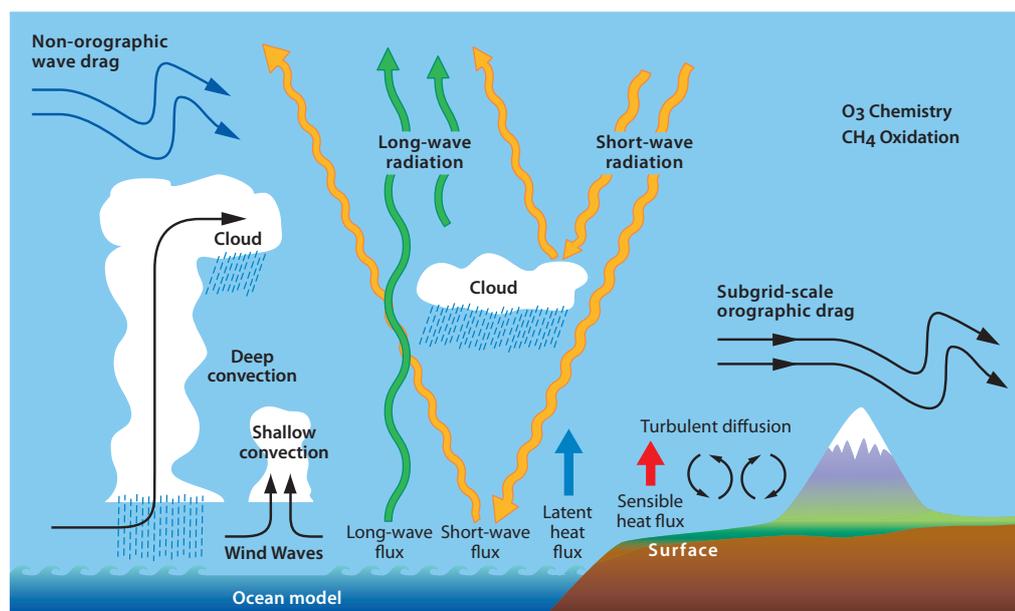


Figure 1.1 Schematic diagram of the different physical processes represented in the IFS model.

to obtain the fluxes is computationally very expensive, so depending on the model configuration, full radiation calculations are performed on a reduced (coarser) radiation grid and/or on a reduced time frequency. The results are then interpolated back to the original grid. However, the short-wave fluxes are updated at every grid point and timestep using values of the solar zenith angle.

The turbulent diffusion scheme ([Chapter 3](#)) represents the vertical exchange of heat, momentum and moisture through sub-grid scale turbulence. The vertical turbulent transport is treated differently in the surface layer and above. In the surface layer, the turbulence fluxes are computed using a first order K-diffusion closure based on the Monin–Obukhov (MO) similarity theory. Above the surface layer a K-diffusion turbulence closure is used everywhere, except for unstable boundary layers where an Eddy-Diffusivity Mass-Flux (EDMF) framework is applied, to represent the non-local boundary layer eddy fluxes ([Köhler \*et al.\*, 2011](#)). The scheme is written in moist conserved variables (liquid static energy and total water) and predicts total water variance. A total water distribution function is used to convert from the moist conserved variables to the prognostic cloud variables (liquid/ice water content and cloud fraction), but only for the treatment of stratocumulus. Convective clouds are treated separately by the shallow convection scheme.

The effects of unresolved orography on the atmospheric flow are parametrized as a sink of momentum (drag). The turbulent diffusion scheme includes a parametrization in the lower atmosphere to represent the turbulent orographic form drag induced by small scale ( $< 5$  km) orography ([Beljaars \*et al.\*, 2004b](#)). In addition, in stably stratified flow, the orographic drag parametrization ([Chapter 4](#)) represents the effects of low-level blocking due to unresolved orography (blocked flow drag) and the absorption and/or reflection of vertically propagating gravity waves (gravity wave drag) on the momentum budget ([Lott and Miller, 1997](#)).

The non-orographic gravity wave drag parametrization ([Chapter 5](#)) accounts for the effects of unresolved non-orographic gravity waves. These waves are generated in nature by processes like deep convection, frontal disturbances, and shear zones. Propagating upward from the troposphere the waves break in the middle atmosphere, comprising the stratosphere and the mesosphere, where they exert a strong drag on the flow. The parametrization uses a globally uniform wave spectrum, and propagates it vertically through changing horizontal winds and air density, thereby representing the wave breaking effects due to critical level filtering and non-linear dissipation. A description of the scheme and its effects on the middle atmosphere circulation can be found in [Orr \*et al.\* \(2010\)](#).

The moist convection scheme ([Chapter 6](#)) is based on the mass-flux approach and represents deep (including congestus), shallow and mid-level (elevated moist layers) convection. The distinction between deep and shallow convection is made on the basis of the cloud depth ( $\leq 200$  hPa for shallow). For deep convection the mass-flux is determined by assuming that convection removes Convective Available Potential Energy (CAPE) over a given time scale. The intensity of shallow convection is based on the budget of the moist static energy, i.e. the convective flux at cloud base equals the contribution of all other physical processes when integrated over the subcloud layer. Finally, mid-level convection can occur for elevated moist layers, and its mass flux is set according to the large-scale vertical velocity. The scheme, originally described in [Tiedtke \(1989\)](#), has evolved over time and amongst many changes includes a modified entrainment formulation leading to an improved representation of tropical variability of convection ([Bechtold \*et al.\*, 2008](#)), and a modified CAPE closure leading to a significantly improved diurnal cycle of convection ([Bechtold \*et al.\*, 2014](#)).

Clouds and large-scale precipitation ([Chapter 7](#)) are parametrized with a number of prognostic equations for cloud liquid, cloud ice, rain and snow water contents and a sub-grid fractional cloud cover. The cloud scheme represents the sources and sinks of cloud and precipitation due to the major generation and destruction processes, including cloud formation by detrainment from cumulus convection, condensation, deposition, evaporation, collection, melting and freezing. The scheme is based on ([Tiedtke, 1993](#)) but with an enhanced representation of mixed-phase clouds and prognostic precipitation ([Forbes and Tompkins, 2011](#); [Forbes \*et al.\*, 2011](#)). Supersaturation with respect to ice is commonly observed in the upper troposphere and is also represented in the parametrization ([Tompkins \*et al.\*, 2007](#)).

The surface parametrization scheme ([Chapter 8](#)) represents the surface fluxes of energy and water and corresponding sub-surface quantities. The scheme is based on a tiled approach (TESSEL) representing

different sub-grid surface types for vegetation, bare soil, snow and open water. Each tile has its own properties defining separate heat and water fluxes used in an energy balance equation which is solved for the tile skin temperature. Four soil layers are represented as well as snow mass and density. The evaporative fluxes consider separately the fractional contributions from snow cover, wet and dry vegetation and bare soil. An interception layer collects water from precipitation and dew fall, and infiltration and run-off are represented depending on soil texture and subgrid orography (HTESSEL, Balsamo *et al.*, 2009, Balsamo *et al.*, 2011a). A carbon cycle is included and land-atmosphere exchanges of carbon dioxide are parametrized to respond to diurnal and synoptic variations in the water and energy cycles (CHTESSEL, Boussetta *et al.*, 2013a).

Chapter 9 ‘Methane oxidation’ describes a simple parametrization of the upper-stratospheric moisture source due to methane oxidation. A parametrization representing photolysis of vapour in the mesosphere is also included.

Chapter 10 ‘Ozone chemistry parametrization’ gives a brief description of the ozone parametrization, based on Cariolle and Déqué (1986). The parametrization assumes that chemical changes in ozone can be described by a linear relaxation towards a photochemical equilibrium. The relaxation rates and the equilibrium values have been determined from a photochemical model, including a representation of the heterogeneous ozone hole chemistry.

Chapter 11 ‘Climatological data’ describes the distributions of climatological fields, including land sea mask, orography (mean, standard deviation, anisotropy, orientation and slope), land use, soil type, vegetation type, leaf area index, albedo, aerosols, ozone and trace gases.

## 1.2 OVERVIEW OF THE CODE

The physics parametrization package is called by the IFS after the explicit dynamical computations. The physics computations are performed only in the vertical. The input information for the physics consists of the values of the gridbox mean prognostic variables (wind components  $u/v$ , temperature  $T$ , specific humidity  $q$ , cloud fraction  $a$ , water contents for cloud liquid  $q_l$ , cloud ice  $q_i$ , rain  $q_r$ , and snow  $q_s$ ), the provisional dynamical tendencies for the same variables and various surface fields, both fixed and variable.

The time integration of the physics is based on the following:

- (i) It has to be compatible with the adiabatic part of the IFS.
- (ii) The tendencies from the different physical processes are computed in separate routines.
- (iii) As a general approach, the value of a prognostic variable is updated with the tendency from one process and the next process starts from this updated value, in what is usually referred to as the ‘method of fractional steps’ (details are different for different processes).
- (iv) Implicit schemes are used when needed for stability.

**CALLPAR** is the routine that controls the physical parametrization package with the exception of the main radiation routine **RADINTG**. **RADINTG** controls the computation of the short-wave transmissivities and the long-wave fluxes. **RADINTG** is called outside **CALLPAR** because of the need to make the radiation space interpolation compatible with the distributed memory version of the IFS. **GP\_MODEL** is a high level routine that controls all computations in grid-point space. It calls both **CALLPAR** via interface routines **EC\_PHYS\_DRV**, **EC\_PHYSG**, **EC\_PHYS** and **RADINTG** via driver **RADDRV**.

In **CALLPAR** the physics routines are called in the following order:

<b>SURFRAD_LAYER</b>	Computes radiative properties of the surface.
<b>CLDPRG_LAYER</b>	Calls <b>CLDPP</b> to compute cloud parameters required for the post processing (e.g. total cloud cover).
<b>RADFLUX_LAYER</b>	Calls <b>RADHEATN</b> to compute the temperature tendencies and the downward radiation fluxes at the surface with updated (every time-step) values for the zenith angle.
<b>GWDRAG_LAYER</b>	Computes the tendencies for $u$ , $v$ and $T$ due to the parametrization of subgrid-scale orographic drag. It also computes subgrid orographic coefficients for use in <b>VDFMAIN</b> .
<b>TURBULENCE_LAYER</b>	Calls <b>VDFOUTER/VDFMAIN</b> in two sub time steps for numerical stability. <b>VDFMAIN</b> computes the vertical exchange of $u$ , $v$ , $T$ , $q$ , $a$ , $q_l$ , $q_i$ by turbulence.
<b>CLOUD_LAYER</b>	Calls <b>CLOUDSC</b> as a first guess call of cloud scheme to determine preliminary entry profiles for convection.
<b>CONVECTION_LAYER</b>	Interface to call <b>CUCALLN/CUMASTRN</b> that controls the computation of the tendencies for $u$ , $v$ , $T$ , $q$ , chemical tracers and the cloud detrainment term due to the parametrization of moist convective processes.
<b>CLOUD_LAYER</b>	Calls <b>CLOUDSC</b> to compute tendencies for $T$ , $q$ , $a$ , $q_l$ , $q_i$ , $q_r$ and $q_s$ due to the parametrization of cloud and precipitation processes.
<b>GWDRAGWMS_LAYER</b>	Calls <b>GWDRAG_WMS</b> to compute the tendencies for $u$ , $v$ and $T$ due to the parametrization of non-orographic gravity waves.
<b>METHOX</b>	Computes tendencies for $q$ due to methane oxidation and water vapour photolysis.
<b>SURFTSTP_LAYER</b>	Calls <b>SURFTSTP</b> to control the soil/surface scheme.
<b>STOCHPERT_LAYER</b>	Optionally add stochastic perturbations to physics tendencies.
<b>O3CHEM</b>	Computes tendencies for $O_3$ due to ozone chemistry.
<b>SLTEND_LAYER</b>	Optionally average tendencies from radiation, convection and cloud in time and space along the semi-Lagrangian trajectory.

## Chapter 2

# Radiation

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## 2.1 RADIATIVE HEATING

[Table 2.1](#) gives the timeline of the major changes affecting the representation of the radiation transfer (RT) in the ECMWF model over the last twenty years.

Since 5 June 2007, with Cy32r2 of the ECMWF IFS operational libraries, a new approach to the inclusion of the cloud effects on radiation fields (the Monte-Carlo Independent Column Approximation, McICA) has been introduced in the ECMWF radiation schemes (McRad, [Morcrette \*et al.\*, 2008a](#)), which are based on the Rapid Radiation Transfer Model (RRTM), originally developed at AER, Inc. ([Mlawer \*et al.\*, 1997](#); [Iacono \*et al.\*, 2008](#)). The McICA approach is described in [Section 2.4](#) together with a description of the long-wave ([Section 2.5](#)) and short-wave ([Section 2.6](#)) parts of the radiation scheme.

The previous radiation schemes based on [Morcrette \(1991\)](#) operational in the high-resolution 10-day forecasts from May 1989 to June 2000 are still described in this documentation as they form the basis for

**Table 2.1** Major changes in the representation of radiation transfer in the ECMWF forecasting system.

Cycle	Implementation date	Description
SPM 32	02/05/1989	RT schemes from Univ.Lille
SPM 46	01/02/1993	Optical properties for ice and mixed phase clouds
IFS 14R3	13/02/1996	Revised LW and SW absorption coefficients from HITRAN'92
IFS 16R2	15/05/1997	Voigt profile in long-wave RT scheme
IFS 16R4	27/08/1997	Revised ocean albedo from ERBE
IFS 18R3	16/12/1997	Revised LW and SW absorption coefficients from HITRAN'96
IFS 18R5	01/04/1998	Seasonal land albedo from ERBE
IFS 22R3	27/06/2000	RRTM <sub>LW</sub> as long-wave RT scheme
IFS 23R4	12/06/2001	short-wave RT scheme with 4 spectral intervals Hourly, instead of 3-hourly, calls to RT code during data assimilation cycle
IFS 25R1	09/04/2002	Short-wave RT scheme with 6 spectral intervals
IFS 26R3	07/10/2003	New aerosol climatology adapted from Tegen et al. (1997), new radiation grid
IFS 28R3	28/09/2004	Radiation called hourly in high resolution forecasts
IFS 32R2	05/06/2007	McICA approach to RT with RRTM <sub>LW</sub> and RRTM <sub>SW</sub> revised cloud optical properties, MODIS-derived land albedo

the linearized schemes for short-wave (Section 2.2) and long-wave (Section 2.3) radiation transfer used in the assimilation.

The solution of the radiative transfer equation to obtain the fluxes is unfortunately very expensive, so depending on the model configuration (high resolution 10-day forecast, medium-resolution EPS, low resolution "climate" simulation or seasonal forecast), savings in computer time spent in the radiation calculations are obtained by using a reduced (coarser) radiation grid (Morcrette *et al.*, 2008b) and/or by calling the full radiation with a reduced time frequency. These reduced radiation configurations are described in Section 2.7 together with the interpolation scheme used for obtaining the radiative fluxes at every grid point and every time step for the relevant instantaneous temperature profile and solar zenith angle.

The radiative heating rate is computed as the divergence of net radiation fluxes  $\mathcal{F}$  so that

$$\left(\frac{\partial T}{\partial t}\right)_{\text{rad}} = -\frac{g}{c_p} \frac{\partial \mathcal{F}}{\partial p} \quad (2.1)$$

where  $c_p$  is the specific heat at constant pressure of moist air

$$c_p = c_{p_{\text{dry}}} \{1 + (c_{p_{\text{vap}}} - c_{p_{\text{dry}}})q/c_{p_{\text{dry}}}\}$$

and  $c_{p_{\text{dry}}}$  and  $c_{p_{\text{vap}}}$  are the specific heats at constant pressure of dry air and water vapour, respectively.

A description of the inputs, in particular the climatologically defined quantities of radiative importance is given in Section 2.8. Finally, an alphabetical list of the subroutines of the radiation scheme is given in Section 2.9.

## 2.2 THE PRE-CY32R2 SHORT-WAVE RADIATION SCHEME

The rate of atmospheric heating by absorption and scattering of short-wave radiation is

$$\frac{\partial T}{\partial t} = \frac{g}{c_p} \frac{\partial \mathcal{F}_{\text{SW}}}{\partial p} \quad (2.2)$$

where  $\mathcal{F}_{\text{SW}}$  is the net total short-wave flux (the subscript SW will be omitted in the remainder of this section).

$$\mathcal{F}(\delta) = \int_0^\infty d\nu \left[ \int_0^{2\pi} d\phi \left\{ \int_{-1}^{+1} \mu \mathcal{L}_\nu(\delta, \mu, \phi) d\mu \right\} \right] \quad (2.3)$$

is the diffuse radiance at wavenumber  $\nu$ , in a direction given by the azimuth angle,  $\phi$ , and the zenith angle,  $\theta$ , with  $\mu = \cos \theta$ . In (2.3), we assume a plane parallel atmosphere, and the vertical coordinate is the optical depth  $\delta$ , a convenient variable when the energy source is outside the medium

$$\delta(p) = \int_p^0 \beta_v^{\text{ext}}(p') dp' \quad (2.4)$$

$\beta_v^{\text{ext}}(p)$  is the extinction coefficient, equal to the sum of the scattering coefficient  $\beta_v^{\text{sca}}$  of the aerosol (or cloud particle absorption coefficient  $\beta_v^{\text{abs}}$ ) and the purely molecular absorption coefficient  $k_\nu$ . The diffuse radiance  $\mathcal{L}_\nu$  is governed by the radiation transfer equation

$$\begin{aligned} \mu \frac{d\mathcal{L}_\nu(\delta, \mu, \phi)}{d\delta} = & \mathcal{L}_\nu(\delta, \mu, \phi) - \frac{\varpi_\nu(\delta)}{4} P_\nu(\delta, \mu, \phi, \mu_0, \phi_0) \mathcal{E}_\nu^0 \exp(-\delta/\mu_r) \\ & - \frac{\varpi_\nu(\delta)}{4} \int_0^{2\pi} d\phi' \left\{ \int_{-1}^{+1} \Phi_\nu(\delta, \mu, \phi, \mu', \phi') \mathcal{L}_\nu(\delta, \mu', \phi') d\mu' \right\} \end{aligned} \quad (2.5)$$

$\mathcal{E}_\nu^0$  is the incident solar irradiance in the direction  $\mu_0 = \cos \theta_0$ ,  $\varpi_\nu$ , is the single scattering albedo ( $= \beta_v^{\text{sca}}/k_\nu$ ) and  $\Phi(\delta, \mu, \phi, \mu', \phi')$  is the scattering phase function which defines the probability that radiation coming from direction  $(\mu', \phi')$  is scattered in direction  $(\mu, \phi)$ . The short-wave part of the scheme, originally developed by [Fouquart and Bonnel \(1980\)](#) solves the radiation transfer equation and integrates the fluxes over the whole short-wave spectrum between 0.2 and 4  $\mu\text{m}$ . Upward and downward fluxes are obtained from the reflectances and transmittances of the layers, and the photon-path-distribution method allows to separate the parametrization of the scattering processes from that of the molecular absorption.

### 2.2.1 Spectral integration

Solar radiation is attenuated by absorbing gases, mainly water vapour, uniformly mixed gases (oxygen, carbon dioxide, methane, nitrous oxide) and ozone, and scattered by molecules (Rayleigh scattering), aerosols and cloud particles. Since scattering and molecular absorption occur simultaneously, the exact amount of absorber along the photon path length is unknown, and band models of the transmission function cannot be used directly as in long-wave radiation transfer (see [Section 2.2](#)). The approach of the photon path distribution method is to calculate the probability  $\Pi(\mathcal{U}) d\mathcal{U}$  that a photon contributing to the flux  $\mathcal{F}_{\text{cons}}$  in the conservative case (i.e., no absorption,  $\omega_\nu = 1$ ,  $k_\nu = 0$ ) has encountered an absorber amount between  $\mathcal{U}$  and  $\mathcal{U} + d\mathcal{U}$ . With this distribution, the radiative flux at wavenumber  $\nu$  is related to  $\mathcal{F}_{\text{cons}}$  by

$$\mathcal{F}_\nu = \mathcal{F}_{\text{cons}} \int_0^\infty \Pi(\mathcal{U}) \exp(-k_\nu \mathcal{U}) d\mathcal{U} \quad (2.6)$$

and the flux averaged over the spectral interval  $\Delta\nu$  can then be calculated with the help of any band model of the transmission function  $t_{\Delta\nu}$

$$\mathcal{F} = \frac{1}{\Delta\nu} \int_{\Delta\nu} \mathcal{F}_\nu d\nu = \mathcal{F}_{\text{cons}} \int_0^\infty \Pi(\mathcal{U}) t_{\Delta\nu}(\mathcal{U}) d\mathcal{U} \quad (2.7)$$

To find the distribution function  $\Pi(\mathcal{U})$ , the scattering problem is solved first, by any method, for a set of arbitrarily fixed absorption coefficients  $k_1$ , thus giving a set of simulated fluxes  $\mathcal{F}_{k_1}$ . An inverse Laplace transform is then performed on (2.6) ([Fouquart, 1974](#)). The main advantage of the method is that the actual distribution  $\Pi(\mathcal{U})$  is smooth enough that (2.6) gives accurate results even if  $\Pi(\mathcal{U})$  itself is not known accurately. In fact,  $\Pi(\mathcal{U})$  needs not be calculated explicitly as the spectrally integrated fluxes are

$$\begin{aligned} \mathcal{F} &= \mathcal{F}_{\text{cons}} t_{\Delta\nu}(\langle \mathcal{U} \rangle) && \text{in the limiting case of weak absorption} \\ \mathcal{F} &= \mathcal{F}_{\text{cons}} t_{\Delta\nu}(\langle \mathcal{U}^{1/2} \rangle) && \text{in the limiting case of strong absorption} \end{aligned}$$

where  $\langle \mathcal{U} \rangle = \int_0^\infty \Pi(\mathcal{U}) \mathcal{U} d\mathcal{U}$  and  $\langle \mathcal{U}^{1/2} \rangle = \int_0^\infty \Pi(\mathcal{U}) \mathcal{U}^{1/2} d\mathcal{U}$ .

The atmospheric absorption in the water vapour bands is generally strong, and the scheme determines an effective absorber amount  $\mathcal{U}_e$  between  $\langle \mathcal{U} \rangle$  and  $\langle \mathcal{U}^{1/2} \rangle$  derived from

$$\mathcal{U}_e = \ln(\mathcal{F}_{k_e}/\mathcal{F}_{\text{cons}})/k_e \quad (2.8)$$

where  $k_e$  is an absorption coefficient chosen to approximate the spectrally averaged transmission of the clear sky atmosphere

$$k_e = \frac{1}{\mathcal{U}_{\text{tot}}/\mu_0} \ln(t_{\Delta\nu}(\mathcal{U}_{\text{tot}}/\mu_0)) \quad (2.9)$$

where  $\mathcal{U}_{\text{tot}}$  is the total amount of absorber in a vertical column and  $\mu_0 = \cos \theta_0$ . Once the effective absorber amounts of  $\text{H}_2\text{O}$  and uniformly mixed gases are found, the transmission functions are computed using Pade approximants

$$t_{\Delta\nu}(\mathcal{U}) = \frac{\sum_{i=0}^N a_i \mathcal{U}^{i-1}}{\sum_{j=0}^N b_j \mathcal{U}^{j-1}} \quad (2.10)$$

Absorption by ozone is also taken into account, but since ozone is located at low pressure levels for which molecular scattering is small and Mie scattering is negligible, interactions between scattering processes and ozone absorption are neglected. Transmission through ozone is computed using (2.7) where  $\mathcal{U}_{\text{O}_3}$  the amount of ozone is

$$\begin{aligned} \mathcal{U}_{\text{O}_3}^{\text{d}} &= M \int_p^0 d\mathcal{U}_{\text{O}_3} && \text{for the downward transmission of the direct solar beam} \\ \mathcal{U}_{\text{O}_3}^{\text{u}} &= r \int_{p_s}^0 d\mathcal{U}_{\text{O}_3} + \mathcal{U}_{\text{O}_3}^{\text{d}}(p_{\text{surf}}) && \text{for the upward transmission of the diffuse radiation} \end{aligned}$$

$r = 1.66$  is the diffusivity factor (see Section 2.2), and  $M$  is the magnification factor (Rodgers, 1967) used instead of  $r$  to account for the sphericity of the atmosphere at very small solar elevations

$$M = 35/\sqrt{\mu_0^2 + 1} \quad (2.11)$$

To perform the spectral integration, it is convenient to discretize the solar spectrum into subintervals in which the surface reflectance, molecular absorption characteristics, and cloud optical properties can be considered as constants. One of the main causes for such a spectral variation is the sharp increase in the reflectivity of the vegetation in the near-infrared. Also, water vapour does not absorb below  $0.69 \mu\text{m}$  nor do liquid water clouds. Till June 2000, the ECMWF short-wave scheme considered only two spectral intervals, one for the visible ( $0.2\text{--}0.69 \mu\text{m}$ ), one for the near-infrared ( $0.69\text{--}4.00 \mu\text{m}$ ) parts of the solar spectrum. From June 2000 to April 2002, the near-infrared interval was sub-divided into three intervals ( $0.69\text{--}1.19\text{--}2.38\text{--}4.00 \mu\text{m}$ ) to account better for the spectral variations of the cloud optical properties. Till April 2002, all the molecular absorption coefficients (for  $\text{O}_3$ ,  $\text{H}_2\text{O}$ , uniformly mixed gases) were derived from statistical models of the transmission function using spectroscopic parameters derived from various versions of the HITRAN database (Rothman *et al.*, 1986, 1992). In April 2002, following the recomputation of all the molecular absorption coefficients from an updated version of the short-wave line-by-line model of Dubuisson *et al.* (1996) using spectroscopic data from HAWKS (2000), the ultraviolet and visible part of the spectrum are now considered in three spectral intervals ( $0.20\text{--}0.25\text{--}0.69 \mu\text{m}$ ) making the scheme having a total of six spectral intervals over which the aerosol and cloud optical properties are also defined. The cut-off at  $0.69 \mu\text{m}$  allows the scheme to be more computational efficient, in as much as the interactions between gaseous absorption (by water vapour and uniformly mixed gases) and scattering processes are accounted for only in the near-infrared interval(s).

### 2.2.2 Vertical integration

Considering an atmosphere where a fraction  $C_{\text{cld}}^{\text{tot}}$  (as seen from the surface or the top of the atmosphere) is covered by clouds (the fraction  $C_{\text{cld}}^{\text{tot}}$  depends on which cloud-overlap assumption is assumed for the calculations), the final fluxes are given as a weighted average of the fluxes in the clear sky and in the cloudy fractions of the column

$$\mathcal{F}^-(j) = C_{\text{cld}}^{\text{tot}} \mathcal{F}_{\text{cld}}^-(j) + (1 - C_{\text{cld}}^{\text{tot}}) \mathcal{F}_{\text{clr}}^-$$

where the subscripts ‘clr’ and ‘cld’ refer to the clear-sky and cloudy fractions of the layer, respectively. In contrast to the scheme of Geleyn and Hollingsworth (1979), the fluxes are not obtained through the solution of a system of linear equations in a matrix form. Rather, assuming an atmosphere divided into

homogeneous layers, the upward and downward fluxes at a given layer interface  $j$  are given by

$$\begin{aligned}\mathcal{F}^-(j) &= \mathcal{F}_0 \prod_{k=j}^N \mathcal{T}_{\text{bot}}(k) \\ \mathcal{F}^+(j) &= \mathcal{F}^-(j) \mathcal{R}_{\text{top}}(j-1)\end{aligned}\quad (2.12)$$

where  $\mathcal{R}_{\text{top}}(j)$  and  $\mathcal{T}_{\text{bot}}(j)$  are the reflectance at the top and the transmittance at the bottom of the  $j$ th layer. Computation of the values of  $\mathcal{R}_{\text{top}}$  starts at the surface and works upwards, whereas determining values of  $\mathcal{T}_{\text{bot}}$  starts at the top of the atmosphere and works downward.  $\mathcal{R}_{\text{top}}$  and  $\mathcal{T}_{\text{bot}}$  account for the presence of cloud in the layer by using

$$\begin{aligned}\mathcal{R}_{\text{top}} &= C_{\text{cld}} \mathcal{R}_{\text{cld}} + (1 - C_{\text{cld}}) \mathcal{R}_{\text{clr}} \\ \mathcal{T}_{\text{bot}} &= C_{\text{cld}} \mathcal{T}_{\text{cld}} + (1 - C_{\text{cld}}) \mathcal{T}_{\text{clr}}\end{aligned}\quad (2.13)$$

where  $C_{\text{cld}}$  is the cloud fractional coverage of the layer within the cloudy fraction  $C_{\text{cld}}^{\text{tot}}$  of the column.

(a) *Cloudy fraction layer*

$\mathcal{R}_{t_{\text{cldy}}}$  and  $\mathcal{R}_{b_{\text{cldy}}}$  are the reflectance at the top and transmittance at the bottom of the cloudy fraction of the layer calculated with the Delta-Eddington approximation. Given  $\delta_c$ ,  $\delta_a$ , and  $\delta_g$ , the optical thicknesses for the cloud, the aerosol and the molecular absorption of the gases ( $= k_e U$ ), respectively, and  $g_c$  and  $g_a$  the cloud and aerosol asymmetry factors,  $\mathcal{R}_{t_{\text{cldy}}}$  and  $\mathcal{R}_{b_{\text{cldy}}}$  are calculated as functions of the total optical thickness of the layer

$$\delta = \delta_c + \delta_a + \delta_g \quad (2.14)$$

of the total single scattering albedo

$$\varpi^* = \frac{\delta_c + \delta_a}{\delta_c + \delta_a + \delta_g} \quad (2.15)$$

of the total asymmetry factor

$$g^* = \frac{\delta_c}{\delta_c + \delta_a} g_c + \frac{\delta_a}{\delta_c + \delta_a} g_a \quad (2.16)$$

of the reflectance  $\mathcal{R}_-$  of the underlying medium (surface or layers below the  $j$ th interface), and of the cosine of an effective solar zenith angle  $\mu_{\text{eff}}(j)$  which accounts for the decrease of the direct solar beam and the corresponding increase of the diffuse part of the downward radiation by the upper scattering layers

$$\mu_{\text{eff}}(j) = [(1 - C_{\text{cld}}^{\text{eff}}(j))/\mu + r C_{\text{cld}}^{\text{eff}}(j)]^{-1} \quad (2.17)$$

with  $C_{\text{cld}}^{\text{eff}}(j)$  the effective total cloudiness over level  $j$

$$C_{\text{cld}}^{\text{eff}}(j) = 1 - \prod_{i=j+1}^N (1 - C_{\text{cld}}(i) E(i)) \quad (2.18)$$

and

$$E(i) = 1 - \exp\left[-\frac{(1 - \varpi_c(i) g_c(i)^2) \delta_c(i)}{\mu}\right] \quad (2.19)$$

$\delta_c(i)$ ,  $\varpi_c(i)$  and  $g_c(i)$  are the optical thickness, single scattering albedo and asymmetry factor of the cloud in the  $i$ th layer, and  $r$  is the diffusivity factor. The scheme follows the Eddington approximation first proposed by [Shettle and Weinman \(1970\)](#), then modified by [Joseph \*et al.\* \(1976\)](#) to account more accurately for the large fraction of radiation directly transmitted in the forward scattering peak in case of highly asymmetric phase functions. Eddington's approximation assumes that, in a scattering medium of optical thickness  $\delta^*$ , of single scattering albedo  $\omega$ , and of asymmetry factor  $g$ , the radiance  $\mathcal{L}$  entering [\(2.58\)](#) can be written as

$$\mathcal{L}(\delta, \mu) = \mathcal{L}_0(\delta) + \mu \mathcal{L}_1(\delta) \quad (2.20)$$

In that case, when the phase function is expanded as a series of associated Legendre functions, all terms of order greater than one vanish when [\(2.3\)](#) is integrated over  $\mu$  and  $\phi$ . The phase function is therefore

given by

$$P(\Theta) = 1 + \beta_1(\Theta)\mu$$

where  $\Theta$  is the angle between incident and scattered radiances. The integral in (2.3) thus becomes

$$\int_0^{2\pi} d\phi' \left\{ \int_{-1}^{+1} p(\mu, \phi, \mu', \phi') \mathcal{L}(\mu', \phi') d\mu' \right\} = 4\pi(L_0 + \pi\mathcal{L}_1) \quad (2.21)$$

where

$$g = \frac{\beta_1}{3} = \frac{1}{2} \int_{-1}^{+1} P(\Theta)\mu d\mu$$

is the asymmetry factor.

Using (2.21) in (2.3) after integrating over  $\mu$  and dividing by  $2\pi$ , we get

$$\mu \frac{d}{d\delta} (\mathcal{L}_0 + \mu\mathcal{L}_1) = -(\mathcal{L}_0 + \mu\mathcal{L}_1) + \varpi(\mathcal{L}_0 + g\mu\mathcal{L}_1) + 1/4\varpi\mathcal{F}_0 \exp(-\delta/\mu_0)(1 + 3g\mu_0\mu) \quad (2.22)$$

We obtain a pair of equations for  $\mathcal{L}_0$  and  $\mathcal{L}_1$  by integrating (2.22) over  $\mu$

$$\begin{aligned} \frac{d\mathcal{L}_0}{d\delta} &= -3(1 - \varpi)\mathcal{L}_0 + \frac{3}{4}\varpi\mathcal{F}_0 \exp(-\delta/\mu_0) \\ \frac{d\mathcal{L}_1}{d\delta} &= -(1 - \varpi g)\mathcal{L}_1 + \frac{3}{4}\varpi g\mu_0\mathcal{F}_0 \exp(-\delta/\mu_0) \end{aligned} \quad (2.23)$$

For the cloudy layer assumed non-conservative ( $\varpi < 1$ ), the solutions to (2.22) and (2.23), for  $0 \leq \delta \leq \delta^*$ , are

$$\begin{aligned} \mathcal{L}_0(\delta) &= C_1 \exp(-K\delta) + C_2 \exp(+K\delta) - \alpha \exp(-\delta/\mu_0) \\ \mathcal{L}_1(\delta) &= P\{C_1 \exp(-K\delta) - C_2 \exp(+K\delta) - \beta \exp(-\delta/\mu_0)\} \end{aligned} \quad (2.24)$$

where

$$\begin{aligned} K &= \{3(1 - \varpi)(1 - \varpi g)\}^{1/2} \\ P &= \{3(1 - \varpi)/(1 - \varpi g)\}^{1/2} \\ \alpha &= 3\varpi F_0 \mu_0 \{1 + 3g(1 - \varpi)\} / \{4(1 - K^2 \mu_0^2)\} \\ \beta &= 3\varpi F_0 \mu_0 \{1 + 3g(1 - \varpi)\mu_0^2\} / \{4(1 - K^2 \mu_0^2)\} \end{aligned}$$

The two boundary conditions allow to solve the system for  $C_1$  and  $C_2$ ; the downward directed diffuse flux at the top of the atmosphere is zero, that is

$$\mathcal{F}^-(0) = \left[ \mathcal{L}_0(0) + \frac{2}{3}\mathcal{L}_1(0) \right] = 0$$

which translates into

$$(1 + 2P/3)C_1 + (1 - 2P/3)C_2 = \alpha + 2\beta/3 \quad (2.25)$$

The upward directed flux at the bottom of the layer is equal to the product of the downward directed diffuse and direct fluxes and the corresponding diffuse and direct reflectance ( $\mathcal{R}_d$  and  $\mathcal{R}_-$ , respectively) of the underlying medium

$$\begin{aligned} \mathcal{F}^+(\delta^*) &= \left\{ \mathcal{L}_0(\delta^*) - \frac{2}{3}\mathcal{L}_1(\delta^*) \right\} \\ &= \mathcal{R}_- \left\{ \mathcal{L}_0(\delta^*) + \frac{2}{3}\mathcal{L}_1(\delta^*) \right\} + \mathcal{R}_d \mu_0 \mathcal{F}_0 \exp(-\delta^*/\mu_0) \end{aligned}$$

which translates into

$$\begin{aligned} \{1 - \mathcal{R}_- - 2(1 + \mathcal{R}_-)P/3\}C_1 \exp(-K\delta^*) + \{1 - \mathcal{R}_- + 2(1 + \mathcal{R}_-)P/3\}C_2 \exp(+K\delta^*) \\ = \{(1 - \mathcal{R}_-)\alpha - 2(1 + \mathcal{R}_-)\beta/3 + \mathcal{R}_d \mu_0 \mathcal{F}_0\} \exp(-\delta^*/\mu_0) \end{aligned} \quad (2.26)$$

In the Delta-Eddington approximation, the phase function is approximated by a Dirac delta function forward-scatter peak and a two-term expansion of the phase function

$$P(\theta) = 2f(1 - \mu) + (1 - f)(1 + 3g'\mu)$$

where  $f$  is the fractional scattering into the forward peak and  $g'$  the asymmetry factor of the truncated phase function. As shown by [Joseph et al. \(1976\)](#), these parameters are

$$\begin{aligned} f &= g^2 \\ g' &= g/(g + 1) \end{aligned} \quad (2.27)$$

The solution of the Eddington's equations remains the same provided that the total optical thickness, single scattering albedo and asymmetry factor entering (2.22) and (2.26) take their transformed values

$$\begin{aligned} \delta'^* &= (1 + \varpi f)\delta^* \\ \omega' &= \frac{(1 - f)\varpi}{1 - \varpi f} \end{aligned} \quad (2.28)$$

Practically, the optical thickness, single scattering albedo, asymmetry factor and solar zenith angle entering (2.22)–(2.26) are  $\delta^*$ ,  $\varpi^*$ ,  $g^*$  and  $\mu_{\text{eff}}$  defined in (2.16) and (2.17).

(b) *Clear-sky fraction of the layers*

In the clear-sky part of the atmosphere, the short-wave scheme accounts for scattering and absorption by molecules and aerosols. The following calculations are practically done twice, once for the clear-sky fraction ( $1 - C_{\text{cld}}^{\text{tot}}$ ) of the atmospheric column  $\mu$  with equal to  $\mu_0$ , simply modified for the effect of Rayleigh and aerosol scattering, the second time for the clear-sky fraction of each individual layer within the fraction  $C_{\text{cld}}^{\text{tot}}$  of the atmospheric column containing clouds, with  $\mu$  equal to  $\mu_e$ .

As the optical thickness for both Rayleigh and aerosol scattering is small,  $\mathcal{R}_{\text{clr}}(j - 1)$  and  $\mathcal{T}_{\text{clr}}(j)$ , the reflectance at the top and transmittance at the bottom of the  $j$ th layer can be calculated using respectively a first- and a second-order expansion of the analytical solutions of the two-stream equations similar to that of [Coakley Jr. and Chylek \(1975\)](#). For Rayleigh scattering, the optical thickness, single scattering albedo and asymmetry factor are respectively  $\delta_{\text{R}}$ ,  $\varpi_{\text{R}} = 1$  and  $g_{\text{R}} = 0$ , so that

$$\begin{aligned} \mathcal{R}_{\text{R}} &= \frac{\delta_{\text{R}}}{2\mu + \delta_{\text{R}}} \\ \mathcal{T}_{\text{R}} &= \frac{2\mu}{(2\mu + \delta_{\text{R}})} \end{aligned} \quad (2.29)$$

The optical thickness  $\delta_{\text{R}}$  of an atmospheric layer is simply

$$\delta_{\text{R}} = \delta^* \{p(j) - p(j - 1)\} / p_{\text{surf}} \quad (2.30)$$

where  $\delta_{\text{R}}^*$  is the Rayleigh optical thickness of the whole atmosphere parametrized as a function of the solar zenith angle ([Deschamps et al., 1983](#))

For aerosol scattering and absorption, the optical thickness, single scattering albedo and asymmetry factor are respectively  $\delta_{\text{a}}$ ,  $\varpi_{\text{a}}$ , with  $1 - \varpi_{\text{a}} \ll 1$  and  $g_{\text{a}}$ , so that

$$\begin{aligned} \text{den} &= 1 + \{1 - \varpi_{\text{a}} + \text{back}(\mu_e)\varpi_{\text{a}}\}(\delta_{\text{a}}/\mu_e) \\ &\quad + (1 - \varpi_{\text{a}})\{1 - \varpi_{\text{a}} + 2 \text{back}(\mu_e)\varpi_{\text{a}}\}(\delta_{\text{a}}^2/\mu_e^2) \end{aligned} \quad (2.31)$$

$$\begin{aligned} \mathcal{R}(\mu_e) &= \frac{(\text{back}(\mu_e)\varpi_{\text{a}}\delta_{\text{a}})/\mu_{\text{a}}}{\text{den}} \\ \mathcal{T}(\mu_e) &= 1/\text{den} \end{aligned} \quad (2.32)$$

where  $\text{back}(\mu_e) = (2 - 3\mu_e g_{\text{a}})/4$  is the backscattering factor.

Practically,  $\mathcal{R}_{\text{clr}}$  and  $\mathcal{T}_{\text{clr}}$  are computed using (2.32) and the combined effect of aerosol and Rayleigh scattering comes from using modified parameters corresponding to the addition of the two scatterers with provision for the highly asymmetric aerosol phase function through Delta-approximation of the forward scattering peak (as in (2.23) and (2.24)).

$$\begin{aligned}\delta^+ &= \delta_{\text{R}} + \delta_{\text{a}}(1 - \varpi_{\text{a}}g_{\text{a}}^2) \\ g^+ &= \frac{g_{\text{a}}}{1 + g_{\text{a}}} \frac{\delta_{\text{a}}}{(\delta_{\text{R}} + \delta_{\text{a}})} \\ \varpi^+ &= \frac{\delta_{\text{R}}}{\delta_{\text{a}} + \delta_{\text{a}}} \varpi_{\text{R}} + \frac{\delta_{\text{a}}}{\delta_{\text{R}} + \delta_{\text{a}}} \frac{\varpi_{\text{a}}(1 - g_{\text{a}}^2)}{1 - \varpi_{\text{a}}g_{\text{a}}^2}\end{aligned}\quad (2.33)$$

As for their cloudy counterparts,  $\mathcal{R}_{\text{clr}}$  and  $\mathcal{T}_{\text{clr}}$  must account for the multiple reflections due to the layers underneath

$$\mathcal{R}_{\text{clr}} = \mathcal{R}(\mu_{\text{e}}) + \mathcal{R}_{-} \mathcal{T}(\mu_{\text{e}})(1 - \mathcal{R}^* \mathcal{R}_{-}) \quad (2.34)$$

and  $\mathcal{R}_{-}$  is the reflectance of the underlying medium  $\mathcal{R}_{-} = \mathcal{R}_t(j - 1)$  and  $r$  is the diffusivity factor.

Since interactions between molecular absorption and Rayleigh and aerosol scattering are negligible, the radiative fluxes in a clear-sky atmosphere are simply those calculated from (2.10) and (2.28) attenuated by the gaseous transmissions (2.8).

### 2.2.3 Multiple reflections between layers

To deal properly with the multiple reflections between the surface and the cloud layers, it should be necessary to separate the contribution of each individual reflecting surface to the layer reflectance and transmittances in as much as each such surface gives rise to a particular distribution of absorber amount. In the case of an atmosphere including  $N$  cloud layers, the reflected light above the highest cloud consists of photons directly reflected by the highest cloud without interaction with the underlying atmosphere, and of photons that have passed through this cloud layer and undergone at least one reflection on the underlying atmosphere. In fact, (2.5) should be written

$$\mathcal{F} = \sum_{i=0}^N \mathcal{F}_{\text{cl}} \int_0^{\infty} \mathcal{P}_1(\mathcal{U}) t_{\Delta v}(\mathcal{U}) dv \quad (2.35)$$

where  $\mathcal{F}_{\text{cl}}$  and  $\mathcal{P}_1(\mathcal{U})$  are the conservative fluxes and the distributions of absorber amount corresponding to the different reflecting surfaces.

Fouquart and Bonnel (1980) have shown that a very good approximation to this problem is obtained by evaluating the reflectance and transmittance of each layer (using (2.22) and (2.28)) assuming successively a non-reflecting underlying medium ( $\mathcal{R}_{-} = 0$ ), then a reflecting underlying medium ( $\mathcal{R}_{-} \neq 0$ ). First calculations provide the contribution to reflectance and transmittance of those photons interacting only with the layer into consideration, whereas the second ones give the contribution of the photons with interactions also outside the layer itself.

From those two sets of layer reflectance and transmittances ( $\mathcal{T}_{t0}$ ,  $\mathcal{T}_{b0}$ ) and ( $\mathcal{R}_{t\neq}$ ,  $\mathcal{T}_{b\neq}$ ) respectively, effective absorber amounts to be applied to computing the transmission functions for upward and downward fluxes are then derived using (2.6) and starting from the surface and working the formulas upward

$$\begin{aligned}\mathcal{U}_{\text{e}0}^- &= \ln(\mathcal{T}_{b0}/\mathcal{T}_{bc})/k_{\text{e}} \\ \mathcal{U}_{\text{e}\neq}^- &= \ln(\mathcal{T}_{b\neq}/\mathcal{T}_{bc})/k_{\text{e}} \\ \mathcal{U}_{\text{e}0}^+ &= \ln(\mathcal{R}_{t0}/\mathcal{R}_{tc})/k_{\text{e}} \\ \mathcal{U}_{\text{e}\neq}^+ &= \ln(\mathcal{R}_{t\neq}/\mathcal{R}_{tc})/k_{\text{e}}\end{aligned}\quad (2.36)$$

where  $\mathcal{R}_{tc}$  and  $\mathcal{T}_{bc}$  are the layer reflectance and transmittance corresponding to a conservative scattering medium.

Finally the upward and downward fluxes are obtained as

$$\mathcal{F}^+(j) = \mathcal{F}_0 \{ \mathcal{R}_{t0} t_{\Delta\nu}(\mathcal{U}_{e0}^+) + (\mathcal{R}_{t\neq} - \mathcal{R}_{t0}) t_{\Delta\nu}(\mathcal{U}_{e\neq}^+) \} \quad (2.37)$$

$$\mathcal{F}^-(j) = \mathcal{F}_0 \{ \mathcal{T}_{b0} t_{\Delta\nu}(\mathcal{U}_{e0}^+) + (\mathcal{T}_{b\neq} - \mathcal{T}_{b0}) t_{\Delta\nu}(\mathcal{U}_{e\neq}^-) \} \quad (2.38)$$

### 2.2.4 Pre-CY32R2 cloud short-wave optical properties

As seen in [Subsection 2.2.2\(a\)](#), the cloud radiative properties depend on three different parameters: the optical thickness  $\delta_c$ , the asymmetry factor  $g_c$ , and the single scattering albedo  $\varpi_c$ .

In this older scheme the cloud optical properties were derived from [Fouquart \(1987\)](#) for the water clouds, and [Ebert and Curry \(1992\)](#) for the ice clouds.

The optical thickness  $\delta_c$  is related to the cloud liquid water amount  $U_{\text{LWP}}$  by

$$\delta_c = \frac{3U_{\text{LWP}}}{2r_e}$$

where  $r_e$  is the mean effective radius of the size distribution of the cloud water droplets.

In an older formulation of this scheme,  $r_e$  was parametrized as a linear function of height from 10  $\mu\text{m}$  at the surface to 45  $\mu\text{m}$  at the top of the atmosphere, in an empirical attempt at dealing with the variation of water cloud type with height. Smaller water droplets are observed in low-level stratiform clouds whereas larger droplets are found in mid-level cumuliform water clouds. This parametrisation was then revised, using the formulaion of [Martin \*et al.\* \(1994\)](#) to compute the effective radius of the liquid water cloud particles from the cloud liquid water content and specified concentrations of cloud condensation nuclei over land and ocean. For ice clouds, the effective dimension of the cloud particles was diagnosed from temperature using a revision of the formulation by [Ou and Liou \(1995\)](#).

In the two-, four-, and six-spectral interval versions of the short-wave radiation scheme, the optical properties of liquid water clouds were defined from [Fouquart \(1987\)](#) and those for ice clouds from [Ebert and Curry \(1992\)](#). Alternative optical properties were also available for liquid water clouds ([Slingo, 1989](#)) and ice clouds ([Fu, 1996](#)).

## 2.3 THE PRE-CY22R3 LONG-WAVE RADIATION SCHEME

As already noted, since cycle Cy22r3, two long-wave radiation schemes have been available in the ECMWF model, the pre-cycle Cy22r3 by [Morcrette \(1991\)](#), and the current long-wave radiation transfer scheme, the Rapid Radiation Transfer Model (RRTM) (see [Section 2.5](#)).

The rate of atmospheric cooling by emission-absorption of long-wave radiation is

$$\frac{\partial T}{\partial t} = \frac{g}{c_p} \frac{\partial \mathcal{F}_{\text{LW}}}{\partial p} \quad (2.39)$$

where  $\mathcal{F}_{\text{LW}}$  is the net long-wave radiation flux (the subscript ‘LW’ is omitted in the remainder of this section).

Assuming a non-scattering atmosphere in local thermodynamic equilibrium,  $\mathcal{F}$  is given by

$$\mathcal{F} = \int_{-1}^1 \mu \, d\mu \left[ \int_0^\infty dv \left\{ \mathcal{L}_v(p_{\text{surf}}, \mu) t_v(p_{\text{surf}}, p, \mu) + \int_{p'=p_{\text{surf}}}^0 \mathcal{L}_v(p', \mu) dt_v \right\} \right] \quad (2.40)$$

where  $\mathcal{L}_v(p, \mu)$  is the monochromatic radiance at wavenumber  $v$  at level  $p$ , propagating in a direction  $\theta$  (the angle that this direction makes with the vertical), where  $\mu = \cos \theta$  and  $t_v(p, p'; r)$  is the monochromatic transmission through a layer whose limits are at  $p$  and  $p'$  seen under the same angle  $\theta$ , with  $r = \sec \theta$ . The subscript ‘surf’ refers to the earth’s surface.

### 2.3.1 The pre-cycle Cy22r3 scheme

After separating the upward and downward components (indicated by superscripts + and –, respectively), and integrating by parts, we obtain the radiation transfer equation as it is actually estimated in the long-wave part of the radiation code

$$\begin{aligned}\mathcal{F}_v^+(p) &= [B_v(T_{\text{surf}}) - B_v(T_{0+})]t_v(p_{\text{surf}}, p; r) + B_v(T(p)) + \int_{p'=p_{\text{surf}}}^p t_v(p, p'; r) dB_v \\ \mathcal{F}_v^-(p) &= [B_v(T_{\infty}) - B_v(T_{\text{top}})]t_v(p, 0; r) + B_v(T(p)) + \int_{p'=p}^0 t_v(p', p; r) dB_v\end{aligned}\quad (2.41)$$

where, taking benefit of the isotropic nature of the long-wave radiation, the radiance  $L_v$  of (2.40) has been replaced by the Planck function  $B_v(T)$  in units of flux,  $\text{Wm}^{-2}$  (here, and elsewhere,  $B_v$  is assumed to always include the  $\pi$  factor).  $T_{\text{surf}}$  is the surface temperature,  $T_{0+}$  that of the air just above the surface,  $T(p)$  is the temperature at pressure-level  $p$ ,  $T_{\text{top}}$  that at the top of the atmospheric model. The transmission  $t_v$  is evaluated as the radiance transmission in a direction  $\theta$  to the vertical such that  $r = \sec \theta$  is the diffusivity factor (Elsasser, 1942). Such an approximation for the integration over the angle is usual in radiative transfer calculations, and tests on the validity of this approximation have been presented by Rodgers and Walshaw (1966) and Liu and Schmetz (1988) among others. The use of the diffusivity factor gives cooling rates within 2% of those obtained with a 4-point Gaussian quadrature.

### 2.3.2 Vertical integration

The integrals in (2.41) are evaluated numerically, after discretization over the vertical grid, considering the atmosphere as a pile of homogeneous layers. As the cooling rate is strongly dependent on local conditions of temperature and pressure, and energy is mainly exchanged with the layers adjacent to the level where fluxes are calculated, the contribution of the distant layers is simply computed using a trapezoidal rule integration, but the contribution of the adjacent layers is evaluated with a 2-point Gaussian quadrature, thus at the  $i$ th level

$$\int_{p'=p_{\text{surf}}}^{p_i} t_v(p, p'; r) dB_v = \sum_{l=1}^2 dB_v(l)w_l t_v(p_i, p_l; r) + \frac{1}{2} \sum_{j=1}^{i-2} dB_v(j)[t_v(p_i, p_j; r) + t_v(p_i, p_{j-1}; r)] \quad (2.42)$$

where  $p_l$  is the pressure corresponding to the Gaussian root and  $w_l$  is the Gaussian weight.  $dB_v(j)$  and  $dB_v(l)$  are the Planck function gradients calculated between two interfaces, and between mid-layer and interface, respectively.

### 2.3.3 Spectral integration

The integration over wavenumber  $\nu$  is performed using a band emissivity method, as first discussed by Rodgers (1967). The long-wave spectrum is divided into six spectral regions.

- (i) 0–350  $\text{cm}^{-1}$  and 1450–1880  $\text{cm}^{-1}$
- (ii) 500–800  $\text{cm}^{-1}$
- (iii) 800–970  $\text{cm}^{-1}$  and 1110–1250  $\text{cm}^{-1}$
- (iv) 970–1110  $\text{cm}^{-1}$
- (v) 350–500  $\text{cm}^{-1}$
- (vi) 1250–1450  $\text{cm}^{-1}$  and 1880–2820  $\text{cm}^{-1}$

corresponding to the centres of the rotation and vibration-rotation bands of  $\text{H}_2\text{O}$ , the 15  $\mu\text{m}$  band of  $\text{CO}_2$ , the atmospheric window, the 9.6  $\mu\text{m}$  band of  $\text{O}_3$ , the 25  $\mu\text{m}$  “window” region, and the wings of the vibration-rotation band of  $\text{H}_2\text{O}$ , respectively. Over these spectral regions, band fluxes are evaluated with the help of band transmissivities precalculated from the narrow-band model of Morcrette and Fouquart (1985) – See Appendix of Morcrette *et al.* (1986) for details.

Integration of (2.41) over wavenumber  $\nu$  within the  $k$ th spectral region gives the upward and downward fluxes as

$$\begin{aligned} \mathcal{F}_k^+(p) &= \{B_k(T_{\text{surf}}) - B_k(T_{0+})\}t_{B_k}\{r\mathcal{U}(p_{\text{surf}}, p), T_{\mathcal{U}}(p_{\text{surf}}, p)\} + B_k(T_p) \\ &\quad + \int_{p'=p_{\text{surf}}}^p t_{dB_k}\{r\mathcal{U}(p, p'), T_{\mathcal{U}}(p, p')\} dB_k \end{aligned} \quad (2.43)$$

$$\begin{aligned} \mathcal{F}_k^-(p) &= \{B_k(T_0) - B_k(T_{\infty})\}t_{B_k}\{r\mathcal{U}(p, 0), T_{\mathcal{U}}(p, 0)\} - B_k(T_p) \\ &\quad - \int_{p'=p}^0 t_{dB_k}\{r\mathcal{U}(p', p), T_{\mathcal{U}}(p', p)\} dB_k \end{aligned} \quad (2.44)$$

The formulation accounts for the different temperature dependencies involved in atmospheric flux calculations, namely that on  $T_p$ , the temperature at the level where fluxes are calculated, and that on  $T_{\mathcal{U}}$ , the temperature that governs the transmission through the temperature dependence of the intensity and half-widths of the lines absorbing in the concerned spectral region. The band transmissivities are non-isothermal accounting for the temperature dependence that arises from the wavenumber integration of the product of the monochromatic absorption and the Planck function. Two normalized band transmissivities are used for each absorber in a given spectral region: the first one for calculating the first right-hand-side term in (2.41), involving the boundaries; it corresponds to the weighted average of the transmission function by the Planck function

$$t_B(\overline{\mathcal{U}p}, T_p, T_{\mathcal{U}}) = \frac{\int_{v_1}^{v_2} B_v(T_p)t_v(\overline{\mathcal{U}p}, T_{\mathcal{U}}) dv}{\int_{v_1}^{v_2} B_v(T_p) dv} \quad (2.45)$$

the second one for calculating the integral term in (2.41) is the weighted average of the transmission function by the derivative of the Planck function

$$t_{dB}(\overline{\mathcal{U}p}, T_p, T_{\mathcal{U}}) = \frac{\int_{v_1}^{v_2} \{dB(T_p)/dT\}t_v(\overline{\mathcal{U}p}, T_{\mathcal{U}}) dv}{\int_{v_1}^{v_2} \{dB(T_p)/dT\} dv} \quad (2.46)$$

where  $\overline{\mathcal{U}p}$  is the pressure weighted amount of absorber.

The effect on absorption of the Doppler broadening of the lines (important only for pressure lower than 10 hPa) is included simply using the pressure correction method of Fels (1979). A finite line width (assumed to represent the Doppler half-width of the line) is retained under low pressure conditions where the pure Lorentz line width (proportional to pressure) would normally become negligible (Giorgetta and Morcrette, 1995).

In the scheme, the actual dependence on  $T_p$  is carried out explicitly in the Planck functions integrated over the spectral regions. Although normalized relative to  $B(T_p)$  or  $dB(T_p)/dT$ , the transmissivities still depend on  $T_{\mathcal{U}}$ , both through Wien's displacement of the maximum of the Planck function with temperature and through the temperature dependence of the absorption coefficients. For computational efficiency, the transmissivities have been developed into Pade approximants

$$t(\overline{\mathcal{U}p}, T_u) = \frac{\sum_{i=0}^2 c_i \mathcal{U}_{\text{eff}}^{i/2}}{\sum_{j=0}^2 d_j \mathcal{U}_{\text{eff}}^{j/2}} \quad (2.47)$$

where  $\mathcal{U}_{\text{eff}} = r(\overline{\mathcal{U}p})\Psi(T_{\mathcal{U}}, \overline{\mathcal{U}p})$  is an effective amount of absorber which incorporates the diffusivity factor  $r$ , the weighting of the absorber amount by pressure  $\overline{\mathcal{U}p}$ , and the temperature dependence of the absorption coefficients. The function  $\Psi(T_{\mathcal{U}}, \overline{\mathcal{U}p})$  takes the form

$$\Psi(T_{\mathcal{U}}, \overline{\mathcal{U}p}) = \exp[a(\overline{\mathcal{U}p})(T_{\mathcal{U}} - 250) + b(\overline{\mathcal{U}p})(T_{\mathcal{U}} - 250)^2] \quad (2.48)$$

The temperature dependence due to Wien's law is incorporated although there is no explicit variation of the coefficients  $c_i$  and  $d_j$  with temperature. These coefficients have been computed for temperatures between 187.5 and 312.5 K with a 12.5 K step, and transmissivities corresponding to the reference temperature the closest to the pressure weighted temperature  $T_{\mathcal{U}}$  are actually used in the scheme.

### 2.3.4 The incorporation of the effects of clouds

The incorporation of the effects of clouds on the long-wave fluxes follows the treatment discussed by [Washington and Williamson \(1977\)](#). Whatever the state of the cloudiness of the atmosphere, the scheme starts by calculating the fluxes corresponding to a clear-sky atmosphere and stores the terms of the energy exchange between the different levels (the integrals in (2.41)). Let  $\mathcal{F}_0^+(i)$  and  $\mathcal{F}_0^-(i)$  be the upward and downward clear-sky fluxes. For any cloud layer actually present in the atmosphere, the scheme then evaluates the fluxes assuming a unique overcast cloud of emissivity unity. Let  $\mathcal{F}_n^+(i)$  and  $\mathcal{F}_n^-(i)$  the upward and downward fluxes when such a cloud is present in the  $n$ th layer of the atmosphere. Downward fluxes above the cloud, and upward fluxes below the cloud, are assumed to be given by the clear-sky values

$$\begin{aligned}\mathcal{F}_n^+(i) &= \mathcal{F}_0^+(i) \quad \text{for } i \leq n \\ \mathcal{F}_n^-(i) &= \mathcal{F}_0^-(i) \quad \text{for } i > n\end{aligned}\tag{2.49}$$

Upward fluxes above the cloud ( $\mathcal{F}_n^+(k)$  for  $k \leq n+1$ ) and downward fluxes below it ( $\mathcal{F}_n^-(k)$  for  $k > n$ ) can be expressed with expressions similar to (2.42) provided the boundary terms are now replaced by terms corresponding to possible temperature discontinuities between the cloud and the surrounding air

$$\begin{aligned}\mathcal{F}_n^+(k) &= \{\mathcal{F}_{\text{cld}}^+ - B(n+1)\}t(p_k, p_{n+1}; r) + B(k) + \int_{p'=p_{n-1}}^{p_k} t(p_k, p'; r) dB \\ \mathcal{F}_n^-(k) &= \{\mathcal{F}_{\text{cld}}^- - B(n)\}t(p_k, p_n; r) + B(k) + \int_{p'=p_k}^{p_n} t(p_k, p'; r) dB\end{aligned}\tag{2.50}$$

where  $B(i)$  is now the total Planck function (integrated over the whole long-wave spectrum) at level  $i$ , and  $\mathcal{F}_{\text{cld}}^+$  and  $\mathcal{F}_{\text{cld}}^-$  are the long-wave fluxes at the upper and lower boundaries of the cloud. Terms under the integrals correspond to exchange of energy between layers in clear-sky atmosphere and have already been computed in the first step of the calculations. This step is repeated for all cloudy layers. The fluxes for the actual atmosphere (with semi-transparent, fractional and/or multi-layered clouds) are derived from a linear combination of the fluxes calculated in previous steps with some cloud overlap assumption in the case of clouds present in several layers. Let  $N$  be the index of the layer containing the highest cloud,  $C_{\text{cld}}(i)$  the fractional cloud cover in layer  $i$ , with  $C_{\text{cld}}(0) = 1$  for the upward flux at the surface, and with  $C_{\text{cld}}(N+1) = 1$  and  $\mathcal{F}_{N+1}^- = \mathcal{F}_0^-$  to have the right boundary condition for downward fluxes above the highest cloud.

Whereas the maximum and random overlap assumptions are also available in the code ([Morcrette and Fouquart, 1986](#)), the maximum-random overlap assumption is operationally used in the ECMWF model, and the cloudy upward  $\mathcal{F}^+$  and downward  $\mathcal{F}^-$  fluxes are obtained as

$$\begin{aligned}\mathcal{F}^+(i) &= \mathcal{F}_0^+(i) && \text{for } i = 1 \\ \mathcal{F}^-(i) &= C_{\text{cld}}(i-1)\mathcal{F}_{i-1}^+(i-) + \sum_{n=0}^{i-2} C_{\text{cld}}(n)\mathcal{F}_n^+(i) \prod_{l=n+1}^{i-1} \{1 - C_{\text{cld}}(l)\} && \text{for } 2 \leq i \leq N+1 \\ \mathcal{F}^+(i) &= C_{\text{cld}}(N)\mathcal{F}_N^+(i) + \sum_{n=0}^{N-1} C_{\text{cld}}(n)\mathcal{F}_n^+(i) \prod_{l=n+1}^N \{1 - C_{\text{cld}}(l)\} && \text{for } i \geq N+2\end{aligned}\tag{2.51}$$

In the case of semi-transparent clouds, the fractional cloudiness entering the calculations is an effective cloud cover equal to the product of the emissivity due to the condensed water and the gases in the layer by the horizontal coverage of the cloud layer, with the emissivity,  $\varepsilon_{\text{cld}}$ , related to the condensed water amount by

$$\varepsilon_{\text{cld}} = 1 - \exp(-k_{\text{abs}}\mathcal{M}_{\text{LWP}})\tag{2.52}$$

where  $k_{\text{abs}}$  is the condensed water mass absorption coefficient (in  $\text{m}^2\text{kg}^{-1}$ ) following [Smith and Shi \(1992\)](#).

## 2.4 THE MONTE-CARLO INDEPENDENT COLUMN APPROXIMATION, McICA

The McICA approach is an approximation to the full Independent Column Approximation (ICA). As discussed by [Barker \*et al.\* \(2003\)](#) and [Pincus \*et al.\* \(2003\)](#), for the , the average monochromatic radiative

flux, over a domain sub-divided in  $N$  columns, in which each layer can only have a cloud fraction of 0 or 1, is

$$\langle F \rangle = \frac{1}{N} \sum_{n=1}^N F_n \quad (2.53)$$

In sub-column  $n$ , using a radiation parametrisation (plane-parallel, and considering a homogeneous cloud water distribution in all overcast layers) with a correlated k-distribution (CKD) approach to deal with absorption, the total flux  $F_n$  is

$$F_n = \sum_{k=1}^K c_k F_{n,k} \quad (2.54)$$

Combining (2.53) and (2.54) gives

$$\langle F \rangle = \frac{1}{N} \sum_{n=1}^N \sum_{k=1}^K c_k F_{n,k} \quad (2.55)$$

A radiation code explicitly integrating the double sum in (2.55) would be far too expensive for GCM applications. The McICA solution to this problem is to approximate (3) as

$$\langle F \rangle_M = \sum_{k=1}^K F_{n_k,k} \quad (2.56)$$

where  $F_{n_k,k}$  is the monochromatic radiative flux for a single randomly selected sub-column  $n_k$ .

From this definition, the McICA solution (2.56) equals the ICA solution only when all  $N$  sub-columns are identical or  $N = 1$ . As discussed in Räisänen and Barker (2004), McICA's incomplete pairing of sub-columns and spectral intervals ensures that its solution will contain random, but unbiased, errors.

McICA can in principle be used within any radiation transfer scheme provided a cloud generator is used to define how the cloud information is distributed over each spectral element in the radiation spectrum. However, to take full benefit of the McICA approach, radiation schemes with a sufficiently large number of transmission calculations are required. With the version of the ECMWF model used for this study, the long-wave radiation fluxes are computed using the Rapid Radiation Transfer Model (RRTM<sub>LW</sub>), already in use at ECMWF since June 2000. For consistency in terms of radiative transfer solution and database of spectroscopic parameters, the short-wave radiation fluxes are now computed with RRTM<sub>SW</sub>. The application of the McICA approach involves using a cloud generator together with slightly modified but otherwise standard radiation schemes.

Table 2.2 summarizes the main features of the McRad radiation package. The ECMWF version of RRTM<sub>LW</sub> (Mlawer *et al.*, 1997, Morcrette *et al.*, 2001) describes the long-wave spectrum with 16 spectral intervals, corresponding to a total of  $K = 140$  g-points, with a g-point being a point in the space of the line intensities within a given spectral interval (Lacis and Oinas, 1991). RRTM<sub>SW</sub> (Mlawer and Clough, 1997) describes the short-wave spectrum with 14 spectral intervals, corresponding to a total of  $K = 112$  g-points. Each of the long-wave or short-wave spectral intervals have a different number of g-points, depending how much the absorption coefficient varies within the spectral interval, but also how much the spectral interval contributes overall to the total flux, and this over the whole depth of the atmosphere represented by the atmospheric model. For each of these g-points, an essentially monochromatic type radiation transfer is carried out using a two-stream method using an approximate provision for the long-wave scattering and using a Delta two-stream method with scattering in the short-wave.

The McICA versions of RRTM<sub>LW</sub> and RRTM<sub>SW</sub> differ from the original versions in two respects:

**Table 2.2** Characteristics of the long-wave and short-wave radiation schemes in McRad.

(\*) refer to the configuration operational with McRad.

 Note: CFC11 is  $CFCl_3$  and CFC12 is  $CF_2Cl_2$ .

	RRTM <sub>LW</sub>	RRTM <sub>SW</sub>
Solution of RT Equation	two-stream method	two-stream method
Number of spectral intervals	16 (140 g-points)	14 (112 g-points)
Absorbers	$H_2O$ , $CO_2$ , $O_3$ , $CH_4$ , $N_2O$ , $CFC11$ , $CFC12$ , aerosols	$H_2O$ , $CO_2$ , $O_3$ , $O_2$ , $CH_4$ , $N_2O$ , $CFC11$ , $CFC12$ , aerosols
Spectroscopic database	HITRAN, 1996	HITRAN, 1996
Absorption coefficients	from LBLRTM line-by-line model	from LBLRTM line-by-line model
Cloud handling	true cloud fraction	true cloud fraction
Cloud optical properties		
method	16-band spectral emissivity	14-band $\tau$ , $g$ , $\omega$
Data: ice clouds	Ebert & Curry, 1992 Fu et al., 1998 (*)	Ebert & Curry, 1992 Fu, 1996 (*)
water clouds	Smith & Shi, 1992 Lindner & Li, 2000 (*)	Fouquart, 1987 Slingo, 1989 (*)
Cloud overlap assumption set up in cloud generator	maximum-random or generalized (*)	maximum-random or generalized (*)
Reference	Mlawer et al., 1997 Morcrette et al., 2001	Mlawer and Clough, 1997

- (i) Avoiding any explicit reference to cloud fraction greatly simplifies the part of the algorithms devoted to the vertical integration, which now deals simply with optical thicknesses. For a given g-point, a cloud when present fully occupies a model layer. Therefore any cloudy calculation only involves modifying the optical parameters.
- (ii) This enables the removal of the 0.7 correction factor multiplying the cloud optical thickness, which had been introduced in December 1997 (Cahalan *et al.*, 1994; Tiedtke, 1996) in the ECMWF IFS to account approximately for the effect of cloud inhomogeneities at the sub-grid level.

As stated in the Introduction, the McICA representation of cloud-radiation interactions requires the cloud information to be distributed by a cloud generator over the vertical with the constraint that the total cloudiness and cloud water loading for a grid-point is conserved.

The purpose of the cloud generator is, starting from a cloud profile (cloud fraction and cloud water content) provided by a traditional cloud scheme (Tiedtke, 1993 for the ECMWF model), to distribute randomly the cloud information (in terms of presence (1) or absence (0)) into each of the layers covered by the original cloud profile. This distribution is done  $N$  times (McICA with  $N$  going to infinity would be equal to ICA) with the constraint that a summation over the  $N$  profiles would recreate the original vertical distribution of partial cloudiness. In the ECMWF model, for each radiation time-step (every one hour of model time for the T<sub>L</sub>799L91 and T<sub>L</sub>399L62 forecasts, every three hours for seasonal simulations at T<sub>L</sub>159L91 and each radiation grid-point, the cloud generator is used twice, to produce two cloud distributions relevant, respectively, to the 140 g-points of the LW- and 112 g-points of SW radiation schemes. We use the cloud generator of Räisänen *et al.*, 2004, which can distribute vertically either the cloud cover according to a maximum-random overlap assumption or both the cloud cover and cloud water assuming a generalized overlap (Hogan and Illingworth, 2000, 2003). The results presented hereafter corresponds to the operational McRad configuration with a generalized overlap with decorrelation lengths of 2 km for cloud cover and 1 km for cloud water, and a normalized standard deviation of the cloud condensate of 1.

Clouds when present occupy the full horizontal extent of the layer, and the vertical distribution of such clouds (of 0 or 1 cloud cover) is defined independently for each of the 140 (112) g-points of the long-wave (short-wave) scheme by the cloud generator, with the constraint that the total cloudiness and cloud water loading for a grid-point is conserved when  $N$  tends to infinity.

**Table 2.3** *Spectral distribution of the absorption by atmospheric gases in RRTM<sub>LW</sub>.*

Note: CCl<sub>4</sub> and CFC22 (CHClF<sub>2</sub>) are presently not accounted for in the ECMWF model.

Spectral intervals cm <sup>-1</sup>	Number of g-points	Gases included	
		Troposphere	Stratosphere
10–350	8	H <sub>2</sub> O	H <sub>2</sub> O
350–500	14	H <sub>2</sub> O	H <sub>2</sub> O
500–630	16	H <sub>2</sub> O, CO <sub>2</sub> , N <sub>2</sub> O	H <sub>2</sub> O, CO <sub>2</sub> , N <sub>2</sub> O
630–700	14	H <sub>2</sub> O, CO <sub>2</sub>	O <sub>3</sub> , CO <sub>2</sub>
700–820	16	H <sub>2</sub> O, CO <sub>2</sub> , O <sub>3</sub> , CCl <sub>4</sub>	O <sub>3</sub> , CO <sub>2</sub> , CCl <sub>4</sub>
820–980	8	H <sub>2</sub> O, CO <sub>2</sub>	CFC11, CFC12
980–1080	12	H <sub>2</sub> O, O <sub>3</sub> , CO <sub>2</sub>	O <sub>3</sub> , CO <sub>2</sub>
1080–1180	8	H <sub>2</sub> O, CFC12, CFC22, CO <sub>2</sub> , O <sub>3</sub> , N <sub>2</sub> O	O <sub>3</sub> , CFC12, CFC22, CO <sub>2</sub> , N <sub>2</sub> O
1180–1390	12	H <sub>2</sub> O, CH <sub>4</sub> , N <sub>2</sub> O	CH <sub>4</sub> , N <sub>2</sub> O
1390–1480	6	H <sub>2</sub> O	H <sub>2</sub> O
1480–1800	8	H <sub>2</sub> O, O <sub>2</sub>	H <sub>2</sub> O, O <sub>2</sub>
1800–2080	8	H <sub>2</sub> O	
2080–2250	4	H <sub>2</sub> O, N <sub>2</sub> O	O <sub>3</sub>
2250–2380	2	CO <sub>2</sub>	CO <sub>2</sub>
2380–2600	2	N <sub>2</sub> O, CO <sub>2</sub>	
2600–3250	2	H <sub>2</sub> O, CH <sub>4</sub>	CH <sub>4</sub>

## 2.5 THE RAPID RADIATION TRANSFER MODEL: LONG-WAVE (RRTM<sub>LW</sub>)

Since cycle Cy22r3, two long-wave radiation schemes are available in the ECMWF model, the pre-cycle Cy22r3 by [Morcrette \(1991\)](#) discussed in [Section 2.3](#), and the current long-wave radiation transfer scheme, the Rapid Radiation Transfer Model, which has been used in a standard way from June 2000 to May 2007, and within the McICA framework since 5 June 2007. In the current operational implementation the spectral bands definition and the active gases in the long-wave spectrum follow the version 4.85 of RRTMG-LW.

As stated in [Mlawer \*et al.\* \(1997\)](#), the objective in the development of RRTM has been to obtain an accuracy in the calculation of fluxes and heating rates consistent with the best line-by-line models. It utilizes the correlated-k method and shows its filiation to the Atmospheric and Environmental Research, Inc. (AER) line-by-line model (LBLRTM, [Clough \*et al.\*, 1989, 1992](#), [Clough and Iacono, 1995](#)) through its use of absorption coefficients for the relevant k-distributions derived from LBLRTM. In the original implementation of RRTM the k-coefficients included the effect of the CKD2.2 water vapour continuum ([Clough \*et al.\*, 1989](#)), upgraded to CKD2.4 in the current operational implementation. [Table 2.3](#) presents the spectral intervals of the RRTM<sub>LW</sub> scheme and the absorbers taken into account in the spectral computations.

The main point in the correlated-k method ([Lacis and Oinas, 1991](#); [Fu and Liou, 1992](#)) is the mapping of the absorption coefficient  $k(\nu)$  from the spectral space (where it varies irregularly with wavenumber  $\nu$ ) to the  $g$ -space (where  $g(k)$  is the probability distribution function, i.e. the fraction of the absorption coefficients in the set smaller than  $k$ ). The effect of this reordering is a rearrangement of the sequence of terms in the integral over wavenumber in the radiative transfer equation (RTE), which makes it equivalent to what would be done for monochromatic radiation.

In the ECMWF model, no provision is presently taken for scattering in the long-wave. Therefore, in order to get the downward radiance, the integration over the vertical dimension is simply done starting from the top of the atmosphere, going downward layer by layer. At the surface, the boundary condition (in terms of spectral emissivity, and potential reflection of downward radiance) is computed, then, in order to get the upward radiance, the integration over the vertical dimension is repeated, this time from the surface upward.

The spectrally averaged radiance (between  $\nu_1$  and  $\nu_2$ ) emerging from an atmospheric layer is

$$\bar{R} = \frac{1}{(\nu_1 - \nu_2)} \int_{\nu_2}^{\nu_1} d\nu \left\{ R_0(\nu) + \int_{t_\nu}^1 [B(\nu, T(t'_\nu)) - R_0(\nu)] dt' \right\} \quad (2.57)$$

where  $R_0$  is the incoming radiance to the layer,  $B(\nu, T)$  is the Planck function at wavenumber  $\nu$  and temperature  $T$ ,  $t_\nu$  is the transmittance for the layer optical path, and  $t'_\nu$  is the transmittance at a point along the optical path in the layer. Under the mapping  $\nu \rightarrow g$ , this becomes

$$\bar{R} = \int_0^1 dg \left\{ B_{\text{eff}}(g, T_g) + [R_0(g) - B_{\text{eff}}(g, T_g)] \exp \left[ -k(g, P, T) \frac{\rho \Delta z}{\cos \phi} \right] \right\} \quad (2.58)$$

where  $B_{\text{eff}}(g, T)$  is an effective Planck function for the layer that varies with the layer's transmittance such as to ensure continuity of flux across layer boundaries for opaque conditions. The dependence of the transmittance is now written in terms of the absorption coefficient  $k(g, P, T)$  at layer pressure  $P$  and temperature  $T$ , the absorber density  $\rho$ , the vertical thickness of the layer  $\Delta z$ , and the angle  $\phi$  of the optical path.

For a given spectral interval, the domain of the variable  $g$  is partitioned into subintervals (see [Table 2.3](#), number of  $g$ -points), each corresponding to a limited range of  $k(g)$  values and for which a characteristic value  $\kappa_j$  of the absorption coefficient is chosen. These  $\kappa_j$  are then used to compute the outgoing radiance

$$\bar{R} = \sum_j W_j \left[ B_{\text{eff},j} + (R_{0,j} - B_{\text{eff},j}) \exp \left( -\kappa_j \frac{\rho \Delta z}{\cos \phi} \right) \right] \quad (2.59)$$

where  $W_j$  is the size of the sub-intervals ( $\sum W_j = 1$ ).

The accuracy of these absorption coefficients has been established by numerous and continuing high-resolution validations of LBLRTM with spectroscopic measurements, in particular those from the Atmospheric Radiation Measurement program (ARM). Compared to the original RRTM ([Mlawer \*et al.\*, 1997](#)), the version used at ECMWF has been slightly modified to account for cloud optical properties and surface emissivity defined for each of the 16 bands over which spectral fluxes are computed. For efficiency reason, the original number of  $g$ -points ( $256 = 16 \times 16$ ) has been reduced to 140 (see [Table 2.3](#)). Other changes are the use of a diffusivity approximation (instead of the three-angle integration over the zenith angle used in the original scheme) to derive upward and downward fluxes from the radiances, and the modification of the original cloud random overlapping assumption to include (to the same degree of approximation as used in the operational SW scheme) a maximum-random overlapping of cloud layers. Given the monochromatic form of the RTE, the vertical integration is simply carried out one layer at a time from the top-of-the-atmosphere to the surface to get the downward fluxes. The downward fluxes at the surface are then used with the spectral surface emissivities and the surface temperature to get the upward long-wave fluxes in each of the 140 subintervals. Then the upward fluxes are obtained in a similar fashion from the surface to the ToA.

For the relevant spectral intervals of the RRTM schemes, ice cloud optical properties are derived from [Ebert and Curry \(1992\)](#), and water cloud optical properties from [Fouquart \(1987\)](#). Whereas in the previous operational scheme the cloud emissivity used to compute the effective cloud cover is defined over the whole LW spectrum from spectrally averaged mass absorption coefficients and the relevant cloud water and/or ice paths (following [Smith and Shi, 1992](#)), in RRTM, the cloud optical thickness is defined as a function of spectrally varying mass absorption coefficients and relevant cloud water and ice paths, and is used within the true cloudy fraction of the layer. Alternate sets of cloud optical properties are also available for RRTM, based on [Savijarvi and Räisänen \(1997\)](#) for liquid water clouds, and [Fu \*et al.\* \(1998\)](#) for ice clouds.

## 2.6 THE RAPID RADIATION TRANSFER MODEL: SHORT-WAVE (RRTM<sub>SW</sub>)

As its long-wave counterpart RRTM<sub>LW</sub>, RRTM<sub>SW</sub> uses absorption coefficient data for the  $g$ -points obtained directly from the line-by-line radiative transfer model LBLRTM ([Clough \*et al.\*, 2005](#)), which has

**Table 2.4** *Spectral distribution of the absorption by atmospheric gases in RRTM<sub>SW</sub>.*

Spectral intervals cm <sup>-1</sup>	Number of g-points	Gases included	
		Troposphere	Stratosphere
800–2600	12	H <sub>2</sub> O	CO <sub>2</sub>
2600–3250	6	H <sub>2</sub> O, CH <sub>4</sub>	
3250–4000	12	H <sub>2</sub> O, CO <sub>2</sub>	H <sub>2</sub> O, CO <sub>2</sub>
4000–4650	8	H <sub>2</sub> O, CH <sub>4</sub>	CH <sub>4</sub>
4650–5150	8	H <sub>2</sub> O, CO <sub>2</sub>	CO <sub>2</sub>
5150–6150	10	H <sub>2</sub> O, CH <sub>4</sub>	H <sub>2</sub> O, CH <sub>4</sub>
6150–7700	10	H <sub>2</sub> O, CO <sub>2</sub>	H <sub>2</sub> O, CO <sub>2</sub>
7700–8050	2	H <sub>2</sub> O, O <sub>2</sub>	O <sub>2</sub>
8050–12850	10	H <sub>2</sub> O	
12850–16000	8	H <sub>2</sub> O, O <sub>2</sub>	O <sub>2</sub>
16000–22650	6	H <sub>2</sub> O	
22650–29000	6		
29000–38000	8	O <sub>3</sub>	O <sub>3</sub>
38000–50000	6	O <sub>3</sub> , O <sub>2</sub>	O <sub>3</sub> , O <sub>2</sub>

been extensively validated against observations, principally at the ARM-South Great Plains site. Fluxes and heating rates are calculated over 14 contiguous bands (see [Table 2.4](#)) in the short-wave. Modelled sources of extinction are water vapour, carbon dioxide, ozone, methane, oxygen, nitrogen, aerosols and Rayleigh scattering. A two-stream algorithm (similar to the one described in [Section 2.2](#)) is used to perform scattering calculations (see also [Oreopoulos and Barker, 1999](#)). [Table 2.4](#) presents the spectral intervals of the RRTM<sub>SW</sub> scheme and the absorbers taken into account in the spectral computations.

## 2.7 HORIZONTAL AND TEMPORAL INTERPOLATIONS

As stated in the introduction, the cost of the radiation scheme described in the previous sections is prohibitive if it were used to compute the radiative fluxes at every time step and every grid point of the model.

A new interface for radiation computations was developed and implemented in October 2003 with Cy26r3. Full radiation computations are now performed using the so-called halo configuration that can be defined according to needs for the various spatial resolutions.

The previous spatial sampling (operational till Cy26r1), was done only in the longitudinal direction. It was going from one out of four points prevalent in sub-tropical and tropical latitudes and reduced gradually to every point in polar areas. On output, Lagrangian cubic interpolation was used. The scheme worked efficiently on vector systems with less than 100 processors and scalar systems with about 1000 processors. The only real problem was the complexity of the message passing, a direct result of the use of a non-standard grid for radiation calculations.

The new interface for radiation computations was developed to address this complexity, and uses a standard IFS model grid, but with a coarser resolution than the current model grid. Further, interpolation between model and radiation grids are performed using the interfaces already existing within the IFS for the semi-Lagrangian interpolation, and as a result should reduce future code maintenance. By using such a standard grid for radiation computations, there is no longer a load balance issue, as each processor is given an equal number of grid points for model and radiation grids.

A new coarser resolution grid is computed, independent of that for the rest of the physics. Interpolation between model and radiation grids are performed using interfaces existing within the IFS libraries and as a result helps reduce code maintenance. Then radiation computations are done, and output fluxes are interpolated back to the reduced grid, at times of full radiation computations. This halo-related grid can be chosen differently with the forecast application (seasonal runs, EPS, high-resolution 10-day forecasts).

**Table 2.5** Impact of the McRad radiation package on the timing of the ECMWF model forecasts for different configurations and different horizontal resolutions. *Dyn* is the resolution for the dynamics, *Rad* that for the radiation. *Freq* is the frequency (hour) for calling the full radiation scheme, *%Rad* is the fraction of computer time taken by the radiative transfer calculations. *Ratio* is the factor by which McRad increases the computer cost relative to the previous operational configuration (ref31R2).

Configuration	Dyn	Rad	Freq	%Rad	Ratio
T <sub>L</sub> 799L91					
ref31R2	799	399	1	7.3	1.000
McRad	799	511	1	36.4	1.456
	799	399	1	26.5	1.262
	799	<b>319</b>	1	19.2	1.147
	799	255	1	13.8	1.076
	799	159	1	6.7	0.994
	799	95	1	3.4	0.960
T <sub>L</sub> 399L62					
ref31R2	399	159	3	4.1	1.000
McRad	399	255	3	31.6	1.403
	399	159	3	16.4	1.148
	399	<b>95</b>	3	7.7	1.039
	399	63	3	3.8	0.998
	399	47	3	3.0	0.989
	399	31	3	2.1	0.980
T <sub>L</sub> 159L91					
ref31R2	159	63	3	8.0	1.000
McRad	159	159	3	67.5	2.831
	159	95	3	45.1	1.675
	159	<b>63</b>	3	27.7	1.273
	159	47	3	19.5	1.143
	159	31	3	11.0	1.034

This radiation grid had been used between October 2003 and June 2007, with a coarsening factor of two in both latitude and longitude w.r.t. the rest of the model (see [Table 2.3](#) for Cy31r2).

The implementation of the more computer-intensive McRad, through its use of RRTM<sub>SW</sub> with an increased number of spectral intervals, has led to the search for an optimal radiation grid for the different weather forecasting applications run at ECMWF. Depending on the model resolution, associated time-step, and the frequency for calling the full radiation schemes, the cost of the model integration drastically increased. However, comparisons of results with the different radiation grids (from R399 to R95 for the T<sub>L</sub>799L91 high-resolution model, from R255 to R31 for the T<sub>L</sub>399L62 model run in the Ensemble Prediction System, from R159 to R31 for the T<sub>L</sub>159L91 model used for seasonal forecasts, were systematically carried out. For the radiation grid, a best compromise was chosen (R319 for T<sub>L</sub>799, R95 for T<sub>L</sub>399, R63 for T<sub>L</sub>159), which allows the maximum benefit of McRad within the time constraints for delivering the various operational products.

As seen in [Table 2.5](#), the full radiation scheme is called with a frequency that depends on the applications. Before Cy23r4, the full radiation computations were only performed every three hours. With Cy23r4, the frequency was changed to one-hour during the first 12 hours used for data assimilation. Since Cy28r3, the frequency was increased to every one-hour during the 10 days of the high-resolution forecast (presently T<sub>L</sub>799L91). Other applications (Ensemble Prediction System, EPS, and seasonal forecasts) use a three-hour frequency for calling the full radiation computations. A temporal interpolation then provides the relevant interaction of the short-wave radiative fluxes with the solar zenith angle at every time step and every grid point.

To do so, a short-wave transmissivity is defined at each model level such that

$$\mathcal{F}_s = \tau_c \mathcal{S}_0 \quad (2.60)$$

where  $\mathcal{F}_s$  is the net solar (short-wave) flux and  $\mathcal{S}_0$  is the solar flux at the top of the atmosphere.  $\mathcal{F}_s$  is defined only for a full radiation time step. At every time step, the net solar fluxes are computed therefore from the transmissivity derived for the last full radiation time step, using (2.60) with the correct solar angle for every grid point. The net long-wave fluxes are kept at the values given by the full radiation calculation.

## 2.8 INPUT TO THE RADIATION SCHEME

### 2.8.1 Model variables

Temperature values are needed at the boundaries of the layers, where the fluxes are computed. They are derived from the full level temperatures with a pressure weighted interpolation

$$T_{k+1/2} = T_k \frac{p_k(p_{k+1} - p_{k+1/2})}{p_{k+1/2}(p_{k+1} - p_k)} + T_{k+1} \frac{p_{k+1/2}(p_{k+1/2} - p_k)}{p_{k+1/2}(p_{k+1} - p_k)} \quad (2.61)$$

At the bottom of the atmosphere, either the surface temperature or the temperature at 2 m is used, while at the top of the atmosphere the temperature is extrapolated from the first full level and second half level temperatures.

### 2.8.2 Clouds

Cloud fraction, liquid, ice and snow water contents are provided in all layers by the cloud scheme. Wherever there is ice cloud present, the snow water content is added to the ice to form a total ice water content as input to the scheme. So the same optical properties are assumed for both ice and snow.

The mean effective radius of the cloud water droplets size distribution is parametrised following the diagnostic formulation of [Martin \*et al.\* \(1994\)](#). The parametrisation uses the cloud liquid water content and prescribed concentration of cloud condensation nuclei (CCN). The CCN concentration is parametrised as function of the 10m wind and it decreases with height simulating the vertical distribution of CCN in the boundary layer. Two different parametrisations are used over land and over sea, the switch is given by the land-sea mask.

For ice clouds, the effective dimension of the cloud particles is diagnosed from the temperature and ice water content using a revised version of the formulation by [Ou and Liou \(1995\)](#)

### 2.8.3 Aerosols

The aerosol climatology used in the operational model up to Cy26r1 was given as annual mean geographical distributions defined from T5 spectral coefficients, for different aerosol types, respectively, maritime, continental, urban and desert, plus a uniformly distributed stratospheric background aerosols, with fixed vertical distributions, following [Tanre \*et al.\* \(1984\)](#). In the last fifteen years, chemical and/or transport models have addressed the life cycles of various aerosol types and attempted an inventory of their spatio-temporal distributions. Out of these studies, a new climatology for the annual cycle of the aerosol distribution of various aerosol types has been compiled by [Tegen \*et al.\* \(1997\)](#), which has been implemented in the ECMWF forecast system from Cy26r3 onwards. [Table 2.6](#) describes the characteristics of the aerosol components for each tropospheric aerosol type and [Table 2.7](#) compares the maximum optical thicknesses in the old and new climatologies.

### 2.8.4 Carbon dioxide, ozone and trace gases

Nitrous oxide, CFC-11 and CFC-12 have constant volume concentrations of 0.31 ppm, 280 ppt, and 484 ppt, respectively (IPCC/SACC, 1990), except in ERA-40 for the variation in concentrations is derived from (IPCC/SACC, 1995).

Before 2009 carbon dioxide and methane were prescribed as globally averaged values. With Cy35r3 CO<sub>2</sub>, CH<sub>4</sub> and O<sub>3</sub> are described each by a bi-dimensional (latitude/height) monthly mean climatology derived from one year of GEMS reanalysis. From Cy40r3 the GEM-based climatology has been updated with a new zonal mean climatology derived from the MACC reanalysis 2003-2011 (see [Section 11.8](#)).

**Table 2.6** Characteristics of the aerosol components for each tropospheric aerosol type in the new climatology for cycle Cy26r3 of the ECMWF model (adapted from [Hess et al. \(1998\)](#)).

Type	RH(%)	Component	Number ( $\text{cm}^{-1}$ )	Volume ( $\mu\text{m}^3/\text{m}^3$ )	Mass ( $\mu\text{g}^3/\text{m}^3$ )	Density ( $\text{g}/\text{cm}^3$ )
“Continental” organic	80	Insoluble	4.00E-01	4.75E+06	9.49E+00	2.00
		Water soluble	7.00E+03	1.57E+07	1.99E+01	1.27
		soot	8.30E+03	4.96E+05	4.96E-01	1.00
“Maritime” sulphate	95	Water soluble	1.50E+03	7.45E+06	8.35E+00	1.12
		sea salt (accum.)	2.00E+01	1.64E+08	1.72E+02	1.05
		sea salt (coarse)	3.20E-03	9.85E+05	1.04E+00	1.05
“Desert” dust-like	50	Water soluble	2.00E+03	2.81E+06	4.00E+00	1.42
		Mineral (nuclei)	2.70E+02	2.88E+06	7.49E+00	2.60
		Mineral (accum.)	3.05E+01	6.47E+07	1.69E+02	2.60
		Mineral (coarse)	1.42E-01	1.77E+07	4.60E+01	2.60
“Urban” black carbon	80	Insoluble	1.50E+00	1.78E+07	3.56E+01	2.00
		Water soluble	2.80E+04	6.28E+07	7.97E+01	1.27
		Soot	1.30E+05	7.78E+06	7.78E+01	1.00

*Type*: First definition (e.g. continental) is the aerosol component as known within both the ECMWF model and the OPAC software; second definition (e.g. organic) is the 3D distribution to which it is linked in the climatology of [Tegen et al. \(1997\)](#). RH is the relative assumed for the computations of the relevant optical properties. The *nuclei*, *accumulation*, and *coarse* modes refer to various size ranges for the component particles.

**Table 2.7** Maximum optical thickness in the two aerosol climatologies.

OLD	Annual	January	July	NEW
Continental	0.2	0.235	0.231	Organic
Maritime	0.05	0.099	0.232	Sulphate
Desert	1.9	0.184	1.01	Dust-like
Urban	0.1	0.039	0.039	Black carbon
Background trop.	0.03			
Background stratos	0.045	0.045	0.045	Background stratos.

Aerosol types of the new and old climatologies are paired according to the dominant components in each mix.

**Table 2.8** *Diffuse and parallel albedo and window emissivity for each tile.*

Tile	1	2	3	4	5	6	7	8
Description	Open sea	Sea ice	Interception layer	Low vegetation	Exposed snow	High vegetation	Shaded snow	Bare ground
Diffuse albedo	0.06	Ebert and Curry (1993)	$\alpha_{sb}$	$\alpha_{sb}$	$\alpha_{sn}$	$\alpha_{sb}$	see <a href="#">Table 8.6</a>	$\alpha_{sb}$
Parallel albedo	Taylor <i>et al.</i> (1996)	Ebert and Curry (1993)	$\alpha_{sb}$	$\alpha_{sb}$	$\alpha_{sn}$	$\alpha_{sb}$	see <a href="#">Table 8.6</a>	$\alpha_{sb}$
Window emissivity	0.99	0.98	0.96	0.93–0.96	0.98	0.93–0.96	0.93–0.96	0.93–0.96

Two more climatologies are available for the ozone distribution. In the first one ( $\text{NOZOCL} = 0$ ), the ozone mixing ratio  $q_{\text{O}_3}$  depends on height, latitude, longitude and season. Its vertical distribution is assumed to be such that its integral from 0 to the pressure  $p$  is

$$\int_0^p q_{\text{O}_3} dp = \frac{a}{1 + (b/p)^{3/2}} \quad (2.62)$$

The constants  $a$  and  $b$  are related to the total amount of ozone and the height of its maximum mixing ratio. They are imposed in terms of a limited series of spherical harmonics (T10) for the geographical distribution and a Fourier series for the seasonal variation. The total amount of ozone was taken from [London \*et al.\* \(1976\)](#) and the altitude of the maximum concentration was derived from [Wilcox and Belmont \(1977\)](#). Plots of these values can be found in the Appendix. In the second climatology ( $\text{NOZOCL} = 1$ ), the ozone mixing ratio  $q_{\text{O}_3}$  depends on height, latitude and month, and is taken from [Fortuin and Langematz \(1994\)](#).

### 2.8.5 Ground albedo and emissivity

The background land albedo,  $\alpha_{sb}$ , is interpolated to the model grid from the monthly mean values of a snow-free albedo produced for the combined 1982–1990 years. The albedo for that dataset was computed using the method of [Sellers \*et al.\* \(1996\)](#), but with new maps of soil reflectance, new values of vegetation reflectance and the biophysical parameters described in [Los \*et al.\* \(2000\)](#). More information on the original data and plots of the monthly mean albedo are shown in [Chapter 11](#).

Spectral albedos for parallel and diffuse radiation are needed by the radiative code. In addition, the surface energy balance equation (see [Chapter 3](#) on vertical diffusion) needs a spectrally integrated parallel+diffused albedo, specified for each independent surface functional unit, tile. The procedure is summarized in [Table 2.8](#). Over open water, the surface albedo for direct parallel radiation is a fit to low-flying aircraft measurements over the ocean given by [Taylor \*et al.\* \(1996\)](#)

$$\alpha_{sp} = \frac{0.037}{1.1\mu_0^{1.4} + 0.15} \quad (2.63)$$

For sea ice, monthly values based on [Ebert and Curry \(1993\)](#) albedos for the Arctic Ocean are interpolated to the forecast time. The bare sea ice albedo value in Ebert and Curry is taken as a representative value for summer, and the dry snow albedo value is used for the winter months. Values for the Antarctic are shifted by six months. Separate values for visible and near-infrared spectral bands are used. The time-varying snow albedo ( $\alpha_{sn}$ , see [Chapter 8](#)), is used for the exposed snow tile only. Finally, the average of the diffuse and parallel albedos are spectrally integrated for each tile.

The thermal emissivity of the surface outside the 800–1250  $\text{cm}^{-1}$  spectral region is assumed to be 0.99 everywhere. In the window region, the spectral emissivity is constant for open water, sea ice, the interception layer and exposed snow tiles. For low and high vegetation and for shaded snow the emissivity depends on the water content in the top soil layer. Emissivity decreases linearly from 0.96 for soils at or above field capacity to 0.93 for soils at or below permanent wilting point. The same formulation is used for bare ground, except for desert areas ( $\alpha_{sb} > 0.3$ ), where a value of 0.93 is used independently of the soil water content. Finally, a broadband emissivity is obtained by convolution of the spectral emissivity and the Planck function at the skin temperature.

### 2.8.6 Solar zenith angle

Equations to compute the annual variation of the solar constant  $I$ , the solar declination  $\delta_s$  and the difference between solar time and official time can be found in [Paltridge and Platt \(1976\)](#). These equations are used to give the cosine of the solar angle at the ground. Because of the curvature of the earth, the zenith angle is not quite constant along the path of a sun ray. Hence the correction applied to  $\mu^a_0$  to give an average  $\mu_0$  for the atmosphere is

$$\mu_0 = \frac{\frac{H}{a}}{(\mu_0^a)^2 + \frac{H}{a} \left( 2 + \frac{H}{a} \right) - (\mu_0^a)^2} \quad (2.64)$$

where  $a$  is the earth radius and  $H$  is the atmospheric equivalent height.  $H/a$  is fixed at 0.001277.

## 2.9 THE RADIATION CODE

Routine **RADHEAT** or **RADHEATN** (depending whether the diagnostic or prognostic cloud scheme is used) is called at every time step to compute the radiative fluxes and heating using the solar zenith angle computed in **CPGLAG** and emissivities and transmissivities (**PEMTU**, **PTRSOL**) computed at full radiation time steps in **RADINT**. or **RADINTG** (see [Subsection 2.9.2](#)). The other routines are called either once at the beginning of the run (**SUECRAD** and below) or once per full radiation step at the first row (**ECRADFR** and below), or at every full radiation time step for all rows. In this section, we briefly describe the function of each routine.

### 2.9.1 Set-up routines

- **SUECRAD** provides the interface with the user, via the namelist NAERAD. It defines the constants of [Table 2.5](#) and sets the configuration for the radiative computations (from **SUPHEC**).
- **ECRADFR** modifies the frequency of full radiative computations (from **CNT4**).
- **SUAERL** and **SUAERSN** set up the long-wave and short-wave radiative characteristics of the aerosols (from **SUECRAD**).
- **SUECRAD** defines the geographical distribution of aerosols, in terms of spectral coefficients (from **UPDTIER**).
- **SUAERV** defines the globally averaged vertical distribution of the aerosols (from **SUECRAD**).
- **SUCLOP** sets up the long-wave and short-wave radiative properties of the ice and water clouds (from **SUECRAD**).
- **SUECOZO** computes the Legendre coefficients for the ozone distribution according to the time of the year, using the Fourier coefficients defined in DATA statements (from **UPDTIER**).
- **SULWN** sets up the coefficients for the pre-Cy22r3 long-wave radiative computations (from **SUECRAD**).
- **SURDI** sets up the concentrations of radiatively active gases and security parameters for the radiative computations (from **SUECRAD**).
- **SUSWN** sets up the coefficients for the pre-Cy32r2 short-wave radiative computations (from **SUECRAD**).
- **UPDTIER** updates the time for full radiative computations (from **ECRADFR**).
- The routines **SUAERH**, **SUECOZO** are called only once per full radiation step, at the first row.
- **SURRTAB** precomputes the array linking gaseous optical thickness and the transmission function (RRTM). (called from **SUECRAD**).
- **SURRTFTR** includes all coefficients related to the  $g$ -point configuration (RRTM). (called from **SUECRAD**).
- **SURRTPK** defines the limits of the spectral intervals, and the coefficients of the spectrally defined and spectrally integrated Planck functions (RRTM). (called from **SUECRAD**).
- **SURTRF** defines the pressure and temperature reference profiles used for the tabulation of the absorption coefficients (RRTM). (called from **SUECRAD**).
- **SUSRTAER** defines the optical properties of aerosols for the RRTM<sub>SW</sub> scheme.
- **SUSRTALB** defines the coefficients used for dealing with the surface albedo in the RRTM<sub>SW</sub> scheme.
- **SUSRTCOP** defines the short-wave radiative properties of the ice and water clouds for use in the RRTM<sub>SW</sub> scheme.

- **SUSRTM** includes all coefficients related to the  $g$ -point configuration in the  $\text{RRTM}_{\text{SW}}$  scheme.
- **RRTM\_CMBGBn**, for each of the 16 spectral intervals of  $\text{RRTM}_{\text{LW}}$ , remaps the absorption coefficients from 16 to the final number of  $g$ -points (called from **RRTM\_INIT\_140GP**).
- **RRTM\_INIT\_140GP** performs the  $g$ -point reduction from 16 per band to a band-dependent number (column 2 in [Table 2.3](#)). It also computes the relative weighting for the new  $g$ -point combinations (called from **SUECRAD**).
- **RRTM\_KGBn** contain the various absorption coefficients for all gases relevant to the different spectral bands of  $\text{RRTM}_{\text{LW}}$ .
- **SRTM\_CMBGBn**, for each of the 14 spectral intervals of  $\text{RRTM}_{\text{SW}}$ , remaps the absorption coefficients from 14 to the final number of  $g$ -points (called from **SRTM\_INIT**).
- **SRTM\_KGBn** contain the various absorption coefficients for all gases relevant to the different spectral bands of  $\text{RRTM}_{\text{SW}}$ .

### 2.9.2 Main routines

- **RADINT** or **RADINTG** is called by **RADDRV** to launch the full radiation computations, depending on whether the pre-Cy26r1 sampling configuration or the Cy26r1 halo configuration is used for spatial interpolation (see [Section 2.7](#)). Zonal mean diagnostic of the temperature, clouds and albedo are computed. Temperature is vertically interpolated. Depending on the value of the variable **NRINT** an interpolation of all input variables to a coarser grid may be carried out. It may be necessary to subdivide the latitude belt in a few parts for the actual calculation of radiative fluxes because of storage space limitations. For this reason a loop over these parts follows. Inside this loop a call to routine **RADLSW** provides solar and thermal fluxes for a subset of points of that latitude row. These fluxes are converted into transmissivities and emissivities and after completion of the whole latitude circle they are transferred to the full grid when the calculations are carried out with the coarse resolution ( $\text{NRINT} > 1$ ).
- **RADLSW** is the driver routine of the solar and thermal fluxes by calling specialized routines **SW** for short-wave radiation and either **RRTM\_RRTM\_140GP** or **LW** for long-wave radiation.

### 2.9.3 Specialized routines

- **RADSRF** is called from **RADPAR/CALLPAR** to compute surface albedo and emissivity. It computes the gridpoint diffuse and parallel spectral albedos and a spectrally integrated albedo (for postprocessing). It also computes the emissivity inside and outside the window region, and the spectrally integrated emissivity. Finally, it computes spectrally integrated tile albedos to be used by the surface energy balance routine (see [Chapter 3](#) on vertical diffusion).
- **LW** organizes the long-wave computation by calling in turn **LWU**, **LWBV**, **LWC**.
- **LWU** computes the effective absorber amounts including the pressure and temperature dependencies in the spectral intervals of the long-wave radiation scheme.
- **LWBV** calls **LWB** and **LWV**.
- **LWB** computes the Planck function with relation to temperature for all levels and spectral intervals.
- **LWV** organizes the vertical integration by calling **LWVN** which deals with the contribution to the flux of the layers adjacent to the level of computation of flux, **LWVD** which deals with the contribution from the more distant layers, and **LWVB** which computes the contribution of the boundary terms.
- **LWTT** and **LWTTM** compute the relevant transmission functions needed in **LWVN**, **LWVD**, and **LWVB**.
- **LWC** introduces the effect of clouds on the long-wave fluxes.
- **SW** organizes the short-wave computation by calling in turn **SWU**, **SW1S**, and **SW2S**.
- **SWU** computes the effective absorber amounts including the pressure and temperature dependencies of the absorption.
- **SW1S** and **SW2S** deal with the short-wave radiation transfer in the two spectral intervals used to describe the solar spectrum. They both call **SWCLR**, which deals with the conservative scattering processes (Rayleigh) and the scattering/absorption by aerosols in the totally clear sky part of the atmospheric column, then **SWR** which deals with the same processes for the clear sky layers in an

otherwise cloudy column, and **SWDE** which computes the reflectivity and transmissivity of a layer including non-conservative scatterers (cloud particles) with the Delta-Eddington approximation.

- **SWTT** and **SWTT1**, computes the relevant transmission functions.
- **RRTM.RRTM\_140GP** organizes the long-wave computation by calling in turn, within a loop on the individual vertical columns, **RRTM\_ECRT\_140GP**, **RRTM\_SETCOEF\_140GP**, **RRTM.GASABS1A\_140GP** and **RRTM.RTRN1A\_140GP**.
- **RRTM\_ECRT\_140GP** defines the surface spectral emissivity, and the spectral aerosol thickness, and the layer absorber amounts and cloud quantities as used in RRTM.
- **RRTM.SETCOEF\_140GP** computes the indices and fractions related to the pressure and temperature interpolations. It also calculates the values of the integrated Planck function for each spectral band at the level and layer temperatures.
- **RRTM.GASABS1A\_140GP** launches the calculation of the spectrally defined optical thickness for gaseous absorption. It calls **RRTM\_TAUMOLn**.
- **RRTM.RTRN1A\_140GP** computes the downward then upward fluxes, using a diffusivity-type approximation for the angle integration. Cloud overlap is treated with a generalized maximum/random overlap method. Adjacent layers are treated with maximum overlap, non-adjacent cloud groups are treated with random overlap. For adjacent cloud layers, cloud information is carried from the previous two layers.
- **RRTM.RRTM\_140GP\_MCICA**, **RRTM\_ECRT\_140GP\_MCICA**, **RRTM.RTRN1A\_140GP\_MCICA** are the McICA equivalent of the routines defined above.
- **SU\_MCICA** includes coefficients required to run the Räisänen and Barker (2004) cloud generator.
- **McICA.CLD\_GEN** prepares in and out of the cloud generator for use in the McICA version of the radiation scheme.
- **McICA.CLD\_GENERATOR** is the stochastic generation of sub-grid scale cloudy columns for use in the McICA version of the radiation scheme.
- **SRTM.SRTM\_224GP** organizes the short-wave computation by calling in turn, within a loop on the individual vertical columns, **SRTM.SETCOEF** and **SRTM.SPCVRT**, **SRTM.VRTQDR** and **SRTM.REFTRA**.
- **SRTM.SETCOEF** computes the indices and fractions related to the pressure and temperature interpolations.
- **SRTM.SPCVRT** computes the short-wave radiation fluxes using a two-stream method, calling first the routines setting up the spectral coefficients (**SRTM\_TAUMOLn**), then the vertical quadrature (**SRTM\_VRTQDR**).
- **SRTM.REFTRA** computes the reflectivity and transmissivity of a layer of given optical thickness using a two-stream approximation.
- **SRTM.SRTM\_224GP\_MCICA** and **SRTM.SPCVRT\_MCICA** are the McICA equivalent of the routines defined above.

#### 2.9.4 Heating rate computation

- **RADHEAT** or **RADHEATN**, depending whether the diagnostic or the prognostic cloud scheme is used, recomputes at each time step the net radiative fluxes from the layers' effective emissivity and transmissivity, using the actual temperature and solar zenith angle. It also computes the downward long-wave and short-wave radiation at the surface.

## APPENDIX A. LIST OF SYMBOLS

$B_\nu$	Planck function integrated over the half sphere with the factor involving $\pi$ absorbed: in units of flux ( $\text{Wm}^{-2}$ )
$C_{\text{cld}}$	fractional cloud cover
$c_p$	specific heat at constant pressure of moist air
$c_{p_{\text{dry}}}$	specific heat at constant pressure of dry air
$c_{p_{\text{vap}}}$	specific heat at constant pressure of water vapour
$\mathcal{E}_\nu^0$	incident solar radiance in the direction $\theta_0$
$\mathcal{F}$	radiative flux

$f$	fractional scattering into the forward peak
$g$	acceleration of gravity
$g$	asymmetry factor for aerosol scattering
$k$	absorption coefficient
$\mathcal{L}_\nu$	monochromatic radiance at wavenumber $\nu$
$M$	magnification factor ( $= 35/\sqrt{(\mu_0^2 + 1)}$ )
$m_{O_3}$	ozone mixing ratio
$P$	scattering phase function
$p$	pressure
$\Pi(\mathcal{U}) d\mathcal{U}$	probability of a photon encountering an absorber amount between $\mathcal{U}$ and $\mathcal{U} + d\mathcal{U}$
$q$	specific humidity
$r$	diffusivity factor ( $= \sec \theta$ )
$r_e$	mean effective radius of cloud water droplets
$\mathcal{R}$	reflectance
$S_0$	solar flux at the top of the atmosphere
$\mathcal{T}$	transmittance
$T$	temperature
$t_\nu$	monochromatic transmission at wavenumber $\nu$
$\mathcal{U}$	absorber amount
$\alpha$	surface albedo
$\beta_\nu^{\text{abs}}$	cloud particle absorption coefficient
$\beta_\nu^{\text{ext}}$	extinction coefficient
$\beta_\nu^{\text{sca}}$	scattering coefficient
$\delta_g$	molecular absorption of gases
$\delta$	optical depth
$\varepsilon_{\text{cld}}$	cloud emissivity
$\mu$	$= \cos \theta$
$\nu$	wavenumber
$\varpi_\nu$	single scattering albedo ( $= \beta_\nu^{\text{sca}}/k_\nu$ )
$\Phi$	scattering phase function
$\varphi$	azimuth angle
$\theta$	zenith angle
$\theta_0$	direction of incident solar beam
$\Theta$	angle between incident and scattered radiances



## Chapter 3

# Turbulent transport and interactions with the surface

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## 3.1 INTRODUCTION

The parametrization scheme described in this chapter represents the turbulent transfer of heat, momentum and moisture between the surface and the lowest model level (i.e. within the surface layer) and the turbulent transport of the same quantities plus liquid and ice water between model levels (i.e. in the outer layer). The scheme computes the physical tendencies of the six prognostic variables ( $u$ ,  $v$ ,  $T$ ,  $q$ ,  $q_l$  and  $q_i$ ) due to the vertical exchange by turbulent processes. These tendencies are obtained as the difference between the results of an implicit time-step from  $t$  to  $t + 1$ . All the diagnostic computations (such as the calculation of the exchange coefficients, etc.) are done at time  $t$ . The surface boundary condition is formulated separately for eight different tiles: water, ice, wet skin, low vegetation, exposed snow, high vegetation, snow under vegetation, and bare soil. The different tiles have their own surface energy balance and their own skin temperature. In this version of the IFS, the mixture of land and ocean tiles is still not used, i.e. a grid box is either 100% ocean (water + ice) or 100% land (tile 3 to 8). Details about tiles are given in [Chapter 8](#).

The vertical turbulent transport is treated differently in the surface layer and in the outer layer. In the surface layer, the turbulence fluxes are computed using a first order K-diffusion closure. In the outer

layer, a K-diffusion turbulence closure is used everywhere (i.e. stable or unstable layers), except in the well-mixed part of unstable boundary layers (i.e. further referred to as the mixed layer) where an Eddy-Diffusivity Mass-Flux (EDMF) framework is applied.

The equation for the vertical turbulent transport of any conservative quantity  $\phi$  is described by

$$\frac{\partial \phi}{\partial t} = \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho K_\phi \frac{\partial \phi}{\partial z} - M(\phi_u - \bar{\phi}) \right) = \frac{1}{\rho} \frac{\partial J_\phi}{\partial z} \quad (3.1)$$

The vertical turbulent flux  $J_\phi$  (positive downwards) is written using a first-order turbulence closure, where  $K_\phi$  is the exchange coefficient, to which a mass-flux component is added in the mixed layer to describe the strongest eddies. Because the mass-flux is zero at the surface and the top of the mixed layer, boundary conditions are satisfied by vertical diffusion

$$\begin{aligned} K_\phi \frac{\partial \phi}{\partial z} &= 0 \quad \text{at } p = p_{\text{top}} \\ K_\phi \frac{\partial \phi}{\partial z} &\rightarrow \sum_{i=1}^{N_T} F_i C_{\phi i} |U(z)| (\phi(z) - \phi_{\text{surf}i}) \quad \text{as } z \rightarrow 0 \end{aligned} \quad (3.2)$$

where  $p_{\text{top}}$  is the pressure at the top of the atmosphere. For heat and moisture the surface boundary condition is provided tile by tile and fluxes are averaged over the  $N_T$  tiles, weighted by their fraction  $F_i$ . The transfer coefficient  $C_{\phi i}$  at the lowest model level depends upon the static stability. The variable  $\phi_{\text{surf}}$  represents the value of  $\phi$  at the surface. For heat and moisture, eight tiles are used (see [Chapter 10](#)). For wind, a single tile is used with a no slip condition at the surface.

The vertical turbulent transport processes are applied to the two horizontal wind components,  $u$  and  $v$ , the specific total water  $q_t$  and the generalized liquid water static energy  $s_l$ ,

$$q_t = q + q_l + q_i \quad (3.3)$$

$$s_l = gz + c_p T - L_c q_l - L_d q_i \quad (3.4)$$

where  $q$ ,  $q_l$ ,  $q_i$  are the specific humidity, specific liquid water and specific ice water,  $c_p$  is the specific heat at constant pressure of dry air,  $g$  is the constant of gravity, and  $L_c$  and  $L_d$  are the latent heats of condensation and deposition respectively. Note that the influence of water vapor on  $c_p$  is neglected throughout the formulation of vertical turbulent fluxes (as is also done in the other physics components).

The problem is simplified by assuming that  $z$  remains constant with respect to time during the turbulent transport process (even if in reality  $T$  variations would modify  $z(p)$ ). Exchange coefficients (with the dimension of a pressure thickness) are then computed for momentum and for heat (sensible plus latent) (the subscripts ‘M’, ‘H’ and ‘Q’ are used to identify the exchange coefficient for momentum, heat and total water), with different formulations for the stable and the unstable cases. The mass-flux term is calculated based on an entraining plume model closed at the surface. The implicit linear equations for the fluxes of momentum, firstly for  $u$  and  $v$  and secondly for  $s_l$  and  $q_t$ , are solved by a Gaussian-elimination/back-substitution method.

The surface boundary condition is applied between the downward scanning elimination and the upward scanning back substitution. It involves a no-slip condition for  $u$  and  $v$  and the tile-by-tile solution of the surface energy balance for the boundary condition of  $q$  and dry static energy

$$s = gz + c_p T \quad (3.5)$$

The water tile is an exception as it ignores the surface energy balance and uses the specified SST and the saturation specific humidity as boundary conditions.

Finally, the tendency of temperature is computed, modified by the effects of local dissipation (it is assumed that there is no storage of turbulence kinetic energy). The tiled surface fluxes of heat and moisture are also computed for later use by the surface scheme.

### 3.2 THE SURFACE LAYER

The surface layer approximation is applied between the lowest model level (about 10 m above the surface in the 60 and 91 level models) and the surface and for each tile separately. It is assumed that the turbulent fluxes are constant with height and equal to the surface values. They can be expressed, using Monin–Obukhov similarity theory, in terms of the gradients of wind, dry static energy and specific humidity, which are assumed to be proportional to universal gradient functions of a stability parameter:

$$\begin{aligned}\frac{\kappa z}{u_*} \frac{\partial u}{\partial z} &= \Phi_M \left( \frac{z}{\mathcal{L}} \right) \\ \frac{\kappa z}{s_*} \frac{\partial s}{\partial z} &= \Phi_H \left( \frac{z}{\mathcal{L}} \right) \\ \frac{\kappa z}{q_*} \frac{\partial q}{\partial z} &= \Phi_Q \left( \frac{z}{\mathcal{L}} \right)\end{aligned}\quad (3.6)$$

The scaling parameters  $u_*$ ,  $s_*$  and  $q_*$  are expressed in terms of surface fluxes  $J_\phi$  by

$$\begin{aligned}\rho u_*^2 &= J_M \\ \rho u_* s_* &= J_s \\ \rho u_* q_* &= J_q\end{aligned}\quad (3.7)$$

The stability parameter  $\mathcal{L}$  is the Obukhov length defined as

$$\mathcal{L} = -u_*^3 / \left( \frac{\kappa g}{T_n} Q_{0v} \right) \quad \text{with} \quad Q_{0v} = \frac{u_* s_* - (c_{p\text{vap}} - c_{p\text{dry}}) T_n u_* q_*}{c_p} + \varepsilon T_n u_* q_* \quad (3.8)$$

$Q_{0v}$  is the virtual temperature flux in the surface layer,  $\kappa$  is the Von Kármán constant ( $= 0.4$ ),  $T_n$  is a reference temperature taken as a near-surface temperature (the temperature of the lowest atmospheric level  $n$ ) and  $\varepsilon = (R_{\text{vap}}/R_{\text{dry}}) - 1$ , where  $R_{\text{vap}}$  and  $R_{\text{dry}}$  are the gas constants for water vapour and dry air, respectively.

In the surface layer, the gradient functions (3.6) can be integrated to profiles

$$u = \frac{\tau_x}{\kappa \rho u_*} \left\{ \log \left( \frac{z_n + z_{0M}}{z_{0M}} \right) - \Psi_M \left( \frac{z_n + z_{0M}}{\mathcal{L}} \right) + \Psi_M \left( \frac{z_{0M}}{\mathcal{L}} \right) \right\} \quad (3.9)$$

$$v = \frac{\tau_y}{\kappa \rho u_*} \left\{ \log \left( \frac{z_n + z_{0M}}{z_{0M}} \right) - \Psi_M \left( \frac{z_n + z_{0M}}{\mathcal{L}} \right) + \Psi_M \left( \frac{z_{0M}}{\mathcal{L}} \right) \right\} \quad (3.10)$$

$$s - s_{\text{surf}} = \frac{J_s}{\kappa \rho u_*} \left\{ \log \left( \frac{z_n + z_{0M}}{z_{0H}} \right) - \Psi_H \left( \frac{z_n + z_{0M}}{\mathcal{L}} \right) + \Psi_H \left( \frac{z_{0H}}{\mathcal{L}} \right) \right\} \quad (3.11)$$

$$q - q_{\text{surf}} = \frac{J_q}{\kappa \rho u_*} \left\{ \log \left( \frac{z_n + z_{0M}}{z_{0Q}} \right) - \Psi_H \left( \frac{z_n + z_{0M}}{\mathcal{L}} \right) + \Psi_H \left( \frac{z_{0Q}}{\mathcal{L}} \right) \right\} \quad (3.12)$$

$z_{0M}$ ,  $z_{0H}$  and  $z_{0Q}$  are the roughness lengths for momentum, heat and moisture. The stability profile functions  $\Psi$  are derived from the gradient functions (3.6) with the help of the relationship  $\Phi = 1 - \zeta(\partial\Psi/\partial\zeta)$ . These profiles are used for the surface atmosphere interaction as explained in the following sections and also for the interpolation between the lowest model level and the surface (post-processing of 10 m wind and 2 m temperature and moisture).

In extremely stable situations, i.e. for very small positive  $\mathcal{L}$ , the ratio  $z/\mathcal{L}$  is large, resulting in unrealistic profile shapes with standard stability functions. Therefore the ratio  $z/\mathcal{L}$  is limited to 5 by defining a height  $h$  such that  $h/\mathcal{L} = 5$ . If  $z < h$ , then the profile functions described above, are used up to  $z = h$  and the profiles are assumed to be uniform above that. This modification of the profiles for exceptionally stable situations (no wind) is applied to the surface transfer formulation as well as to the interpolation for post-processing.

#### 3.2.1 Surface fluxes

Surface fluxes for heat and moisture are computed separately for the different tiles, so most of the surface layer computations loop over the tile index. Here a general description is given of the aerodynamic aspects

of the transfer between the surface and the lowest model level. The description of the individual tiles can be found in [Chapter 8](#).

Assuming that the first model level above the surface is located in the surface boundary layer at a specified height  $z_n$ , the gradient functions (3.6) can be integrated to profiles for wind, dry static energy and specific humidity. The surface fluxes are expressed in terms of differences between parameters at level  $z_n$  and surface quantities (identified by the subscript ‘surf’; the tile index has been omitted in this general description) so

$$\begin{aligned} J_M &= \rho C_M |U_n|^2 \\ J_s &= \rho C_H |U_n| (s_n - s_{\text{surf}}) \\ J_q &= \rho C_Q |U_n| (\alpha_n q_n - \alpha_{\text{surf}} q_{\text{surf}}) \end{aligned} \quad (3.13)$$

where  $q_{\text{surf}} = q_{\text{sat}}(T_{\text{surf}})$ ,  $\alpha_n$  and  $\alpha_{\text{surf}}$  are provided by the land scheme, and  $s_{\text{surf}} = c_p T_{\text{surf}}$ , (the humidity equation simplifies over water where  $\alpha_n = 1$  and  $\alpha_{\text{surf}} = 1$ ).

The transfer coefficients can be expressed as

$$C_M = \frac{\kappa^2}{\left[ \log\left(\frac{z_n + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_n + z_{0M}}{\mathcal{L}}\right) + \Psi_M\left(\frac{z_{0M}}{\mathcal{L}}\right) \right]^2} \quad (3.14)$$

$$C_H = \frac{\kappa^2}{\left[ \log\left(\frac{z_n + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_n + z_{0M}}{\mathcal{L}}\right) + \Psi_M\left(\frac{z_{0M}}{\mathcal{L}}\right) \right] \left[ \log\left(\frac{z_n + z_{0M}}{z_{0H}}\right) - \Psi_H\left(\frac{z_n + z_{0M}}{\mathcal{L}}\right) + \Psi_H\left(\frac{z_{0H}}{\mathcal{L}}\right) \right]} \quad (3.15)$$

$$C_Q = \frac{\kappa^2}{\left[ \log\left(\frac{z_n + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_n + z_{0M}}{\mathcal{L}}\right) + \Psi_M\left(\frac{z_{0M}}{\mathcal{L}}\right) \right] \left[ \log\left(\frac{z_n + z_{0M}}{z_{0Q}}\right) - \Psi_Q\left(\frac{z_n + z_{0M}}{\mathcal{L}}\right) + \Psi_Q\left(\frac{z_{0Q}}{\mathcal{L}}\right) \right]} \quad (3.16)$$

The wind speed  $|U_n|$  is expressed as

$$|U_n|^2 = u_n^2 + v_n^2 + w_*^2 \quad (3.17)$$

with  $w_*$  the free convection velocity scale defined by

$$w_* = \left( z_i \frac{g}{T_n} Q_{\text{ov}} \right)^{1/3} \quad (3.18)$$

The parameter  $z_i$  is a scale height of the boundary layer depth and is set to constant value of 1000 m, since only the order of magnitude matters. The additional term in equation (3.17) represents the near surface wind induced by large eddies in the free-convection regime. When the surface is heated, this term guarantees a finite surface wind-forcing in the transfer law even for vanishing  $u_n$  and  $v_n$ , and prevents  $|U_n|$  and  $\mathcal{L}$  from becoming zero. [Beljaars \(1994\)](#) showed that this empirical term, when added into the standard Monin–Obukhov scaling, is in agreement with scaling laws for free convection. When used with the roughness lengths defined below, it provides a good fit to observational data, both over land and over sea.

### 3.2.2 Stability functions

The empirical forms of the dimensionless gradient functions  $\Phi$  (given by (3.6)) have been deduced from field experiments over homogeneous terrain.

- (i) *Unstable conditions* ( $\zeta = z/\mathcal{L} < 0$ ). The gradient functions proposed by Dyer and Hicks are used ([Dyer, 1974](#); [Hogström, 1988](#)). Therefore

$$\begin{aligned} \Phi_M(\zeta) &= (1 - 16\zeta)^{-1/4} \\ \Phi_H(\zeta) &= \Phi_Q(\zeta) = (1 - 16\zeta)^{-1/2} \end{aligned} \quad (3.19)$$

These functions can be integrated to the universal profile stability functions,  $\Psi$ , (Paulson, 1970) so that

$$\begin{aligned}\Psi_M(\zeta) &= \frac{\pi}{2} - 2 \operatorname{atan}(x) + \log \frac{(1+x)^2 \cdot (1+x^2)}{8} \\ \Psi_H(\zeta) &= \Psi_Q(\zeta) = 2 \log \left\{ \frac{1+x^2}{2} \right\}\end{aligned}\quad (3.20)$$

with  $x = (1 - 16\zeta)^{1/4}$ . The  $\Psi$ -functions are used in the surface layer and the  $\Phi$ -functions for unstable stratification are used above the surface layer for local closure.

- (ii) *Stable conditions* ( $\zeta = z/\mathcal{L} > 0$ ). The code contains gradient function  $\Phi_M$  as documented by Hogström (1988), and  $\Phi_H$  as derived from the Ellison and Turner relation for the ratio  $\Phi_M/\Phi_H$  giving

$$\begin{aligned}\Phi_M(\zeta) &= 1 + 5\zeta \\ \Phi_H(\zeta) &= \Phi_Q(\zeta) = (1 + 4\zeta)^2\end{aligned}\quad (3.21)$$

These functions were meant to be used for local closure above the surface layer, but are not used at all in the current model version, because Richardson number dependent functions are used instead (see section on exchange coefficients above the surface layer).

The stable profile functions as used in the surface layer, are assumed to have the empirical forms proposed by Holtslag and Bruin (1988), with a modification to allow for the effects of a critical flux Richardson number for large  $\zeta$ : The profiles are given by

$$\begin{aligned}\Psi_M(\zeta) &= -b \left( \zeta - \frac{c}{d} \right) \exp(-d\zeta) - a\zeta - \frac{bc}{d} \\ \Psi_H(\zeta) &= \Psi_Q(\zeta) = -b \left( \zeta - \frac{c}{d} \right) \exp(-d\zeta) - \left( 1 + \frac{2}{3}a\zeta \right)^{1.5} - \frac{bc}{d} + 1\end{aligned}\quad (3.22)$$

where  $a = 1$ ,  $b = 2/3$ ,  $c = 5$ , and  $d = 0.35$ .

### 3.2.3 Computation of the Obukhov length

The transfer coefficients needed for the surface fluxes require the estimation of stability parameter  $\zeta$ , itself a function of the surface fluxes. Therefore, an implicit equation, relating  $\zeta$  to bulk Richardson number  $Ri_{\text{bulk}}$ , is solved using

$$Ri_{\text{bulk}} = \zeta \cdot \frac{\left[ \log \left( \frac{z_n + z_{0M}}{z_{0H}} \right) - \Psi_H \left( \frac{z_n + z_{0M}}{\mathcal{L}} \right) + \Psi_H \left( \frac{z_{0H}}{\mathcal{L}} \right) \right]}{\left[ \log \left( \frac{z_n + z_{0M}}{z_{0M}} \right) - \Psi_M \left( \frac{z_n + z_{0M}}{\mathcal{L}} \right) + \Psi_M \left( \frac{z_{0M}}{\mathcal{L}} \right) \right]^2}\quad (3.23)$$

with

$$Ri_{\text{bulk}} = \left( \frac{g}{\theta_v} \right) \frac{z_n (\theta_{vn} - \theta_{\text{vsurf}})}{|U_n|^2}\quad (3.24)$$

where  $\theta_{vn}$  and  $\theta_{\text{vsurf}}$  are the virtual potential temperatures at level  $z_n$  and at the surface, and  $\theta_v$  is a virtual potential temperature within the surface layer. Equation (3.24) can be expressed in terms of dry static energy:

$$Ri_{\text{bulk}} = \frac{gz_n}{|U_n|^2} \left[ \frac{2(s_n - s_{\text{surf}})}{(s_n + s_{\text{surf}} - gz_n)} + \varepsilon(q_n - q_{\text{surf}}) \right]\quad (3.25)$$

Knowing  $Ri_{\text{bulk}}$  at time  $t$ , a first guess of the Obukhov length is made from fluxes computed at the previous time step. Equation (3.23) is solved numerically using the Newton iteration method to retrieve  $\zeta$ .

In contrast to the previous formulation used in the model (Louis *et al.*, 1982), the present scheme allows a consistent treatment of different roughness lengths for momentum, heat and moisture. The revised stability functions also reduce diffusion in stable situations resulting in more shallow stable boundary layers.

### 3.2.4 Roughness lengths

The integration constants  $z_{0M}$ ,  $z_{0H}$  and  $z_{0Q}$ , in the equations for the transfer coefficients  $C_M$ ,  $C_H$  and  $C_Q$  ((3.14) to (3.16)) are called roughness lengths because they are related to the small scale inhomogeneities of the surface that determine the air–surface transfer.

- (i) *Over land.* The roughness lengths over land are assumed to be fixed and related to the land surface cover. Every time step the dominant tile is determined and the roughness lengths are set according to a table that relates the roughness length to vegetation type or to land cover (bare soil and exposed snow). The roughness length for momentum is different from the one for heat, but the ones for heat and moisture are assumed to be the same (See Chapter 8 for the tables).
- (ii) *Over sea.* The specification of surface roughness lengths is particularly important over the sea. Because of the fixed boundary conditions for temperature and moisture the sea is, in principle, an infinite source of energy to the model. Following Beljaars (1994) the surface roughness lengths are expressed by

$$\begin{aligned} z_{0M} &= \alpha_M \frac{\nu}{u_*} + \alpha_{Ch} \frac{u_*^2}{g} \\ z_{0H} &= \alpha_H \frac{\nu}{u_*} \\ z_{0Q} &= \alpha_Q \frac{\nu}{u_*} \end{aligned} \quad (3.26)$$

These expressions account for both low and high wind regimes.

- At low wind speed the sea surface becomes aerodynamically smooth and the sea surface roughness length scales with the kinematic viscosity  $\nu$  ( $= 1.5 \times 10^{-5} \text{m}^2 \text{s}^{-1}$ ).
- At high wind speed the Charnock relation is used. The chosen constants are  $\alpha_M = 0.11$ ,  $\alpha_H = 0.40$ , and  $\alpha_Q = 0.62$  (Brutsaert, 1982). The Charnock coefficient,  $\alpha_{Ch}$ , is set equal to 0.018 for the uncoupled model, and is provided by the wave model in coupled mode.

The smooth-surface parametrization is retained in high wind speed regimes for heat and moisture because observations indicate that the transfer coefficients for heat and moisture have very little wind-speed dependence above  $4 \text{ms}^{-1}$  (Miller *et al.*, 1992; Godfrey and Beljaars, 1991). In (3.26) friction velocity  $u_*$ , is calculated from

$$u_* = C_M^{1/2} (u_n^2 + v_n^2 + w_*^2)^{1/2} \quad (3.27)$$

with  $w_*$  from (3.18) using fluxes from the previous time step.

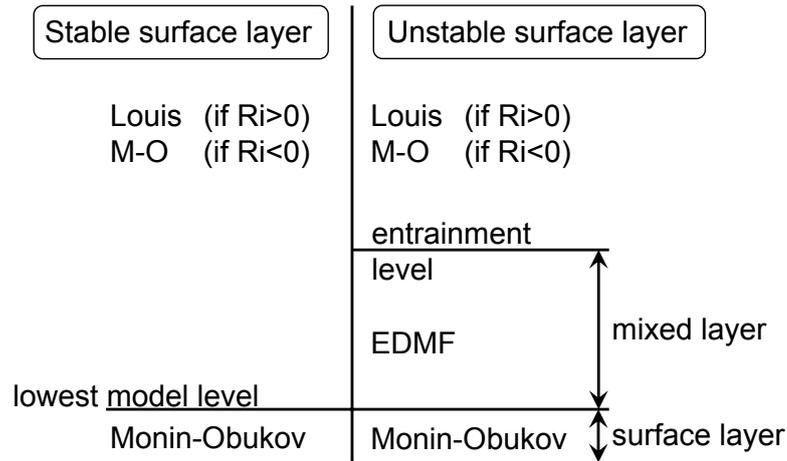
## 3.3 THE OUTER LAYER

The computation of the exchange coefficients depends on the stability regimes (locally and at the surface) and on the vertical location above the surface. Fig. 3.1 summarizes the various areas where each scheme (EDMF with non-local K-profiles in the mixed layer, local diffusion dependent on the Richardson number following Louis *et al.*, local diffusion with Monin–Obukhov functions) is applied.

### 3.3.1 The exchange coefficients in the mixed layer

The Eddy-Diffusivity Mass-Flux (EDMF) approach is used to compute the exchange coefficients within the well mixed part of unstable (convective) boundary layers (Tompkins *et al.*, 2004; Köhler, 2005; Köhler *et al.*, 2011). The decision whether the boundary layer is stable or unstable is based on the stability at the surface: if the bouyancy flux at the surface is positive the boundary layer is unstable, otherwise it is stable. If the unstable boundary layer is dry (not cloudy) or topped by a stratocumulus cloud, the mixed layer where the EDMF approach is applied spans from the first model level to the top of the boundary layer. If the boundary layer is cloudy, but contains shallow cumulus or shallow cumulus under stratocumulus, the mixed layer spans from the first model level to the base of the shallow cumulus.

Distinguishing between cumulus and well-mixed stratocumulus is therefore a crucial component of the system. Klein and Hartmann (1993) showed empirically that the stratus cloud cover increases with the



**Figure 3.1** Schematic diagram of the different boundary layer regimes.

lower tropospheric stability (LTS) of the atmosphere defined as  $\theta_{700\text{hPa}} - \theta_{\text{sfc}}$ . Wood and Bretherton (2006) defined a new formulation of the inversion strength called the estimated inversion strength EIS that is a better predictor of the stratus cloud amount than is the LTS. If a cloud is detected in an unstable boundary layer, a threshold value of 7 K is adopted for distinguishing whether it is a stratocumulus or a shallow cumulus. That is, if  $\text{EIS} > 7 \text{ K}$ , the boundary layer is considered to be well mixed and topped by a stratocumulus, and the shallow convection scheme is turned off. If  $\text{EIS} < 7 \text{ K}$ , the shallow cumulus regime is considered, which means that the shallow convection scheme is active and that the EDMF is applied only between the surface and the cloud base. Moreover, in this case the mass-flux part of EDMF is turned off because it is considered that the non-local mixing will be taken care off in this case by the shallow convection scheme.

### EDMF concept

The concept behind the EDMF approach is to describe the strong large-scale organized updraughts with mass fluxes and the remaining small-scale turbulent part with diffusion. The up/down-draughts described by the mass flux term allow for non-local mixing, while the local mixing described by the diffusion term is limited to down gradient transports. Siebesma and Cuijpers (1995) arbitrarily define a strong updraught as a fixed small fractional area  $a_u$  containing the strongest upward vertical motions. The horizontal distribution of a field  $\phi$  can then be described with perturbation terms in both updraught and environment areas separately

$$\phi_u = \phi'_u + \overline{\phi}_u^u \quad \text{and} \quad \phi_e = \phi'_e + \overline{\phi}_e^e \quad (3.28)$$

where u and e refer to the updraught and environment areas both for the field and the averaging operator. The domain average then becomes

$$\overline{\phi} = a_u \overline{\phi}_u^u + (1 - a_u) \overline{\phi}_e^e \quad (3.29)$$

After some manipulation the vertical turbulent flux breaks into three terms

$$\overline{w'\phi'} = a_u \overline{w'\phi'_u^u} + (1 - a_u) \overline{w'\phi'_e^e} + \frac{M}{\rho} (\phi_u - \phi_e) \quad (3.30)$$

where  $M = \rho a_u w_u$  is the mass flux of the strongest updraughts. It is assumed  $a_u \ll 1$  while  $\overline{w'\phi'_u^u}$  and  $\overline{w'\phi'_e^e}$  are of same order of magnitude. This permit the first term on the RHS to be neglected and  $\phi_e \simeq \overline{\phi}$ . The second term on the RHS can be approximated by diffusion with a coefficient of  $K_\phi$ . Then

$$\overline{\rho w'\phi'} = -\rho K_\phi \frac{\partial \overline{\phi}}{\partial z} + M(\phi_u - \overline{\phi}) \quad (3.31)$$

Equation (3.31) is the basic equation for vertical turbulent transport of  $\phi$  and will be considered for the moist conserved variables of generalized liquid water static energy  $s_l$  and total water mass mixing ratio  $q_t$ .

### Mass flux component and plume model

The mass flux employs a single bulk plume model and is closed near the surface as a fraction of the tail of the vertical velocity distribution.

An air parcel is traced through its updraught to find cloud base, cloud/PBL top and the updraught properties. This is done with an entraining/detraining bulk plume model for the variables  $\phi = \{s_l, q_t\}$  (e.g. Betts, 1973) and updraught kinetic energy  $\frac{1}{2}w_u^2$  (Simpson and Wiggert, 1969)

$$\frac{\partial \phi_u}{\partial z} = -\epsilon(\phi_u - \bar{\phi}) \quad (3.32)$$

$$\frac{1}{2} \frac{\partial w_u^2}{\partial z} = -\epsilon(w_u^2 - \bar{w}^2) + g \frac{\theta_{v,u} - \bar{\theta}_v}{\bar{\theta}_v} \quad (3.33)$$

where  $\epsilon$  is the fractional entrainment rate. The last term represents the buoyancy acceleration. The environmental kinetic energy  $\frac{1}{2}\bar{w}^2$  can be neglected. The top of the boundary layer  $z_i$  (inversion height) is found when the vertical velocity drops to zero.  $z_i$  is interpolated between model levels. If condensation occurs, a cloud base height  $z_{cb}$  is defined, which is also interpolated between model layers.

The parcel is initialised by taking mean fields at the lowest model level  $z_n$  and adding an excess that scales with the surface fluxes (Troen and Mahrt, 1986), i.e. for variables  $\phi = \{s_l, q_t\}$

$$\phi_u(z_n) = \overline{\phi(z_n)} + b \frac{\overline{w'\phi'}^{\text{surf}}}{\sigma_w(z_n)} \quad (3.34)$$

where surf refers to the surface mean and  $b$  is a parameter estimated as 1.0 based on LES experiments (Siebesma and Teixeira, 2000). For the standard deviation of vertical velocity  $\sigma_w$  an empirical expression based on atmospheric data, tank measurements and LES data is used (Holtslag and Moeng, 1991)

$$\sigma_w \simeq 1.2 \left( u_*^3 + 1.5\kappa w_*^3 \frac{z}{z_i} \right)^{1/3} \left( 1 - \frac{z}{z_i} \right)^{1/2} = 1.2 \left( u_*^3 + 1.5\kappa \frac{g}{\bar{\theta}_v^{\text{surf}}} \overline{w'\theta'_v}^{\text{surf}} z \right)^{1/3} \left( 1 - \frac{z}{z_i} \right)^{1/2} \quad (3.35)$$

where  $\kappa = 0.4$  is the Von Kármán's constant,  $u_* \equiv (\overline{u'w'}^{\text{surf}2} + \overline{u'w'}^{\text{surf}2})^{1/4}$  is the friction velocity, and  $w_* \equiv (z_i \frac{g}{\bar{\theta}_v^{\text{surf}}} \overline{w'\theta'_v}^{\text{surf}})^{1/3}$  is the free convective velocity scale. Because  $z_i$  is not known yet when the updraught properties are computed and since  $\sigma_w$  at the lowest model level is not very sensitive to  $z_i$ , the last term  $(1 - \frac{z}{z_i})^{1/2}$  in equation (3.35) is neglected.

The parcel entrainment  $\epsilon$  is written as

$$\epsilon = a + \frac{b}{z} \quad (3.36)$$

where  $a = 2 \times 10^{-4}$  and  $b = 0.8$ . This is the same formulation as the one used for the test parcel in the convection scheme.

With the different height scales defined and the updraught properties known, it is still necessary to define the profiles of mass flux and diffusion coefficients. The mass flux  $M$  is directly calculated from the entrainment  $\epsilon$  and the detrainment  $\delta$

$$\frac{\partial M}{\partial z} = (\epsilon - \delta)M. \quad (3.37)$$

$M$  is initialised at the lowest level as  $\rho a_u \bar{w}_u$ , where  $a_u$  represents the updraught fraction in the surface layer. This is a free parameter in the parametrization that can be set to any fraction of the vertical velocity spectrum that one intends to parametrise with the mass flux. Here  $a_u = 0.05$ . Assuming a Gaussian distribution of vertical velocities, the mean of the upper 5% tail of the distribution corresponds to a

$$\bar{w}_u(a_u, \sigma_w) = b\sigma_w \quad (3.38)$$

with  $b = 2.05$ . This  $\overline{w}_u(a_u, \sigma_w)$  is also used to initialize the parcel. For simplicity, the temperature and moisture perturbations are assumed to be correlated with the vertical velocity perturbations. Therefore, the parameter  $b$  is also used in (3.34). Detrainment is set to  $3 \cdot 10^{-4} m^{-1}$  above cloud base and zero below. This corresponds to a bulk plume model.

The updraught equation is integrated from the surface to the top using backward finite differences. All finite difference equations are written in conservative form.

### Diffusion component

A first-order closure specifies the turbulent flux of a given quantity  $\phi$  at a given model level proportional to the vertical gradient of that quantity. Therefore the diffusion component in the EDMF framework can be written as

$$J_\phi^K = \rho K_\phi \frac{\partial \phi}{\partial z}. \quad (3.39)$$

The exchange coefficients  $K_\phi$  are estimated at half model levels.

In unstable surface conditions (positive bouyancy flux), the exchange coefficients are expressed as integral profiles for the entire convective mixed layer. This  $K$ -profile closure is based on the form proposed by Troen and Mahrt (1986). This approach is more suitable than the local diffusion one when the length scale of the largest transporting turbulent eddies have a similar size as the boundary layer height itself (unstable and convective conditions). It also allows for an explicit entrainment parametrization in the capping inversion (Beljaars and Viterbo, 1999).

In a dry boundary layer the mixing is due to surface driven eddies, while in a stratocumulus capped boundary layer, the turbulent mixing is also due to cloud top driven eddies. So in these cases the eddy-diffusion coefficients are the sum of the surface and a cloud top driven  $K$ -profile, and are overwritten with an explicit BL top entrainment at cloud top. For the surface driven diffusion a simple  $K$ -profile is specified (Troen and Mahrt, 1986; Holtslag, 1998):

$$\begin{aligned} K_H^{\text{sfc}} &= \kappa u_* \Phi_{H0}^{-1} \left(1 - \frac{z}{z_i}\right)^2 \\ K_M^{\text{sfc}} &= \kappa u_* \Phi_{M0}^{-1} \left(1 - \frac{z}{z_i}\right)^2 \end{aligned} \quad (3.40)$$

where  $\Phi_{H0}$  and  $\Phi_{M0}$  are stability functions given by

$$\begin{aligned} \Phi_{H0} &= \left(1 - 39 \frac{z}{\mathcal{L}}\right)^{-1/3} \\ \Phi_{M0} &= \left(1 - 15 \frac{z}{\mathcal{L}}\right)^{-1/3} \end{aligned} \quad (3.41)$$

and  $\mathcal{L} = -u_*^3 (\kappa \overline{w' \theta'_v} / \theta_v)^{-1}$  is the Obukhov length. Cloud top driven diffusion is described after Lock *et al.* (2000) as

$$K_H^{\text{top}} = 0.85 \kappa v_{\text{cld}} \frac{(z - z_{\text{mb}})^2}{z_i - z_{\text{mb}}} \left(1 - \frac{z - z_{\text{mb}}}{z_i - z_{\text{mb}}}\right)^{1/2} \quad (3.42)$$

where  $z_i$  is the inversion height or cloud top and  $z_{\text{mb}}$  is the level below cloud base to which the top driven mixing extends. A value of  $z_{\text{mb}} = 0$  is assumed, implying a strong coupling between the cloud layer and the sub-cloud layer. Parameter  $v_{\text{cld}}$  represents a top entrainment velocity scale encompassing a radiative cooling and a buoyancy reversal term

$$v_{\text{cld}}^3 = v_{\text{rad}}^3 + v_{\text{br}}^3 \quad (3.43)$$

The buoyancy reversal term is neglected and the radiation term is written as

$$v_{\text{rad}}^3 = \frac{g}{\theta_0} z_i \Delta R / (\rho c_p) \quad (3.44)$$

after Lock (1998), where  $\Delta R$  is the radiative flux jump at cloud top.

Boundary layer top entrainment at  $z = z_i$  is explicitly specified from surface and cloud top driven components

$$\begin{aligned} \overline{w'\theta_v'}^{\text{entr}} &= -0.2\overline{w'\theta_v'^s} - 0.2\Delta R/(\rho c_p) = -K_H^e \frac{\partial \theta_v}{\partial z} \approx -K_H^e \frac{\Delta \theta_v}{\Delta z} \equiv w_e \Delta \theta_v, \text{ with} \\ (\Delta \theta_v)_{k+1/2} &= \frac{1}{c_{p\text{dry}}} \{s_k - s_{k+1} - 0.5(\delta - \varepsilon)(q_k - q_{k+1})(s_k + s_{k+1})\} \end{aligned} \quad (3.45)$$

and  $w_e$  representing the top entrainment velocity.

For both top driven and entrainment diffusion coefficients the turbulent Prantl number is  $Pr = K_M/K_H = 0.75$ :

$$\begin{aligned} K_M^{\text{top}} &= 0.75 K_H^{\text{top}} \\ K_M^e &= 0.75 K_H^e \end{aligned} \quad (3.46)$$

Within the PBL the total diffusion coefficient for  $s_l$  and  $q_t$  is specified as

$$K_H = K_H^{\text{sf}} + K_H^{\text{top}} \quad (3.47)$$

while at the top of the PBL

$$K_H = \max(K_H^{\text{sf}} + K_H^{\text{top}}, K_H^e) \quad (3.48)$$

is used.

### Cloud

In order to provide coupling of the boundary layer scheme to the cloud scheme, the variance of total water is computed from the following equation (Deardorff, 1974)

$$\frac{\partial \sigma_{qt}^2}{\partial t} = -2\overline{w'q_t'} \frac{\partial q_t}{\partial z} - \frac{\partial (\overline{w'\sigma_{qt}^2})}{\partial z} - \frac{\sigma_{qt}^2}{\tau_{\sigma_{qt}}} \quad (3.49)$$

The first term is the generation term in the presence of a vertical  $q_t$  gradient, the second term represents transport which is neglected and the third term is the decay term. The associated decay time scale is written as

$$\tau_{\sigma_{qt}} = \frac{z_i}{\overline{w_u}^{\text{PBL}}} \quad (3.50)$$

where the updraught velocity  $\overline{w_u}^{\text{PBL}}$  is averaged over the height of the boundary layer  $z_i$ .

The change in total water and total water variance needs to be transformed into the Tiedtke (1993) cloud scheme's prognostic variables of cloud water and cloud cover. At the beginning of the boundary layer parametrization time step, the model variables,  $q_v$ ,  $q_l$ ,  $q_i$ ,  $T$ , and  $f_c$  (cloud cover), are converted to  $q_t$ ,  $q_l$ , and  $\sigma_{qt}$  ( $q_v$ ,  $q_l + q_i$ ,  $q_{\text{sat}}(T)$ ), assuming a Beta distribution as used by Tompkins (2002). The variance of the total water  $\sigma_{qt}$  is diagnosed that gives the specified liquid water and cloud cover. Then, (3.49) is integrated for one time step and the total water distribution function is used to diagnose the equivalent change in cloud cover and cloud water/ice. The differences between cloud variables before and after the time step are then used as a tendency (from boundary layer processes) for the cloud scheme.

### 3.3.2 The exchange coefficients in stable conditions and in unstable layers situated above the mixed layer

#### General

In all stable layers (i.e. within or above the boundary layer) and in unstable layers situated above the mixed layer (free-shear layers), the turbulent transports are determined based on local stability with a first order closure (see Fig. 3.1). The diffusion coefficients depend on shear, on a mixing length and some stability functions of the local Richardson number. The local Richardson number is computed in each

vertical layer using

$$\begin{aligned}
 |\Delta U|_{k+1/2}^2 &= (u_k - u_{k+1})^2 + (v_k - v_{k+1})^2 \\
 \left(\frac{\Delta s_v}{c_p T}\right)_{k+1/2} &= \frac{2(s_k - s_{k+1})}{(s_k - gz_k + s_{k+1} - gz_{k+1})} + \varepsilon(q_k - q_{k+1}) \\
 Ri_{k+1/2} &= (gz_k - gz_{k+1}) \frac{\{(\Delta s_v)/(c_p T)\}_{k+1/2}}{|\Delta U|_{k+1/2}^2}
 \end{aligned} \tag{3.51}$$

In the outer layer, the mixing length used in the surface layer ( $l = \kappa z$ ) is bounded by introducing an asymptotic length scale  $\lambda$  (Blackadar, 1962):

$$\frac{1}{l} = \frac{1}{\kappa z} + \frac{1}{\lambda}. \tag{3.52}$$

The underlying idea is that vertical extent of the boundary layer, or of the free-shear unstable layer, limits the turbulence length scale. The asymptotic length scale has different values in stable and in unstable layers (see below).

### The exchange coefficients

(a) *M–O similarity for unstable layers with  $Ri < 0$*

In these statically unstable layers situated above the mixed layer, the exchange coefficients  $K_\phi$  are based on local similarity (Nieuwstadt, 1984) stating that the expressions of the surface layer similarity can be used in the outer layer (strictly speaking only valid for stable conditions):

$$\begin{aligned}
 K_M &= \frac{l^2}{\Phi_M^2} \cdot \left| \frac{\partial U}{\partial z} \right| \\
 K_H &= \frac{l^2}{\Phi_M \Phi_H} \left| \frac{\partial U}{\partial z} \right|
 \end{aligned} \tag{3.53}$$

with  $\Phi_M$  and  $\Phi_H$  specified in (3.19), where  $\zeta$  is set to the local Richardson number  $R_i$ . The asymptotic mixing length is equal to 150m.

(b) *Revised Louis scheme for stable layers with  $Ri > 0$*

Since November 2013, the treatment of the diffusion in stable layers was simplified and the degree of mixing maintained in stable layers near the surface was reduced. This was mainly achieved by changing the formulation of the asymptotic mixing length and by removing a term that was added up to CY38R2 to the computation of shear (used to compute both the local  $R_i$  and the exchange coefficients) in order to account for non-resolved shear (Sect. 3.4 of the CY38R1 IFS documentation). This term was removed as its formulation is theoretically not very satisfactory as it remains unclear how it scales with wind speed and model resolution, and how it should vary vertically (Mahrt and Vickers, 2006).

The exchange coefficients are now given by

$$K = \left| \frac{\partial U}{\partial z} \right| l^2 f_{LTG}(Ri) \tag{3.54}$$

where  $f_{LTG}(Ri)$  are the Louis, Tiedtke, Geleyn stability functions of  $Ri$  (Louis *et al.*, 1982), revised by Beljaars (1995); Beljaars and Viterbo (1999); Viterbo *et al.* (1999) in order to enhance turbulent transport in stable layers. These functional dependencies with  $Ri$  for momentum  $f_{LTG,M}$  and heat/moisture  $f_{LTG,H}$  are given by:

$$\begin{aligned}
 f_{LTG,M}(Ri) &= \frac{1}{1 + 2bRi(1 + dRi)^{-1/2}} \\
 f_{LTG,H}(Ri) &= \frac{1}{1 + 2bRi(1 + dRi)^{1/2}}
 \end{aligned} \tag{3.55}$$

with  $b = 5$  and  $d = 1$ .

The asymptotic mixing length is now equal to 10% of the boundary layer height in stable boundary layers; which means that it is small in very stable and shallow boundary layers and it is bigger in weakly stable deeper boundary layers. (The boundary layer height is diagnosed as the height where the bulk Richardson number becomes superior to 0.25). In stable layers situated above the boundary layer, the asymptotic mixing length is constant and equal to 30m, a value closer to that used in other NWP models (i.e. UK Metoffice, NCEP GFS) and to observational evidence (Tjernstrom, 1993).

### 3.4 TURBULENT OROGRAPHIC FORM DRAG (TOFD)

With the introduction of CY31R1, the orographic contribution to the aerodynamic roughness length has been replaced by an explicit specification of stress on model levels due to turbulent orographic form drag (TOFD). The TOFD scheme is based on the work of Wood and Mason (1993) in which the orographic surface drag is parametrized for sinusoidal hills and on the suggestion by Wood *et al.* (2001) to distribute this drag explicitly in the vertical. It is further inspired by the notion that fine scale data sets with sufficient horizontal resolution to compute slope or silhouette parameters on a global scale, are not available. The TOFD scheme, as described in detail by Beljaars *et al.* (2004b), has three key aspects.

First, the orographic spectrum is parametrized and the effect of all the scales is obtained by integrating over the spectrum (5 km down to 10 m). The standard deviation of filtered orography  $\sigma_{flt}$  is used as input for the scheme. It is defined in such a way that it can be measured from the available data at 1 km resolution (see appendix describing the climatological fields).

Secondly, the total drag is represented as a spectral integral over all wave numbers contributing to the variance of the slope. Drag, due to small horizontal orography scales is distributed over a shallow layer, whereas large scales affect deep layers. Convergence problems associated with the variance of the slope when computed as the integral over the spectrum, have been alleviated by including the wind forcing level in the spectral integral. Physically, it means that smaller horizontal scales have a wind forcing at a lower level than the large horizontal scales.

Thirdly, simplifications are applied to avoid explicit evaluation of the integral over the orographic spectrum. For numerical stability it is also necessary to have an implicit formulation that can be solved as part of the vertical diffusion tridiagonal solver.

The parametrization results in the following additional tendency (stress divergence) in the equations for the horizontal wind vector  $\vec{U}$

$$\frac{\partial}{\partial z} \vec{\tau}_o / \rho = -2\alpha_{tofd} \beta_{tofd} C_{md} C_{corr} |\vec{U}(z)| \vec{U}(z) \int_{k_o}^{k_\infty} \frac{k^2}{l_w} F_o(k) e^{-z/l_w} dk \quad (3.56)$$

with

$$\begin{aligned} l_w &= \min(2/k, 2/k_1), \\ F_o(k) &= a_1 k^{n_1}, \text{ for } k_o < k < k_1, \\ F_o(k) &= a_2 k^{n_2}, \text{ for } k_1 < k < k_\infty, \\ n_1 &= -1.9, & n_2 &= -2.8, \\ a_1 &= \sigma_{flt}^2 (I_H k_{flt}^{n_1})^{-1}, & a_2 &= a_1 k_1^{n_1 - n_2}, \\ k_o &= 0.000628 \text{ m}^{-1}, & k_1 &= 0.003 \text{ m}^{-1}, \\ k_{flt} &= 0.00035 \text{ m}^{-1}, & k_\infty &= 2\pi c_m / z_o, \\ I_H &= 0.00102 \text{ m}^{-1}, & c_m &= 0.1, \\ \alpha_{tofd} &= 35, & \beta_{tofd} &= 1, \\ C_{md} &= 0.005, & C_{corr} &= 0.6. \end{aligned} \quad (3.57)$$

The parameter  $\alpha_{tofd}$  was determined from LES data as 12. But during extensive testing an optimal value of  $12(1.7)^2 = 35$  was determined that reflects a 70% increase in the standard deviation of filtered

orography  $\sigma_{\text{ft}}$ . Spectrum  $F_o(k)$  of the sub-grid orography is represented with empirical power laws (power  $n_1$  and  $n_2$ ) in two different scale ranges. The integral of the right hand side of (3.56) can be pre-computed for different heights, without giving a computational burden. However, with hybrid vertical coordinates (as in the ECMWF model), model level heights vary with surface pressure, and therefore it is more convenient to have an analytical expression. A good approximation of (3.56) is

$$\frac{\partial \vec{U}}{\partial t} = \frac{\partial}{\partial z} \vec{\tau}_{\text{xo}} / \rho = -\alpha_{\text{tofd}} \beta_{\text{tofd}} C_{\text{md}} C_{\text{corr}} |\vec{U}(z)| \vec{U}(z) 2.109 e^{-(z/1500)^{1.5}} a_2 z^{-1.2} \quad (3.58)$$

The use of (3.58) rather than (3.56) gives virtually identical results in single column simulations.

The two components of the stress divergence are included in the momentum equations and solved together with the turbulent transport equations. An implicit formulation is needed for stability. The standard way of time stepping a non-linear problem with implicit equations is by evaluating the non-linear part at the old time level and keeping the linear part for the new time level. In this case it means that the absolute wind speed  $|U|$  is taken from the old time level and that the  $U(z)$  and  $V(z)$  components are evaluated implicitly.

With (3.58), and the constants in (3.57), the entire parametrization depends on a single geographical parameter namely the standard deviation of the filtered orography  $\sigma_{\text{ft}}$ . Equation (3.58) is written as

$$\begin{aligned} \frac{\partial \vec{U}}{\partial t} &= -C_{\text{tofd}} |\vec{U}(z)| \vec{U}(z), \\ C_{\text{tofd}} &= -\alpha_{\text{tofd}} \beta_{\text{tofd}} C_{\text{md}} C_{\text{corr}} 2.109 e^{-(z/1500)^{1.5}} a_2 z^{-1.2} \end{aligned} \quad (3.59)$$

The expression in (3.59) is computed in subroutine VDFTOFDC. Output  $C_{\text{tofd}}^* = C_{\text{tofd}} \alpha \Delta t |\vec{U}(z)|$  is passed to the implicit solver in VDFDIFM.

### 3.5 SOLUTION OF THE EDMF EQUATIONS

The equations for turbulent transfer are solved simultaneously for diffusion, mass fluxes, TOFD, the implicit and explicit terms from the sub-grid orography scheme ( $\beta_{\text{so}}\phi$  and  $\alpha_{\text{so}}$ ), and the tendencies from the adiabatic (subscript ‘dyn’) and radiative processes (subscript ‘rad’) as source terms in the right-hand side:

$$\frac{\partial \phi}{\partial t} = -g \frac{\partial J_\phi}{\partial p} - C_{\text{tofd}} |\vec{U}| \phi - \beta_{\text{so}} \phi + \alpha_{\text{so}} + \left. \frac{\partial \phi}{\partial t} \right|_{\text{dyn}} + \left. \frac{\partial \phi}{\partial t} \right|_{\text{rad}} \quad (3.60)$$

Flux  $J_\phi$  (defined down-ward) has a diffusive part and a massflux term:

$$J_\phi = \rho K \frac{\partial \phi}{\partial z} - M(\phi_{\text{u}} - \phi) \quad (3.61)$$

Since the thickness of the model layers  $\Delta z$  is small near the ground and the time step is long, the time-stepping procedure must be implicit in order to avoid numerical instability ( $K\Delta t/(\Delta z)^2 > 1$ ). The advantage of doing the implicit computation with as many processes as possible in a single equation (3.60) is to maintain balance between processes which avoids time step dependence for long time steps. (Beljaars, 1991, Janssen *et al.*, 1992, Beljaars *et al.*, 2004a). Several time discretizations including explicit and implicit solutions can be written by defining a general time level  $\hat{\phi}$  and the implicitness factor  $\alpha$ .

$$\hat{\phi} \equiv \alpha \phi^{t+1} + (1 - \alpha) \phi^t \quad (3.62)$$

For  $\alpha = 0$  the scheme is explicit, for  $\alpha = 0.5$  we have a Crank–Nicholson and for  $\alpha = 1$  we have an implicit backward scheme. In the model,  $\alpha = 1.5$ , to avoid non-linear instability from the  $K$ -coefficients. The diffusion coefficients, mass fluxes and other coefficients are computed from the mean variables at  $t - 1$  making them explicit. The time tendency can then be written as

$$\frac{\partial \phi}{\partial t} = \frac{\phi^{t+1} - \phi^t}{\Delta t} = \frac{\hat{\phi} - \phi^t}{\alpha \Delta t} = f(\hat{\phi}) + g(\phi^t) \quad (3.63)$$

with implicit terms  $f(\hat{\phi})$  and explicit terms  $g(\phi^t)$ . Equation (3.60) is written in discrete form for  $1 < k < n$  using upwind differencing in the mass-flux term:

$$\begin{aligned} \frac{\hat{\phi} - \phi^t}{\alpha \Delta t} &= \frac{\Delta \phi_{\text{dyn}}}{\Delta t} + \frac{\Delta \phi_{\text{rad}}}{\Delta t} - C_{\text{tofd}} |\vec{U}| \hat{\phi}_k - \beta_{\text{so}} \hat{\phi}_k + \alpha_{\text{so}} \\ &+ \frac{g}{\Delta p_k} \left( \rho_{k-\frac{1}{2}} K_{k-\frac{1}{2}} \frac{\hat{\phi}_{k-1} - \hat{\phi}_k}{\Delta z_{k-\frac{1}{2}}} + M_{k-1} (\phi_{u,k-1} - \hat{\phi}_{k-1}) \right. \\ &\quad \left. - \rho_{k+\frac{1}{2}} K_{k+\frac{1}{2}} \frac{\hat{\phi}_k - \hat{\phi}_{k+1}}{\Delta z_{k+\frac{1}{2}}} - M_k (\phi_{u,k} - \hat{\phi}_k) \right) \end{aligned} \quad (3.64)$$

with  $\Delta z_{k+\frac{1}{2}} = z_k - z_{k+1}$  and  $\Delta p_k = p_{k+\frac{1}{2}} - p_{k-\frac{1}{2}}$ . This can be rewritten as

$$\begin{aligned} \frac{\hat{\phi}_{k-1}}{\alpha} A + \frac{\hat{\phi}_k}{\alpha} B + \frac{\hat{\phi}_{k+1}}{\alpha} C &= RHS \quad (3.65) \\ A &= \frac{1}{\Delta p_k} \left( -K_{k-\frac{1}{2}}^* + M_{k-1}^* \right) \\ B &= \frac{1}{\Delta p_k} \left( +K_{k-\frac{1}{2}}^* + K_{k+\frac{1}{2}}^* - M_k^* \right) + 1 + C_{\text{tofd}} |\vec{U}| \alpha \Delta t + \beta_{\text{so}} \alpha \Delta t \\ C &= \frac{1}{\Delta p_k} \left( -K_{k+\frac{1}{2}}^* \right) \\ RHS &= \frac{\phi_k^t}{\alpha} + \frac{1}{\Delta p_k} M_{k-1}^* \frac{\phi_{u,k-1}}{\alpha} - \frac{1}{\Delta p_k} M_k^* \frac{\phi_{u,k}}{\alpha} + \Delta \phi_{\text{dyn}} + \Delta \phi_{\text{rad}} + \alpha_{\text{so}} \Delta t \end{aligned}$$

leading to the inversion of a tridiagonal matrix to solve for  $\hat{\phi}/\alpha$ . The coefficients  $K$  and  $M$  are rescaled for convenience as  $K_{k+1/2}^* = K_{k+1/2} \alpha \rho_{k+1/2} \left( \frac{g \Delta t}{\Delta z_{k+1/2}} \right)$  and  $M^* = M g \alpha \Delta t$ .

At the lowest level ( $k = n$ ) equation (3.65) is modified to include the surface fluxes which are obtained in the surface energy balance routine by averaging over  $N_T$  tiles:

$$\begin{aligned} A &= \frac{1}{\Delta p_n} \left( -K_{n-\frac{1}{2}}^* + M_{n-1}^* \right) \\ B &= \frac{1}{\Delta p_n} \left( +K_{n-\frac{1}{2}}^* - M_n^* \right) + 1 + C_{\text{tofd}} |\vec{U}| \alpha \Delta t + \beta_{\text{so}} \alpha \Delta t \\ C &= 0 \\ RHS &= \frac{\phi_n^t}{\alpha} + \frac{1}{\Delta p_n} M_{n-1}^* \frac{\phi_{u,n-1}}{\alpha} - \frac{1}{\Delta p_n} M_n^* \frac{\phi_{u,n}}{\alpha} + \Delta \phi_{\text{dyn}} + \Delta \phi_{\text{rad}} + \alpha_{\text{so}} \Delta t - \frac{g \Delta t}{\Delta p_n} \bar{J}_\phi \end{aligned} \quad (3.66)$$

where the surface flux is a weighted average over the tiles

$$\frac{g}{\Delta p_n} \bar{J}_\phi = + \sum_{i=1}^{N_T} F_i \frac{C_{\phi i}^*}{\alpha \Delta p_n} \{ A_{ni} \hat{\phi}_n - A_{\text{surf}i} \hat{\phi}_{\text{surf}i} \} \quad (3.67)$$

with  $C_{\phi i}^* = C_{\phi i}^{t-1} |U_n| g \rho \alpha \Delta t$  and

$$\begin{array}{llll} \phi = 0 & A_n = 1 & A_{\text{surf}} = 1 & N_T = 1 \text{ for } \phi = u, v \\ \phi_{\text{surf}i} = s_{\text{skin}i} & A_{ni} = 1 & A_{\text{surf}i} = 1 & N_T = 8 \text{ for } \phi = s \\ \phi_{\text{surf}i} = q_{\text{sat}}(T_{\text{skin}i}) & A_{ni} = \alpha_{ni} & A_{\text{surf}i} = \alpha_{\text{surf}i} & N_T = 8 \text{ for } \phi = q \end{array} \quad (3.68)$$

The downward elimination of the tridiagonal matrix results in linear relations between the lowest model level dry static energy and specific humidity and their fluxes

$$\begin{aligned} \hat{s}_n &= A_s \bar{J}_s + B_s \\ \hat{q}_n &= A_q \bar{J}_q + B_q \end{aligned} \quad (3.69)$$

Coefficients ( $A$ 's and  $B$ 's) are passed to the surface energy balance computation for the different tiles, and the resulting weighted fluxes are returned to the tridiagonal solver for upward back-substitution (see [Section 3.6](#)).

At the top of the atmosphere ( $k = 1$ ) turbulent fluxes are set to zero resulting in a modification of equation (3.65) as

$$\begin{aligned}
 A &= 0 \\
 B &= \frac{1}{\Delta p_1} \left( K_{1+\frac{1}{2}}^* - M_1^* \right) + 1 + C_{\text{tofd}} |\vec{U}| \alpha \Delta t + \beta_{\text{so}} \alpha \Delta t \\
 C &= \frac{1}{\Delta p_1} \left( -K_{1+\frac{1}{2}}^* \right) \\
 RHS &= \frac{\phi_1^t}{\alpha} - \frac{1}{\Delta p_1} M_1^* \frac{\phi_{u,1}}{\alpha} + \Delta \phi_{\text{dyn}} + \Delta \phi_{\text{rad}} + \alpha_{\text{so}} \Delta t
 \end{aligned} \tag{3.70}$$

The tridiagonal matrix equation is solved by a downward elimination scan followed by back substitution in an upward scan ([Press \*et al.\*, 1992](#), pp 42–43).

To safeguard against instabilities for the mass-flux term a relaxed CFL criteria for  $M$  is enforced:

$$M < f \rho \frac{\Delta z}{\Delta t} \tag{3.71}$$

As there is no strict stability analysis available for the EDMF equations, the factor  $f=2$  is determined by experimentation.

### 3.6 THE SURFACE ENERGY BALANCE

The surface energy balance is satisfied independently for each tile by calculating its skin temperature. The skin layer represents the vegetation layer, the top layer of the bare soil, or the top layer of the snow pack, has no heat capacity and therefore responds instantaneously to changes in, e.g. radiative forcing. In order to calculate the skin temperature, the surface energy balance equation is linearized for each tile leading to an expression for the skin temperature. This procedure is equivalent to the Penmann–Monteith approach which can be derived by eliminating the skin temperature from the surface energy balance equation assuming that the net radiation minus ground heat flux is known (e.g. [Brutsaert, 1982](#)). The approach followed here is an extension to the Penmann–Monteith derivation in the sense that it allows for coupling with the underlying soil (or snow, ice). Because of the short time scale associated with the skin layer, the equation for its temperature is solved implicitly together with the vertical turbulent transport in the boundary layer. In a fully implicit approach, the skin temperatures depend on each other and can not be solved independently. We follow the approach suggested by [Best \*et al.\* \(2004\)](#) which allows for such a solution. The coupling strategy of [Best \*et al.\* \(2004\)](#) also provides a well defined (universal) interface between atmosphere and land surface models, making it possible to have a stand alone library of the land surface code.

The following general discussion applies to each tile but the parameters are tile dependent as discussed in the land surface part of the documentation ([Chapter 8](#)). The surface energy balance equation can be written as:

$$\mathfrak{R}_{\text{SW}} + \mathfrak{R}_{\text{LW}} + H + LJ_q = \Lambda_{\text{skin}} (T_{\text{sk}} - T_s) \tag{3.72}$$

where  $\mathfrak{R}_{\text{SW}}$  and  $\mathfrak{R}_{\text{LW}}$  are the net short-wave and long-wave radiation fluxes at the surface and the right-hand side represents the ground heat flux through coupling with the underlying soil, snow or ice with temperature  $T_s$ . The turbulent sensible and latent heat fluxes are

$$H = J_s \tag{3.73}$$

$$J_s = \rho C_H |U_n| \{s_n - s_{\text{sk}}\} \tag{3.74}$$

$$J_q = \rho C_Q |U_n| \{\alpha_n q_n - \alpha_{\text{surf}} q_{\text{sat}}(T_{\text{sk}})\} \tag{3.75}$$

The equations for  $J_s$  and  $J_q$  are linearized

$$J_s = \rho C_H |U_n| \{(\hat{s}_n - \hat{s}_{sk})\} \quad (3.76)$$

$$J_q = \rho C_Q |U_n| \left\{ \alpha_n \hat{q}_n - \alpha_s q_{sat}(T_{sk}^t) - \alpha_s \frac{dq_{sat}}{dT} \left( \frac{\hat{s}_{sk}}{c_p^t} - T_{sk}^t \right) \right\} \quad (3.77)$$

and written as

$$\begin{aligned} J_s &= C_{Js1} S_n + C_{Js2} q_n + C_{Js3} S_{sk} + C_{Js4} \\ J_q &= C_{Jq1} S_n + C_{Jq2} q_n + C_{Jq3} S_{sk} + C_{Jq4} \end{aligned} \quad (3.78)$$

with coefficients

$$\begin{aligned} C_{Js1} &= \rho C_H |U_n|, & C_{Jq1} &= 0 \\ C_{Js2} &= 0, & C_{Jq2} &= \rho C_Q |U_n| \alpha_n \\ C_{Js3} &= -\rho C_H |U_n|, & C_{Jq3} &= \rho C_Q |U_n| \alpha_s \frac{dq_{sat}}{dT} c_p^{-1}, \\ C_{Js4} &= 0, & C_{Jq4} &= -\rho C_Q |U_n| \alpha_s (q_{sat}(T_{sk}^t) - \frac{dq_{sat}}{dT} T_{sk}^t) \end{aligned}$$

Substitution of the expression for dry static energy and moisture fluxes in the surface energy balance equation, and linearization of the long wave radiation leads to the following expressions

$$\mathfrak{R}_{SW} + \mathfrak{R}_{LW} + J_s + L J_q = \Lambda_{skin} (\hat{T}_{sk} - T_s) \quad (3.79)$$

$$\begin{aligned} R_{SW} + R_{LW}^{trad} + \frac{dR_{LW}}{dT_{sk}} (\hat{s}_{sk}/c_p^t - T_{sk}^{trad}) + C_{Js1} \hat{s}_n + C_{Js3} \hat{s}_{sk} + L(C_{Jq2} \hat{q}_n + C_{Jq3} \hat{s}_{sk} + C_{Jq4}) \\ = \Lambda_{skin} (\hat{T}_{sk} - T_s) \end{aligned} \quad (3.80)$$

which is written in the following form

$$\hat{s}_{sk} = D_{ss1} \hat{s}_n + D_{ss2} \hat{q}_n + D_{ss4} \quad (3.81)$$

The coefficients are (using  $\hat{s}_{sk} = c_p^t \hat{T}_{sk}$ )

$$\begin{aligned} D_{ss1} &= -C_{Js1} Z^{-1} \\ D_{ss2} &= -C_{Jq2} L Z^{-1} \\ D_{ss4} &= (-R_{SW} - R_{LW}^{trad} + \frac{dR_{LW}}{dT_{sk}} T_{sk}^{trad} - L C_{Jq4} - \Lambda_{skin} T_s) Z^{-1} \\ Z &= \left( \frac{dR_{LW}}{dT_{sk}} - \Lambda_{skin} \right) c_p^{-1} + C_{Js3} + L C_{Jq3} \end{aligned} \quad (3.82)$$

With (3.80),  $\hat{s}_{sk}$  can be eliminated and the flux equations can be written in the following form.

$$\begin{aligned} J_s &= D_{Js1} \hat{s}_n + D_{Js2} \hat{q}_n + D_{Js4} \\ J_q &= D_{Jq1} \hat{s}_n + D_{Jq2} \hat{q}_n + D_{Jq4} \end{aligned} \quad (3.83)$$

with

$$\begin{aligned} D_{Js1} &= C_{Js1} + C_{Js3} D_{ss1}, & D_{Jq1} &= C_{Jq1} + C_{Jq3} D_{ss1} \\ D_{Js2} &= C_{Js2} + C_{Js3} D_{ss2}, & D_{Jq2} &= C_{Jq2} + C_{Jq3} D_{ss2} \\ D_{Js4} &= C_{Js3} D_{ss4} + C_{Js4}, & D_{Jq4} &= C_{Jq3} D_{ss4} + C_{Jq4} \end{aligned}$$

With equation (3.83), a linear expression of fluxes is available in terms of lowest model level variables. The grid box average can be obtained by taking the weighted average of the coefficients over the tiles.

$$\begin{aligned} \bar{J}_s &= \hat{s}_n \sum_i F r^i D_{Js1}^i + \hat{q}_n \sum_i F r^i D_{Js2}^i + \sum_i F r^i D_{Js4}^i, \\ \bar{J}_q &= \hat{s}_n \sum_i F r^i D_{Jq1}^i + \hat{q}_n \sum_i F r^i D_{Jq2}^i + \sum_i F r^i D_{Jq4}^i. \end{aligned} \quad (3.84)$$

The over-bar indicates the grid box average of the fluxes. Equation (3.84) can be written as

$$\begin{aligned}\bar{J}_s &= E_{Js1}\hat{s}_n + E_{Js2}\hat{q}_n + E_{Js4}, \\ \bar{J}_q &= E_{Jq1}\hat{s}_n + E_{Jq2}\hat{q}_n + E_{Jq4}.\end{aligned}\tag{3.85}$$

where the  $E$ -coefficients are the grid box averages of the  $D$ -coefficients. After the downward elimination of the tridiagonal solver of the vertical turbulent transport a linear relation exists between the lowest model level  $\hat{s}_n, \hat{q}_n$  and the surface fluxes in the form of equation (3.69).

Together with equation (3.85) it is straightforward to solve for  $\bar{J}_s, \bar{J}_q, \hat{s}_n$ , and  $\hat{q}_n$ . With the latter two, back substitution can be started in an upward scan of the vertical turbulent transport equation.

With the lowest model level dry static energy and specific humidity known, it is also possible to solve for all the tile dependent fluxes and skin temperatures using (3.82) and (3.83).

### 3.7 TRACER DIFFUSION

Tracers are diffused in the same way as heat and moisture, but no mass flux term is used. The surface boundary condition consists of an externally specified flux. The implicitness factor is set to 1, because a higher value is not necessary for stability. As for momentum, heat and moisture the implicit solver uses the dynamics term as source terms to obtain balance and small time step dependence for long time steps. It can be demonstrated that implicitness factors larger than 1 can lead to negative tracer concentrations due to the combination with the dynamics source term.

### 3.8 TENDENCY CALCULATIONS AND ENERGY DISSIPATION

Total tendencies for wind and total water after the vertical transport (including diffusion and mass-flux terms and also dynamics, radiation, TOFD, and sub-grid orography tendencies) are

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{u^{t+1} - u^t}{\Delta t} \\ \frac{\partial v}{\partial t} &= \frac{v^{t+1} - v^t}{\Delta t} \\ \frac{\partial q_t}{\partial t} &= \frac{q_t^{t+1} - q_t^t}{\Delta t}\end{aligned}\tag{3.86}$$

The tendencies and model level fluxes are also computed separately for each process for diagnostic purposes. The surface fluxes of turbulent diffusion plus total TOFD (vertically integrated) are post-processed as turbulent surface drag. The vertically integrated tendency of the sub-grid orography scheme is post-processed as gravity wave stress.

The kinetic energy loss by the mean flow through the diffusion process, and TOFD ( $E_{\text{diss}}$ ), is

$$E_{\text{turb}} = 2\Delta t \frac{\partial u}{\partial t} \Big|_{\text{turb}+\text{TOFD}} \left( \frac{u^{t+1} + u^t}{2} \right) + 2\Delta t \frac{\partial v}{\partial t} \Big|_{\text{turb}+\text{TOFD}} \left( \frac{v^{t+1} + v^t}{2} \right)\tag{3.87}$$

The kinetic energy lost is assumed to be transformed locally into internal energy. This procedure bypasses the sub-grid scale energy cascade, but it allows a closed energy cycle in the model (the term is generally small). Therefore

$$\frac{\partial s_l}{\partial t} \Big|_{\text{turb}+\text{dyn}+\text{rad}} = \frac{s_l^{t+1} + E_{\text{diss}} - s_l^t}{\Delta t}\tag{3.88}$$

### 3.9 SHORTER TIME STEP IN THE VERTICAL TURBULENT TRANSPORT SCHEME

The vertical turbulent transport scheme is called two times in every physics time step, with a time step of 1/2 of the standard time step.

### 3.10 DIAGNOSTIC COMPUTATIONS FOR POST-PROCESSING

#### 3.10.1 Diagnostic boundary layer height

Because of its importance for applications (e.g. in air pollution modelling), the boundary layer height is diagnosed and made available for post-processing. The parametrization of the mixed layer uses a boundary layer height from an entraining parcel model. But in order to get a continuous field, also in neutral and stable situations a bulk Richardson method is used as a diagnostic, independent of the turbulence parametrization. This method follows the conclusions of the recent study by [Seidel \*et al.\* \(2012\)](#).

[Seidel \*et al.\* \(2012\)](#) evaluated a large number of methods proposed in the literature for estimating the boundary layer height and found that an algorithm based on the bulk Richardson number, originally proposed by [Vogelezang and Holtslag \(1996\)](#), is the most appropriate for application to radiosondes, reanalysis and climate model data sets. Several approximations are applied to the original algorithm, so that it can be consistently applied for both radiosondes and model output. Thus, since the friction velocity is not known from radiosonde data, the surface frictional effects are ignored in the computation of the bulk shear. Similarly, because radiosonde observations do not include winds close to the surface (at 2m), winds at 2m are set to zero. With these assumptions, the boundary layer height  $h_{BL}$  is defined as the lowest level at which the bulk  $R_i$  reaches the critical value of 0.25. [Seidel \*et al.\* \(2012\)](#) showed that this algorithm is suitable for both convective and stable boundary layers, identifies a nonnegative height in all cases, and is not strongly dependent on the sounding vertical resolution.

In IFS, the bulk Richardson number is computed as follows.

$$\begin{aligned}
 |\Delta U|^2 &= u_{hbl}^2 + v_{hbl}^2 \\
 s_{vn} &= c_p T_n (1 + \varepsilon q_n) + g z_n \\
 s_{vhbl} &= c_p T_{hbl} (1 + \varepsilon q_{hbl}) + g h_{bl} \\
 Ri_b &= h_{bl} \frac{2g(s_{vhbl} - s_{vn})}{(s_{vhbl} + s_{vn} - g h_{bl} - g z_n) |\Delta U|^2}
 \end{aligned} \tag{3.89}$$

where index  $n$  indicates the lowest model level and  $h_{bl}$  indicates the boundary layer height, i.e the level where  $Ri_b = 0.25$ . The virtual dry static energy from the lowest level  $s_{vn}$  is compared to the virtual dry static energy at the boundary layer height  $h_{bn}$ . For the buoyancy parameter  $g/T$ ,  $T$  is computed from  $s$  and averaged between the lowest model level and the boundary layer height. The boundary layer height is found by a vertical scan from the surface upwards. If the boundary layer height is found to be between two levels a linear interpolation is done to find the exact position.

#### 3.10.2 Wind at 10 m level and friction velocity

Wind at the 10 m level is computed for post-processing because it is the standard level for SYNOP observations. It can be obtained rather easily by vertical interpolation between the lowest model level and the surface, making use of profile functions (3.9) and (3.10). This procedure is appropriate over the ocean or in areas where the surface is smooth and homogeneous. However, the postprocess-ed field is meant to be comparable to wind from SYNOP observations and for observations over land WMO requires SYNOP stations to be in open terrain in order to be well exposed to wind. So the SYNOP wind observations are not necessarily compatible with the wind that is representative for a large area (i.e. a grid box from the model). Over inhomogeneous terrain, the problem can be particularly serious, because the ‘‘aerodynamic roughness length’’ in the model is adjusted to provide sufficient drag at the surface which is dominated by the rough elements. This approach leads to a low area-averaged wind speed which is not comparable to the ‘‘open-terrain’’ wind speed as observed by WMO stations.

In order to make the postprocess-ed wind compatible with SYNOP observations, the concept of exposure correction is introduced. The open-terrain wind is obtained by taking the wind information from such a height above the surface that it is less influenced by the underlying terrain. This height is called the blending height  $h_{blend}$  and for the interpolation to 10 m an aerodynamic roughness length is used that is typical for open terrain with grassland.

The interpolation procedure is as follows. First the blending height and the interpolation roughness length are set dependent on the model roughness length field using

$$\begin{aligned} h_{\text{blend}} &= 40 & z_{0\text{MWMO}} &= 0.03 & F_{\text{blend}} &= (u_{\text{blend}}^2 + v_{\text{blend}}^2)^{1/2} & \text{if } z_{0M} > 0.03 \\ h_{\text{blend}} &= z_n & z_{0\text{MWMO}} &= z_{0M} & F_{\text{blend}} &= (u_n^2 + v_n^2)^{1/2} & \text{if } z_{0M} < 0.03 \end{aligned} \quad (3.90)$$

$$F_{10} = F_{\text{blend}} \frac{\log\left(\frac{z_{10} + z_{0\text{MWMO}}}{z_{0\text{MWMO}}}\right) - \Psi_M\left(\frac{z_{10} + z_{0\text{MWMO}}}{\mathcal{L}}\right) + \Psi_M\left(\frac{z_{0\text{MWMO}}}{\mathcal{L}}\right)}{\log\left(\frac{z_{\text{blend}} + z_{0\text{MWMO}}}{z_{0\text{MWMO}}}\right) - \Psi_M\left(\frac{z_{\text{blend}} + z_{0\text{MWMO}}}{\mathcal{L}}\right) + \Psi_M\left(\frac{z_{0\text{MWMO}}}{\mathcal{L}}\right)} \quad (3.91)$$

where  $z_{10} = 10$  m,  $F_{\text{blend}}$  is the horizontal wind speed at the blending height either interpolated from model levels to 75 m or copied from the lowest model level, and  $F_{10}$  is the resulting horizontal wind speed at 10 m.

The 10-metre equivalent-neutral wind is calculated as:

$$F_{10}^n = F_{\text{blend}} \frac{\log\left(\frac{z_{10} + z_{0\text{MWMO}}}{z_{0\text{MWMO}}}\right)}{\log\left(\frac{z_{\text{blend}} + z_{0\text{MWMO}}}{z_{0\text{MWMO}}}\right) - \Psi_M\left(\frac{z_{\text{blend}} + z_{0\text{MWMO}}}{\mathcal{L}}\right) + \Psi_M\left(\frac{z_{0\text{MWMO}}}{\mathcal{L}}\right)}, \quad (3.92)$$

and the friction velocity  $u_*$  as:

$$u_* = F_{\text{blend}} \frac{\kappa}{\log\left(\frac{z_{\text{blend}} + z_{0\text{MWMO}}}{z_{0\text{MWMO}}}\right) - \Psi_M\left(\frac{z_{\text{blend}} + z_{0\text{MWMO}}}{\mathcal{L}}\right) + \Psi_M\left(\frac{z_{0\text{MWMO}}}{\mathcal{L}}\right)}. \quad (3.93)$$

The wind speed from (3.91, 3.92) is converted to components by making use of the wind direction from the lowest model level.

### 3.10.3 Temperature and humidity at the 2 m level

Computation of temperature and moisture at the 2 m level is based on interpolation between the lowest model level and the surface making use of the same profile functions as in the parametrization of the surface fluxes. The expressions derived from equations (3.11) and (3.12) are

$$s_2 = s_{\text{surf}} + (s_n - s_{\text{surf}}) \frac{\log\left(\frac{z_2 + z_{0\text{MWMO}}}{z_{0\text{HWMO}}}\right) - \Psi_H\left(\frac{z_2 + z_{0\text{MWMO}}}{\mathcal{L}}\right) + \Psi_H\left(\frac{z_{0\text{HWMO}}}{\mathcal{L}}\right)}{\log\left(\frac{z_n + z_{0\text{MWMO}}}{z_{0\text{HWMO}}}\right) - \Psi_H\left(\frac{z_n + z_{0\text{MWMO}}}{\mathcal{L}}\right) + \Psi_H\left(\frac{z_{0\text{HWMO}}}{\mathcal{L}}\right)} \quad (3.94)$$

$$q_2 = q_{\text{surf}} + (q_n - q_{\text{surf}}) \frac{\log\left(\frac{z_2 + z_{0\text{MWMO}}}{z_{0\text{QWMO}}}\right) - \Psi_H\left(\frac{z_2 + z_{0\text{MWMO}}}{\mathcal{L}}\right) + \Psi_H\left(\frac{z_{0\text{QWMO}}}{\mathcal{L}}\right)}{\log\left(\frac{z_n + z_{0\text{MWMO}}}{z_{0\text{QWMO}}}\right) - \Psi_H\left(\frac{z_n + z_{0\text{MWMO}}}{\mathcal{L}}\right) + \Psi_H\left(\frac{z_{0\text{QWMO}}}{\mathcal{L}}\right)} \quad (3.95)$$

with  $z_2 = 2$  m,  $z_{0\text{HWMO}} = z_{0\text{QWMO}} = 0.003$  if  $z_{0M} > 0.03$ , and otherwise  $z_{0\text{HWMO}} = z_{0H}$  and  $z_{0\text{QWMO}} = z_{0Q}$ . Temperature  $T_2$  is derived from  $s_2$  with (3.5). Also the dew point is computed from  $q_2$  and surface pressure. The dew point uses the saturation formulation with respect to water to be consistent with WMO reporting practise. If the resulting dew point is lower than temperature  $T_2$ , the dew point is set equal to temperature.

### 3.10.4 Wind gusts

The computation of gusts is intended to be compatible with WMO observing practise for wind extremes. In order to get uniform observations, WMO defines a wind gust as the maximum of the wind averaged over 3 second intervals.

To simulate gusts, the standard deviation of the horizontal wind is estimated on the basis of the similarity relation by Panofsky *et al.* (1977)

$$\begin{aligned} \sigma_u &= 2.29u_* \left(1 - \frac{0.5}{12} \frac{z_i}{\mathcal{L}}\right)^{1/3} & \text{for } \mathcal{L} < 0 \\ \sigma_u &= 2.29u_* & \text{for } \mathcal{L} > 0 \end{aligned} \quad (3.96)$$

with  $z_i = 1000$  m. The difference between the gust and  $F_{10}$  is proportional to  $\sigma_u$ , where the multiplier has been determined from universal turbulence spectra for a 50% exceeding probability of the three-second wind gust (see [Beljaars, 1987](#)). The resulting wind gust is

$$F_{\text{gust}} = F_{10} + C_{\text{ugn}} u_* \quad (3.97)$$

with parameter  $C_{\text{ugn}} = 7.71$  and  $u_*$  from the surface stress as computed in the vertical turbulent transport code.

From the controlling parameters it is clear that the effects of surface friction (through surface roughness) and stability are captured. However, the approach is not adequate for gusts in baroclinic situations and where gusts are due to strong convective events. Therefore, in the presence of deep convection, a convective contribution as a function of the vertical wind shear ([Bechtold and Bidlot, 2009](#)) is added to the turbulence gustiness (3.97) so that the total gustiness becomes

$$F_{\text{gust}} = F_{10} + C_{\text{ugn}} u_* + C_{\text{conv}} \max(0, U_{850} - U_{950}) \quad (3.98)$$

where the convective mixing parameter  $C_{\text{conv}} = 0.6$ , and  $U_{850}$  and  $U_{950}$  are the wind speeds at 850 and 950 hPa, respectively. Parameter  $F_{\text{gust}}$  is computed every time step and its maximum since the last post-processing is written out for archiving.

### 3.11 CODE

Vertical turbulent transports, which affect temperature, velocities and specific humidity, is performed in subroutine **VDFMAIN** called by **VDFOUTER** which, in turn, is called by **CALLPAR**. **VDFOUTER** calls **VDFMAIN** two times with 1/2 of the normal time step (these two routines have identical arguments so **VDFMAIN** can be called directly if the vertical turbulent transports is only needed once per time step). All the routines for the surface energy balance and for the interaction between the lowest model level and the surface, are in the SURF library. Transfer of information between the IFS and the SURF library is limited to a few interaction routines. The other SURF routines can not be called by the IFS.

At the start of the model integration the following setup routines are called to initialize modules specific to the vertical transport code:

- **SUVDF**. Setup routine for a number of parametrization constants.
- **SUVDFS**. Setup routine for constants and tables related to the stability functions. Stability functions are included as statement functions from `fcvds.h`.
- **SUSVEG**. Is part of the SURF library and sets a number of vegetation and tile parameters.

The main subroutine (**VDFMAIN**) does a sequence of computations and subroutine calls:

- **SURFEXCDRIVER**. This is the first call to the SURF library to prepare all the necessary parameters for exchange with the surface. The tiled surface fluxes and tiled skin temperatures are cycled from time step to time step (fluxes are needed for the first guess of stability parameters), but are not available at the start of the forecast. For the first time step, neutral transfer coefficients are used to estimate momentum fluxes, the tiled skin temperatures are set equal to the grid box averaged skin temperature from the initial condition, and the sensible and latent heat fluxes needed as a first guess for the Obukhov length computation are set to zero. **SURFEXCDRIVER** calls a number of subroutines from the SURF library.
  - **VUPDZ0**. This routine computes roughness lengths for momentum, heat and moisture over ocean surfaces according to (3.26). It also computes surface buoyancy flux and Obukhov length from the fluxes of the previous time level.
  - **VSURF**. This routine prepares the surface boundary conditions for temperature and humidity and is called for every tile. The following quantities are computed: the surface specific humidity at saturation, the derivative of the saturation humidity curve at the surface, surface dry static energy, and vegetation stomatal resistances (see [Chapter 8](#)).

- **VEXCS**. This routine determines the transfer coefficients between the surface and the lowest model level with the thermal stability expressed as function of the Obukhov length. It is called for every tile. The implicit relation between  $z/\mathcal{L}$  and the Richardson number  $Ri_{\text{bulk}}$  is solved iteratively (using the Newton method with the derivative approximated in finite differences). Pre-computed tables defined in subroutine **SUVDFS** are used to obtain the first guess in stable conditions ( $Ri > 0$ ) at the first time step. Transfer coefficients are multiplied by a constant factor  $\alpha \rho g \frac{\Delta t}{\Delta z}$ .
- **VEVAP**. This routine computes for each tile the equivalent evapo-transpiration efficiency and the corresponding parameters  $a_n$  and  $a_{\text{surf}}$  defined by the land surface scheme (see chapter 8). Dry static energy at the surface at time level  $t$  is estimated as well.
- **VSFLX**. This routine computes surface fluxes for each tile (heat flux, evaporation, momentum flux and virtual heat flux) at time  $t$  for later use in similarity functions and for the estimation of the diagnostic boundary layer depth.
- **CLOUDVAR**. This routine calculates the variance of total water from the humidity and cloud water using a symmetrical beta distribution. This value of total water variance is used as input value for the vertical turbulent transport.
- **COVER**. This routine diagnoses cloud fraction, water vapor, cloud liquid water and cloud ice from total water, total water variance and temperature. This first call diagnoses input reference values of cloud parameters.
- **VDFDPBL**. This routine diagnoses the boundary layer height for time level  $t$ . This boundary layer height is for post-processing only and is not used by the parametrization.
- **VDFHGTN**. This routine uses an entraining plume model to determine updraught properties, mass-flux, cloud base and PBL top.
- **VDFEXCU**. This routine determines the turbulent diffusion coefficients between the model levels above the surface layer.
- **VDFTOCD**. This routine computes coefficients for turbulent orographic form drag.
- **VDFDIFM**. This routine solves the diffusion equation for momentum, by Gaussian elimination of the tridiagonal matrices.
- **VDFDIFH**. This routine solves the EDMF equations for total water and liquid water static energy. A downward elimination scan is done through the tridiagonal matrices, and coefficients  $A_s$ ,  $B_s$ ,  $A_q$ , and  $B_q$  are computed. Then, a call is made to SURF routine **SURFSEB** to compute the surface fluxes for heat and moisture. Also the tiled fluxes are returned.
- **VDFDIFC**. Routine solves the diffusion equations for passive tracers. A specified flux at the surface is used as a boundary condition.
- **VDFINCR**. This routine computes the tendencies of the prognostic variables and estimates the kinetic energy dissipation.
- **VDFBLEND**. This routine computes the blending height.
- **SURFPP**. This is the routine from the SURF library for the interpolation of SYNOP parameters. It calls:
  - **SPPCFL**. This routine computes the surface 2 metre temperature and humidity (dew point and specific humidity), the wind at 10 m, the equivalent-neutral wind at 10m, and the friction velocity.
  - **SPPGUST**. This routine computes wind gusts as they are typically observed by standard WMO SYNOP stations.
- **COVER**. This routine diagnoses cloud fraction, water vapor, cloud liquid water and cloud ice from total water, total water variance and temperature. This second call diagnoses final values of cloud variables. Differences of final and initial values provide the output tendencies.

## APPENDIX A. LIST OF SYMBOLS

$a_u$	updraught fraction
$C_H$	transfer coefficient for heat
$C_M$	transfer coefficient for momentum (drag coefficient)
$C_Q$	transfer coefficient for moisture

$c_p$	specific heat at constant pressure of moist air
$E_{\text{diss}}$	kinetic energy lost by the diffusion process
$f$	Coriolis parameter
$F_{\text{blend}}$	horizontal wind speed at blending height (for pp of 10 m wind)
$F_{10}$	horizontal wind speed at 10 m level (for pp)
$g$	acceleration of gravity
$h_{\text{BL}}$	diagnosed boundary layer height
$h_{\text{blend}}$	blending height (for pp of 10 m wind)
$J_\phi$	vertical turbulent flux of $\phi$
$J_q$	surface humidity flux
$J_s$	surface flux of dry static energy
$J_M$	surface momentum flux
$K_\phi$	turbulent exchange coefficient for $\phi$
$K_H$	turbulent exchange coefficient for heat
$K_M$	turbulent exchange coefficient for momentum
$K_Q$	turbulent exchange coefficient for moisture
$\mathcal{L}$	Obukhov length
$L_c$	latent heat of condensation
$L_d$	latent heat of deposition
$l_H$	mixing length for heat
$l_M$	mixing length for momentum
$M$	mass flux
$N_T$	number of tiles
$p$	pressure
$Pr$	Prandtl number
$q$	specific humidity
$q_l$	specific liquid water
$q_i$	specific ice water
$q_t$	specific total water = $q + q_l + q_i$
$q_{\text{sat}}$	saturated specific humidity
$q_*$	= $J_q / (\rho u_*)$
$\Delta R$	radiative flux jump at cloud top
$Q_{0v}$	virtual temperature flux in the surface layer
$R_{\text{dry}}$	gas constant for dry air
$R_{\text{vap}}$	gas constant for water vapour
$\mathfrak{R}_{\text{LW}}$	net long-wave radiation at the surface
$\mathfrak{R}_{\text{SW}}$	net short-wave radiation at the surface
$RH_{\text{surf}}$	relative humidity at the surface
$Ri$	local Richardson number
$Ri_{\text{bulk}}$	bulk Richardson number for the surface layer
$s$	dry static energy
$s_l$	generalized liquid water static energy
$s_v$	virtual dry static energy
$s_*$	= $J_s / (\rho u_*)$
$T$	temperature
$t$	time
$ U $	horizontal wind speed
$u, v$	horizontal wind components
$u_*$	friction velocity = $(J_M / \rho)^{1/2}$
$w_*$	free convection velocity scale
$z_{0M}$	roughness length for momentum (aerodynamic roughness length)
$z_{0H}$	roughness length for heat
$z_{0Q}$	roughness length for moisture
$z_i$	scale height of the boundary layer
$z_n$	height of the lowest model level $n$

$z_{0\text{MWMO}}$	roughness length for momentum at SYNOP station
$z_{0\text{HWMO}}$	roughness length for heat at SYNOP station
$z_{0\text{QWMO}}$	roughness length for moisture at SYNOP station
$z_2$	height of screen level observation (2 m)
$z_{10}$	height of surface wind observation (10 m)
$\alpha$	implicitness factor for diffusion equation
$\alpha_{\text{Ch}}$	Charnock parameter
$\beta$	scaling parameter for asymptotic mixing length
$\Delta t$	time step
$\Delta z$	vertical grid length
$\varepsilon$	$= (R_{\text{vap}}/R_{\text{dry}}) - 1$
$\varepsilon$	parcel entrainment
$\theta_v$	virtual potential temperature
$\kappa$	Von Kármán's constant
$\lambda$	asymptotic mixing length
$\Lambda_{\text{skin}}$	conductivity of
$\nu$	kinematic viscosity
$\rho$	density
$\sigma_{\text{qt}}$	standard deviation of total water
$\sigma_{\text{u}}$	standard deviation of horizontal wind
$\sigma_{\text{w}}$	standard deviation of vertical wind
$\zeta$	$= z/\mathcal{L}$
$\phi$	symbolic reference to a conservative quantity
$\Phi_{\text{M}}$	universal gradient stability function for wind
$\Phi_{\text{H}}$	universal gradient stability function for temperature
$\Phi_{\text{Q}}$	universal gradient stability function for moisture
$\Psi_{\text{M}}$	universal profile stability function for wind
$\Psi_{\text{H}}$	universal profile stability function for temperature
$\Psi_{\text{Q}}$	universal profile stability function for moisture

## Subscripts:

$i$	tile index
$k$	level index (counted from model top downwards)
$n$	referring to lowest model level
skin	referring to the skin layer
surf	referring to the surface
u	referring to the updraught
e	referring to the environment

## Superscripts:

$t$	index for old time level, indicating beginning of time step
$t + 1$	index for new time level, indicating end of time step
trad	index referring to the latest full radiation time step

## Special symbols:

$\hat{\phi}$	implicit variable $\phi$ defined by equation (3.62)
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## Chapter 4

# Subgrid-scale orographic drag

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### Appendix A. List of symbols

## 4.1 GENERAL PRINCIPLES

The subgrid-scale orography intersects model levels, and consequently influences the momentum of the atmosphere, and hence other parts of the physics. In the model stably stratified flow over the subgrid-scale orography creates drag through a combination of low-level flow blocking (i.e. blocked-flow drag) and the absorption and/or reflection of vertically propagating gravity waves (i.e. gravity-wave drag). The parameterisation scheme is described in detail in [Lott and Miller \(1997\)](#).

The scheme is based on ideas presented by [Baines and Palmer \(1990\)](#), combined with ideas from bluff-body dynamics. The assumption is that the mesoscale-flow dynamics can be described by two conceptual models, whose relevance depends on the non-dimensional height of the mountain via

$$H_n = \frac{NH}{|U|} \quad (4.1)$$

where  $H$  is the maximum height of the obstacle,  $U$  is the wind speed and  $N$  is the Brunt–Väisälä frequency of the incident flow.

At small  $H_n$  most of the flow goes over the mountain and gravity waves are forced by the vertical motion of the fluid. Suppose that the mountain has an elliptical shape and a height variation determined by a parameter  $b$  in the along-ridge direction and by a parameter  $a$  in the cross-ridge direction, such that the mountain anisotropy  $\gamma = a/b \leq 1$ , then the geometry of the mountain can be written in the form

$$h(x, y) = \frac{H}{1 + x^2/a^2 + y^2/b^2} \quad (4.2)$$

In the simple case when the incident flow is at right angles to the ridge the surface stress due to the gravity wave has the magnitude

$$\tau_{\text{wave}} = \rho_0 b G B(\gamma) N U H^2 \quad (4.3)$$

provided that the Boussinesq and hydrostatic approximations apply. In (4.3)  $G$  is a function of the mountain sharpness ([Phillips, 1984](#)), and for the mountain given by (4.2),  $G \approx 1.23$ . The term  $B(\gamma)$  is a function of  $\gamma$ , and can vary from  $B(0) = 1$  for a two-dimensional ridge to  $B(1) = \pi/4$  for a circular mountain.

At large  $H_n$ , the vertical motion of the fluid is limited and part of the low-level flow is blocked and goes around the mountain. The depth,  $Z_{\text{blk}}$ , of this blocked layer, when  $U$  and  $N$  are independent of height, can be expressed as

$$Z_{\text{blk}} = H \times \max\left(0, \frac{H_n - H_{n_{\text{crit}}}}{H_n}\right) \quad (4.4)$$

where  $H_{n_{\text{crit}}}$  is a critical non-dimensional mountain height of order unity. The depth  $Z_{\text{blk}}$  can be viewed as the upstream elevation of the isentropic surface that is raised exactly to the mountain top. At each level below  $Z_{\text{blk}}$  the flow streamlines divide around the obstacle, and it is supposed that flow separation occurs on the obstacle's flanks. Then, the drag,  $D_{\text{blk}}(z)$ , exerted by the obstacle on the flow at these levels can be written as

$$D_{\text{blk}}(z) = -\rho_0 C_d l(z) \frac{U|U|}{2} \quad (4.5)$$

Here  $l(z)$  represents the horizontal width of the obstacle as seen by the flow at an upstream height  $z < Z_{\text{blk}}$ , and for an elliptical mountain is given by,

$$l(z) = 2b \left( \frac{Z_{\text{blk}} - z}{z} \right)^{1/2} \quad (4.6)$$

$C_d$  represents the drag coefficient, which according to the free streamline theory of jets in ideal fluids is a constant having a value close to 1 (Kirchoff, 1876; Gurevitch, 1965). However, observations show  $C_d$  can be nearer 2 in value when suction effects occur in the rear of the obstacle (Batchelor, 1967). Here, this drag is applied to the flow, level by level, and will be referred to as the 'blocked-flow drag',  $D_{\text{blk}}$ . Unlike the gravity-wave drag computation (4.3), the total stress exerted by the mountain on the 'blocked' flow does not need to be known *a priori*.

In (4.6), it is assumed that the level  $Z_{\text{blk}}$  is raised up to the mountain top, with each layer below  $Z_{\text{blk}}$  raised by a factor  $H/Z_{\text{blk}}$ . This leads, effectively, to a reduction of the obstacle width, as seen by the flow when compared with the case in which the flow does not experience vertical motion as it approaches the mountain. Then applying (4.5) to the fluid layers below  $Z_{\text{blk}}$ , the stress due to the blocked-flow drag is obtained by integrating from  $z = 0$  to  $z = Z_{\text{blk}}$ , viz.

$$\tau_{\text{blk}} \approx C_d \pi b \rho_0 Z_{\text{blk}} \frac{U|U|}{2} \quad (4.7)$$

Moreover, the blocked layer results in a reduction of the mountain height which produces gravity waves, meaning the mountain height used in (4.3) is replaced with a lower effective (or cut-off) mountain height, i.e.

$$H_{\text{eff}} = 2(H - Z_{\text{blk}}) \quad (4.8)$$

The factor 2 was added later (in Cy32r2) because diagnostics indicated that without the factor 2, the gravity wave activity was too weak. In the present scheme the value of  $C_d$  is allowed to vary with the aspect ratio of the obstacle, as in the case of separated flows around immersed bodies (Landweber, 1961), while at the same time setting the critical number  $H_{n_{\text{crit}}}$  equal to 0.5 as a constant intermediate value.

## 4.2 DESCRIPTION OF THE SCHEME

Following Baines and Palmer (1990), the subgrid-scale orography over one grid-point region is represented by four parameters  $\mu$ ,  $\gamma$ ,  $\sigma$  and  $\theta$  which stand for the standard deviation, the anisotropy, the slope and the geographical orientation of the orography, respectively. These four parameters have been calculated from the GTOPO30 data set at 30'' (about 1000 m) resolution (Gesch and Larson, 1998), averaged to 2'30'' resolution so as to remove scales less than 5000m. At sub-grid horizontal scales less than 5000 m, small surface obstacles generate additional turbulence or turbulent orographic form drag (TOFD).

The scheme uses values of wind velocity,  $U_H$ , Brunt-Väisälä frequency,  $N_H$ , and fluid density,  $\rho_H$ , which are evaluated by averaging between  $\mu$  and  $2\mu$  above the model mean orography, i.e. representative of flow incident to the subgrid-scale orography. Following Wallace *et al.* (1983),  $2\mu$  is interpreted as the envelope

of the subgrid-scale mountain peaks above the model orography. The evaluation of the blocking height  $Z_{\text{blk}}$  is based on a very simple interpretation of the non-dimensional mountain height  $H_n$ . To first order in the mountain amplitude, the obstacle excites a wave, and the sign of the vertical displacement of a fluid parcel is controlled by the wave phase. If a fluid parcel ascends the upstream mountain flank over a height large enough to significantly modify the wave phase, its vertical displacement can become zero, and it will not cross the mountain summit. In this case  $Z_{\text{blk}}$  is the highest level located below the mountain top for which the phase change between  $Z_{\text{blk}}$  and the mountain top exceeds a critical value  $H_{n_{\text{crit}}}$ , i.e.

$$\int_{Z_{\text{blk}}}^{3\mu} \frac{N}{U_p} dz \geq H_{n_{\text{crit}}} \quad (4.9)$$

In the inequality (4.9), the wind speed,  $U_p(z)$ , is calculated by resolving the wind,  $U(z)$ , in the direction of the flow  $U_H$ . Then, if the flow veers or backs with height, (4.9) will be satisfied when the flow becomes normal to  $U_H$ . Levels below this ‘critical’ altitude define the low-level blocked flow. The inequality (4.9) will also be satisfied below inversion layers, where the parameter  $N$  is very large. These two properties allow the new parametrization scheme to mimic the vortex shedding observed when pronounced inversions occur (Etling, 1989). The upper limit in the equality (4.9) was chosen to be  $3\mu$ , which is above the subgrid-scale mountain tops. This ensures that the integration in equality (4.9) does not lead to an underestimation of  $Z_{\text{blk}}$ , which can occur because of the limited vertical resolution when using  $2\mu$  as an upper limit (a better representation of the peak height), but this upper limit could be relaxed given better vertical resolution.

In the following subsection the drag amplitudes will be estimated combining formulae valid for elliptical mountains with real orographic data. Considerable simplifications are implied and the calculations are, virtually, scale analyses relating the various amplitudes to the sub-grid parameters.

#### 4.2.1 Blocked-flow drag

Within a given layer located below the blocking level  $Z_{\text{blk}}$ , the drag is given by (4.5). At a given altitude  $z$ , the intersection between the mountain and the layer approximates to an ellipse with eccentricity of

$$(a', b') \approx (a, b) \left( \frac{Z_{\text{blk}} - z}{z + \mu} \right)^{\frac{1}{2}} \quad (4.10)$$

where, by comparison with (4.6), it is also supposed that the level  $z = 0$  (i.e. the model mean orography) is at an altitude  $\mu$  above the mountain valleys. If the flow direction is taken into account, the length  $l(z)$  can be written approximately as

$$l(z) \approx 2 \max(b \cos \psi, a \sin \psi) \left( \frac{Z_{\text{blk}} - z}{z + \mu} \right)^{\frac{1}{2}} \quad (4.11)$$

where  $\psi$  is the angle between the incident flow direction  $\theta$ . For one grid-point region and for uniformly distributed subgrid-scale orography, the incident flow encounters  $L/(2a)$  obstacles is normal to the ridge ( $\psi = 0$ ), whereas if it is parallel to the ridge ( $\psi = \pi/2$ ) it encounters  $L/(2b)$  obstacles, where  $L$  is the length scale of the grid-point region. If we sum up these contributions, the dependence of (4.11) on  $a$  and  $b$  can be neglected, and the length  $l(z)$  becomes

$$l(z) = L \left( \frac{Z_{\text{blk}} - z}{z + \mu} \right)^{\frac{1}{2}} \quad (4.12)$$

Furthermore, the number of consecutive ridges (i.e. located one after the other in the direction of the flow) depends on the obstacle shape: there are approximately  $L/(2b)$  successive obstacles when the flow is along the ridge, and  $L/(2a)$  when it is normal to the ridge. If we take this into account, together with the flow direction, then

$$l(z) = \frac{L^2}{2} \left( \frac{Z_{\text{blk}} - z}{z + \mu} \right)^{\frac{1}{2}} \max \left( \frac{\cos \psi}{a}, \frac{\sin \psi}{b} \right) \quad (4.13)$$

Relating the parameters  $a$  and  $b$  to the subgrid-scale orography parameters  $a \approx \mu/\sigma$  and  $a/b \approx \gamma$  and, allowing the drag coefficient to vary with the aspect ratio of the obstacle as seen by the incident flow, we

have

$$r = \frac{\cos^2 \psi + \gamma \sin^2 \psi}{\gamma \cos^2 \psi + \sin^2 \psi} \quad (4.14)$$

and the drag per unit area and per unit height can be written as

$$D_{\text{blk}}(z) = -C_d \max\left(2 - \frac{1}{r}, 0\right) \rho \frac{\sigma}{2\mu} \left(\frac{Z_{\text{blk}} - z}{z + \mu}\right)^{\frac{1}{2}} \max(\cos \psi, \gamma \sin \psi) \frac{U|U|}{2} \quad (4.15)$$

The drag coefficient is modulated by the aspect ratio of the obstacle to account for the fact that  $C_d$  is twice as large for flow normal to an elongated obstacle as it is for flow round an isotropic obstacle. The drag tends to zero when the flow is nearly along a long ridge because flow separation is not expected to occur for a configuration of that kind. It can be shown that the term  $\max(\cos \psi, \gamma \sin \psi)$  is similar to a later form used for the directional dependence of the gravity-wave stress. For simplicity, this later form has been adopted, so that

$$D_{\text{blk}}(z) = C_d \max\left(2 - \frac{1}{r}, 0\right) \rho \frac{\sigma}{2\mu} \left(\frac{Z_{\text{blk}} - z}{z + \mu}\right)^{\frac{1}{2}} (B \cos^2 \psi + C \sin^2 \psi) \frac{U|U|}{2} \quad (4.16)$$

with the constants (Phillips, 1984)

$$B = 1 - 0.18\gamma - 0.04\gamma^2, \quad C = 0.48\gamma + 0.3\gamma^2, \quad C_d = 2 \quad (4.17)$$

The difference between (4.15) and (4.16) has been shown to have only a negligible impact on all aspects of the model's behaviour.

In practice, (4.16) is suitably resolved and applied to the component from of the horizontal momentum equations. This equation is applied level by level below  $Z_{\text{blk}}$  and, to ensure numerical stability, a quasi-implicit treatment is adopted whereby the wind velocity  $U$  in (4.16) is evaluated at the updated time  $t + dt$ , while the wind amplitude,  $|U|$ , is evaluated at the previous time step.

#### 4.2.2 Gravity-wave drag

This gravity-wave part of the scheme is based on the work of Miller *et al.* (1989) and Baines and Palmer (1990), and takes into account some three-dimensional effects in the wave stress amplitude and orientation. For clarity and convenience, a brief description is given here. On the assumption that the subgrid-scale orography has the shape of one single elliptical mountain, the gravity wave surface stress (4.3) can be written as (Phillips, 1984)

$$(\tau_1, \tau_2) = \rho_H U_H N_H H_{\text{eff}}^2 b G (B \cos^2 \psi_H + C \sin^2 \psi_H, (B - C) \sin \psi \cos \psi_H) \quad (4.18)$$

where  $\psi_H$  is the mean value between  $z = \mu$  and  $z = 2\mu$ . Furthermore, when  $b$  or  $a$  are significantly smaller than the length  $L$ , characteristic of the gridpoint region size, there are, typically,  $L^2/(4ab)$  ridges inside the grid-point region. Summing all the associated forces we find the stress per unit area, viz.

$$(\tau_1, \tau_2) = \rho_H U_H N_H (H_{\text{eff}}^2/4)(\sigma/\mu) G \{B \cos^2 \psi_H + C \sin^2 \psi_H, (B - C) \sin \psi_H \cos \psi_H\} \quad (4.19)$$

where  $a$  has been replaced by  $\mu/\sigma$ .

It is worth noting that, since the basic parameters  $\rho_H$ ,  $U_H$ ,  $N_H$  are evaluated for the layer between  $\mu$  and  $2\mu$  above the mean orography that defines the model's lower boundary, there will be much less diurnal cycle in the stress than in previous formulations that used the lowest model levels for this evaluation. The vertical distribution of the gravity-wave stress will determine the levels at which the waves break and slow down the synoptic flow. Since this part of the scheme is active only above the blocked flow, this stress is now constant from the bottom model level to  $Z_{\text{blk}}$ . Above this height, up to the top of the model, the stress is constant until the waves break (by the top of the model the gravity wave must have broken completely). This occurs when the total Richardson number,  $Ri$ , falls below a critical value  $Ri_{\text{crit}}$ , which is of order unity. When the non-dimensional mountain height is close to unity, this algorithm will usually predict wave breaking at relatively low levels (i.e. immediately above the 'blocked' layer).

This is not surprising since the linear theory of mountain gravity waves predicts low-level breaking waves at large non-dimensional mountain heights (Miles and Huppert, 1969). In reality, the depth over which gravity-wave breaking occurs is more likely to be related to the vertical wavelength of the waves. For this reason, when low-level wave breaking occurs in the scheme, the corresponding drag is distributed (above the blocked flow), over a layer of thickness  $\Delta z$ , equal to a quarter of the vertical wavelengths of the waves, i.e.

$$\int_{Z_{\text{blk}}}^{Z_{\text{blk}}+\Delta z} \frac{N}{U_p} dz \approx \frac{\pi}{2} \quad (4.20)$$

Above the height  $Z_{\text{blk}} + \Delta z$  are waves with an amplitude such that  $Ri > Ri_{\text{crit}}$ . The remaining part of the momentum flux above 9.9 Pa is spread between this level and the top of the model.

### 4.3 SPECIFICATION OF SUBGRID-SCALE OROGRAPHY

For completeness, the following describes how the subgrid-scale orography fields were computed by Baines and Palmer (1990). The mean topographic height above mean sea level over the grid-point region is denoted by  $\bar{h}$ , and the coordinate  $z$  denotes elevation above this level. Then the topography relative to this height  $h(x, y) - \bar{h}$  is represented by four parameters, as follows.

- (i) The net variance, or standard deviation,  $\mu$ , of  $h(x, y)$  in the grid-point region. This gives a measure of the amplitude and  $2\mu$  approximates the physical envelope of the peaks.
- (ii) A parameter  $\gamma$  which characterizes the anisotropy of the topography within the grid-point region.
- (iii) An angle  $\psi$ , which denotes the angle between the direction of the low-level wind and that of the principal axis of the topography.
- (iv) A parameter  $\sigma$  which represents the mean slope within the grid-point region.

The parameters  $\gamma$  and  $\psi$  may be defined from the topographic gradient correlation tensor

$$H_{ij} = \frac{\partial h}{\partial x_i} \frac{\partial h}{\partial x_j}$$

where  $x_1 = x$ , and  $x_2 = y$ , and where the terms be calculated (from the USN data-set) by using all relevant pairs of adjacent gridpoints within the grid-point region. This symmetric tensor may be diagonalized to find the directions of the principal axes and the degree of anisotropy. If

$$K = \frac{1}{2} \left\{ \overline{\left( \frac{\partial h}{\partial x} \right)^2} + \overline{\left( \frac{\partial h}{\partial y} \right)^2} \right\}, \quad L = \frac{1}{2} \left\{ \overline{\left( \frac{\partial h}{\partial x} \right)^2} - \overline{\left( \frac{\partial h}{\partial y} \right)^2} \right\} \quad \text{and} \quad M = \overline{\frac{\partial h}{\partial x} \frac{\partial h}{\partial y}} \quad (4.21)$$

the principal axis of  $H_{ij}$  is oriented at an angle  $\theta$  to the  $x$ -axis, where  $\theta$  is given by

$$\begin{aligned} \theta &= \frac{1}{2} \arctan(M/L) \quad \text{if} \quad L > 0 \\ \theta &= \frac{1}{2} \arctan(M/L) + \pi/2 \quad \text{if} \quad L < 0 \text{ and } M > 0 \\ \theta &= \frac{1}{2} \arctan(M/L) - \pi/2 \quad \text{if} \quad L < 0 \text{ and } M < 0 \end{aligned} \quad (4.22)$$

This gives the direction where the topographic variations, as measured by the mean-square gradient, are largest. The corresponding direction for minimum variation is at right angles to this. Changing coordinates to  $x', y'$  which are oriented along the principal axes  $x' = x \cos \theta + y \sin \theta$  and  $y' = y \cos \theta - x \sin \theta$ , the new values of  $K$ ,  $L$ , and  $M$  relative to these axes, denoted  $K'$ ,  $L'$  and  $M'$ , are given by

$$K' = K, \quad L' = (L^2 + M^2)^{\frac{1}{2}} \quad \text{and} \quad M' = 0$$

where  $K$ ,  $L$  and  $M$  are given by (4.21). The anisotropy of the orography or ‘aspect ratio’.  $\gamma$  is then defined by the equations

$$\gamma^2 = \frac{\overline{\left( \frac{\partial h}{\partial y'} \right)^2}}{\overline{\left( \frac{\partial h}{\partial x'} \right)^2}} = \frac{K' - L'}{K' + L'} = \frac{K - (L^2 + M^2)^{1/2}}{K + (L^2 + M^2)^{1/2}} \quad (4.23)$$

If the low-level wind vector is directed at an angle  $\varphi$  to the  $x$ -axis, then the angle  $\psi$  is given by

$$\psi = \theta - \varphi \quad (4.24)$$

The slope parameter,  $\sigma$ , is defined as

$$\sigma^2 = \overline{\left(\frac{\partial h}{\partial x'}\right)^2} \quad (4.25)$$

which is the mean-square gradient along the principal axis.

## 4.4 CODE

The principal routine is **GWDRAG**, which is called from **CALLPAR**. **GWDRAG** first calls **GWSETUP** to define all the basic input values required for the evaluation of the blocking drag and gravity wave stress. It then computes the surface gravity wave stress and calls **GWPROFIL** to calculate its vertical distribution. **GWDRAG** then computes the momentum tendency coefficients (rather than the actual momentum tendencies). The coefficients are passed back to **CALLPAR** where they are solved in the vertical diffusion scheme as a joint implicit calculation with TOFD momentum tendency coefficients. The joint implicit calculation introduces some degree of dependency into these coupled processes, and so reduces the time step sensitivity which would have existed if each scheme had evaluated its tendencies independently (Beljaars *et al.*, 2004b).

### 4.4.1 GWSETUP

This routine defines various reference model levels for controlling the vertical structure of the calculations, and sets up a number of derived atmospheric variables and geometric calculations required to run the scheme.

- (i) The definition of the Brunt–Väisälä frequency on half levels

$$N_{k-1/2}^2 = \frac{2g^2}{c_{p\text{dry}}(T_k + T_{k-1})} \left\{ 1 - c_{p\text{dry}} \rho_{k-1/2} \frac{(T_k - T_{k-1})}{(p_k - p_{k-1})} \right\} \quad (4.26)$$

- (ii) The definition of the mean wind components in the layer  $\mu < z < 2\mu$

$$U_{\text{LOW}} = \frac{\sum_{k=\mu}^{k=2\mu} U_k \Delta p_k}{\sum_{k=\mu}^{k=2\mu} \Delta p_k} \quad (4.27)$$

and similarly for  $V_{\text{LOW}}$ ; likewise the mean static stability,  $N_{\text{LOW}}$ , and the mean density,  $\rho_{\text{LOW}}$  are calculated.

- (iii) The calculation of necessary geometry pertaining to geographical orientation of subgridscale orography and wind direction,

$$\varphi_k = \tan^{-1} \left( \frac{V_k}{U_k} \right) \quad (4.28)$$

$$\psi_k = \theta - \varphi_k \quad (4.29)$$

$$\bar{\varphi} = \tan^{-1} \left( \frac{V_{\text{LOW}}}{U_{\text{LOW}}} \right) \quad (4.30)$$

and  $\bar{\psi} = \theta - \bar{\varphi}$ . Also computed are the parameters  $B$  and  $C$  (4.17).

- (iv) The calculation of the vertical wind-profile in the plane of the gravity wave stress. Defining

$$\hat{U}_k = \frac{U_{\text{LOW}}}{|\mathbf{V}_{\text{LOW}}|} U_k + \frac{V_{\text{LOW}}}{|\mathbf{V}_{\text{LOW}}|} V_k$$

and similarly for  $\hat{V}_k$ , where  $\mathbf{V}_{\text{LOW}} = (U_{\text{LOW}}, V_{\text{LOW}})$ , then the wind profile is defined level-by-level as

$$V_k^{\text{G}} = \frac{(\hat{U}_k D_1 + \hat{V}_k D_2)}{\sqrt{(D_1^2 + D_2^2)}} \quad (4.31)$$

where  $D_1 = B - (B - C) \sin^2 \bar{\psi}$  and  $D_2 = (B - C) \sin \bar{\psi} \cos \bar{\psi}$ ; the values of  $V_k^G$  are also used to compute half level values  $V_{k+1/2}^G$  etc. by linear interpolation in pressure.

- (v) The calculation of basic flow Richardson Number

$$\overline{Ri}_{k-1/2} = N_{k-1/2}^2 \left\{ \frac{p_k - p_{k-1}}{g \rho_{k-1/2} (V_k^G - V_{k-1}^G)} \right\}^2$$

- (vi) The calculation of the blocked layer depth (4.4), given by the value of  $Z_{\text{blk}}$  that is the solution to the finite-difference form of the equation

$$\int_{Z_{\text{blk}}}^{3\mu} \frac{N_k}{\hat{U}_k} dz \geq H_{\text{ncrit}} \quad (4.32)$$

- (vii) The calculation of the layer in which low-level wave-breaking occurs (if any). This is given by the value of  $\Delta z$  that is the solution to the finite difference form of the equation

$$\int_{Z_{\text{blk}}}^{Z_{\text{blk}} + \Delta z} \frac{N_k}{\hat{U}_k} dz = \frac{\pi}{2} \quad (4.33)$$

the value of  $(Z_{\text{blk}} + \Delta z)$  is not allowed to be less than  $4\mu$ .

- (viii) The calculation of the assumed vertical profile of the sub-grid scale orography needed for the 'blocking' computations (4.10), for  $z < Z_{\text{blk}}$ ,

$$z_k^{\text{DEP}} = \sqrt{\frac{Z_{\text{blk}} - z_k}{z_k + \mu}} \quad (4.34)$$

#### 4.4.2 GWPROFIL

This routine computes the vertical profile of gravity-wave stress by constructing a local wave Richardson number which attempts to describe the onset of turbulence due to the gravity waves becoming convectively unstable or encountering critical layers. This local Richardson number can be written in the form

$$\tilde{Ri} = \overline{Ri} \left\{ \frac{1 - \alpha}{(1 + \overline{Ri}^{1/2} \alpha)^2} \right\}$$

where  $\overline{Ri}$  is the Richardson number of the basic flow. The parameter  $\alpha = N|\delta z|/V_k^G$  in which  $|\delta z|$  represents the amplitude. By requiring that  $\tilde{Ri}$  never falls below a critical value  $\tilde{Ri}_{\text{crit}}$  (currently equal to 0.25), values of wave stress are defined progressively from the top of the blocked layer upwards.

When low-level breaking occurs the relevant depth is assumed to be related to the vertical wavelength. Hence a linear (in pressure) decrease of stress is included over a depth  $\Delta z$  given by the solution of (4.33). The linear decrease of gravity wave stress is written as

$$\tau_{\text{wave}_{p_k}} = \tau_{\text{wave}_{z_{\text{blk}}}} + (\tau_{\text{wave}_{z_{\text{blk}} + \Delta z}} - \tau_{\text{wave}_{z_{\text{blk}}}}) \frac{p_k - p_{z_{\text{blk}}}}{p_{z_{\text{blk}} + \Delta z} - p_{z_{\text{blk}}}} \quad (4.35)$$

#### 4.4.3 GWDRAG

This is the main routine. The total (TOT) tendency due the dynamics (DYN), vertical diffusion (VDF), wave drag, and blocking drag is given by

$$\left( \frac{\partial u}{\partial t} \right)_{\text{TOT}} = \left( \frac{\partial u}{\partial t} \right)_{\text{wave}} + \left( \frac{\partial u}{\partial t} \right)_{\text{blk}} + \left( \frac{\partial u}{\partial t} \right)_{\text{DYN+VDF}} = \alpha_u - \beta u^{n+1} + \left( \frac{\partial u}{\partial t} \right)_{\text{DYN+VDF}} \quad (4.36)$$

where  $\alpha_u$  and  $\beta$  are the explicit gravity wave drag tendency coefficient and implicit blocking drag tendency coefficient respectively. A similar equation is apparent for the  $v$  component. As stated above, these tendencies are computed in the vertical diffusion routine.

(a) *Gravity wave drag component*

Using (4.19) the surface gravity-wave stress is computed in the form,

$$\tau_{\text{wave}} = \rho_{\text{LOW}}(H_{\text{eff}}^2/9)(\sigma/\mu)G(U_{\text{LOW}}^2 + V_{\text{LOW}}^2)^{1/2}(D_1^2 + D_2^2)^{1/2}N_{\text{LOW}} \quad (4.37)$$

where  $G$  is a function of the mountain sharpness. As  $Z_{\text{blk}}$  is able to reach a maximum height of  $3\mu$  (4.32), we must choose  $H = 3\mu$ . However, the surface stress must be scaled to a mountain height of  $2\mu$ , so the denominator of (4.37) is divided by 9 rather than 4 (4.19) (i.e. for  $Z_{\text{blk}} = 0$  both equations are identical). Following this, **GWPROFIL** is called to compute the vertical profile of gravity wave stress.

For  $z > Z_{\text{blk}}$  the gravity wave tendency coefficient is defined level by level as,

$$\left(\frac{\partial u}{\partial t}\right)_{\text{wave}} = -g \frac{\tau_{\text{wave}_{k+1}} - \tau_{\text{wave}_k}}{p_{k+1} - p_k} f_u(\psi) = \alpha_u \quad (4.38)$$

where  $f_u(\psi)$  is the necessary geometric function to generate components. A similar equation exists for  $\alpha_v$ . Here  $\beta = 0$ .

To avoid excessive tendencies at the top of the model, the remaining momentum flux above 9.9 Pa is spread over the layer between 9.9 Pa and the top of the model according to the following expression

$$\left(\frac{\partial u}{\partial t}\right)_{\text{wave}} = -g \frac{\tau_{\text{wave}_{k\text{top}}} - \tau_{\text{wave}_1}}{p_{k\text{top}} - p_1} (p_{k+1} - p_k) f_u(\psi) \quad (4.39)$$

where  $k\text{top}$  is the index of the first level above 9.9 Pa.

(b) *Blocking drag component*

For  $z \leq Z_{\text{blk}}$  the blocking drag tendency is defined level by level as

$$\left(\frac{\partial u}{\partial t}\right)_{\text{blk}} = -C_d \max\left(2 - \frac{1}{r}, 0\right) \frac{\sigma}{2\mu} \sqrt{\frac{Z_{\text{blk}} - z}{z + \mu}} (B \cos^2 \psi + C \sin^2 \psi) \frac{U|U|}{2} \quad (4.40)$$

Here  $\alpha_{u,v} = 0$ . This equation is evaluated in the following partially implicit manner by writing it in the form

$$\left(\frac{\partial u}{\partial t}\right)_{\text{blk}} = \frac{U^{n+1} - U^n}{\Delta t} = -A|U^n|U^{n+1} = -\beta_u U^{n+1}$$

with  $U^{n+1} = U^n/(1 + \beta)$  and  $\beta = \beta_u \Delta t$ , with  $\beta_u = A|U^n|$ . Here

$$\beta_u = C_d \max\left(2 - \frac{1}{r}, 0\right) \frac{\sigma}{2\mu} \sqrt{\frac{Z_{\text{blk}} - z}{z + \mu}} (B \cos^2 \psi + C \sin^2 \psi) \frac{|U|}{2} \quad (4.41)$$

(c) *Evaluation of tendencies*

The tendency coefficients are passed to **CALLPAR** where they are jointly implicitly computed in the vertical diffusion code with momentum tendency coefficients from the TOFD scheme. The actual tendencies are given as

$$\left(\frac{\partial u}{\partial t}\right)_{\text{wave+blk}} = \left(\frac{\partial u}{\partial t}\right)_{\text{TOT}} - \left(\frac{\partial u}{\partial t}\right)_{\text{DYN+VDF}} \quad (4.42)$$

Finally the tendencies are incremented. Local dissipation heating is calculated in the form

$$\left(\frac{\partial u}{\partial t}\right)_{\text{wave+blk}} = \frac{1}{c_p} \frac{DISS}{\Delta t}$$

with  $DISS = 0.5((U^n)^2 + (V^n)^2 - \hat{U}^2 - \hat{V}^2)$ , where  $\hat{U} = U^n + \Delta t(\partial u/\partial t)_{\text{wave+blk}}$  and  $\hat{V} = V^n + \Delta t(\partial v/\partial t)_{\text{wave+blk}}$ .

## APPENDIX A. LIST OF SYMBOLS

$a$	half mountain width in the cross-ridge or $x$ -direction
$B, C$	functions of the mountain anisotropy
$b$	half mountain width in the along-ridge or $y$ -direction
$C_d$	drag coefficient
$c_{p_{\text{dry}}}$	specific heat capacity
$D_{\text{blk}}$	blocked-flow drag
$g$	gravitational acceleration
$G$	function of the mountain sharpness
$H$	maximum mountain height
$H_{\text{eff}}$	effective mountain height
$h(x, y)$	mountain height profile
$H_n$	non-dimensional mountain height ( $= NH/ U $ )
$H_{n_{\text{crit}}}$	critical non-dimensional mountain height
$L$	length scale of the grid-point region
$l(z)$	horizontal width of mountain seen by the upstream flow
$N$	Brunt–Väisälä frequency
$N_H$	( $=N_{\text{LOW}}$ ) mean Brunt–Väisälä frequency of low-level flow between $z = \mu$ and $z = 2\mu$
$p_k$	model level air pressure
$\overline{Ri}$	Richardson number of the basic flow
$\tilde{Ri}$	local Richardson number
$\tilde{Ri}_{\text{crit}}$	critical Richardson number
$T$	temperature
$U$	wind speed
$U_{\text{LOW}}, V_{\text{LOW}}$	horizontal components of mean low-level flow between $z = \mu$ and $z = 2\mu$
$U_H$	mean wind speed of low-level flow between $z = \mu$ and $z = 2\mu$
$U_p$	component of the wind speed in the direction of $U_H$
$U_\tau$	component of wind speed in the direction of the stress $\tau$
$u, v$	horizontal wind components
$U_k, V_k$	level-by-level horizontal wind components
$V_k^G$	level-by-level wind profile in the plane of gravity wave stress
$Z_{\text{blk}}$	depth of blocked layer
$\alpha_{u,v}$	explicit gravity wave tendency coefficient
$\beta$	( $= \beta_u \Delta t$ ) implicit blocking drag tendency coefficient
$\gamma$	anisotropy of the orography ( $= a/b \leq 1$ )
$\theta$	orientation of the orography
$\mu$	standard deviation of orography
$\rho_k$	model level air density
$\rho_0$	density of air at the surface
$\rho_H$	( $=\rho_{\text{LOW}}$ ) mean density of low-level flow between $z = \mu$ and $z = 2\mu$
$\sigma$	slope of the orography
$\tau_{\text{blk}}$	stress due to blocked flow
$\tau_{\text{wave}}$	surface stress due to gravity waves
$\psi$	angle between incident flow and orographic principal axis
$\overline{\psi}$	( $= \psi_H$ ) mean value of $\psi$ between $z = \mu$ and $z = 2\mu$
$\varphi$	angle between low-level wind and the $x$ -axis
$\overline{\varphi}$	mean value of $\varphi$ between $z = \mu$ and $z = 2\mu$



## Chapter 5

# Non-orographic gravity wave drag

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### 5.1 INTRODUCTION

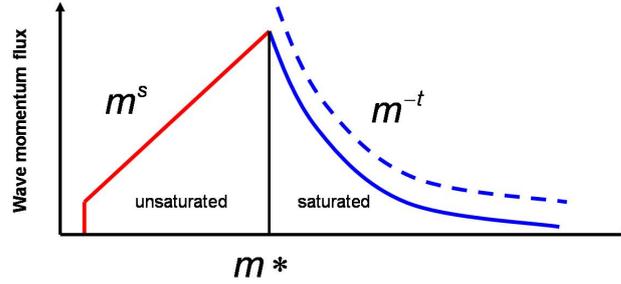
Gravity waves have an important influence on the climate of the middle atmosphere, comprising the stratosphere and the mesosphere. The middle atmosphere is dominated by a westerly jet in the winter hemisphere and an easterly jet in the summer hemisphere, and a meridional circulation with upwelling in the tropics and downwelling over the winter pole, referred to as the Brewer-Dobson circulation. This circulation is driven by the momentum deposited by breaking Rossby and small-scale non-orographic gravity waves. Furthermore, non-orographic gravity waves form the dominant source of momentum in the mesosphere and the thermosphere, and are also important in driving the variability of the tropical stratosphere, most prominent are the quasi-biennial oscillation and the semi-annual oscillation.

Non-orographic gravity waves are forced by dynamical motions such as convection, frontogenesis, and jet stream activity (e.g. [Fritts and Nastrom, 1980](#)). They have vertical wavelengths which vary from less than one to many tens of kilometres and horizontal wavelengths which vary from tens to thousands of kilometres ([Ern \*et al.\*, 2004](#)). Thus these waves are generally unresolved or under-resolved by the model as the generating process is often poorly represented, and therefore have to be parametrized.

Prior to Cy35r3 (September 2009) the effect of small-scale non-orographic gravity waves has been parametrized in the simplest possible manner by Rayleigh friction above the stratopause, which is formulated as a drag force proportional to the mean flow. The parametrization scheme adopted since then is that of [Scinocca \(2003\)](#). The description is kept short and the effects of the parametrization on the middle atmosphere circulation in the IFS are not discussed as all material is described in detail, including an extensive literature list, in [Orr \*et al.\* \(2010\)](#).

### 5.2 HISTORY

The [Scinocca \(2003\)](#) scheme, hereafter referred to as S03 scheme, is a spectral scheme that follows from the [Warner and McIntyre \(1996\)](#) scheme which represents the three basic wave mechanisms that are conservative propagation, critical level filtering, and non-linear dissipation. However, the full non-hydrostatic and rotational wave dynamics considered by [Warner and McIntyre \(1996\)](#) is too costly for operational models, and therefore only hydrostatic and non-rotational wave dynamics is employed.



**Figure 5.1** Schematic of the gravity wave spectrum as a function of vertical wavenumber at constant intrinsic frequency  $\tilde{\omega}$  showing the different spectral shapes, separated by  $m_*$  for large and small  $m$ . The saturation spectrum is denoted by the dashed line.

### 5.3 LAUNCH SPECTRUM

The dispersion relation for a hydrostatic gravity wave in the absence of rotation is

$$m^2 = \frac{k^2 N^2}{\tilde{\omega}^2} = \frac{N^2}{\tilde{c}^2} \quad (5.1)$$

where  $k$ ,  $m$  are horizontal and vertical wavenumbers,  $\tilde{\omega} = \omega - kU$  and  $\tilde{c} = c - U$  are the intrinsic frequency and phase speed (with  $c$  the ground based phase speed and  $U$  the background wind speed in the direction of propagation), and  $N$  is the Brunt-Väisälä frequency (here expressed in height coordinate  $z$  for a better tangent-linear approximation and not in pressure coordinates as in the original code)

$$N^2 = \frac{g^2}{c_p T} + \frac{g \partial T}{T \partial z}; \quad N \geq 10^{-12} \quad (5.2)$$

with  $g$  the gravity constant and  $c_p$  the specific heat of dry air at constant volume.

The launch spectrum is given by the total wave energy per unit mass in each azimuth  $\phi$  following [Fritts and Nastrom \(1993\)](#), and is globally uniform and constant

$$\tilde{E}(m, \tilde{\omega}, \phi) = B \left( \frac{m}{m_*} \right)^s \frac{N^2 \tilde{\omega}^{-p}}{1 - \left( \frac{m}{m_*} \right)^{s+3}} \quad (5.3)$$

where  $B$ ,  $s$  and  $p$  are constants, and  $m_* = 2\pi/L$  is a transitional wavelength (see [Section 5.8](#) for an overview of the parameter settings).

Observation and theory suggest  $p$  values in the range  $1 \leq p \leq 2$ . The spectrum is separable in terms of both  $m$  and  $\tilde{\omega}$  and is displayed in [Fig. 5.1](#) for a given value of  $\tilde{\omega}$ .

However, instead of the total wave energy the momentum flux spectral density  $\rho \tilde{F}(m, \tilde{\omega}, \phi)$  is required, where  $\rho$  is the air density. It is obtained through the group velocity rule

$$\rho \tilde{F}(m, \tilde{\omega}, \phi) = \rho c_g \frac{k}{\tilde{\omega}} E(m, \tilde{\omega}, \phi) \quad (5.4)$$

where  $c_g = \partial \tilde{\omega} / \partial m = \tilde{\omega} / m$  is the group velocity for hydrostatic dynamics. However as shown in S03, in the  $(m, \tilde{\omega})$  coordinate framework the wave momentum flux is not conserved in the absence of dissipative processes as the spectrum propagates vertically through height varying background winds and buoyancy frequencies, but it is conserved in the  $(k, \omega)$  coordinate framework. Performing a coordinate transformation on (5.4) and using the dispersion relation (5.1), one obtains the expression of the unscaled momentum flux density or Eliassen-Palm flux density as a function of the independent variable  $c$

$$\rho F^*(\hat{c}, \phi) = \rho \frac{\hat{c} - \hat{U}}{N} \left( \frac{\hat{c} - \hat{U}}{\hat{c}} \right)^{2-p} \frac{1}{1 + \left( \frac{m_*(\hat{c} - \hat{U})}{N} \right)^{s+3}} \quad (5.5)$$

where  $\hat{U} = U^{(\phi)} - U_0^{(\phi)}$  and  $\hat{c} = c - U_0^{(\phi)}$ , with  $U^{(\phi)} = u \cos(\phi) + v \sin(\phi)$  the velocity in the direction of the azimuth  $\phi$  and the subscript 0 referring to the launch level. Note that at the launch level  $\hat{U} = 0$  so that the transformation renders the wave flux independent of azimuth. In the current formulation the departure spectrum is globally constant, therefore identical for each model column.

The final result for the scaled momentum or Eliassen-Palm flux density (having units  $\rho \text{ m}^2 \text{ s}^{-2} / d\hat{c} = \rho \text{ ms}^{-1}$ ) at the launch level  $\hat{F}$  is obtained through scaling with  $\rho_0 F_{\text{launch}}$  which is the imposed launch momentum flux, and the most important free model parameter

$$\rho \hat{F}(\hat{c}, \phi) = \rho F^*(\hat{c}, \phi) * A; \quad A = (\rho_0 F_{\text{launch}}) / \int (\rho F^*(\hat{c}, \phi_1)) d\hat{c} \quad (5.6)$$

Prior to cycle 38r2 the launch momentum flux has been set to a globally uniform value  $\rho_0 F_{\text{launch}} = \rho_0 F_{\text{launch}}^0$ . However, it was recognized that lower values are preferable for the tropics while the higher latitudes require larger values to control the polar night jets. Therefore, with cycle 38r2 a latitude dependent launch momentum flux has been introduced

$$\rho_0 F_{\text{launch}} = \rho_0 F_{\text{launch}}^0 \left[ 1 - a_\lambda \exp \left( \frac{-\lambda^2}{\lambda_0^2} \right) \right] \quad (5.7)$$

with  $\lambda$  the latitude.

## 5.4 DISCRETISATION

The wave fluxes are defined at half model levels. Therefore, all variables like  $U$ ,  $N$  (temperature) etc. required in the computation of the fluxes have to be interpolated to half-levels.

The wave spectrum is discretised in  $n_c$  phase speed bins, and  $n_\phi$  equally spaced azimuths. The wave momentum flux is initialised at the model level just below the launch level height  $p_{\text{launch}}$ .

As it is the small- $m$  portion of the spectrum which is associated with large values of momentum flux it is important to apply a coordinate stretch in order to obtain higher resolution and better accuracy at large phase speeds (i.e. small  $m$ ). The problem is solved in the space of the transformed variable  $\bar{X}$  having uniform resolution  $d\bar{X}$ . Taking as untransformed variable

$$X = \frac{1}{\hat{c}} \quad (5.8)$$

the coordinate stretch under the constraint  $X_{\min} \leq X \leq X_{\max}$  and  $X_{\min} \leq \bar{X} \leq X_{\max}$  is defined by

$$X = B_1 e^{(\bar{X} - X_{\min})/\Gamma} + B_2 \quad (5.9)$$

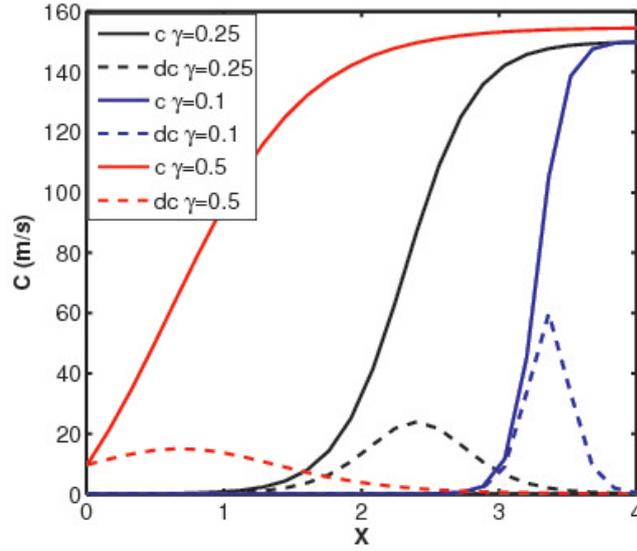
and

$$B_1 = \frac{X_{\max} - X_{\min}}{e^{(X_{\max} - X_{\min})/\Gamma} - 1}; \quad B_2 = X_{\min} - B_1 \quad (5.10)$$

The free parameters are the half-width of the stretch  $\Gamma$ , as well as  $\hat{c}_{\min}$  with  $X_{\max} = 1/\hat{c}_{\min}$ , and  $\hat{c}_{\max}$  with  $X_{\min} = 1/\hat{c}_{\max}$ . In discretised space this means one uses  $\bar{X}_i = X_{\min} + (X_{\max} - X_{\min})(i/n_c)$  instead of  $\hat{c}_i = \hat{c}_{\min} + (\hat{c}_{\max} - \hat{c}_{\min})(i/n_c)$ , and the space element  $d\hat{c}$  then becomes

$$d\hat{c} = \frac{d\hat{c}}{dX} \frac{dX}{d\bar{X}} d\bar{X} = X^{-2} \frac{B_1}{\Gamma} e^{(\bar{X} - X_{\min})/\Gamma} d\bar{X} \quad (5.11)$$

The coordinate stretch is displayed in [Fig. 5.2](#) for different values of  $\Gamma$ .



**Figure 5.2** Schematic of the coordinate stretch with phase speed  $\hat{c}$  and phase element  $\hat{dc}$  as a function of  $\bar{X}$  for different values of  $\Gamma$ .

### 5.5 CRITICAL LEVEL FILTERING

Waves that encounter critical levels are filtered from the wave spectrum, depositing their momentum to the mean flow in this layer. In practice one checks for each azimuth bin in each layer  $z$  above the departure level and for each phase speed. Critical level filtering is encountered if  $U$  increases such with height that  $\hat{U}_1 > \hat{U}_0$  and  $\hat{U}_0 \leq \hat{c} \leq \hat{U}_1$ . Then all momentum flux in the corresponding  $(\hat{c}, \phi)$  bin is removed from  $\rho\hat{F}(\hat{c}, \phi)$ . Note that at the departure level  $\hat{U}_0 = 0$  which sets an absolute lower bound for critical filtering of  $\hat{c} = \hat{c}_{\min}$ .

The consequence of critical level filtering is that, as the waves propagate vertically through stratospheric westerlies (easterlies) as in the middle latitude winter (summer) hemisphere, the spectrum becomes more and more asymmetric with height leaving only the easterly (westerly) phase speed component. In contrast, in the tropical atmosphere winds change from easterlies to westerlies or vice versa and both wind directions are filtered, leaving little zonal momentum flux.

The portions of the wave spectrum that survive critical level filtering undergo conservative propagation to the next model level where they are checked for nonlinear dissipation.

### 5.6 NONLINEAR DISSIPATION

Nonlinear dissipation is simply modelled by assuming that waves are dissipative in nature and employing 'saturation' theory (Lindzen, 1981) such that the amplitude of the parametrized wave field is limited to some threshold value (thought to be associated with the onset of instability). This is dealt with empirically by limiting the growth of the gravity wave spectrum at large- $m$  (short wavelengths) so as not to exceed the observed  $m^{-3}$  dependence. The saturation momentum flux is written as

$$\rho\hat{F}^{\text{sat}}(\hat{c}, \phi) = \rho C^* A \frac{\hat{c} - \hat{U}}{N} \left( \frac{\hat{c} - \hat{U}}{\hat{c}} \right)^{2-p} \tag{5.12}$$

where  $C^*$  is a tuning parameter introduced by McLandress and Scinocca (2005). The saturation momentum flux is depicted in Fig. 5.1. It decreases with height as a consequence of decreasing density, it is therefore not conserved. Nonlinear dissipation is formulated as the constraint  $\rho\hat{F}(\hat{c}, \phi) \leq \rho\hat{F}^{\text{sat}}(\hat{c}, \phi)$ . When the wave momentum flux exceeds the saturated value  $\rho\hat{F}(\hat{c}, \phi)$  is set equal to  $\rho\hat{F}^{\text{sat}}$  meaning that the excess momentum flux is deposited to the flow. Increasing the parameter  $C^*$  means pushing the onset of nonlinear dissipation to higher amplitudes and therefore to greater heights.

Finally, at the model top momentum conservation is achieved by depositing any remaining momentum from the wave field to the mean flow, i.e. the upper boundary condition is zero wave momentum flux.

## 5.7 TENDENCIES

The tendencies for the  $u, v$  wind components are given by the divergence of the net eastward  $\rho\bar{F}_E$ , and northward,  $\rho\bar{F}_N$  momentum fluxes, which are obtained through summation of the total momentum flux (i.e. integrated over all phase speed bins) in each azimuth  $\phi_i$  projected onto the east and north directions, respectively.

$$\left. \begin{aligned} \frac{\partial u}{\partial t} &= g \frac{\partial(\rho\bar{F}_E)}{\partial p}; & \bar{F}_E &= \sum_{i=1}^{n_\phi} (\rho\hat{F}(\phi_i)) \cos \phi_i \\ \frac{\partial v}{\partial t} &= g \frac{\partial(\rho\bar{F}_N)}{\partial p}; & \bar{F}_N &= \sum_{i=1}^{n_\phi} (\rho\hat{F}(\phi_i)) \sin \phi_i \end{aligned} \right\} \quad (5.13)$$

The dissipation of the non-orographic gravity waves (an external wave source) is also taken into account as a heat source for the model

$$\frac{\partial T}{\partial t} = -\frac{1}{c_p} \left( u \frac{\partial u}{\partial t} + v \frac{\partial v}{\partial t} \right) \quad (5.14)$$

## 5.8 PARAMETER SETTINGS

- $\hat{c}_{\min} = 0.25 \text{ m s}^{-1}$ ,  $\hat{c}_{\max} = 100 \text{ m s}^{-1}$ : minimum and maximum intrinsic phase speed
- $\Gamma = 0.25$ : half width of the stretch in (5.9)
- $L = 2000 \text{ m}$ : transitional wavelength
- $p = 1$ : the  $\tilde{\omega}$  exponent in (5.5) and (5.12)
- $s = 1$ : the small  $m$  spectral slope in (5.5) and (5.12)
- $p_{\text{launch}} = 450 \text{ hPa}$ : launch elevation (Pa)
- $n_\phi = 4$  (=N, S, E, W): number of equally spaced azimuths
- $n_c = 25$ : number of spectral intervals for phase speed
- $C^* = 1$ : non-dimensional constant for saturation spectrum in (5.12)
- $\rho_0 F_{\text{launch}}^0 = 3.75 \times 10^{-3} \text{ (Pa)}$ : launch momentum flux in (5.7)
- $\lambda_0 = 20^\circ$  in (5.7)
- $a_\lambda = 0.25$  in (5.7)

The most important tuning for future applications (in particular higher horizontal resolutions) would require a reduction of the amplitude of the launch momentum flux with increasing resolution so that as the resolved gravity wave momentum flux increases the parametrised flux decreases, and the total flux remains fairly constant. A reduction of the launch momentum flux would also increase the period of the models 'Quasi-Biennial Oscillation' which is currently too short. Further tuning could also include a time-dependent and horizontally variable amplitude of the launch momentum flux, but this is probably fairly difficult to achieve.

## 5.9 CODE

The principal routine is **GWDRAG\_WMS**, which is called from **CALLPAR**. The tunable parameters are defined in the setup routine **SUGWWMS** which is called from **SUPHEC**. To save computing time routine **GWDRAG\_WMS** is not called every time step, but typically every few time steps (i.e. every hour for spectral truncations  $>255$  and every 2 hours for truncations  $<255$ ). Therefore, the tendencies due to non-orographic gravity wave dissipation are stored in buffers (GFL arrays), and the total physics tendencies are incremented every time step inside routine **GWDRAG\_WMS**.

## APPENDIX A. LIST OF SYMBOLS

$A$       scaling factor for momentum flux density

$B$	formal dimensional constant, not actually used
$c$	phase speed
$\tilde{c}$	intrinsic phase speed = $c - U$
$\hat{c}$	intrinsic phase speed using departure level Doppler shift = $c - U_0$
$c_g$	group velocity
$\hat{c}_{\max}$	maximum intrinsic phase speed
$\hat{c}_{\min}$	minimum intrinsic phase speed
$c_p$	specific heat at constant pressure
$C_*$	parameter
$F$	wave momentum flux (without density factor)
$F^{\text{sat}}$	saturation value of wave momentum flux
$g$	gravity constant
$\Gamma$	parameter
$k$	horizontal wavenumber
$L$	transitional wavelength
$m$	vertical wavenumber
$n_c$	number of phase speed bins
$n_\phi$	number of azimuth bins
$N$	Brunt–Väisälä frequency
$p$	constant, the $\omega$ exponent
$p$	pressure
$s$	constant, the small $m$ spectral slope
$t$	time
$T$	temperature
$u$	west-east component of wind speed
$U$	wind speed in direction of wave
$v$	south-north component of wind speed
$z$	height
$\phi$	azimuthal direction
$\lambda$	latitude
$\omega$	frequency

# Chapter 6

## Convection

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### 6.1 INTRODUCTION

Cumulus convection is parametrized by a bulk mass flux scheme which was originally described in [Tiedtke \(1989\)](#). The scheme considers deep, shallow and mid-level convection. Clouds are represented by a single pair of entraining/detraining plumes which describes updraught and downdraught processes. Momentum and tracer transport is also included.

## 6.2 LARGE-SCALE BUDGET EQUATIONS

The contributions from cumulus convection to the large-scale budget equations of heat moisture, momentum, and chemical tracers are

$$\left. \begin{aligned}
 \left(\frac{\partial \bar{s}}{\partial t}\right)_{\text{cu}} &= g \frac{\partial}{\partial p} [M_{\text{up}} s_{\text{up}} + M_{\text{down}} s_{\text{down}} - (M_{\text{up}} + M_{\text{down}}) \bar{s}] \\
 &\quad + L(c_{\text{up}} - e_{\text{down}} - e_{\text{subcl}}) - (L_{\text{subl}} - L_{\text{vap}})(M_{\text{elt}} - F_{\text{rez}}) \\
 \left(\frac{\partial \bar{q}}{\partial t}\right)_{\text{cu}} &= g \frac{\partial}{\partial p} [M_{\text{up}} q_{\text{up}} + M_{\text{down}} q_{\text{down}} - (M_{\text{up}} + M_{\text{down}}) \bar{q}] \\
 &\quad - (c_{\text{up}} - e_{\text{down}} - e_{\text{subcl}}) \\
 \left(\frac{\partial \bar{u}}{\partial t}\right)_{\text{cu}} &= g \frac{\partial}{\partial p} [M_{\text{up}} u_{\text{up}} + M_{\text{down}} u_{\text{down}} - (M_{\text{up}} + M_{\text{down}}) \bar{u}] \\
 \left(\frac{\partial \bar{v}}{\partial t}\right)_{\text{cu}} &= g \frac{\partial}{\partial p} [M_{\text{up}} v_{\text{up}} + M_{\text{down}} v_{\text{down}} - (M_{\text{up}} + M_{\text{down}}) \bar{v}] \\
 \left(\frac{\partial \bar{C}^i}{\partial t}\right)_{\text{cu}} &= g \frac{\partial}{\partial p} [M_{\text{up}} C_{\text{up}}^i + M_{\text{down}} C_{\text{down}}^i - (M_{\text{up}} + M_{\text{down}}) \bar{C}^i]
 \end{aligned} \right\} \quad (6.1)$$

where  $M_{\text{up}}$ ,  $M_{\text{down}}$  are the net contributions from all clouds to the updraught and downdraught mass fluxes,  $c_{\text{up}}$  and  $e_{\text{down}}$  are the condensation/sublimation in the updraughts, and the evaporation in the downdraughts.  $s_{\text{up}}$ ,  $s_{\text{down}}$ ,  $q_{\text{up}}$ ,  $q_{\text{down}}$ ,  $u_{\text{up}}$ ,  $u_{\text{down}}$ ,  $v_{\text{up}}$ ,  $v_{\text{down}}$ ,  $C_{\text{up}}^i$  and  $C_{\text{down}}^i$  are the weighted averages of the dry static energy  $\bar{s}$ , the specific humidity  $\bar{q}$ , the horizontal wind components  $\bar{u}$  and  $\bar{v}$  and the passive chemical tracer  $\bar{C}^i$  from all updraughts and downdraughts within a grid box (although individual convective elements are not considered) obtained from the bulk cloud model described below.  $L_{\text{subl}}$  and  $L_{\text{vap}}$  are latent heats of sublimation and vaporization, and  $L$  is the effective latent heat for an ice–water mix (an empirical function of temperature).  $e_{\text{subcl}}$  is the evaporation of precipitation in the unsaturated sub-cloud layer,  $M_{\text{elt}}$  is the melting rate of snow and  $F_{\text{rez}}$  is the freezing rate of condensate in the convective updraught. In addition to (6.1) the precipitation fluxes are defined as

$$P^{\text{rain}}(p) = \int_{P_{\text{top}}}^p (G^{\text{rain}} - e_{\text{down}}^{\text{rain}} - e_{\text{subcl}}^{\text{rain}} + M_{\text{elt}}) \frac{dp}{g}; \quad P^{\text{snow}}(p) = \int_{P_{\text{top}}}^p (G^{\text{snow}} - e_{\text{down}}^{\text{snow}} - e_{\text{subcl}}^{\text{snow}} - M_{\text{elt}}) \frac{dp}{g} \quad (6.2)$$

where  $P^{\text{rain}}$  and  $P^{\text{snow}}$  are the fluxes of precipitation in the forms of rain and snow at level  $p$ .  $G^{\text{rain}}$  and  $G^{\text{snow}}$  are the conversion rates from cloud water into rain and cloud ice into snow, and  $M_{\text{elt}}$  denotes melted precipitation. The evaporation of precipitation in the downdraughts  $e_{\text{down}}$ , and below cloud base  $e_{\text{subcl}}$ , have been split into water and ice components,  $e_{\text{down}}^{\text{rain}}$ ,  $e_{\text{down}}^{\text{snow}}$ ,  $e_{\text{subcl}}^{\text{rain}}$ , and  $e_{\text{subcl}}^{\text{snow}}$ . The microphysical terms in (6.1) and (6.2) referring to the updraught are explained in detail in Section 6.6, those referring to the downdraught are defined in (6.17).

## 6.3 CLOUD MODEL EQUATIONS

### 6.3.1 Updraughts

The updraught of the cloud ensemble is assumed to be in a steady state. Then the bulk equations for mass, heat, moisture, cloud water content, momentum and tracers are

$$\left. \begin{aligned}
 -g \frac{\partial M_{\text{up}}}{\partial p} &= E_{\text{up}} - D_{\text{up}} \\
 -g \frac{\partial (M_{\text{up}} s_{\text{up}})}{\partial p} &= E_{\text{up}} \bar{s} - D_{\text{up}} s_{\text{up}} + L c_{\text{up}}, & -g \frac{\partial (M_{\text{up}} q_{\text{up}})}{\partial p} &= E_{\text{up}} \bar{q} - D_{\text{up}} q_{\text{up}} - c_{\text{up}} \\
 -g \frac{\partial (M_{\text{up}} l_{\text{up}})}{\partial p} &= -D_{\text{up}} l_{\text{up}} + c_{\text{up}} - G, & -g \frac{\partial (M_{\text{up}} r_{\text{up}})}{\partial p} &= -D_{\text{up}} r_{\text{up}} + G - S_{\text{fallout}} \\
 -g \frac{\partial (M_{\text{up}} u_{\text{up}})}{\partial p} &= E_{\text{up}} \bar{u} - D_{\text{up}} u_{\text{up}}, & -g \frac{\partial (M_{\text{up}} v_{\text{up}})}{\partial p} &= E_{\text{up}} \bar{v} - D_{\text{up}} v_{\text{up}} \\
 & & -g \frac{\partial (M_{\text{up}} C_{\text{up}}^i)}{\partial p} &= E_{\text{up}} \bar{C}^i - D_{\text{up}} C_{\text{up}}^i
 \end{aligned} \right\} \quad (6.3)$$

where  $E_{\text{up}}$  and  $D_{\text{up}}$  are the rates of mass entrainment and detrainment,  $l_{\text{up}}$  is the updraught cloud water/ice content, and  $r_{\text{up}}$  is precipitating rain and snow. The vertical integration of (6.3) requires knowledge of the cloud-base mass flux and of the mass entrainment and detrainment rates. The cloud-base mass flux is determined for the various types of convection from the closure assumptions discussed in Section 6.4.

Entrainment of mass into convective plumes is assumed to occur (1) through turbulent exchange or inflow of mass through the cloud edges; and detrainment is assumed to occur (1) through turbulent exchange and (2) through organized outflow at cloud top. The superscripts (1) and (2) are used to denote the different components of the entrainment and detrainment processes

$$E_{\text{up}} = E_{\text{up}}^{(1)}, \quad D_{\text{up}} = D_{\text{up}}^{(1)} + D_{\text{up}}^{(2)} \quad (6.4)$$

(a) *Entrainment rates*

Entrainment rates ( $\text{s}^{-1}$ ) are parametrized as

$$E_{\text{up}}^{(1)} \cong \varepsilon_{\text{up}}^{(1)} \frac{M_{\text{up}}}{\bar{\rho}} f_{\text{scale}}^{(1)} \quad (6.5)$$

where the fractional entrainment ( $\text{m}^{-1}$ ) traditionally inversely depends on cloud radius ( $R_{\text{up}}$ ) following (Simpson and Wiggert, 1969; Simpson, 1971), i.e.

$$\varepsilon_{\text{up}}^{(1)} \sim \frac{0.2}{R_{\text{up}}} \quad (6.6)$$

With Cy36r4 the updraught entrainment formulation has been simplified to retain only one entrainment process/formulation englobing both "turbulent" and "organized" mass exchanges. Entrainment above cloud base is applied to positively buoyant convection only. Observations show that mid-tropospheric relative humidity strongly controls the cloud top heights, and it could be even shown through cloud resolving simulations (Stirling and Derbyshire, private communication) that dry environments lead to larger entrainment, probably through evaporative cooling and inflow effects. The simplest way to represent this sensitivity and to increase the mass fluxes in unstable buoyant situations, is a formulation depending on the environmental relative humidity  $RH$

$$E_{\text{up}}^{(1)} = \varepsilon_{\text{up}}^{(1)} f_{\varepsilon} \frac{M_{\text{up}}}{\bar{\rho}} \left(1.3 - RH\right) f_{\text{scale}}^{(1)}, \quad \varepsilon_{\text{up}}^{(1)} = 1.75 \times 10^{-3} \text{m}^{-1}, \quad f_{\text{scale}}^{(1)} = \left(\frac{q_{\text{sat}}(\bar{T})}{q_{\text{sat}}(\bar{T}_{\text{base}})}\right)^3 \quad (6.7)$$

This entrainment formulation is able to reasonably represent the tropical variability of convection (Bechtold *et al.*, 2008). It is applied to all types of convection and only distinguishes between the different types by the factor  $f_{\varepsilon}$  which takes the value of one for deep convection and 2 for shallow and mid-level convection. The vertical scaling function  $f_{\text{scale}}^{(1)}$  in (6.7) is supposed to mimick the effects of a cloud ensemble and/or the effect of increasing ( $R_{\text{up}}$ ) with height. As the scaling function strongly decreases with height the detrainment rate will become eventually larger than the entrainment rate, and the mass flux starts to decrease with height. Together with the detrainment (see below) the formulation produces on average a vertical distribution of the convective mass flux that broadly follows that of the large-scale ascent which is partly supported by diagnostic studies for tropical convection (e.g. Cheng *et al.*, 1980; Johnson, 1980). Finally, note that in cycles prior to 32r3 the "organized" entrainment has been linked to the the large-scale moisture convergence as first advocated by Lindzen (1981). However, the imposed strong coupling between the large-scale and the convection had a detrimental effect on the forecasts ability to represent tropical variability. Only since Cy32r3, using entrainment rates scaled by a vertical function together with a relative humidity based organized entrainment, and a variable convective adjustment time-scale (see below), the model is able to maintain a realistic level of tropical variability.

(b) *Detrainment rates*

Turbulent detrainment rates ( $\text{s}^{-1}$ ) for deep convection are parametrized as

$$D_{\text{up}}^{(1)} = \delta_{\text{up}}^{(1)} \frac{M_{\text{up}}}{\bar{\rho}} \left(1.6 - RH\right); \quad \delta_{\text{up}}^{(1)} = 0.75 \times 10^{-4} \text{m}^{-1} \quad (6.8)$$

whereas turbulent detrainment rates for shallow convection are parametrized as

$$D_{\text{up}}^{(1)} = E_{\text{up}}^{(1)} \left( 1.6 - RH \right) \quad (6.9)$$

Organized detrainment is set proportional to the decrease with height of the updraught kinetic energy  $K_{\text{up}}$  which is obtained from

$$\frac{\partial K_{\text{up}}}{\partial z} = -\frac{\mu_{\text{up}}}{M_{\text{up}}} (1 + \beta C_d) 2K_{\text{up}} + \frac{1}{f(1 + \gamma)} g \frac{T_{\text{v,up}} - \bar{T}_{\text{v}}}{\bar{T}_{\text{v}}} \quad (6.10)$$

with

$$K_{\text{up}} = \frac{w_{\text{up}}^2}{2} \quad (6.11)$$

where  $w_{\text{up}}\bar{\rho}$  is the updraught vertical velocity,  $T_{\text{v,up}}$  is the virtual temperature of the updraught and  $\bar{T}_{\text{v}}$  the virtual temperature of the environment.  $\mu_{\text{up}}$  is a mixing coefficient which is equal to the entrainment rate ( $E_{\text{up}}\bar{\rho}$ ), or the detrainment rate ( $D_{\text{up}}\bar{\rho}$ ) if this is larger. As entrainment is set to zero in the upper part of the cloud layer, use of detrainment in this region better represents the effect of mixing and vertical pressure gradients in the upper part of deep convective clouds, reducing vertical velocity and reducing overshoot of convective towers into the lower stratosphere.

$\gamma = 0.5$  is the virtual mass coefficient (Simpson and Wiggert, 1969), the factor  $f = 2$  is introduced because the flow is highly turbulent (Cheng *et al.*, 1980) and for the drag coefficient a value of  $C_d = 0.506$  is used (Simpson and Wiggert, 1969). The value for  $\beta$  is 1.875. The cloud base value of the updraught velocity is chosen as  $1 \text{ m s}^{-1}$ .

$w_{\text{up}}$  enters the scheme in several ways: (i) for the generation and fallout of rain (Section 6.6), (ii) to determine the top of the updraught (cloud) as the level where  $w_{\text{up}}$  reduces to zero, and (iii) to specify organized detrainment below the top of the updraught.

The organized detrainment is set proportional to the decrease in updraught vertical velocity with height

$$\frac{M_{\text{up}}(z)}{M_{\text{up}}(z + \Delta z)} = (1.6 - RH) \sqrt{\frac{K_{\text{up}}(z)}{K_{\text{up}}(z + \Delta z)}}; \quad D_{\text{up}}^{(2)} = \left( 1.6 - RH \right) \sqrt{\frac{K_{\text{up}}(z)}{K_{\text{up}}(z + \Delta z)}} \frac{M_{\text{up}}}{(\bar{\rho}\Delta z)} \quad (6.12)$$

where a relative humidity dependent factor has been introduced assuring a more gradual detrainment, in particular in the vicinity of inversions. The mean value of this proportionality factor roughly corresponds to the value derived by Derbyshire *et al.* (2011) from cloud resolving model data.

### 6.3.2 Downdraughts

Downdraughts are considered to be associated with convective precipitation from the updraughts and originate from cloud air influenced by the injection of environmental air. Following Fritsch and Chappell (1980) and Foster (1958), the Level of Free Sinking (LFS) is assumed to be the highest model level (below the level of minimum moist static energy) where a mixture of equal parts of cloud and saturated environmental air at the wet-bulb temperature becomes negative buoyant with respect to the environmental air. The downdraught mass flux is assumed to be directly proportional to the upward mass flux. Following Johnson (1976, 1980) the mass flux at the LFS is specified from the updraught mass flux at cloud base as

$$(M_{\text{down}})_{\text{LFS}} = -\eta(M_{\text{up}})_{\text{base}} \quad \text{with} \quad \eta = 0.3 \quad (6.13)$$

The vertical distribution of the downdraught mass flux, dry static energy, moisture, horizontal momentum and passive tracers below the LFS are determined by entraining/detraining plume equations similar to

those for the updraught:

$$\left. \begin{aligned}
 g \frac{\partial M_{\text{down}}}{\partial p} &= E_{\text{down}} - D_{\text{down}} \\
 g \frac{\partial (M_{\text{down}} s_{\text{down}})}{\partial p} &= E_{\text{down}} \bar{s} - D_{\text{down}} s_{\text{down}} + L e_{\text{down}} \\
 g \frac{\partial (M_{\text{down}} q_{\text{down}})}{\partial p} &= E_{\text{down}} \bar{q} - D_{\text{down}} q_{\text{down}} - e_{\text{down}} \\
 g \frac{\partial (M_{\text{down}} u_{\text{down}})}{\partial p} &= E_{\text{down}} \bar{u} - D_{\text{down}} u_{\text{down}} \\
 g \frac{\partial (M_{\text{down}} v_{\text{down}})}{\partial p} &= E_{\text{down}} \bar{v} - D_{\text{down}} v_{\text{down}} \\
 g \frac{\partial (M_{\text{down}} C_{\text{down}}^i)}{\partial p} &= E_{\text{down}} \bar{C}^i - D_{\text{down}} C_{\text{down}}^i
 \end{aligned} \right\} \quad (6.14)$$

$e_{\text{down}}$  is the evaporation of convective rain to maintain a saturated descent; the moistening and cooling of the environmental air injected at the LFS is also due to evaporating rain.

Entrainment and detrainment in downdraughts are highly uncertain as relevant data are not available. As for the updraught, both turbulent and organized entrainment/detrainment are considered.

(a) *Turbulent entrainment and detrainment*

For turbulent mixing

$$\varepsilon_{\text{down}}^{(1)} = \delta_{\text{down}}^{(1)} = 2 \times 10^{-4} \text{ m}^{-1} \quad (6.15)$$

(b) *Organized entrainment and detrainment*

Organized entrainment for the downdraught is based upon a formulation suggested by Nordeng (1994) so that

$$\varepsilon_{\text{down}}^{(2)} = \frac{\left\{ g \frac{T_{v,\text{down}} - T_{\text{down}} r_{\text{down}} - \bar{T}_v}{T} \right\}}{(w_{\text{down}}^{\text{LFS}})^2 - \int_{z_{\text{LFS}}}^z \left\{ g \frac{T_{v,\text{down}} - T_{\text{down}} r_{\text{down}} - \bar{T}_v}{T} \right\} dz} \quad (6.16)$$

where  $w_{\text{down}}^{\text{LFS}}$  is the vertical velocity in the downdraught at the LFS (set to  $-1 \text{ ms}^{-1}$ ). The total evaporation rate in the downdraft corresponds to the total downdraft precipitation rate that is simply given as

$$\sum_{k=\text{LFS}}^{\text{nlev}} e_{\text{down}} = \sum_{k=\text{LFS}}^{\text{nlev}} \frac{g}{\Delta p} (q_{\text{down},k} - \hat{q}_{\text{down},k}) M_{\text{down},k} \quad (6.17)$$

where  $q_{\text{down},k}$  is the value of the downdraft humidity computed from (6.14) without saturation adjustment, and  $\hat{q}_{\text{down},k}$  is the humidity after the saturation adjustment. The value of the rain water content in the downdraft used in (6.16) is estimated as  $r_{\text{down}} = e_{\text{down}} \Delta p / (g M_{\text{up}})$ , for the definition of the pressure thickness  $\Delta p$  of layer  $k$  see (6.52).

Organized detrainment from the downdraught occurs when either the downdraught becomes positively buoyant or approaches the surface. If the downdraught remains negatively buoyant until it reaches the surface then the mass flux is decreased linearly over the lowest 60 hPa of the atmosphere. However, if a downdraught becomes positively buoyant during its descent, it is detrained over one level, except where this occurs at cloud base. In this case the downdraught fluxes are decreased linearly (deep convection) or quadratically (mid-level convection) to zero at the surface.

## 6.4 CONVECTION INITIATION AND CONVECTIVE TYPES

The first important task of a convection parameterization is to decide if convection is active or not in a model grid column. This is done in a very simplified ‘‘first-guess’’ updraught computation that implies the determination of the cloud base level, i.e. the Lifting Condensation Level (LCL), and of the properties

of the cloud (updraught) at cloud base. Furthermore, in using a bulk mass flux scheme, as opposed to a scheme which considers an ensemble of convective clouds (such as that of [Arakawa and Schubert, 1974](#)), some determination of convective cloud type must be made so that appropriate choices can be made for the cloud properties.

The scheme first tests for the occurrence of shallow convection by computing the ascent of a surface parcel. The following simplified updraught equation is applied

$$\frac{\partial \phi_{\text{up}}}{\partial z} = \varepsilon_{\text{up}}^{\text{ini}} (\bar{\phi} - \phi_{\text{up}}) \quad (6.18)$$

where  $\phi$  stands either for the dry static energy or the total water specific humidity. Similar to [Jakob and Siebesma \(2003\)](#) the entrainment rate for the test parcel for shallow convection is set to  $\varepsilon_{\text{up}}^{\text{ini}} = (\frac{0.8}{z} + 2 \times 10^{-4})$  and is now the same as that used for the updraught parcel in the boundary-layer scheme (Chapter 3). Additionally, a temperature  $\Delta T_{\text{up}}$  and moisture excess  $\Delta q_{\text{up}}$  with respect to the environment is given to the test parcel at the lowest model level depending on the surface sensible and latent turbulent heat fluxes

$$\Delta T_{\text{up}}^{\text{shal}} = -1.5 \frac{J_s}{\rho c_p w_*} \quad \text{and} \quad \Delta q_{\text{up}}^{\text{shal}} = -1.5 \frac{J_q}{\rho L w_*} \quad (6.19)$$

where the convective-scale velocity  $w_*$  is given as

$$w_* = 1.2 \left( u_*^3 - 1.5 \frac{gz\kappa}{\rho \bar{T}} \left[ \frac{J_s}{c_p} + 0.61 \bar{T} \frac{J_q}{L} \right] \right)^{\frac{1}{3}} \quad (6.20)$$

with  $\kappa = 0.4$  the von Kármán constant; the friction velocity  $u_*$  is set to a constant value of  $0.1 \text{ ms}^{-1}$ . The convective-scale velocity  $w_*$  is also used to initialise the updraft vertical velocity at the first model level. A grid column is then identified as shallow convective if a LCL is found for the surface parcel, if the updraft vertical velocity at the LCL (obtained by solving the kinetic energy equation (6.10)) is positive, and if the cloud thickness is smaller than 200 hPa.

Next, the occurrence of deep convection is tested for by repeating the updraught computations but starting at the next higher model level. However, the entrainment rate is now set as similar to the first full updraught computation, i.e.  $\varepsilon_{\text{up}}^{\text{ini}} = 0.4 \times \varepsilon_{\text{up}}^{(1)} \left( q_{\text{sat}}(\bar{T}) / q_{\text{sat}}(\bar{T}_{\text{surf}}) \right)^3$ . The simplified microphysics is taken into account by removing at each level 50% of the condensed water; the initial parcel perturbations are specified as

$$\Delta T_{\text{up}}^{\text{deep}} = 0.2 \text{ K} \quad \text{and} \quad \Delta q_{\text{up}}^{\text{deep}} = 1 \times 10^{-4} \text{ kg kg}^{-1} \quad (6.21)$$

and the updraught vertical velocity at the departure level is initialised to  $1 \text{ ms}^{-1}$ . Furthermore, in the lowest 60 hPa of the atmosphere that typically correspond to the mixed-layer depth over oceanic regions, the updraught values of the dry static energy (or humidity) at the departure level  $k$  are initialised as  $s_{\text{up},k} = \tilde{s}_k + c_p \Delta T_{\text{up}}^{\text{deep}}$ , where the tilde symbol represents a 50 hPa layer average, instead of  $s_{\text{up},k} = \bar{s}_k + c_p \Delta T_{\text{up}}^{\text{deep}}$  as for departure levels above the assumed 60 hPa mixed-layer. The idea behind is that deep convection requires a sufficiently deep source layer, this procedure also avoids spurious convection in the early morning hours when the surface-layer undergoes strong heating. A grid-column is then identified as deep-convective, if a LCL is found and the resulting cloud (the top being defined as the level where the updraught vertical velocity vanishes) is thicker than 200 hPa. If this criterion is verified the cloud is identified as deep and the results obtained for the shallow convective test parcel are ignored (only one cloud type can exist). If no deep convective cloud is found for the given departure level, the procedure is repeated starting from the next higher model level and so on until the departure level of the test parcel is more than 350 hPa above ground. A summary of this procedure, and a discussion of the consequences for the simulation of the diurnal cycle of convection over land is given in [Bechtold \*et al.\* \(2004\)](#).

Finally, if neither deep nor shallow convection has been found, elevated (or mid-level) convection is tested for (see [Subsection 6.4.3](#)). Also, at the end of this procedure and if a column has been identified as convective, the computed values of the updraught vertical velocity, dry static energy, liquid water

and specific humidity at cloud base are used to initialise the following full updraught computation at cloud base. The updraught values of the horizontal wind components at cloud base are simply set to the environmental values at the level just below (see [Section 6.9](#)).

In the following, the determination of the convective activity (as controlled by the cloud-base mass flux) is discussed separately for each type of convection.

#### 6.4.1 Deep convection

Following the derivation in [Bechtold \*et al.\* \(2014\)](#) that included earlier work by [Donner and Philips \(2003\)](#), [Nordeng \(1994\)](#) and [Gregory \*et al.\* \(2000\)](#), an equilibrium is assumed between the large-scale and boundary-layer forcing (generating convective available potential energy CAPE) and convection (reducing the CAPE). As a measure for the CAPE ( $\text{J kg}^{-1}$ ) we use the density weighted buoyancy integral of an entraining ascending air parcel, denoted as PCAPE ( $\text{J m}^{-3}$ )

$$\text{PCAPE} = - \int_{p_{\text{base}}}^{p_{\text{top}}} \left( \frac{T_{v,\text{up}} - \bar{T}_v}{\bar{T}_v} - l_{\text{up}} \right) dp \quad (6.22)$$

The advantage of PCAPE over an entraining CAPE is the density scaling that more readily relates the time derivative of PCAPE to the convective mass flux. Furthermore, assuming that convection reduces PCAPE over a time-scale  $\tau$  toward a reference value  $\text{PCAPE}_{\text{bl}}$ , one can relate the reduction in PCAPE to the convective tendency expressed as the vertical advection of the environmental virtual temperature by the mass flux.

$$\begin{aligned} \left( \frac{\partial \text{PCAPE}}{\partial t} \right)_{\text{cu}} &= - \frac{\text{PCAPE} - \text{PCAPE}_{\text{bl}}}{\tau} = \\ &= - \int_{z_{\text{base}}}^{z_{\text{top}}} \frac{g}{\bar{T}_v} \bar{\rho} \left( \frac{\partial \bar{T}_v}{\partial t} \right)_{\text{cu}} dz \approx - \int_{z_{\text{base}}}^{z_{\text{top}}} M \frac{g}{\bar{T}_v} \left( \frac{\partial \bar{T}_v}{\partial z} + \frac{g}{c_p} \right) dz \end{aligned} \quad (6.23)$$

where

$$M = M_{\text{up}} + M_{\text{down}} = \frac{M_{\text{base}}}{M_{\text{base}}^*} M^* \quad (6.24)$$

The ratio between the actual (final) cloud base mass flux, and the unit (initial) cloud base mass flux  $M_{\text{base}}/M_{\text{base}}^*$  is the convective scaling or closure factor. The initial mass flux profile  $M^*$  is known from the updraught computation starting at cloud base with value  $M_{\text{base}}^* = 0.1 \Delta p_{\text{base}} / (g \Delta t)$ , with  $\Delta t$  the model time step.

Substituting (6.24) in (6.23) results in an expression for the “final” cloud base mass flux given by

$$M_{\text{base}} = M_{\text{base}}^* \frac{\text{PCAPE} - \text{PCAPE}_{\text{bl}}}{\tau} \frac{1}{\int_{z_{\text{base}}}^{z_{\text{top}}} \frac{g}{\bar{T}_v} M^* \left( \frac{\partial \bar{T}_v}{\partial z} + \frac{g}{c_p} \right) dz}; \quad M_{\text{base}} \geq 0 \quad (6.25)$$

The convective adjustment time scale  $\tau$  is determined as follows. In cycles prior to 32r3 it was set close to or larger than the model time step. From cycle 32r3 onward it is set proportional to the convective turnover time scale  $\tau_c$

$$\tau = \frac{H}{\bar{w}_{\text{up}}} \quad \alpha_{n_T} = \tau_c \quad \alpha_{n_T}, \quad \alpha_{n_T} = (1 + 264/n_T), \quad 720 \leq \tau \leq 10800 \text{ s} \quad (6.26)$$

where  $H$  is the cloud depth,  $\bar{w}_{\text{up}}$  is the updraught velocity averaged over the cloud depth, and  $\alpha_{n_T}$  is a proportionality factor depending on horizontal resolution (model truncation  $n_T$ ) so that the adjustment time scale varies by roughly a factor of two between model truncations T799 and T159. An absolute lower bound of 720 s for  $\tau$  also helps to facilitate the transition to more resolved convection at horizontal resolutions  $n_T \geq 1279$  when the model time step becomes smaller than 720 s.

The  $\text{PCAPE}_{\text{bl}}$  term requires further elaboration. In model cycles prior to 40r1 this term has not been considered, implying that deep convection removes all PCAPE [Gregory \*et al.\* \(2000\)](#). This assumption

obviously fails in non-equilibrium situations with rapidly varying boundary-layer forcing such as the diurnal cycle of convection and the convection tied to advective boundary-layers. As shown in [Shutts and Gray \(1999\)](#) deep convection scales with the surface buoyancy flux and therefore for convection rooting in the boundary-layer (lowest 50 hPa). [Bechtold \*et al.\* \(2014\)](#) have expressed the departure from equilibrium by  $PCAPE_{bl}$  but using the integrated tendencies instead of the surface flux

$$PCAPE_{bl} = -\tau_{bl} \frac{1}{T_*} \int_{p_{surf}}^{p_{base}} \left. \frac{\partial \bar{T}_v}{\partial t} \right|_{bl} dp \quad (6.27)$$

The tendency  $\partial \bar{T}_v / \partial t|_{bl}$  includes all boundary-layer processes other than convection, i.e. turbulent diffusion, radiation and advection; therefore the corresponding tendencies must be available before the call of the convection scheme. The temperature scale  $T_* = c_p^{-1} g H$  is set to 1 and different boundary-layer time-scales  $\tau_{bl}$  are used for land and water, i.e. over land the convective turnover time-scale is used and for water the advective time-scale

$$\begin{aligned} \tau_{bl} &= \tau_c \quad \text{land} \\ \tau_{bl} &= \frac{z_{base}}{\bar{u}_{bl}} \quad \text{water} \end{aligned} \quad (6.28)$$

where  $z_{base}$  is the cloud base height and  $\bar{u}_{bl}$  the average horizontal wind speed in the subcloud layer bounded to a lower limit of  $2 \text{ m s}^{-1}$ . With this definition  $PCAPE_{bl}$  can also be seen as an efficient sine filter on PCAPE.  $PCAPE_{bl}$  is set to zero for convection rooting above the boundary-layer.

Once  $M_{base}$  is computed all deep convective updraught and downdraught fluxes of mass, energy, momentum and water are rescaled by the closure factor  $M_{base}/M_{base}^*$ .

#### 6.4.2 Shallow convection

A distinction between deep and shallow convection is made on the basis of the first-guess convective cloud depth. If the cloud extends over more than 200 hPa then convection is classified as deep, and shallow otherwise. This distinction is only necessary for the closure and the specification of the entrainment rates that are a factor of two larger for shallow convection (see [Section 6.3.1](#)). Typical examples of shallow convection are trade-wind cumuli under a subsidence inversion, convection occurring in the ridge region of tropical easterly waves, daytime convection over land and in oceanic regions during cold air outbreaks. This type of convection is effectively controlled by the surface fluxes. In fact, most of the diagnostic studies carried out for trade-wind cumuli show that the net upward moisture flux at cloud-base level is nearly equal to the turbulent moisture flux at the surface ([Le Mone and Pennell, 1976](#)).

As shown in [Bechtold \*et al.\* \(2014\)](#) the shallow convective mass flux can be derived in a similar way to the deep convective mass flux but using a balance assumption for the subcloud-layer. Using the moist static energy budget one obtains

$$\frac{1}{g} \int_{p_{surf}}^{p_{base}} \left. \frac{\partial \bar{h}}{\partial t} \right|_{cu} dp = \left. F_h \right|_{p_{surf}} = M_{base}^* (h_{up} - \bar{h})_{base} = -\frac{1}{g} \int_{p_{surf}}^{p_{base}} \left. \frac{\partial \bar{h}}{\partial t} \right|_{bl} dp; \quad M_{base}^* \geq 0 \quad (6.29)$$

with  $F_h$  the convective moist static energy flux (assumed to be zero at the surface) and

$$\bar{h} = c_p \bar{T} + L \bar{q} + gz \quad (6.30)$$

The rhs of [6.29](#), i.e. the integrated tendency of the moist static energy contains the contributions from the advection, the radiation and the boundary-layer diffusion (surface fluxes), The final mass flux  $M_{base}$  is quasi-identical to the initial mass flux  $M_{base}^*$  apart from a correction due to downdraught effects. It is computed using [\(6.29\)](#) but with replacing the cloud base excess  $(h_{up} - \bar{h})$  by  $(h_{up} - \beta h_{down} - (1 - \beta)\bar{h})$ , with  $\beta = (M_{down}/M^*)_{base}$ .

The mass flux scaling depends on the choice of the variable. Using  $h$  assures that  $(h_{up} - \bar{h})$  is positive but produces larger mass flux than when considering the budget for  $q$  instead. One cannot use the budget for  $T_v$  as the updraught buoyancy excess at cloud base is mostly negative, but we could have also used the scaling in [\(6.27\)](#) to obtain very similar mass fluxes to [\(6.29\)](#).

Again, once  $M_{\text{base}}$  is computed all deep convective updraught and downdraught fluxes of mass, energy, momentum and water are rescaled by the closure factor  $M_{\text{base}}/M_{\text{base}}^*$ .

### 6.4.3 Mid-level convection

Mid-level convection is considered to occur at rain bands at warm fronts and in the warm sector of extratropical cyclones (Browning *et al.*, 1973; Houze *et al.*, 1976; Herzegh and Hobbs, 1980). These cells are probably formed by the lifting of low level air until it becomes saturated (Wexler and Atlas, 1959) and the primary moisture source for the clouds is from low-level large-scale convergence (Houze *et al.*, 1976). This occurs often in connection with mesoscale circulations which might be related to conditionally symmetric instability (Bennets and Hoskins, 1979; Bennets and Sharp, 1982) or a wave-CISK mechanism (Emanuel, 1982).

Here we use a parametrization which in a simple way considers the finding of the diagnostic studies mentioned above. We assume that mid-level convection can be activated in a height range between  $0 < z < 1 \times 10^4$  m when there is a large-scale ascent, and the environmental air is sufficiently moist, i.e. of relative humidity in excess of 80%.

However, in practice mid-level convection is often activated in moist layers near the top of the boundary-layer if no shallow convection is found. In order to assure a continuity with the shallow closure, the convective mass flux is set to the maximum of the contribution by either the large-scale vertical mass transport or the surface enthalpy flux

$$M_{\text{base}} = \max\left(-\bar{\omega}_{\text{base}}, (J_s + J_q)/(gz_{\text{base}})\right); \quad M_{\text{base}} \geq 0 \quad (6.31)$$

with  $\omega$  the large-scale vertical velocity in pressure coordinates. The mid-level closure is the only place in the scheme where the convection is directly linked to the grid-scale vertical velocity field.

## 6.5 SUB-CLOUD LAYER

The first level at which convective mass, momentum and thermodynamic fluxes are estimated is cloud base. To represent the effects of convective updraughts on the sub-cloud layer a simple scaling of cloud base fluxes is applied in which they decrease to zero at the surface through the sub-cloud layer.

Care must be taken to ensure that fluxes of liquid water are zero below cloud base. Through the cloud base level an interpolation of the fluxes of liquid water static energy and total water content is used to estimate fluxes of dry static energy and water vapour mixing ratio in the level immediately below cloud base;

$$\begin{aligned} (Ms)_{\text{up}}^{\text{base}+1} &= (Z^n)(Ms)_{\text{up}}^{\text{base}} - L(Ml)_{\text{up}}^{\text{base}} \\ (Mq)_{\text{up}}^{\text{base}+1} &= (Z^n)(Mq)_{\text{up}}^{\text{base}} + (Ml)_{\text{up}}^{\text{base}} \\ (Ml)_{\text{up}}^{\text{base}+1} &= 0 \end{aligned} \quad (6.32)$$

where  $\phi^{\text{base}+1}$  refers to the value of  $\phi$  at the level immediately below cloud base.  $Z$  is given by

$$Z = \left(\frac{p_{\text{surf}} - p_{\text{base}+1}}{p_{\text{surf}} - p_{\text{base}}}\right)^m \quad (6.33)$$

and  $p_{\text{surf}}$  is the surface pressure.

For deep and shallow convection  $m$  is set to 1 (implying a linear decrease in the flux with pressure below cloud base) while for mid-level convection  $m$  is equal to 2 (implying a quadratic reduction in flux below cloud base).

For the remainder of the sub-cloud layer, fluxes at level 'B + 1' are reduced to zero at the surface using  $Z$  recomputed as

$$Z = \left(\frac{p_{\text{surf}} - p_k}{p_{\text{surf}} - p_{\text{base}+1}}\right)^m \quad (6.34)$$

where  $p_k$  is the pressure at level model  $k$ .

The cloud-mass and momentum fluxes in the sub-cloud layer are treated in a similar manner.

## 6.6 CLOUD MICROPHYSICS

### 6.6.1 Condensation rate in updraughts

The updraught condensation rate  $c_{\text{up}}$  is computed through a saturation adjustment

$$c_{\text{up}} = \frac{g}{\Delta p} (q_{\text{up}} - \hat{q}_{\text{up}}) M_{\text{up}} \quad (6.35)$$

where  $q_{\text{up}}$  is the value of the specific humidity before the saturation adjustment, and  $\hat{q}_{\text{up}}$  is the specific humidity at saturation after the adjustment.

### 6.6.2 Freezing in convective updraughts

We assume that condensate in the convective updraughts freezes in the temperature range  $250.16 \text{ K} < T < 273.16 \text{ K}$  maintaining a mixed phase within that range according to (7.6) (see Chapter 7 ‘Clouds and large-scale precipitation’).

### 6.6.3 Generation of precipitation

The conversion from cloud water/ice to rain/snow is treated in a consistent way with that in the large-scale precipitation scheme by using a formulation following Sundqvist (1978)

$$G^{\text{precip}} = \frac{M_{\text{up}}}{\bar{\rho}} \frac{c_0}{0.75 w_{\text{up}}} l_{\text{up}} [1 - \exp\{-(l_{\text{up}}/l_{\text{crit}})^2\}] \quad (6.36)$$

where  $c_0 = 1.4 \times 10^{-3} \text{ s}^{-1}$  and  $l_{\text{crit}} = 0.5 \text{ g kg}^{-1}$ .  $w_{\text{up}}$  is the updraught vertical velocity and is limited to a maximum value of  $10 \text{ m s}^{-1}$  in (6.36). Conversion only proceeds if  $l_{\text{up}}$  is greater than a threshold liquid water content of  $0.3 \text{ g kg}^{-1}$  over water and  $0.5 \text{ g kg}^{-1}$  over land to prevent precipitation generation from small water contents. With this value the updraft condensate content is probably still overestimated. However, with even larger values of the conversion coefficient the precipitation efficiency of the convection scheme would be too high, and the detrainment of cloud condensate too low.

Sundqvist (1978) takes into account the Bergeron–Findeisen process for temperatures below  $-5^\circ\text{C}$  through a temperature dependent modification of  $c_0$  and  $l_{\text{crit}}$  given by

$$\begin{aligned} c'_0 &= c_0 c_{\text{BF}} \\ l'_{\text{crit}} &= l_{\text{crit}}/c_{\text{BF}} \end{aligned} \quad (6.37)$$

where

$$\begin{aligned} c_{\text{BF}} &= 1 + 0.5 \sqrt{\min(T_{\text{BF}} - T_{\text{up}}, T_{\text{BF}} - T_{\text{ice}})} & \text{for } T < T_{\text{BF}} \\ c_{\text{BF}} &= 1 & \text{for } T > T_{\text{BF}} \end{aligned} \quad (6.38)$$

with  $T_{\text{BF}} = 268.16 \text{ K}$  and  $T_{\text{ice}} = 250.16 \text{ K}$ .

Equation (6.36) is integrated analytically in the vertical using the generic differential equation  $dl/dz = -al + b$ , where  $l$  is the cloud water,  $a = G^{\text{precip}} \bar{\rho} / (l_{\text{up}} M_{\text{up}})$ , and  $b = c_{\text{up}} \Delta t$ . The analytical solution is then given by  $l = l_0 \exp^{-az} + b/a(1 - \exp^{-az})$ .

### 6.6.4 Fallout of precipitation

The fallout of rain water/snow is parametrized as (e.g. Kuo and Raymond, 1980)

$$S_{\text{fallout}} = \frac{g}{\Delta p} M_{\text{up}} \frac{V}{w_{\text{up}}} r_{\text{up}} \quad (6.39)$$

where  $\Delta p$  is the model layer depth. The terminal velocity  $V$  is parametrized as (Liu and Orville, 1969)

$$V = 21.18 r_{\text{up}}^{0.2} \quad (6.40)$$

Since the fall speed of ice particles is smaller than that of water droplets, only half the value of  $V$  calculated with (6.40) is used for ice. In estimating the fallout of precipitation in the mixed phase region of the cloud a weighted mean of the fall speed for ice and water precipitation is used. Equation (6.39) is integrated in the vertical with the same analytical framework as (6.36).

### 6.6.5 Evaporation of rain

The evaporation rate of convective rain below cloud base is activated when the relative humidity  $RH$  in the environment drops below 90% over water and 70% over land. It is parametrized following Kessler (1969), where the evaporation is assumed to be proportional to the saturation deficit ( $\bar{q}_{\text{sat}} - \bar{q}$ ) and to be dependent on the density of rain  $\rho_{\text{rain}}$  ( $\text{gm}^{-3}$ )

$$e_{\text{subcl}} = \alpha_1 (RH\bar{q}_{\text{sat}} - \bar{q}) \rho_{\text{rain}}^{13/20} \quad (6.41)$$

where  $\alpha_1$  is a constant being zero for  $\bar{q} > RH\bar{q}_{\text{sat}}$ .

As the density of rain  $\rho_{\text{rain}}$  is not given by the model it is convenient to express it in terms of the precipitation flux  $P$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ) as

$$P = \rho_{\text{rain}} V_{\text{rain}} \quad (6.42)$$

where  $V_{\text{rain}}$  is the mean fall speed of rain drops which again is parametrized following Kessler (1969).

$$V_{\text{rain}} = \alpha_2 \rho_{\text{rain}}^{1/8} / \sqrt{p/p_{\text{surf}}} \quad (6.43)$$

(Note that this is different from the formulation used in the estimation of the fallout of precipitation.)

Considering that the convective rain takes place only over a fraction  $C_{\text{conv}}$  of the grid area, the evaporation rate at level  $k$  becomes

$$e_{\text{subcl}} = C_{\text{conv}} \alpha_1 (RH\bar{q}_{\text{sat}} - \bar{q}) \left[ \frac{\sqrt{p/p_{\text{surf}}}}{\alpha_2} \frac{P}{C_{\text{conv}}} \right]^{\alpha_3} \quad (6.44)$$

where the constants have the following values (Kessler, 1969)

$$\alpha_1 = 5.44 \times 10^{-4} \text{ s}^{-1} \quad \alpha_2 = 5.09 \times 10^{-3} \quad \alpha_3 = 0.5777$$

and where for the fractional area of precipitating clouds a constant value of  $C_{\text{conv}} = 0.05$  is assumed.

### 6.6.6 Melting and freezing of precipitation

Melting of snow falling across the freezing level  $T_0$  is parameterized by a simple relaxation towards  $T_0$  so that

$$M_{\text{elt}} = \frac{c_p (\bar{T} - T_0)}{L_f \tau} \Delta p \quad (6.45)$$

where  $M_{\text{elt}}$  is the rate of melting and  $\tau_{\text{melt}}$  is a relaxation time scale which decreases with increasing temperature

$$\tau_{\text{melt}} = \frac{\tau_m}{\{1 + 0.5(\bar{T} - \bar{T}_0)\}} \quad (6.46)$$

where  $\tau_m = 11800$  s. The parametrization may produce melting over a deeper layer than observed (Mason, 1971) but this has been intentionally introduced to account implicitly for the effects of vertical mixing which may develop in response to the production of negative buoyancy.

## 6.7 LINK TO CLOUD SCHEME

Before the introduction of the prognostic cloud scheme (see Chapter 7 ‘Clouds and large-scale precipitation’) water detrained from convection ( $D_{\text{up}} l_{\text{up}}$ ) was evaporated instantaneously. However with the prognostic cloud scheme water detrained from convection is a source of cloud mass increasing the water content of clouds. Therefore

$$\frac{\partial \bar{l}}{\partial t} = D_{\text{up}} l_{\text{up}} \quad (6.47)$$

where  $\bar{l}$  is the grid-box mean cloud water.

## 6.8 MOMENTUM TRANSPORT AND KINETIC ENERGY DISSIPATION

Equation set (6.3) includes a treatment of the vertical transport of horizontal momentum by convection. Studies have shown that for deep convection momentum transports are overestimated by the plume models unless the effects of cloud scale horizontal pressure gradients are included (Gregory *et al.*, 1997). For unorganized convection the effects of the pressure gradients are to adjust the in-cloud winds towards those of the large-scale flow. Pressure gradient effects might be represented by an enhanced entrainment rate in the updraught equations. For mass continuity the detrainment rate must then also be increased by an equivalent amount

$$\begin{aligned} E_{\text{up}}^{(u,v)} &= E_{\text{up}} + \lambda D_{\text{up}} \\ D_{\text{up}}^{(u,v)} &= D_{\text{up}} + \lambda D_{\text{up}} \end{aligned} \quad (6.48)$$

where  $D_{\text{up}}$  is given by (6.4).

Prior to Cy40r3  $\lambda = 2$  for deep and mid-level convection and  $\lambda = 0$  otherwise (Gregory, 1997). However, the revision of the organized detrainment in Cy40r3 has allowed to set  $\lambda = 0$  for all types of convection. The momentum formulation limits the momentum transports to be downgradient. Upgradient transports by highly organized convective systems (e.g. African squall lines) are not captured by this method.

The definition of the horizontal wind in the updraught and downdraught at and below cloud base and LFS is not well known. For the updraught, the value at cloud base is set to the environmental value at the departure level. For the downdraught, the initial values at the LFS are set equal to the average values of the winds in the updraught and those of the large-scale flow. The updraught values below cloud base are derived assuming a linear decrease of the fluxes from their cloud base value to zero at the surface. Finally, in order to correct for an apparent low-bias in the near surface wind speeds with the present linear flux relation (quasi-linear in case of an implicit time discretisation see Section 6.10), the updraught velocities are decreased by a constant perturbation  $u_{\text{pert}} = 0.3 \text{ m s}^{-1}$

$$\begin{aligned} u_{\text{up}} &= u_{\text{up}} - u_{\text{pert}} \text{sign}(\bar{u}) \\ v_{\text{up}} &= v_{\text{up}} - u_{\text{pert}} \text{sign}(\bar{v}). \end{aligned} \quad (6.49)$$

Finally, with the introduction of Cy36r4 we have included the dissipation of the kinetic energy as a consequence of the convective momentum transport as an additional large-scale heat source as the convective momentum transport conserves momentum but not energy. The total kinetic energy dissipation  $D_{\text{st}}$  ( $\text{W m}^{-2}$ ) in a model column can be estimated as

$$D_{\text{st}} \approx - \left( \frac{\partial K}{\partial t} \right)_{\text{cu}} \approx \int_{P_{\text{surf}}}^0 \left( \bar{u} \left( \frac{\partial u}{\partial t} \right)_{\text{cu}} + \bar{v} \left( \frac{\partial v}{\partial t} \right)_{\text{cu}} \right) \frac{dp}{g} \quad (6.50)$$

A more precise formulation of the dissipation and discussion is provided in Steinheimer *et al.* (2007). Unfortunately one does not really know where the dissipation actually occurs. But one can reasonably distribute the dissipation over the model column using the module of the tendencies to obtain an additional convective heating due to kinetic energy dissipation as

$$\left( \frac{\partial \bar{T}}{\partial t} \right)_{\text{cu}} = c_p^{-1} D_{\text{st}} g f(p); \quad f(p) = \frac{\sqrt{\left( \frac{\partial u}{\partial t} \right)_{\text{cu}}^2 + \left( \frac{\partial v}{\partial t} \right)_{\text{cu}}^2}}{- \int_{P_{\text{surf}}}^0 \sqrt{\left( \frac{\partial u}{\partial t} \right)_{\text{cu}}^2 + \left( \frac{\partial v}{\partial t} \right)_{\text{cu}}^2} dp} \quad (6.51)$$

## 6.9 VERTICAL DISCRETIZATION OF THE MODEL EQUATIONS

The flux divergence in the large-scale budget equations (6.1) and in the cloud equations (6.3) and (6.14) are approximated by centred finite differences as

$$g \frac{\partial(M\phi)}{\partial p} = \frac{g}{\Delta p} (M_{k+1/2} \phi_{k+1/2} - M_{k-1/2} \phi_{k-1/2}), \quad \Delta p = p_{k+1/2} - p_{k-1/2} \quad (6.52)$$

Furthermore, the updraught/downdraught equations (6.3) and (6.14) including the entrainment/detrainment terms are discretized as

$$\begin{aligned} \frac{g}{\Delta p} (M_{\text{up},k-1/2} \phi_{\text{up},k-1/2} - M_{\text{up},k+1/2} \phi_{\text{up},k+1/2}) &= E_{\text{up}} \bar{\phi}_{k+1/2} - D_{\text{up}} \phi_{\text{up},k+1/2} \\ \frac{g}{\Delta p} (M_{\text{down},k+1/2} \phi_{\text{down},k+1/2} - M_{\text{down},k-1/2} \phi_{\text{down},k-1/2}) &= E_{\text{down}} \bar{\phi}_{k-1/2} - D_{\text{down}} \phi_{\text{down},k-1/2} \end{aligned} \quad (6.53)$$

The updraught equation is solved for  $\phi_{\text{up},k-1/2}$  and the downdraught equation for  $\phi_{\text{down},k+1/2}$ . Note that with the definition (6.5) the terms  $E_{\text{down}}$  and  $D_{\text{down}}$  are negative. For the horizontal wind components and for tracers, the half-level environmental values are defined as shifted full-level values, i.e.  $\bar{\phi}_{k+1/2} = \bar{\phi}_k$  and  $\bar{\phi}_{k-1/2} = \bar{\phi}_{k-1}$ . For temperature (dry static energy) and humidity, the half-level environmental values are determined by downward extrapolation from the next full level above along a cloud-ascent through that level giving

$$\left. \begin{aligned} \bar{T}_{k+1/2} &= \bar{T}_k + \left( \frac{\partial \bar{T}}{\partial p} \right)_{h_{\text{sat}}} (p_{k+1/2} - p_k) \\ \bar{q}_{k+1/2} &= \bar{q}_k + \left( \frac{\partial \bar{q}}{\partial p} \right)_{h_{\text{sat}}} (p_{k+1/2} - p_k) \end{aligned} \right\} \quad (6.54)$$

where  $h_{\text{sat}} = c_p T + gz + Lq_{\text{sat}}$  is the saturation moist static energy. Using an extrapolation like (6.54) for calculating the subsidence of environmental air assures smooth profiles, and is also more consistent with the calculation of the updraughts where cloud air is transported upwards through level  $k+1/2$  with the thermal state below that level and equally with the downdraughts which depend only on values of  $s$  and  $q$  above that level. Similarly, because of (6.54) the subsidence of environmental air through the same level accounts now only for thermal properties above that level. The choice of a moist adiabat for extrapolation is dictated by the property of the moist static energy which is, by convection in the absence of downdraughts, only changed through the fluxes of moist static energy

$$\left( \frac{\partial \bar{h}}{\partial t} \right)_{\text{cu}} = g \frac{\partial}{\partial p} [M_{\text{up}}(h_{\text{up}} - \bar{h})] \quad (6.55)$$

As the lines of the saturation moist static energy  $h_{\text{sat}}$  through point  $(p_{k+1/2}, \bar{T}_{k-1/2})$  and the updraught moist static energy are almost parallel, apart from entrainment effects, the difference  $h_{\text{up}} - \bar{h}$  is little affected by the vertical discretization.

The ascent in the updraughts is obtained by vertical integration of (6.3). Starting at the surface the condensation level (equal to the lowest half-level which is saturated or supersaturated and where updraught velocity is positive) is determined from an adiabatic ascent. The cloud profile above cloud base is determined layer by layer by first doing a dry adiabatic ascent with entrainment and detrainment included and then adjusting temperature and moisture towards a saturated state, taking into account condensation and freezing processes. The buoyancy of the parcel is calculated taking into account the effects of cloud and precipitation water loading so that

$$B = T_{\text{up}}(1 + 0.608q_{\text{up}} - l_{\text{up}} - r_{\text{up}}) - \bar{T}(1 + 0.608q_e) \quad (6.56)$$

Special care has to be taken in the discretization of (6.10) because of overshooting effects. A centred differencing scheme is used so that

$$\begin{aligned} \frac{K_{\text{up},k-1/2} - K_{\text{up},k+1/2}}{z_{k-1/2} - z_{k+1/2}} &= \frac{E_{\text{up},k}}{M_{\text{up},k+1/2}} (1 + \beta C_d) \{K_{\text{up},k-1/2} + K_{\text{up},k+1/2}\} \\ &+ \frac{1}{f(1 + \gamma)} \frac{1}{2} g \left[ \frac{\{T_{\text{v,up}} - \bar{T}_{\text{v}}\}_{k-1/2}}{\{T_{\text{v}}\}_{k-1/2}} + \frac{\{T_{\text{v,up}} - \bar{T}_{\text{v}}\}_{k+1/2}}{\{T_{\text{v}}\}_{k+1/2}} \right] \end{aligned} \quad (6.57)$$

Finally, we mention that for numerical reasons the environmental air must not be convectively unstably stratified so

$$\bar{s}_{k-1/2} \geq \bar{s}_{k+1/2} \quad (6.58)$$

In fact, one of the forecasts with the ECMWF global model became numerically unstable when (6.56) was not imposed.

## 6.10 TEMPORAL DISCRETIZATION

The convective tendencies for the environmental values are obtained by an explicit solution of the advection equation (6.1) written in flux form

$$\left(\frac{\partial \bar{\phi}}{\partial t}\right)_{\text{cu}} = \frac{\bar{\phi}_k^{n+1} - \bar{\phi}_k^n}{\Delta t} = \frac{g}{\Delta p} [M_{\text{up}} \phi_{\text{up}} + M_{\text{down}} \phi_{\text{down}} - (M_{\text{up}} + M_{\text{down}}) \bar{\phi}^n]_{k-1/2}^{k+1/2} \quad (6.59)$$

as the tendency (or the new environmental value  $\bar{\phi}$  at time  $n+1$ ) only depends on quantities known at time step  $n$ . However, in order for the explicit solution to be stable it must satisfy the Courant–Friedrich–Levy (CFL) criterion, and therefore the mass flux values should be limited to

$$M_{\text{up}} + M_{\text{down}} \leq \frac{\Delta p}{g \Delta t} \quad (6.60)$$

It turned out that this mass flux limit is frequently reached in the case of shallow convection and long model time steps of order  $\Delta t > 1800$  s, and that the application of this mass flux limiter contributed to a sensitivity of model results to the model time step. Therefore, from model cycle Cy26r3 onwards it was decided to relax this mass flux limiter to three times the value given by the CFL criterion in the case of shallow convection and for model time steps  $\Delta t > 1800$  s – as a further restriction this relaxed mass flux limiter is only applied to temperature and humidity, but not to the horizontal winds.

With cycle Cy31r1 onwards the convective transports are solved implicitly for chemical tracers and horizontal winds, whereas a semi-implicit formulation is used for specific humidity and dry static energy. The implicit formulation for tracers or momentum reads

$$\left(\frac{\partial \bar{\phi}}{\partial t}\right)_{\text{cu}} = \frac{\bar{\phi}_k^{n+1} - \bar{\phi}_k^n}{\Delta t} = \frac{g}{\Delta p} [M_{\text{up}} \phi_{\text{up}} + M_{\text{down}} \phi_{\text{down}} - (M_{\text{up}} + M_{\text{down}}) \bar{\phi}^{n+1}]_{k-1/2}^{k+1/2} \quad (6.61)$$

With the “shifted” vertical discretization for Tracers and horizontal winds  $\bar{\phi}_{k+1/2} = \bar{\phi}_k$  and  $\bar{\phi}_{k-1/2} = \bar{\phi}_{k-1}$ , this equation constitutes a bi-diagonal linear system with unknowns  $\bar{\phi}_k^{n+1}$  and  $\bar{\phi}_{k-1}^{n+1}$ .

However, the implicit formulation for specific humidity and dry static energy (temperature) is less straightforward, as the half-level values are non-linear functions of the full-level values (6.56). However, expressing the half-level values as a linear function of the full-level values

$$\begin{aligned} \bar{s}_{k-1/2}^{n+1} &= \bar{s}_{k-1}^{n+1} + \alpha_{k-1/2}^{(s)} \bar{s}_k^n \\ \bar{q}_{k-1/2}^{n+1} &= \bar{q}_{k-1}^{n+1} + \alpha_{k-1/2}^{(q)} q_{\text{sat}}(\bar{T}_k^n), \end{aligned} \quad (6.62)$$

with the coefficients  $\alpha^{(s)}$  and  $\alpha^{(q)}$  precomputed from

$$\begin{aligned} \bar{s}_{k-1/2}^n &= \bar{s}_{k-1}^n + \alpha_{k-1/2}^{(s)} \bar{s}_k^n \\ \bar{q}_{k-1/2}^n &= \bar{q}_{k-1}^n + \alpha_{k-1/2}^{(q)} q_{\text{sat}}(\bar{T}_k^n) \end{aligned} \quad (6.63)$$

the same bi-diagonal linear equation system as for tracers and momentum is obtained. Note that only the temperature and not the geopotential term of the dry static energy is formulated implicitly, and that the saturation specific humidity  $q_{\text{sat}}(\bar{T}_k^n)$  has been preferred to  $q_k^n$  as it is smoother and positive definite. Overall the implicit solution provides a stable solution, and smoother and non-local vertical profiles of tendencies through its inherent diffusivity. With Cy32r3 onward the mass flux CFL limit for temperature and humidity is set to 5 for horizontal resolutions below T511, and to 3 for all higher resolutions. For momentum a CFL limit of 1 is retained in order to prevent too strong surface winds.

## 6.11 DIAGNOSTICS FOR POSTPROCESSING: CAPE AND CIN

As the CAPE computed in the convection routines is only computed for convectively active model columns, but taking into account lateral entrainment and liquid water loading (6.22) it was decided to provide to forecasters a CAPE product that is horizontally more homogeneous and close in line with the actual WMO definition (i.e the CAPE corresponding to a pseudo-adiabatic ascent)

$$\text{CAPE} = \int_{z_{\text{dep}}}^{z_{\text{top}}} g \left( \frac{T_{\text{up}} - \bar{T}}{\bar{T}} \right) dz \approx \int_{z_{\text{dep}}}^{z_{\text{top}}} g \left( \frac{\theta_{e,\text{up}} - \bar{\theta}_{\text{esat}}}{\bar{\theta}_{\text{esat}}} \right) dz \quad (6.64)$$

For reasons of numerical efficiency the CAPE has been approximated using the updraught equivalent potential temperature  $\theta_e = T \left( \frac{p_0}{p} \right)^{R/c_p} \exp \left( \frac{Lq}{c_p T} \right)$  which is conserved during pseudo-adiabatic ascent, and the environmental saturated  $\theta_e$  which is a function of the environmental temperature only; a more accurate formulation of  $\theta_e$  could have been used using e.g. the temperature at the LCL and taking into account glaciation processes, but the present simple definition is of sufficient accuracy for the diagnostic purpose.

The above integral is evaluated for parcels ascending from model levels in the lowest 350 hPa initialising  $\theta_{e,\text{up}} = \bar{T}_k \left( \frac{p_0}{\bar{p}_k} \right)^{R/c_p} \exp \left( \frac{L\bar{q}_k}{c_p \bar{T}_k} \right)$  at a given "departing" model level  $k_{\text{dep}}$ ; for parcels ascending in the lowest 30 hpa, mixed layer values are used. The CAPE value retained is the maximum value from the different ascents.

The Convective Inhibition (CIN) is estimated in analogy to CAPE by retaining the negative part of the integral

$$\text{CIN} = - \int_{z_{\text{dep}}}^{z_{\text{LFC}}} g \left( \frac{T_{\text{up}} - \bar{T}}{\bar{T}} \right) dz \approx - \int_{z_{\text{dep}}}^{z_{\text{LFC}}} g \left( \frac{\theta_{e,\text{up}} - \bar{\theta}_{\text{esat}}}{\bar{\theta}_{\text{esat}}} \right) dz; \quad \theta_{e,\text{up}} - \bar{\theta}_{\text{esat}} < 0 \quad (6.65)$$

where LFC is the Level of Free Convection, approximated as the level where CAPE exceeds a small threshold. CIN is therefore positive definit. The CIN value retained is the minimum value from the different ascents with  $\text{CIN} > 0$ .

## 6.12 STRUCTURE OF CODE

The parameterization of cumulus convection is performed in subroutines shown in Fig. 6.1.

**CUCALLN:** Provides interface of routines for cumulus parametrization. It takes the input values through arguments from CALLPAR and returns updated tendencies of  $T$ ,  $q$ ,  $l$ ,  $u$ ,  $v$  and chemical Tracers, as well as convective precipitation rates.

**CUMASTRN:** Master routine for convection scheme. Also performs the convective closure and with Cy32r3 computes the momentum in the convective draughts.

**CUININ:** Initializes variables for convection scheme (including vertical interpolation to the half model levels).

**CUBASEN:** First Guess updraught. Calculates condensation level, and sets updraught base variables and first guess cloud type.

**CUASCN:** Calculates ascent in updraughts. Before Cy32r3 CUASCN has been called twice as part of an iterative procedure. With ccyle 32r3 CUASCN is only called once and the mass flux scaling is done in routine CUMASTRN. Routines CUENTR and CUBASMCN are called from CUASCN.

**CUENTR:** Calculates turbulent entrainment and detrainment rates.

**CUBASMCN:** Calculates cloud base properties of mid-level convection.

**CUDLSFN:** Calculates the level of free sinking for downdraughts.

**CUDDRAFN:** Calculates the downdraught descent.

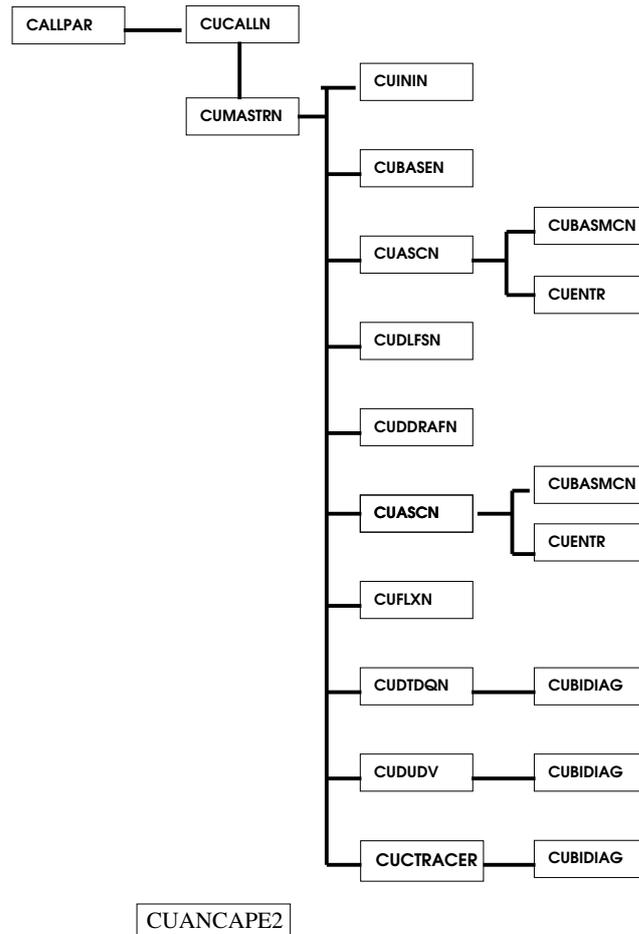


Figure 6.1 Structure of convection scheme.

**CUFLXN:** Calculates final convective fluxes and surface precipitation rates taking into account of melting/freezing and the evaporation of falling precipitation.

**CUDTDQN:** Calculates the tendencies of  $T$  and  $q$  from convection.

**CUDUDV:** Calculates the tendencies of  $u$  and  $v$  from convection.

**CUADJTQ:** Calculates super/sub saturation and adjusts  $T$  and  $q$  accordingly.

**CUCTRACER:** Calculates convective tendencies for chemical Tracers.

**CUBIDIAG:** Solver for bi-diagonal linear equation system.

**CUANCAPE2:** Computes CAPE diagnostics.

EXTERNALS

Subroutine **SATUR** for calculating saturation mixing ratio.

PARAMETERS

Defined in subroutine **SUCUM** called from INIPHY.

## APPENDIX A. LIST OF SYMBOLS

CAPE	Convective available potential energy
CIN	Convective inhibition
PCAPE	Entraining and density weighted convective available potential energy
PCAPE <sub>bl</sub>	Boundary-layer production of PCAPE
$C^i$	Convective chemical Tracer no $i$
$C_{\text{down}}^i$	Convective Tracer concentration in updraught
$C_{\text{down}}^i$	Convective Tracer concentration in downdraught
$C_d$	Drag coefficient
$C_{\text{conv}}$	Fraction of grid square occupied by convection
$c_p$	Specific at constant pressure for dry air
$c_{\text{up}}$	Condensation/sublimation in the updraughts
$c_0$	Autoconversion coefficient
$D_{\text{up}}$	Rate of mass detrainment in the updraughts
$D_{\text{down}}$	Rate of mass detrainment in the downdraughts
$D_{\text{st}}$	Total kinetic energy dissipation in mmodel column
$E_{\text{up}}$	Rate of mass entrainment in the updraughts
$E_{\text{down}}$	Rate of mass entrainment in the downdraughts
$e^{\text{rain}}$	Evaporation of rain
$e^{\text{down}}$	Evaporation of precipitation (rain and snow) in the downdraughts
$e_{\text{down}}^{\text{rain}}$	Evaporation of rain in the downdraughts
$e_{\text{down}}^{\text{snow}}$	Evaporation of snow in the downdraughts
$\tilde{e}_{\text{subcld}}$	Evaporation of precipitation (rain and snow) in the unsaturated sub-cloud layer
$f_{\varepsilon}$	multiplicative entrainment factor for shallow/mid-level convection
$f_{\text{scale}}$	vertical scaling function for the entrainment
$\tilde{e}_{\text{subcld}}^{\text{rain}}$	Evaporation of rain in the unsaturated sub-cloud layer
$\tilde{e}_{\text{subcld}}^{\text{snow}}$	Evaporation of snow in the unsaturated sub-cloud layer
$F_{\text{rez}}$	Freezing rate of condensate in the updraughts
$F_h$	moist static energy flux
$g$	gravity constant
$G^{\text{precip}}$	Conversion rate from cloud (water+ice) into precipitation (rain+snow)
$G^{\text{rain}}$	Conversion rate from cloud water into rain
$G^{\text{snow}}$	Conversion rate from cloud ice into snow
$\bar{h}$	Moist static energy ( $= c_p \bar{T} + L\bar{q} + gz$ ) in the environment
$\bar{h}_{\text{sat}}$	Saturated moist static energy in the environment
$h_{\text{up}}$	Moist static energy in the updraughts
$h_{\text{down}}$	Moist static energy in the downdraughts
$J_s$	Surface turbulent sensible heat flux
$J_q$	Surface turbulent latent heat flux
$k$	model level
$K_{\text{up}}$	Kinetic energy in the updraughts
$L$	Effective latent heat for an ice/water mix
$L_{\text{fus}}$	Latent heat of fusion
$L_{\text{subl}}$	Latent heat of sublimation
$L_{\text{vap}}$	Latent heat of vaporization
CFL	Courant–Friedrich–Levy criterium
LCL	Lifting Condensation Level
LFC	Level of Free Convection
$l_{\text{up}}$	Cloud water/ice content in the updraughts
$l_{\text{crit}}$	Cloud water/ice content above which autoconversion occurs
$M_{\text{elt}}$	Melting rate of snow
$M$	Net mass flux in the convective clouds (updraughts + downdraughts)
$M_{\text{cld}}^*$	First-guess net mass flux
$M_{\text{up}}$	Updraught mass flux
$M_{\text{down}}$	Downdraught mass flux

$n$	index for time dsicretization
$n_T$	horizontal truncation (global wavenumber)
nlev	number of vertical model levels (nlev denotes the first layer above surfac
$P^{\text{rain}}$	Net flux of precipitation in the form of rain
$P^{\text{snow}}$	Net flux of precipitation in the form of snow
$p$	Pressure
$p_0$	Reference pressure=1000 hPa
$\bar{q}$	Specific humidity of the environment
$q_{\text{up}}$	Specific humidity in the updraughts
$q_{\text{down}}$	Specific humidity in the downdraughts
$R$	Rain intensity
$RH$	Relative humidity
$r_{\text{up}}$	Precipitation (rain+snow) in the updraughts
$r_{\text{down}}$	Precipitation (rain+snow) in the downdraughts
$S_{\text{fallout}}$	Fall-out of rain/snow
$\bar{s}$	Dry static energy in the environment
$s_{\text{up}}$	Dry static energy in the updraughts
$s_{\text{down}}$	Dry static energy in the downdraughts
$\bar{T}_v$	Virtual temperature in the environment
$T_{v,\text{up}}$	Virtual temperature in the updraughts
$\bar{u}$	$u$ component of wind in the environment
$u_{\text{up}}$	$u$ component of wind in the updraughts
$u_{\text{down}}$	$u$ component of wind in the downdraughts
$u_{\text{pert}}$	additional updraught perturbation velocity
$V$	Mean terminal velocity of precipitation (rain+snow)
$V_{\text{rain}}$	Mean terminal velocity of rain drops
$\bar{v}$	$v$ component of wind in the environment
$v_{\text{up}}$	$v$ component of wind in the updraughts
$v_{\text{down}}$	$v$ component of wind in the downdraughts
$\bar{w}$	Vertical velocity in the environment
$w_{\text{up}}$	Vertical velocity in the updraughts
$w_*$	Convective velocity scale
$z$	height
$\alpha_1, \alpha_2, \alpha_3$	Microphysical constants
$\alpha_n$	Horizontal resolution dependency of the deep convective adjustment tim
$\alpha^{(s)}, \alpha^{(q)}$	Interpolation coefficients for half-level values
$\delta$	Detrainment per unit length
$\varepsilon$	Entrainment per unit length
$\eta$	Updraught mass flux fraction to initialise downdraught
$\kappa$	von Karman constant
$\rho$	Density of air
$\rho_{\text{rain}}$	Density of rain
$\tau$	Adjustment time scale
$\tau_m$	Melting time scale
$\omega$	Omega (large-scale) vertical velocity
$\Delta p$	Pressure difference between two model half-levels
$\Delta t$	Model time step

## Chapter 7

# Clouds and large-scale precipitation

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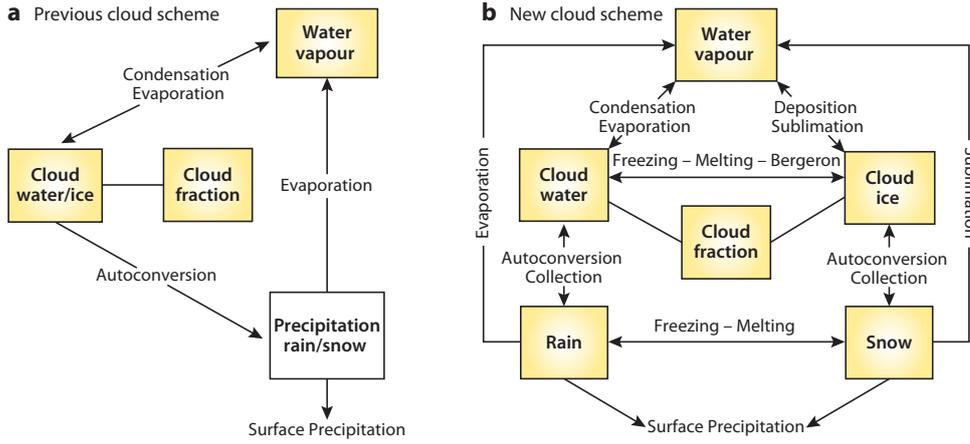
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## 7.1 INTRODUCTION

Cloud and large-scale precipitation processes are described by prognostic equations for cloud liquid water, cloud ice, rain, snow and a grid box fractional cloud cover. This new scheme was implemented in IFS Cycle 36r4 and is significantly modified from the previous version which had just two prognostic cloud variables (cloud condensate and cloud fraction). The philosophy of the original scheme (Tiedtke, 1993) is retained with regards to the prognostic cloud fraction and sources/sinks of all cloud variables due to the major generation and destruction processes, including detrainment from convection. However, liquid and ice water contents are now independent, allowing a more physically realistic representation of super-cooled liquid water and mixed-phase cloud. Rain and snow precipitation are also now able to precipitate with a determined terminal fall speed and can be advected by the three-dimensional wind. A multi-dimensional implicit solver is used for the numerical solution of the cloud and precipitation prognostic equations. Figure 7.1 shows a schematic to highlight the differences between the current and previous (pre-36r4) cloud parametrization schemes. Further discussion of the changes and impacts on the IFS forecasts can be found in Forbes and Tompkins (2011) and Forbes *et al.* (2011).

Section 2 describes the equations used to parametrize the cloud and precipitation processes. Section 3 discusses some of the numerical aspects of the parametrization formulation and Section 4 outlines the code structure. The Appendices provide further information on relevant diagnostics available, the history of key changes to the IFS cloud scheme and a list of symbols used in this documentation.



**Figure 7.1** Schematic of the IFS cloud scheme: (a) the original Tiedtke (1993) scheme with three moisture related prognostic variables, operational from 1995 to 2010 (before IFS Cy36r4) and (b) the current cloud scheme with six moisture related prognostic variables (Cy36r4 onwards). Shaded boxes indicate prognostic variables.

## 7.2 THEORY

### 7.2.1 Definitions

#### (a) Specific water contents and cloud fraction

The grid-mean specific water content for cloud liquid ( $\text{kg kg}^{-1}$ ) is defined as

$$q_l = \frac{1}{V} \int_V \frac{\rho_l}{\rho} dV \quad (7.1)$$

where  $\rho_l$  is the mass of cloud water per unit volume (density,  $\text{kg m}^{-3}$ ),  $\rho$  is the density of moist air ( $\text{kg m}^{-3}$ ) and  $V$  is the volume of the grid box ( $\text{m}^3$ ). The variables for specific humidity ( $q_v$ ), cloud ice ( $q_i$ ), rain ( $q_r$ ) and snow ( $q_s$ ) follow a similar definition. The fraction of the grid box covered by clouds is defined as

$$a = \frac{1}{V} \int_V \delta dV, \quad \delta = \begin{cases} 1, & \text{in clouds} \\ 0, & \text{otherwise} \end{cases} \quad (7.2)$$

Furthermore, the definition of the specific cloud water content per cloud area (in-cloud water/ice content) is

$$q_l^{\text{cld}} = \frac{q_l}{a} \quad (7.3)$$

The above applies also to cloud ice and the precipitation variables, rain and snow, where the cloud fraction,  $a$ , is replaced with precipitation fraction,  $a_p$ .

#### (b) Saturation specific humidity

The saturation specific humidity is expressed as a function of saturation water vapour pressure as

$$q_{\text{sat}} = \frac{\frac{R_{\text{dry}}}{R_{\text{vap}}} e_{\text{sat}}(T)}{p - \left(1 - \frac{R_{\text{dry}}}{R_{\text{vap}}}\right) e_{\text{sat}}(T)} \quad (7.4)$$

where the saturation water vapour pressure is expressed with the Tetens formula

$$e_{\text{sat}}(T) = a_1 \exp\left\{a_3 \left(\frac{T - T_0}{T - a_4}\right)\right\} \quad (7.5)$$

with the parameters set according to Buck (1981) for saturation over water ( $a_1 = 611.21 \text{ Pa}$ ,  $a_3 = 17.502$  and  $a_4 = 32.19 \text{ K}$ ) and to the AERKi formula of Alduchov and Eskridge (1996) for saturation over ice ( $a_1 = 611.21 \text{ Pa}$ ,  $a_3 = 22.587$  and  $a_4 = -0.7 \text{ K}$ ), with  $T_0 = 273.16 \text{ K}$ .

(c) *Mixed phase*

With separate prognostic variables for cloud liquid and cloud ice, both can coexist in cloud at temperatures below freezing in variable amounts dependent on the different sources and sinks (see section 7.2.4). However, there are still some cloud generating processes, such as the detrainment of convective cloud, that diagnose the phase of the cloud source as a function of temperature (as in Tiedtke (1993)). For these processes the fraction of liquid water in the total condensate is described as

$$\begin{aligned} \alpha &= 0 & T &\leq T_{\text{ice}} \\ \alpha &= \left( \frac{T - T_{\text{ice}}}{T_0 - T_{\text{ice}}} \right)^2 & T_{\text{ice}} &< T < T_0 \\ \alpha &= 1 & T &\geq T_0 \end{aligned} \quad (7.6)$$

where  $T_{\text{ice}}$  and  $T_0$  represent the threshold temperatures between which a mixed phase is allowed to exist and are chosen as  $T_{\text{ice}} = 250.16$  K and  $T_0 = 273.16$  K.

**7.2.2 Basic equations**

With the above definitions and the assumption that clouds extend vertically over the whole model layer depth, the equations for the time change of the grid-box averaged cloud liquid, cloud ice, rain and snow water contents are

$$\frac{\partial q_l}{\partial t} = A(q_l) + S_{\text{conv}} + S_{\text{strat}} + S_{\text{melt}}^{\text{ice}} - S_{\text{dep}}^{\text{ice}} - S_{\text{evap}}^{\text{liq}} - S_{\text{auto}}^{\text{rain}} - S_{\text{rime}}^{\text{snow}} \quad (7.7)$$

$$\frac{\partial q_i}{\partial t} = A(q_i) + S_{\text{conv}} + S_{\text{strat}} + S_{\text{dep}}^{\text{ice}} - S_{\text{melt}}^{\text{ice}} - S_{\text{evap}}^{\text{ice}} - S_{\text{auto}}^{\text{snow}} \quad (7.8)$$

$$\frac{\partial q_r}{\partial t} = A(q_r) - S_{\text{evap}}^{\text{rain}} + S_{\text{auto}}^{\text{rain}} + S_{\text{melt}}^{\text{snow}} - S_{\text{frz}}^{\text{rain}} \quad (7.9)$$

$$\frac{\partial q_s}{\partial t} = A(q_s) - S_{\text{evap}}^{\text{snow}} + S_{\text{auto}}^{\text{snow}} - S_{\text{melt}}^{\text{snow}} + S_{\text{frz}}^{\text{rain}} + S_{\text{rime}}^{\text{snow}} \quad (7.10)$$

and for the cloud fraction,

$$\frac{\partial a}{\partial t} = A(a) + \delta a_{\text{conv}} + \delta a_{\text{strat}} - \delta a_{\text{evap}} \quad (7.11)$$

The terms on the right-hand side of Eqns. (7.7) to (7.11) represent the following processes:

- $A(q)$ ,  $A(a)$  – rate of change of water contents and cloud area due to transport through the boundaries of the grid volume (advection, sedimentation).
- $S_{\text{conv}}$ ,  $\delta a_{\text{conv}}$  – rate of formation of cloud water/ice and cloud area by convective processes.
- $S_{\text{strat}}$ ,  $\delta a_{\text{strat}}$  – rate of formation of cloud water/ice and cloud area by stratiform condensation processes.
- $S_{\text{evap}}$  – rate of evaporation of cloud water/ice, rain/snow.
- $S_{\text{auto}}$  – rate of generation of precipitation from cloud water/ice (autoconversion).
- $S_{\text{melt}}$  – rate of melting ice/snow.
- $S_{\text{rime}}$  – rate of riming (collection of cloud liquid drops).
- $S_{\text{frz}}$  – rate of freezing of rain.
- $\delta a_{\text{evap}}$  – rate of decrease of cloud area due to evaporation.

The large-scale budget equations for specific humidity  $q_v$ , and dry static energy  $s = c_p T + gz$  in the cloud scheme are

$$\frac{\partial q_v}{\partial t} = A(q_v) - S_{\text{strat}} + S_{\text{evap}} \quad (7.12)$$

and

$$\frac{\partial s}{\partial t} = A(s) + L_{vap}(S_{strat} - S_{evap}) + L_{fus}(S_{frz} + S_{rime} - S_{melt}) \quad (7.13)$$

where  $A(q_v)$  and  $A(s)$  represent all processes except those related to clouds,  $L_{vap}$  is the latent heat of condensation and  $L_{fus}$  is the latent heat of freezing.

### 7.2.3 Microphysical assumptions

For each of the hydrometeor categories, there are assumptions about the particle size distribution, mass-diameter relationship and terminal velocity that are used in the parametrization of microphysical processes. At present, this is limited to the rain evaporation and snow riming but in subsequent versions they will be consistently used for the other microphysical processes. The general formulations are given below for a particle of diameter  $D$  for hydrometeor  $x$  and the constants for each hydrometeor category are shown in Tables 7.1, 7.2 and 7.3.

#### (a) *Mass-diameter relationship*

The mass of a particle is defined by a power law:

$$m_x(D) = a_x D^{b_x} \quad (7.14)$$

#### (b) *Particle size distribution*

The number concentration ( $\text{m}^{-3}$ ) of particles is:

$$N_x(D) = N_{0x} \exp(-\Lambda D) \quad (7.15)$$

where  $\Lambda$  is the slope of the particle size distribution and  $N_{0x}$  is the intercept parameter, assumed to be either constant or a function of  $\Lambda$ :

$$N_{0x} = n_{ax} \Lambda^{n_{bx}} \quad (7.16)$$

The specific mass integrated over all particle sizes (i.e. the prognostic variable) can therefore be described by:

$$q_x = \frac{1}{\rho} \int_{D=0}^{\infty} N_x(D) m_x(D) dD = \frac{1}{\rho} \int_{D=0}^{\infty} n_{ax} \Lambda^{n_{bx}} a_x D^{b_x} \exp(-\Lambda D) dD \quad (7.17)$$

Integrating and rearranging gives the slope of the particle size distribution  $\Lambda$  as a function of the prognostic specific mass ( $q_x$ ):

$$\Lambda = \left( \frac{n_{ax} a_x \Gamma(b_x + 1)}{q_x \rho} \right)^{1/(b_x + 1 - n_{bx})} \quad (7.18)$$

where the generalised factorial function is defined as:

$$\Gamma(y) = \int_{D=0}^{\infty} D^{y-1} \exp(-D) dD \quad (7.19)$$

#### (c) *Terminal velocity of hydrometeor sedimentation*

The terminal velocity of a hydrometeor particle is defined by a power law:

$$\nu_x(D) = c_x D^{d_x} \left( \frac{\rho_0}{\rho} \right)^{0.5} \quad (7.20)$$

where  $c_x$  and  $d_x$  are constants defined for each hydrometeor in Table 7.3. The last term accounts for the decreased drag (and therefore higher terminal velocities) in less dense air where  $\rho_0$  is a reference air density ( $=1 \text{ kg m}^{-3}$ ). Note that these terminal velocities are not yet used in the sedimentation (see section i).

Hydrometeor	$n_{ax}$	$n_{bx}$	References
Rain	0.22	2.2	Abel and Boutle (2012)
Snow	$2 \times 10^6$	0.0	

**Table 7.1** List of particle size distribution parameters for each hydrometeor.

Hydrometeor	$a_x$	$b_x$
Rain	$\pi/6$	3.0
Snow	0.069	2.0

**Table 7.2** List of mass-diameter relationship parameters for each hydrometeor.

Hydrometeor	$c_x$	$d_x$
Rain	386.8	0.67
Snow	16.8	0.527

**Table 7.3** List of hydrometeor terminal velocity parameters for each hydrometeor.

## 7.2.4 Definition of the source and sink terms

### (a) Convection

Clouds formed by convective processes are parametrized by considering them to be condensates produced in cumulus updraughts and detrained into the environmental air. This approach, besides being part of the cloud parametrization, represents also an important extension of the model's cumulus parametrization. It is applied for all types of convection, namely deep, shallow and mid-level. The source of cloud water/ice content is

$$S_{\text{conv}} = D_{\text{up}} l_{\text{up}} + \frac{M_{\text{up}}}{\rho} \frac{\partial q_l}{\partial z} \quad (7.21)$$

and is partitioned between ice and water diagnostically according to Eq. 7.6. The source of cloud area is described as

$$\delta a_{\text{conv}} = D_{\text{up}} + \frac{M_{\text{up}}}{\rho} \frac{\partial a}{\partial z} \quad (7.22)$$

where  $D_{\text{up}}$  ( $\text{s}^{-1}$ ) is the detrainment of mass from cumulus updraughts,  $l_{\text{up}}$  ( $\text{kg kg}^{-1}$ ) is the specific cloud water/ice content in cumulus updraughts and  $M_{\text{up}}$  is the updraught mass flux (see Chapter 6). The first term in (7.21) and (7.22) represents the detrainment of cloud from the convective updraughts and the second term represents the advection of cloud in the vertical due to compensating subsidence in the environmental air. The evaporation due to this subsidence is represented by term  $E_1$  described in subsection (e) below.

### (b) Formation of stratiform clouds

Here the formation of clouds by non-convective processes (e.g. large-scale lifting of moist air, radiative cooling etc.) is considered. The parametrization is based on the principle that condensation processes are determined by the rate at which the saturation specific humidity decreases. This rate is linked to vertical motions and diabatic cooling through

$$\frac{dq_{\text{sat}}}{dt} = \left( \frac{dq_{\text{sat}}}{dp} \right)_{\text{ma}} (\bar{w} + gM_{\text{Cu}}) + \left( \frac{dq_{\text{sat}}}{dT} \right) \left( \frac{dT}{dt} \right)_{\text{diab}} \quad (7.23)$$

where  $(dq_{\text{sat}}/dp)_{\text{ma}}$  is the change of  $q_{\text{sat}}$  along a moist adiabat through point  $(p, T)$ ,  $\bar{w}$  is the area-mean generalized vertical velocity,  $gM_{\text{Cu}}$  is the cumulus-induced subsidence between the updraughts, and  $(dT/dt)_{\text{diab}}$  is the net temperature tendency due to radiative and turbulent processes. The changes in any existing cloudy part of the grid box (increase in condensate,  $C_1$ ) and in the clear part of the grid box (formation of new cloud,  $C_2$ ) are treated separately.

$$S_{\text{strat}} = C_1 + C_2 \quad (7.24)$$

The condensation rate in already existing clouds ( $C_1$ ) is described as

$$C_1 = -a \frac{dq_{\text{sat}}}{dt} \quad \frac{dq_{\text{sat}}}{dt} < 0 \quad (7.25)$$

The formation of new clouds ( $C_2$ ) is more complex due to the assumption of subgrid heterogeneity of humidity in the clear air. New clouds are assumed to form when the grid-averaged relative humidity exceeds a threshold value, defined as a function of height,

$$\begin{aligned} RH_{\text{crit}} &= RH_c + (1 - RH_c) \left( \frac{\sigma - \sigma_1}{1 - \sigma_1} \right)^2 & \sigma > \sigma_1 \\ RH_{\text{crit}} &= RH_c & \sigma < \sigma_1 \end{aligned} \quad (7.26)$$

where  $RH_c = 0.8$ ,  $\sigma = p/p_{\text{surf}}$  with  $p$  being the pressure and  $p_{\text{surf}}$  the pressure at the surface and  $\sigma_1 = 0.8$ . The increase in cloud cover is determined by how much of the cloud-free area exceeds saturation in one time step which in turn depends on the moisture distribution in the cloud-free area and how fast saturation is approached. The vapour is assumed to be evenly distributed within the range  $[\{q_v^{\text{env}} - (q_{\text{sat}} - q_v^{\text{env}})\}, q_{\text{sat}}]$  around the mean environmental value  $q_v^{\text{env}}$ , while the approach to saturation is determined by  $dq_{\text{sat}}/dt$ . The rate of increase in cloud cover then becomes

$$\delta a_{\text{strat}} = \frac{1}{2} \frac{-(1-a)}{(q_{\text{sat}} - q_v^{\text{env}})} \frac{dq_{\text{sat}}}{dt} \quad \frac{dq_{\text{sat}}}{dt} < 0 \quad (7.27)$$

which can be expressed in terms of grid averages (using the definition  $q_v = aq_{\text{sat}} + (1-a)q_v^{\text{env}}$ ) as

$$\delta a_{\text{strat}} = \frac{-(1-a)^2}{2} \frac{1}{(q_{\text{sat}} - q_v)} \frac{dq_{\text{sat}}}{dt} \quad \frac{dq_{\text{sat}}}{dt} < 0 \quad (7.28)$$

For the application of (7.28) at values of  $q_v$  close to saturation, the constraint  $\delta a_{\text{strat}} < (1-a)/\Delta t$  is imposed to ensure realistic values of  $a$ .

The generation rate of cloud water/ice in newly formed clouds is then as for the generation rate in existing cloud (Eq. 7.25), except for half the total change in cloud fraction ( $\Delta a_{\text{strat}} = \delta a_{\text{strat}} \Delta t$ ) due to the sub-grid variability assumption in the clear sky:

$$C_2 = -\frac{1}{2} \Delta a_{\text{strat}} \frac{dq_{\text{sat}}}{dt} \quad \frac{dq_{\text{sat}}}{dt} < 0 \quad (7.29)$$

### (c) Ice crystal nucleation and supersaturation with respect to ice ( $T < -38^\circ\text{C}$ )

Ice crystal nucleation contrasts sharply with the equivalent process for liquid cloud droplets, since the nucleation process is not activated at small supersaturations with respect to the ice saturation vapour pressure. At cold temperatures, where the difference between the liquid water and ice saturation vapour pressures is large, the relative humidity ( $RH$ ) with respect to ice can exceed 150% before the onset of the nucleation process, and supersaturations with respect to ice are commonly observed by in-situ and remote sensing techniques (e.g. Heymsfield *et al.*, 1998; Gierens *et al.*, 1999, 2000, 2004; Spichtinger *et al.*, 2003).

The formulation for cloud generation outlined above is modified to allow supersaturation with respect to ice in the clear sky portion of the grid-cell. For temperatures below the homogeneous freezing temperature (defined as  $-38^\circ\text{C}$ ), the scheme assumes ice nucleation initiates when the  $RH$  measured with respect to ice saturation *locally* reaches the threshold  $RH_{\text{homo}}$  specified by Kärcher and Lohmann (2002):

$$RH_{\text{homo}} = 2.583 - \frac{T}{207.8} \quad (7.30)$$

where  $T$  is the temperature in Kelvin. However, as in the warm phase, the clear-sky humidity fluctuations are assumed to be uniformly distributed with a fixed constant variance. Thus nucleation can occur when the grid-mean  $RH$  exceeds a threshold that is lower than this local criterion, and is given by  $RH_{\text{crit}} \times RH_{\text{hom}}.$  For warmer temperatures ( $T > -38^\circ\text{C}$ ), the liquid water saturation mixing ratio can be lower than this, at which point liquid water droplets will nucleate (see discussion on mixed phase processes in next subsection). Thus cloud formation occurs when

$$RH > RH_{\text{crit}} \times \text{MIN} \left( RH_{\text{hom}}, \frac{q_{\text{sat(w)}}}{q_{\text{sat(i)}}} \right) \quad (7.31)$$

Once ice cloud is formed, the deposition process is considered to be sufficiently rapid relative to the model time-step that it can be approximated by a diagnostic adjustment to exactly ice saturated conditions inside the cloud. This assumption is necessary, since to allow ice supersaturation both within the pure ice cloud and in the clear sky environment would either require a separate prognostic variable to monitor the evolution of the water vapour inside the cloud, or a diagnostic assumption would have to be used to divide the grid-mean humidity between the two regions, which can generate large artificial horizontal sub-grid humidity fluxes (see [Tompkins \*et al.\*, 2007](#), for more detail). In any case, this assumption appears to be reasonably justified in a wide range of updraught situations by modelling of the homogeneous nucleation process ([Khvorostyanov and Sassen, 1998](#)). The obvious drawback is that pure ice clouds may not exist in a subsaturated or supersaturated state, and no information concerning the ice crystal number concentration is available.

Thus, if  $T < T_0$  and the relative humidity exceeds the threshold given by (7.31), the scheme calculates the increase in cloud fraction from (7.28). The associated change in cloud ice mass is calculated in two stages. The first source term is derived using (7.29). This generation term for ice mass reduces  $RH$  back to the threshold given by (7.31), and leaves the newly generated cloudy region in an ice supersaturated state. This is then corrected by clipping the grid-mean humidity to the limit of

$$q_v^{\text{max}} = q_s(a + (1 - a)RH_{\text{hom}}) \quad (7.32)$$

This clipping term has the effect of reducing the in-cloud humidity to the ice saturated value within one time-step, but allowing supersaturation in the clear air.

[Tompkins \*et al.\* \(2007\)](#) show that the supersaturation scheme, while simple, reproduces very well the climatological PDF of upper tropospheric  $RH$  derived from MOZAIC aircraft observations ([Gierens \*et al.\*, 1999](#)), as well as the geographical distribution of ice supersaturation given by MLS retrievals ([Spichtinger \*et al.\*, 2003](#)).

(d) ***Ice crystal growth by deposition in mixed phase clouds ( $T > -38^\circ\text{C}$ )***

The scheme allows supercooled liquid water to exist at temperatures warmer than the homogeneous nucleation threshold of  $-38^\circ\text{C}$ . At temperatures colder than this water droplets are assumed to freeze instantaneously. When supercooled liquid and ice are coexistent, they are assumed to be well mixed and distributed uniformly through the cloud (see [Rotstajn \*et al.\* \(2000\)](#) for a discussion of alternative assumptions). The ice crystals can then grow at the expense of the water droplets through the Wegener-Bergeron-Findeisen process. If water droplets are present, the ice crystals are in an environment supersaturated with respect to ice and grow by deposition, reducing the water vapour and leading to subsaturation with respect to water. Water droplets then evaporate and the process continues with ice growth until the water droplets are completely evaporated. Thus in mixed phase clouds, the deposition process acts as a sink of cloud liquid and a source of ice cloud. A description of the consequences of various assumptions in the formulation of this parametrization can be found in [Forbes and Ahlgrimm \(2014\)](#).

Following [Pruppacher and Klett \(1997\)](#) and [Rotstajn \*et al.\* \(2000\)](#), the rate of growth of an ice crystal of mass  $M_i$  is:

$$\frac{dM_i}{dt} = \frac{4\pi C(S_i - 1)}{A'' + B''} \quad (7.33)$$

where  $C$  is the capacitance of the particle (related to the shape) and  $S_i = e/e_{si}$  is the saturation ratio with respect to ice. Terms  $A''$  and  $B''$  represent heat conduction and vapour diffusion respectively,

$$A'' = \frac{L_{subl}}{K_a T} \left( \frac{L_{subl}}{R_v T} - 1 \right) \quad (7.34)$$

$$B'' = \frac{R_v T}{\chi e_{si}} \quad (7.35)$$

where  $K_a$  is the heat conductivity of air and  $\chi$  is the diffusivity of water vapour in air, which varies inversely with pressure as  $\chi = 2.21/p$ . If the ice crystal number concentration is  $N_i$  and the ice crystals are assumed to be monodispersed with all particles having equal diameter  $D_i$  and equal mass  $M_i$  (and therefore also equal density  $\rho_i$ ), then the cloud ice specific water content  $q_i = M_i N_i / \rho$ . If the air is at water saturation, then  $S_i = (e_{sl} - e_{si})/e_{si}$ , and from (7.33) the rate of change of  $q_i$  is

$$\frac{dq_i}{dt} = \frac{N_i}{\rho} \frac{4\pi C}{(A'' + B'')} \frac{(e_{sl} - e_{si})}{e_{si}} \quad (7.36)$$

The capacitance term  $C$  assumes ice crystals are spherical ( $C = D_i/2$ ) where  $D_i = (6M_i/\pi\rho_i)^{1/3}$ . Elimination of  $C$  from (7.36) then gives

$$\frac{dq_i}{dt} = c_{vd}^s q_i^{1/3} \quad (7.37)$$

where

$$c_{vd}^s = \left( \frac{N_i}{\rho} \right)^{2/3} \frac{7.8}{\rho_i^{1/3} (A'' + B'')} \frac{(e_{sl} - e_{si})}{e_{si}} \quad (7.38)$$

The analytical treatment of [Rotstayn \*et al.\* \(2000\)](#) is used, which assumes the temperature dependent quantities in (7.38) can be approximated as constant through the timestep. The ice condensate amount at time  $t$  can then be calculated by integrating (7.37) with respect to time, giving

$$q_i^t = \left( \frac{2}{3} c_{vd}^s \Delta t + (q_i^{t-1})^{2/3} \right)^{3/2} \quad (7.39)$$

and the deposition rate,  $S_{dep} = (q_i^t - q_i^{t-1})/\Delta t$ .

As stated in the previous section, no prognostic equation for the ice crystal concentration is introduced, and thus  $N_i$  is given diagnostically assuming the air is at water saturation according to [Meyers \*et al.\* \(1992\)](#):

$$N_i = 1000 \exp[12.96(e_{sl} - e_{si})/e_{si} - 0.639] \quad (7.40)$$

As the formulation uses only the saturation vapour pressures for water and ice, the ice crystal concentration is effectively only a function of temperature.

To initiate the glaciation process, at each gridbox, a minimum ice mass mixing ratio is assumed of

$$q_i^{min} = \frac{m_{i0} N_i}{\rho} \quad (7.41)$$

where  $m_{i0}$  is the initial mass of an ice particle and is set to  $10^{-12}$  kg.

Once the supercooled liquid water reservoir in the cloud is exhausted through the deposition process, there is a complication to consider. At the point the cloud becomes completely glaciated, the in-cloud vapour pressure is equal to  $e_{s1}$ . From this point the ice crystals would continue to grow by deposition of water vapour, until the in-cloud vapour pressure is reduced to the saturation value with respect to ice

(or an equilibrium value exceeding the saturation value in the presence of strong updraughts, see [Ren and Mackenzie, 2005](#), for example). However, unlike the mixed phase situation, where the “memory” for the vapour reservoir is provided by the prognostic cloud liquid water variable, there is no memory for the water vapour content in the cloudy region of the gridbox to model the deposition process in glaciated clouds. So, the new cloud follows the same assumption as before for pure ice cloud (see previous subsection) and the deposition of the remaining water vapour in excess of ice saturation occurs within a single timestep, bringing the cloud to exactly ice saturation.

So, in mixed phase clouds with supercooled liquid water present, the cloud is assumed to be exactly at saturation with respect to water (and therefore supersaturated with respect to ice), and when only ice is present, the cloud is assumed to be exactly at saturation with respect to ice. However, supersaturation is permitted in the clear sky portion of a gridcell in all cases.

It is this process of deposition that largely determines the partition between liquid and ice in mixed phase clouds in the scheme.

A modification to the parametrization of ice deposition rate is additionally included to represent the effects of unresolved microphysical processes in the region of cloud top. The small scale dynamical and microphysical processes are unique at cloud top due to radiative cooling enhancing turbulent production of supercooled water, the rate of ice nucleation controlling the depletion of the liquid water through the Wegener-Bergeron-Findeison mechanism, and the fall out of the growing ice crystals reducing the number of crystals and limiting the amount of ice mass in the shallow layer at cloud top. In the model, if there is sufficient resolved vertical motion or vertical transport by the sub-grid parametrizations, or local longwave radiative cooling for the model grid-box to reach water saturation, then supercooled water is produced. Ice nucleation will then occur and the deposition process will start to evaporate the supercooled water and increase the ice mass. The sedimentation term then removes a proportion of the ice from the grid box, depending on the assumed fallspeed for the ice, the depth of the model layer and time step of the model. For a model with high vertical grid resolution and a well-resolved cloud layer, the sedimentation may reduce the ice content in the layers at the top of the cloud significantly, reducing the deposition rate substantially and allowing supercooled liquid water to persist. For a model with low vertical grid resolution and a poorly resolved cloud layer, only a small portion of the total ice in the grid box may be removed due to sedimentation, and the remaining ice is implicitly redistributed vertically throughout the grid box, as the model cloud is always assumed to homogeneously fill each model layer in the vertical. Thus, the amount of ice at “cloud top” will be much higher in the low resolution case, leading to higher deposition rates than the vertically resolved case near cloud top, and less supercooled liquid water.

One solution is implemented here by finding cloud top and decreasing the deposition rate over a specified depth scale near the top of the cloud. The impact is to reduce the sink of liquid water to ice and increase the amount of supercooled liquid water at cloud top. The deposition rate is multiplied by a factor,  $F_{dep}$ , which reduces the conversion rate from supercooled liquid water to ice near cloud top, defined as

$$F_{dep} = MIN[f_{nuc} + (1 - f_{nuc})(F_{ref} + \Delta z_{cltop}/\Delta z_{refdep}), 1] \quad (7.42)$$

where  $f_{nuc}$  is a function of the activated ice nuclei concentration,  $F_{ref}$  is a reference deposition rate factor set to 0.1,  $\Delta z_{cltop}$  is the distance in metres of the model layer from the cloud top layer (cloud top defined by the presence of supercooled liquid water and a cloud fraction threshold of 1%) and  $\Delta z_{refdep}$  is a reference depth of 500m.

The formulation has two components. The first represents the reduced deposition rate near cloud top due to the unresolved microphysical processes of ice nucleation, growth and fallout, controlled by the parameters  $F_{ref}$  and  $\Delta z_{refdep}$ . The formulation in terms of distance from cloud top gives the parametrization some independence of vertical grid resolution. If the model layer depths at a particular altitude are of the same magnitude as  $\Delta z_{refdep}$ , then only the top cloud layer will be modified, but if the vertical resolution is higher, then the deposition rate factor increases linearly with layer distance from the diagnosed cloud top.

The second component represents the reduced effect of the parametrization as the number of activated ice nuclei increases with higher supersaturation. This is represented by the function  $f_{nuc}$  defined as

$$f_{nuc} = \text{MIN}(N_i/15000, 1) \quad (7.43)$$

where  $N_i$  is the number of activated ice nuclei given by (7.40). As discussed earlier,  $N_i$  in this application is only dependent on temperature and the function  $f_{nuc}$  gives a value increasing from 0.035 at 0°C to 1 at -23°C; the latter defined to be consistent with the previous diagnostic scheme mixed-phase temperature threshold. This term therefore reduces the effect of the deposition rate modification with decreasing temperature (increasing altitude) as the number of activated ice nuclei increases.

(e) **Evaporation of cloud water/ice**

The scheme describes evaporation of clouds by two processes; (i) in connection with large-scale and cumulus-induced descent and diabatic heating and (ii) by turbulent mixing of cloud air with unsaturated environmental air. The evaporation rate is defined as

$$S_{\text{evap}}^{\text{liq/ice}} = E_1 + E_2 \quad (7.44)$$

The first process ( $E_1$ ) is accounted for in the same way as the stratiform cloud formation except that  $dq_{\text{sat}}/dt > 0$ . Hence

$$E_1 = a \frac{dq_{\text{sat}}}{dt} \quad \frac{dq_{\text{sat}}}{dt} > 0 \quad (7.45)$$

Assuming an homogeneous horizontal distribution of liquid water in the cloud, the cloud fraction remains unaltered by this process except at the final stage of dissipation where it reduces to zero.

$$\delta a_{\text{evap}} = \frac{a}{\Delta t} \quad \text{if } q_l \rightarrow 0 \quad (7.46)$$

The second process ( $E_2$ ) represents the parametrization of cloud dissipation as cloud air mixes with environmental air. It is described as a diffusion process proportional to the saturation deficit of the air:

$$E_2 = aK(q_{\text{sat}} - q_v) \quad (7.47)$$

where the diffusion coefficient  $K = 3.10^{-6} \text{ s}^{-1}$ . The rate of decrease in cloud cover is parametrized as

$$\delta a_{\text{evap}} = \frac{E_2}{q^{\text{cld}}} \quad (7.48)$$

where  $q^{\text{cld}}$  is the specific cloud water/ice content per cloud area as defined in (7.3). Note that because of (7.3) the parametrizations (7.47) and (7.48) imply a reduction in cloud area while  $q^{\text{cld}}$  remains unchanged.

(f) **Autoconversion/accretion of cloud water to rain**

Autoconversion describes the formation of rain from cloud droplets through the collision-coalescence process. Accretion represents the collection process where larger rain drops falling through the cloud drops coalesce and grow. Two options are available for the parametrization of autoconversion and accretion and Option 2 is the default.

*Option 1: Parametrization based on Sundqvist (1978)*

Following (Sundqvist, 1978) the rate of generation of precipitation due to autoconversion and accretion is parameterized as

$$S_{\text{auto}}^{\text{rain}} = ac_0 q_l^{\text{cld}} \left[ 1 - \exp \left\{ - \left( \frac{q_l^{\text{cld}}}{q_l^{\text{crit}}} \right)^2 \right\} \right] \quad (7.49)$$

where  $c_0^{-1}$  represents a characteristic time scale for conversion of cloud liquid droplets into rain drops and  $q_l^{\text{crit}}$  is a typical cloud water content at which the generation of precipitation begins to be efficient. The critical liquid water content is different over ocean and land to take account of the differences in cloud

condensation nuclei (CCN) in clean and polluted air. Cleaner air over the ocean has fewer CCN, hence larger drops and an onset of precipitation at lower cloud liquid water contents than in the more polluted air over land. These disposable parameters are defined as

$$c_0 = c_0^* F_1 \quad (7.50)$$

and

$$q_1^{\text{crit}} = \frac{q_1^{\text{crit}*}}{F_1} \quad (7.51)$$

to take into account the effect of collection (accretion) of cloud droplets by raindrops falling through the cloud ( $F_1$ ). Here  $F_1$  is defined as

$$F_1 = 1 + b_1 \sqrt{P_{\text{loc}}} \quad (7.52)$$

where  $P_{\text{loc}}$  is the local cloudy precipitation rate ( $P_{\text{loc}} = P^{\text{clld}}/a_P^{\text{clld}}$ ). For a definition of precipitation fraction overlapping with cloud ( $a_P^{\text{clld}}$ ), see the section on precipitation fraction below. The values for the constants are  $b_1 = 100 \text{ (kg m}^{-2} \text{ s}^{-1})^{-0.5}$  and  $c_0^* = 1.67 \times 10^{-4} \text{ s}^{-1}$ . The critical liquid water content,  $q_1^{\text{crit}*}$  is set to  $0.25 \text{ g kg}^{-1}$  over ocean and  $0.55 \text{ g kg}^{-1}$  over land. Currently, any rain that is formed at temperatures colder than zero is assumed to freeze and is converted to the snow hydrometeor category.

*Option 2: Parametrization based on [Khairoutdinov and Kogan \(2000\)](#) (default)*

The second autoconversion/accretion parameterization is a non-linear function of the mass of both liquid cloud and rain water. The formulation follows [Khairoutdinov and Kogan \(2000\)](#) which is derived from large eddy simulation studies of drizzling stratocumulus clouds. Given the large uncertainties in the autoconversion/accretion process, it is assumed here that this parameterization is at least as valid as the Sundqvist scheme for cloud regimes other than stratocumulus.

The rates for autoconversion ( $S_{\text{aut}}$ ) and accretion ( $S_{\text{acc}}$ ) are parameterized as:

$$S_{\text{aut}} = 1350 q_l^{2.47} N_c^{-1.79} F_{\text{aut}} \quad (7.53)$$

and

$$S_{\text{acc}} = 67 q_l^{1.15} q_r^{1.15} F_{\text{acc}} \quad (7.54)$$

where  $N_c$  is the cloud droplet number concentration and  $q_r$  is the rain water content. The droplet number concentration is considered to be constant within the grid box with a value of  $50 \text{ cm}^{-3}$  over ocean and  $300 \text{ cm}^{-3}$  over land.  $F_{\text{aut}}$  and  $F_{\text{acc}}$  are the inhomogeneity enhancement factors for the autoconversion and accretion processes respectively, which account for the subgrid variance and covariance of the cloud liquid water and cloud rain water content inhomogeneities. At present these are fixed values ( $F_{\text{aut}} = 1.5$  and  $F_{\text{acc}} = 2$ ) in reasonable agreement with observations ([Boutle et al., 2013](#)), but in the future could depend on other prognosed quantities such as cloud fraction.

The rate of conversion of cloud liquid to rain is more non-linear than in the Sundqvist scheme, creating a stronger contrast between the slow production of rain for low liquid water clouds and more rapid conversion to rain for high liquid water clouds. The accretion rate depletes clouds with higher precipitation rates more quickly than clouds with low precipitation rates.

*(g) Autoconversion of ice to snow*

The ice phase is currently represented with two separate prognostic variables for “cloud ice” and “snow”. This formulation requires a parametrization representing the aggregation process where ice particles collide to form larger “snow” particles. The parametrization of this ice to snow autoconversion process follows the [Sundqvist \(1978\)](#) approach, with the rate of generation of precipitation defined as in (7.49) but with parameters appropriate for ice. The rate coefficient ( $c_0$ ) is based on [Lin et al. \(1983\)](#),

$$c_0 = 10^{-3} e^{0.03(T-273.15)} \quad (7.55)$$

where  $T$  is the temperature in Kelvin. Note the exponent multiplier of 0.03 is slightly larger than the 0.025 in [Lin et al. \(1983\)](#).

For this process, [Lin et al. \(1983\)](#) set  $q_i^{\text{crit}}$  to  $10^{-3} \text{kg kg}^{-1}$  in their cloud resolving model (note they were using a Kessler-type scheme rather than the Sundqvist form, but the sensitivities to  $c_0$  and  $q_i^{\text{crit}}$  are likely to be similar in both schemes). A lower value is appropriate for a GCM sized grid box (unless sub-grid cloud variability is explicitly taken into account; [Rotstavn, 2000](#); [Pincus and Klein, 2000](#)), and based on model tuning  $q_i^{\text{crit}}$  is set to  $3 \times 10^{-5} \text{kg kg}^{-1}$ .

#### (h) *Riming*

This process represents the collection and freezing of cloud liquid water drops by falling snow particles. The formulation is based on [Wilson and Ballard \(1999\)](#). The liquid drops are assumed to be stationary, the snow particles are represented by spheres of diameter,  $D$  and the collection efficiency,  $E_{\text{rime}}$  is assumed to be 0.3. The riming term for a single snow particle can be written as

$$\frac{dm}{dt} = E_{\text{rime}} \frac{\pi}{4} D^2 \nu(D) \rho q_l \quad (7.56)$$

Integrating over the snow particle size spectrum gives:

$$S_{\text{rime}} = E_{\text{rime}} \frac{\pi}{4} q_l \int_{D=0}^{\infty} n_{ax} \Lambda^{n_{bx}} \exp(-\Lambda D) c_x D^{d_x} \left(\frac{\rho_0}{\rho}\right)^{0.5} dD \quad (7.57)$$

$$S_{\text{rime}} = E_{\text{rime}} \frac{\pi}{4} q_l \frac{n_{ax} c_x \Gamma(3 + d_s)}{\Lambda^{3+d_s}} \left(\frac{\rho_0}{\rho}\right)^{0.5} \quad (7.58)$$

The riming of ice particles is assumed to be negligible.

#### (i) *Sedimentation*

The numerical formulation of the hydrometeor sedimentation follows an implicit upstream approach. The rain, snow and ice hydrometeor categories are allowed to sediment.

With the potential for hydrometeors to settle through many model layers in a single timestep, using a mass related fall speed formulation can lead to numerical 'shocks' when long timesteps are necessary. Thus the fallspeeds are set to a constant ( $V_i = 0.13 \text{ m s}^{-1}$ ,  $V_s = 1 \text{ m s}^{-1}$ ,  $V_r = 4 \text{ m s}^{-1}$ ) to avoid this. The formulation of the sedimentation term will be a topic for future development.

#### (j) *Precipitation fraction and overlap*

Precipitation processes are treated separately in clear and cloudy skies. This owes to the fact that the microphysical processes in these two regions are very distinct from each other, with conversion, collection and accretion processes being relevant in clouds whereas evaporation of precipitation is the relevant process outside clouds. Therefore the precipitation flux is written as

$$P = P^{\text{cld}} + P^{\text{clr}} \quad (7.59)$$

with

$$P^{\text{cld}} \equiv \frac{1}{A} \int P \cdot H(a) dA \quad (7.60)$$

and

$$P^{\text{clr}} \equiv \frac{1}{A} \int P \cdot (1 - H(a)) dA \quad (7.61)$$

where the step function,  $H(a)$ , marks the portion of the grid-cell containing cloud and  $A$  is the area of the grid-cell.

The precipitation fraction in the grid-box is then described as

$$a_P = a_P^{\text{cld}} + a_P^{\text{clr}} \quad (7.62)$$

with

$$a_P^{\text{cl}} \equiv \frac{1}{A} \int H(a)H(a_P) dA \quad (7.63)$$

and

$$a_P^{\text{clr}} \equiv \frac{1}{A} \int (1 - H(a))H(a_P) dA \quad (7.64)$$

Whereas cloud fraction (for cloud liquid and cloud ice) is a prognostic variable, precipitation fraction (for rain and snow) is still treated diagnostically. The treatment of precipitation fraction in the previous cloud scheme (Jakob and Klein, 2000) in IFS cycles prior to 36r4 needed to be updated for the new prognostic microphysical treatment, since snow and rain are now prognostic variables and can survive from one timestep to the next. So the previous complex two-stream treatment is replaced by a simpler treatment where the total precipitation fraction  $a_P$  is calculated using a maximum-random overlap treatment of the cloud fraction, so that at level  $k$ :

$$a_{P,k} = 1 - \left( \frac{(1 - a_{P,k-1})(1 - \text{MAX}[a_k, a_{k-1}])}{1 - \text{MIN}[a_{k-1}, 1 - \epsilon]} \right) \quad (7.65)$$

where  $\epsilon$  is equal to a small number ( $10^{-6}$ ) and  $a_k$  is the cloud cover at level  $k$ . The clear sky precipitation fraction is then given by

$$a_P^{\text{clr}} = a_P - a \quad (7.66)$$

The precipitation flux is proportionally divided between the clear-sky and the in-cloud component.

Rain and snow are prognostic variables and can be advected by the wind out of the column that they were produced, but precipitation fraction is still a diagnostic. Therefore a precipitation fraction needs to be specified when there is no cloud fraction in the column above. At present, this is set as a minimum precipitation coverage of 10%.

#### (k) *Evaporation of rain*

There are two available options for the evaporation of rain. Option (2) is the default. The treatment of the subgrid heterogeneity of rain is the same for both options and described at the end of this subsection.

##### *Option 1: Following Kessler (1969)*

The parametrization of rain evaporation follows Kessler (1969) with evaporation occurring only in the clear air part of the grid-box.

The evaporation rate for rain is assumed to be proportional to the saturation deficit ( $q_{\text{sat}} - q_v^{\text{env}}$ ) and dependent on the density of rain in the clear air,  $\rho_r^{\text{clr}}$  ( $\text{gm}^{-3}$ ),

$$S_{\text{evap}} = \alpha_1 (q_{\text{sat}} - q_v^{\text{env}}) (\rho_r^{\text{clr}})^{13/20} \quad (7.67)$$

where  $\alpha_1$  is a constant.

Even though the rain density can be calculated directly from the new prognostic specific rain water content, this version of the parametrization calculates the rain density indirectly from the precipitation flux,  $P^{\text{clr}}$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ),

$$\rho_r^{\text{clr}} = P^{\text{clr}} / V_r \quad (7.68)$$

where  $V_r$  is the mean fall speed of rain drops which again is parametrized following Kessler (1969),

$$V_r = \alpha_2 (\rho_r^{\text{clr}})^{1/8} / \sqrt{p/p_0} \quad (7.69)$$

Considering that the evaporation only takes place in the clear-sky precipitation fraction  $a_P^{\text{clr}}$ , the evaporation rate becomes

$$S_{\text{evap}} = a_P^{\text{clr}} \alpha_1 (q_{\text{sat}} - q_v^{\text{env}}) \left[ \frac{\sqrt{p/p_0} P^{\text{clr}}}{\alpha_2 a_P^{\text{clr}}} \right]^{\alpha_3} \quad (7.70)$$

where the constants have the following values (Kessler, 1969),

$$\alpha_1 = 5.44 \times 10^{-4} \text{ s}^{-1}, \quad \alpha_2 = 5.09 \times 10^{-3}, \quad \alpha_3 = 0.5777$$

*Option 2: Following Abel and Boutle (2012) (default)*

In this parametrization, the rate of change of mass of a raindrop is given by:

$$\frac{dm}{dt} = \frac{4\pi C(S_w - 1)F}{A'' + B''} \quad (7.71)$$

where  $C$  is the capacitance of the particle assumed to be spherical ( $C = D/2$ )  $S_w = e/e_{sw}$  is the saturation ratio with respect to water, and  $F$  is the ventilation factor given by Beard and Pruppacher (1971)

$$F = 0.78 + 0.31S_c^{1/3}R_e^{1/2} \quad (7.72)$$

where  $S_c = 0.6$  is the Schmidt Number and the Reynolds number  $R_e = \nu(D)D\rho/\mu$  where  $\mu$  is the kinematic viscosity of air.

Terms  $A''$  and  $B''$  represent heat conduction and vapour diffusion respectively,

$$A'' = \frac{L_{vap}}{K_a T} \left( \frac{L_{vap}}{R_v T} - 1 \right) \quad (7.73)$$

$$B'' = \frac{R_v T}{\chi e_{sw}} \quad (7.74)$$

where  $K_a$  is the heat conductivity of air and  $\chi$  is the diffusivity of water vapour in air (which varies inversely with pressure as  $\chi = 2.21/p$ ),  $L_{vap}$  is the latent heat of vaporization and  $R_v$  is the gas constant for water vapour.

Integrating over the particle size distribution for raindrops (see section 7.2.3) gives the following evaporation rate equation for rain:

$$S_{evap}^{rain} = c_{phase}^r \frac{(S_w - 1)}{\rho(A'' + B'')} \quad (7.75)$$

where

$$c_{phase}^r = \frac{2\pi n_{ar}}{\rho} \left( 0.78 \frac{\Gamma(2 - n_{br})}{\Lambda_r^2} + 0.31 \left( \frac{c_r}{\mu} \right)^{0.5} S_c^{1/3} (\rho\rho_0)^{1/4} \frac{\Gamma(0.5d_r + 2.5)}{\Lambda_r^{(0.5d_r + 2.5 - n_{br})}} \right) \quad (7.76)$$

The formulation implicitly assumes an exponential droplet size distribution, but with the Abel and Boutle (2012) formulation of the intercept as described in Eq. 7.16 that increases the number of smaller particles for lower rain rates. This is in closer agreement to aircraft observations in drizzling cloud, and results in higher evaporation rates for light precipitation.

#### *Sub-grid heterogeneity assumption*

For both option (1) and (2) there is an assumption of sub-grid heterogeneity in the clear-sky precipitation fraction, so that the precipitation fraction is reduced proportionally to the reduction in precipitation flux by the evaporation process.

In addition, there is a limit in moist environments to prevent the gridbox saturating when only part of the gridbox has evaporating precipitation. Therefore evaporation of rain/snow only takes place when the grid mean relative humidity is below a threshold value. The choice of the threshold value is not straightforward for numerical reasons. Here, the assumption is made that the clear-sky relative humidity (which is equal to the grid mean relative humidity in the absence of clouds) that can be reached by

evaporation of precipitation is a function of the fractional coverage with precipitation of the clear sky part of the grid-box. Hence, the threshold value is parametrized as

$$RH_{\text{crit},EP} = 0.7 + 0.3 \frac{a_P^{\text{clr}}}{(1-a)} \quad (7.77)$$

(l) *Sublimation of snow*

The sublimation of snow (i.e. from solid to vapour) currently uses the same [Kessler \(1969\)](#) formulation as for the first the rain evaporation (Option 1) outlined above, except it uses the saturation deficit with respect to ice in [\(7.70\)](#).

(m) *Melting of ice and snow*

The melting of ice and snow hydrometeors is parametrized by allowing the part of the grid box that contains precipitation to cool to  $T_{\text{melt}}$  over a time scale  $\tau$ , that is

$$S_{\text{melt}} = (a_P^{\text{cld}} + a_P^{\text{clr}}) \frac{c_p}{L_{\text{fus}}} \frac{T_w - T_{\text{melt}}}{\tau} \quad (7.78)$$

where  $T_{\text{melt}} = 0^\circ\text{C}$ ,  $T_w$  is the wet-bulb temperature and

$$\tau = \frac{\tau_m}{1 + 0.5(T_w - T_{\text{melt}})}$$

where  $\tau_m = 7200$  s. The wet-bulb temperature is used in order to account for the thermal (cooling) effect of evaporation on the melting process in sub-saturated air. The evaporation counteracts the latent heating due to melting and allows snow particles to survive to slightly warmer temperatures when the relative humidity of the air is low. The wet-bulb temperature is approximated as in the scheme described by [Wilson and Ballard \(1999\)](#)

$$T_w = T - (q_{\text{sat}(w)} - q_v)(A + B(p - C) - D(T - E)) \quad (7.79)$$

where  $A = 1329.31$ ,  $B = 0.0074615$ ,  $C = 0.85 \times 10^5$ ,  $D = 40.637$ , and  $E = 275$ .

(n) *Freezing of rain*

If rain (containing no ice) falls into sub-zero temperatures, it will become supercooled. The probability of a rain drop freezing then depends exponentially on the temperature difference of the air below  $0^\circ\text{C}$  and on the volume of the drop, with larger drops more likely to freeze than smaller drops ([Bigg 1953](#); [Wisner et al. 1972](#)). However, in this version of the model, the rate of freezing of rain is parametrized with a similar functional form to the melting of snow (Eq. [7.78](#)), but with a much slower time scale (increased by a factor of 1000) to give freezing rates closer to those observed.

If the rain drops are not completely melted, they will contain an ice core which leads to rapid freezing if the melting particles fall from an elevated above-freezing layer into a layer of sub-zero temperatures below. In this case, the same form of freezing parametrization is used as above, but with the standard timescale (7200s). It is assumed that at least 20% of the precipitation mass must be in the ice phase at the base of the warm layer for refreezing to be rapid. Otherwise, if more than 80% of the precipitation mass at the base of the warm layer is rain, the slower timescale for supercooled rain drops is applied in the column below and freezing rain can be diagnosed at the surface.

## 7.3 NUMERICS

### 7.3.1 Integration of the equations

The new scheme is a multi-species prognostic microphysics scheme, with  $m = 5$  prognostic equations for water vapour, cloud liquid water, rain, cloud ice and snow (i.e. the single cloud condensate equation in the previous version is replaced by four variables). The equation governing each prognostic cloud variable

within the cloud scheme is

$$\frac{\partial q_x}{\partial t} = A_x + \frac{1}{\rho} \frac{\partial}{\partial z} (\rho V_x q_x) \quad (7.80)$$

where  $q_x$  is the specific water content for category  $x$  (so  $x = 1$  represents cloud liquid,  $x = 2$  for rain, and so on),  $A_x$  is the net source or sink of  $q_x$  through microphysical processes, and the last term represents the sedimentation of  $q_x$  with fall speed  $V_x$ .

The solution to this set of equations uses the upstream approach. Writing the advection term in mass flux form and collecting all fast processes (relative to a GCM timestep) into an implicit term, gives

$$\frac{q_x^{n+1} - q_x^n}{\Delta t} = A_x + \sum_{x=1}^m B_{xy} q_y^{n+1} - \sum_{x=1}^m B_{yx} q_x^{n+1} + \frac{\rho_{k-1} V_x q_{x,k-1}^{n+1} - \rho V_x q_x^{n+1}}{\rho \Delta Z} \quad (7.81)$$

for timestep  $n$ . The subscript " $k - 1$ " refers to a term calculated at the model level above the present level  $k$  for which all other terms are being calculated. The matrix  $\tilde{B}$  (with terms  $B_{xx}, B_{xy}, B_{yx}$ ) represents all the implicit microphysical pathways such that  $B_{xy} > 0$  represents a sink of  $q_y$  and a source of  $q_x$ . Matrix  $\tilde{B}$  is positive-definite off the diagonal, with zero diagonal terms since  $B_{xx} = 0$  by definition. Some terms, such as the creation of cloud through condensation resulting from adiabatic motion or diabatic heating, are more suitable for an explicit framework, and are retained in the explicit term  $A$ .

For cloud fraction, there are no multi-dimensional dependencies, so the equation simplifies to

$$\frac{a^{n+1} - a^n}{\Delta t} = A + B a^{n+1} \quad (7.82)$$

However, for the cloud and precipitation variables in (7.81), a matrix approach is required. Due to the cross-terms  $q_y^{n+1}$ , (7.81) is rearranged to give a straight forward matrix equation which can be solved with standard methods (the scheme currently uses the LU decomposition method). As long as the solution method is robust the choice for solution is not critical as the number of microphysical prognostic equations is small ( $m = 5$ ), in contrast to chemical models with typically  $O(100)$  species. The solution method is simplified by assuming the vertical advection terms due to convective subsidence and sedimentation act only in the downward direction, allowing the solution to be conducted level by level from the model top down.

The matrix on the LHS has the microphysical terms in isolation off the diagonal, with the sedimentation term on the diagonal, thus the matrix equation for a 3-variable system is

$$\begin{pmatrix} 1 + \Delta t \left( \frac{V_1}{\Delta z} + B_{21} + B_{31} \right) & -\Delta t B_{12} & -\Delta t B_{13} \\ -\Delta t B_{21} & 1 + \Delta t \left( \frac{V_2}{\Delta z} + B_{12} + B_{32} \right) & -\Delta t B_{23} \\ -\Delta t B_{31} & -\Delta t B_{32} & 1 + \Delta t \left( \frac{V_3}{\Delta z} + B_{13} + B_{23} \right) \end{pmatrix} \cdot \begin{pmatrix} q_1^{n+1} \\ q_2^{n+1} \\ q_3^{n+1} \end{pmatrix} = \left[ q_1^n + \Delta t \left( A_1 + \frac{\rho_{k-1} V_1 q_{1,k-1}^{n+1}}{\rho \Delta Z} \right), q_2^n + \Delta t \left( A_2 + \frac{\rho_{k-1} V_2 q_{2,k-1}^{n+1}}{\rho \Delta Z} \right), q_3^n + \Delta t \left( A_3 + \frac{\rho_{k-1} V_3 q_{3,k-1}^{n+1}}{\rho \Delta Z} \right) \right] \quad (7.83)$$

There are some aspects that require attention. Firstly, although implicit terms are unable to reduce a cloud category to zero, the explicit can, and often will, achieve this. Thus safety checks are required to ensure that all end-of-timestep variables remain positive definite, in addition to ensuring conservation. Practically, to aid the conservation requirement, the explicit source and sink terms are thus also generalised from a vector  $\vec{A}$  to an anti-symmetric matrix  $\tilde{A}$ ,

$$\tilde{A} = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ -A_{12} & A_{22} & A_{32} \\ -A_{13} & -A_{23} & A_{33} \end{pmatrix} \quad (7.84)$$

Thus  $A_{xy} > 0$  represents a source of  $q_x$  and a sink of  $q_y$ , and the original vector for  $A$  can be obtained by summing over the rows. Although this matrix approach involves a degree of redundancy, it is a simple

method of ensuring conservation properties. The matrix diagonals  $A_{xx}$  contain the 'external' sources of  $q_x$  such as the cloud water detrainment terms from the convection scheme.

In order to simultaneously guarantee conservation and positive-definite properties, the sum of all sinks for a given variable are scaled to avoid negative values. This solution is not accurate, seen from the consideration of the simple case of a variable in equilibrium, with a small initial value subjected to a large source and an equal and opposing sink: the sink will be clipped first and the variable will increase. However, this is deemed preferable to any method that attempts to account for variable sources, which must invoke a sensitivity to the order in which the variables are considered.

The impact on the temperature budget is calculated from the change in cloud variables due to each process after the cloud scheme 'solver' has been applied, and then collecting the terms together that are associated with latent heating/cooling. With the fully implicit solver it is thus easier to use conserved variables to govern the temperature budget; the scheme uses the liquid water temperature  $T_L$  defined as:

$$T_L = T - \frac{L_{vap}}{C_p}(q_l + q_r) - \frac{L_{subl}}{C_p}(q_i + q_s) \quad (7.85)$$

Since  $dT_L/dt = 0$ , the temperature change is thus given by

$$\frac{\partial T}{\partial t} = \sum_{x=1}^m \frac{L(x)}{C_p} \left( \frac{dq_x}{dt} - D_{q_x} - \frac{1}{\rho} \frac{\partial}{\partial z} (\rho V_x q_x) \right) \quad (7.86)$$

The first term on the right in the brackets is the rate of change of species  $q_x$  due to *all* processes, including the convective detrainment term  $D_{q_x}$  and the advective flux terms, which are then subtracted separately since they represent a net  $T_L$  flux not associated with latent heating.

### 7.3.2 Calculation of $dq_{sat}/dt$

Special care has to be taken in the numerical calculation of  $dq_{sat}/dt$  from (7.23). Since the saturation water vapour pressure depends exponentially on temperature, straightforward numerical integration of (7.23) would produce large truncation errors. Therefore the average of  $dq_{sat}/dt$  over the time step is determined by the means of moist adjustment (e.g. Haltiner and Williams, 1980). This is achieved by first extrapolating the cloud temperature to time-level  $t + \Delta t$  and then adjusting temperature and moisture toward saturation conditions.

### 7.3.3 Convective cloud source

The vertical discretisation of (7.21) and (7.22) is achieved with a simple upstream scheme, that is

$$S_{conv} = D_{up,k} l_{up,k+1/2} - gM_{up,k-1/2} \frac{q_{k-1} - q_k}{z_{k-1} - z_k} \quad (7.87)$$

and

$$\delta a_{conv} = D_{up,k} - gM_{up,k-1/2} \frac{a_{k-1} - a_k}{z_{k-1} - z_k} \quad (7.88)$$

The convective source is treated explicitly to ensure conservation. As stated earlier, cloud fraction is treated implicitly.

### 7.3.4 Final moist adjustment

In the case where semi-Lagrangian advection is not used, a final test for supersaturation is performed after the calculation of the cloud tendencies and the corresponding tendencies of temperature and moisture. If any supersaturation is found, the grid box is re-adjusted to saturation (using the moist adjustment formulation) and the moisture excess is converted into precipitation. When the semi-Lagrangian advection scheme is utilized however, this final supersaturation check is performed by a separate routine that is applied on the effective profiles of temperature and humidity after all physical and explicit dynamical contributions have been computed. See Section 3.10 of Chapter 3 "Semi-Lagrangian formulation" for a more detailed description. Note, that small amounts of non-physical supersaturation due to numerical approximations may remain in the postprocessed fields and that supersaturation with respect to ice is allowed as described in Section 7.2.4.

## 7.4 CODE

The parametrization of cloud and large-scale precipitation processes is performed in the following routines.

### CLOUDSC

This routine carries out all calculations necessary to solve (7.7) to (7.11). The calculations are carried out in the following order.

- (i) Section 1: Calculate initial input profiles
- (ii) Section 2: Setup
  - initial setup
- (iii) Section 3: Sources and sinks
  - supersaturation check
  - convective detrainment and subsidence [(7.21) and (7.22)]
  - erosion of clouds by turbulent mixing [(7.47) and (7.48)]
  - calculation of  $dq_{\text{sat}}/dt$  (see Subsection 7.3.2)
  - large-scale evaporation (7.45)
  - large-scale cloud formation [(7.25), (7.28) and (7.29)]
  - growth of ice by vapour deposition
- (iv) Section 4: Precipitation processes
  - ice sedimentation
  - calculation of precipitation overlap
  - autoconversion of cloud ice to snow
  - autoconversion of cloud water to rain
  - riming of snow by cloud water 7.58
  - melting of snow and ice (7.78)
  - evaporation of rain and snow
- (v) Section 5: Numerical solvers
  - analytical integration of the equation for cloud fraction,  $a$
  - truncate explicit sinks to avoid negative values
  - LU decomposition for cloud condensate/precipitation
- (vii) Section 6: Update tendencies
  - final tendency calculations of all thermodynamic quantities
- (viii) Section 7: Flux calculations for diagnostics and budget terms

### CLDPP

This routine calculates total, high, mid-level and low cloud cover for postprocessing diagnostics. These are obtained using a generalised cloud overlap assumption between maximum and random (Hogan and Illingworth, 2000; Mace and Benson-Troth, 2002; Barker, 2008), where the degree of randomness between two cloudy layers increases with increasing vertical separation distance between the layers. The algorithm to apply the generalised overlap is the same as used in the IFS McICA radiation scheme (Chapter 2) and is based on the stochastic cloud generator described in Räisänen *et al.* (2004). The cloud cover diagnostics are calculated over the relevant vertical atmospheric slab. In sigma coordinates these slabs are defined as follows.

- (i) Low clouds:  $p > 0.8p_s$
- (ii) Medium clouds:  $0.45p_s \leq p \leq 0.8p_s$
- (iii) High clouds:  $p < 0.45p_s$

where  $p$  is the pressure and  $p_s$  is the surface pressure.

## APPENDIX A. RELEVANT CLOUD AND PRECIPITATION DIAGNOSTICS

### A.1 Grib definitions

3D fields (Grib Table 128 unless specified):

Grib code	Short name	Description	Units
248	CC	Cloud fraction	(0-1)
246	CLWC	Cloud liquid specific water content	(kg kg <sup>-1</sup> )
247	CIWC	Cloud ice specific water content	(kg kg <sup>-1</sup> )
75	CRWC	Precipitation rain specific water content	(kg kg <sup>-1</sup> )
76	CSWC	Precipitation snow specific water content	(kg kg <sup>-1</sup> )
130	T	Temperature	(K)
133	Q	Specific humidity	(kg kg <sup>-1</sup> )
157	RH	Relative humidity (see A.2)	(%)

2D fields (Grib Table 128 unless specified):

Grib code	Short name	Description	Units
164	TCC	Total cloud cover	(0-1)
186	LCC	Low cloud cover	(0-1)
187	MCC	Mid-level cloud cover	(0-1)
188	HCC	High cloud cover	(0-1)
78	TCLW	Total column liquid water	(kg m <sup>-2</sup> )
79	TCIW	Total column ice water	(kg m <sup>-2</sup> )
228088	TCSLW	Total column supercooled liquid water	(kg m <sup>-2</sup> )
228089	TCRW	Total column rain water	(kg m <sup>-2</sup> )
228090	TCSW	Total column snow water	(kg m <sup>-2</sup> )
142	LSP	Accumulated large-scale (stratiform) precipitation (rain+snow)	(m)
144	SF	Accumulated snowfall (stratiform + convective)	(m)
50	LSPF	Accumulated large-scale surface precipitation fraction	
228217	ILSPF	Instantaneous large-scale surface precipitation fraction	(0-1)
228218	CRR	Convective rain rate	(kg m <sup>-2</sup> s <sup>-1</sup> )
228219	LSRR	Large scale rain rate	(kg m <sup>-2</sup> s <sup>-1</sup> )
228220	CSFR	Convective snowfall rate water equivalent	(kg m <sup>-2</sup> s <sup>-1</sup> )
228221	LSSFR	Large scale snowfall rate water equivalent	(kg m <sup>-2</sup> s <sup>-1</sup> )
228222	MXTPR3	Maximum total precipitation rate in the last 3 hours	(kg m <sup>-2</sup> s <sup>-1</sup> )
228223	MNTPR3	Minimum total precipitation rate in the last 3 hours	(kg m <sup>-2</sup> s <sup>-1</sup> )
228224	MXTPR6	Maximum total precipitation rate in the last 6 hours	(kg m <sup>-2</sup> s <sup>-1</sup> )
228225	MNTPR6	Minimum total precipitation rate in the last 6 hours	(kg m <sup>-2</sup> s <sup>-1</sup> )
228226	MXTPR	Maximum total precipitation rate since previous post-processing	(kg m <sup>-2</sup> s <sup>-1</sup> )
228227	MNTPR	Minimum total precipitation rate since previous post-processing	(kg m <sup>-2</sup> s <sup>-1</sup> )
260015	PTYPE	Precipitation type (WMO Code Table 4.201)	(0-8)
		0 No precipitation	
		1 Rain	
		3 Freezing rain	
		5 Snow	
		6 Wet snow	
		7 Mixture of rain and snow	
		8 Ice pellets	

### A.2 Definition of relative humidity

The relative humidity diagnostic is calculated (in routine GPRH) on model levels from the specific humidity, temperature and pressure fields and then interpolated onto pressure levels (if required). It is

currently defined as relative humidity with respect to water for temperatures warmer than  $0^{\circ}\text{C}$ , with respect to ice for temperatures colder than  $-23^{\circ}\text{C}$ , and a mix of the two in the  $0^{\circ}\text{C}$  to  $-23^{\circ}\text{C}$  temperature range. The definition of relative humidity is the ratio of the partial pressure of water vapour ( $e$ ) to the saturation partial pressure of water vapour over a plane of liquid water/ice at temperature  $T$ , ( $e_{\text{sat}}(T)$ ), given by:

$$\frac{e}{e_{\text{sat}}(T)} = \frac{pq\frac{1}{\epsilon}}{e_{\text{sat}}(T)\left(1 + q\left(\frac{1}{\epsilon} - 1\right)\right)} \quad (7.89)$$

where  $p$  is the atmospheric pressure,  $q$  is the specific humidity and  $\epsilon$  is the ratio of the molar masses of water and dry air ( $= R_{\text{dry}}/R_{\text{vap}} = 0.621981$  in the IFS).

The saturation vapour pressure is defined as

$$e_{\text{sat}}(T) = \alpha e_{\text{sat}(w)}(T) + (1 - \alpha)e_{\text{sat}(i)}(T) \quad (7.90)$$

where  $e_{\text{sat}(w)}(T)$  and  $e_{\text{sat}(i)}(T)$  are the saturation partial pressures of water vapour with respect to water and ice, respectively, given by Tetens formula in Eq. 7.5, and  $\alpha$  is the mixed phase function given by Eq. 7.6.

## APPENDIX B. HISTORY OF KEY CLOUD SCHEME DEVELOPMENTS

There have been a number of modifications to the cloud scheme since the operational implementation of the [Tiedtke \(1993\)](#) scheme in 1995. Here are some of the changes since Cycle 31r1:

- **Cycle 31r1:** Change to an implicit upstream ice sedimentation formulation and separate ice to snow autoconversion process to reduce vertical resolution sensitivity. Allow ice supersaturation in clear air.
- **Cycle 33r1:** The cloud cover diagnostics (total, high, medium, low) changed to use an exponential overlap assumption (consistent with McICA radiation scheme implemented in Cycle 32r2). This corrected low and medium cloud cover diagnostics resulting in an increase in occurrence of medium and low cloud cover in deeper cloud.
- **Cycle 35r1:** Increase melting rate for snow to reduce snowfall at warmer temperatures.
- **Cycle 36r4:** New prognostic microphysics scheme (separate cloud liquid, cloud ice, rain, snow prognostic variables) and multi-phase numerical solver.
- **Cycle 37r2:** Numerical limiter reinstated for cloud condensation in existing cloud, allow humidity heterogeneity close to tropopause by reducing critical relative humidity threshold.
- **Cycle 37r3:** Supersaturation term in temperature range  $0$  to  $-40^{\circ}\text{C}$  changed to source of liquid phase and passed to ice deposition term. Reduction of deposition rate at cloud top and dependence on diagnosed ice nuclei concentration to improve representation of super-cooled liquid water layers in mixed-phase cloud.
- **Cycle 38r1:** Modified sedimentation fallspeed profiles for ice/snow/rain, include a timescale for the freezing of rain to allow supercooled rain in low level sub-zero temperature inversions. Remove excessive clear-air supersaturations in partially cloudy grid boxes. Convert melting ice to rain rather than liquid (ice particles near  $0^{\circ}\text{C}$  assumed to be large enough to form rain droplets), removing unrealistic liquid water cloud generation at the melting level.
- **Cycle 40r3:** New warm-rain autoconversion and accretion parametrization based on [Khairoutdinov and Kogan \(2000\)](#)  
 New rain evaporation formulation with enhanced evaporation for small droplets, based on [Abel and Boutle \(2012\)](#)  
 New riming parametrization for snow (collection of liquid water drops by falling snow particles). Included a slower timescale for the freezing of rain to allow supercooled rain and a faster timescale for the freezing of partially melted particles with an ice core.  
 Corrected the precipitation fraction formulation for rain and snow evaporation.  
 Modified the ice to snow autoconversion to increase ice water content at cold temperatures (tropical upper troposphere).

Additional diagnostics for "instantaneous" precipitation rates, maximum and minimum over different time periods, precipitation area fraction, precipitation type and total column supercooled liquid water.

## APPENDIX C. LIST OF SYMBOLS

$A()$	advective transport through the boundaries of the grid box
$a$	fraction of grid box covered by clouds (0-1)
$a_P$	fraction of grid box covered by precipitation (0-1)
$a_{up}$	fractional area of updraughts (0-1)
$c_p$	specific heat capacity at constant pressure ( $J\ kg^{-2}\ K^{-1}$ )
$D_{up}$	detrainment in the cumulus updraughts ( $s^{-1}$ )
$e_{sat}$	saturation vapour pressure (Pa)
$e_{sl}$	saturation vapour pressure with respect to water (Pa)
$e_{si}$	saturation vapour pressure with respect to ice (Pa)
$g$	acceleration of gravity ( $m\ s^{-2}$ )
$K$	diffusion coefficient
$K_a$	conductivity of heat in air
$L$	latent heat ( $J\ kg^{-1}$ )
$L_{fus}$	latent heat of fusion ( $J\ kg^{-1}$ )
$L_{subl}$	latent heat of sublimation ( $J\ kg^{-1}$ )
$L_{vap}$	latent heat of vaporization ( $J\ kg^{-1}$ )
$l_{up}$	specific cloud water/ice content in the cumulus updraughts ( $kg\ kg^{-1}$ )
$M_{Cu}$	cumulus-induced subsidence mass flux
$P$	precipitation rate ( $kg\ m^{-2}\ s^{-1}$ )
$P_{loc}$	local precipitation rate ( $kg\ m^{-2}\ s^{-1}$ )
$p$	pressure (Pa)
$q^{env}$	environmental specific humidity ( $kg\ kg^{-1}$ )
$q_v$	grid-mean specific humidity ( $kg\ kg^{-1}$ )
$q_l$	grid-mean specific cloud liquid water content ( $kg\ kg^{-1}$ )
$q_l^{cld}$	in-cloud specific cloud liquid water content ( $kg\ kg^{-1}$ ) or per cloud area
$q_i$	grid-mean specific cloud ice water content ( $kg\ kg^{-1}$ )
$q_r$	grid-mean specific precipitating rain water content ( $kg\ kg^{-1}$ )
$q_s$	grid-mean specific precipitating snow water content ( $kg\ kg^{-1}$ )
$q_{sat}$	saturation specific humidity ( $kg\ kg^{-1}$ )
$q_{sat(i)}$	saturation specific humidity with respect to ice ( $kg\ kg^{-1}$ )
$q_{sat(w)}$	saturation specific humidity with respect to water ( $kg\ kg^{-1}$ )
$q_{up}$	specific humidity in the convective updraughts ( $kg\ kg^{-1}$ )
$R_{cld}$	radiative heating rate in cloudy air
$R_{clear}$	radiative heating rate in cloud-free air
$R_{dry}$	gas constant for dry air
$R_{vap}$	gas constant for water vapour
$RH_c$	= 0.8 (in the free troposphere)
$RH_{crit}$	threshold value of the relative humidity for condensation
$RH_{homo}$	threshold relative humidity for homogenous nucleation
$S_{auto}$	generation of precipitation in the form of rain/snow
$S_{evap}$	rate of evaporation of cloud water/ice/rain/snow
$S_{conv}$	formation of cloud water/ice by convective processes
$S_{rime}$	riming of snow by cloud liquid water
$S_{strat}$	formation of cloud water/ice by stratiform condensation processes
$S_{bl}$	formation of cloud water/ice by boundary-layer processes
$S_{frz}$	rate of freezing of rain
$S_{melt}$	rate of melting of ice/snow
$s$	dry static energy
$s_v$	virtual dry static energy
$T$	temperature

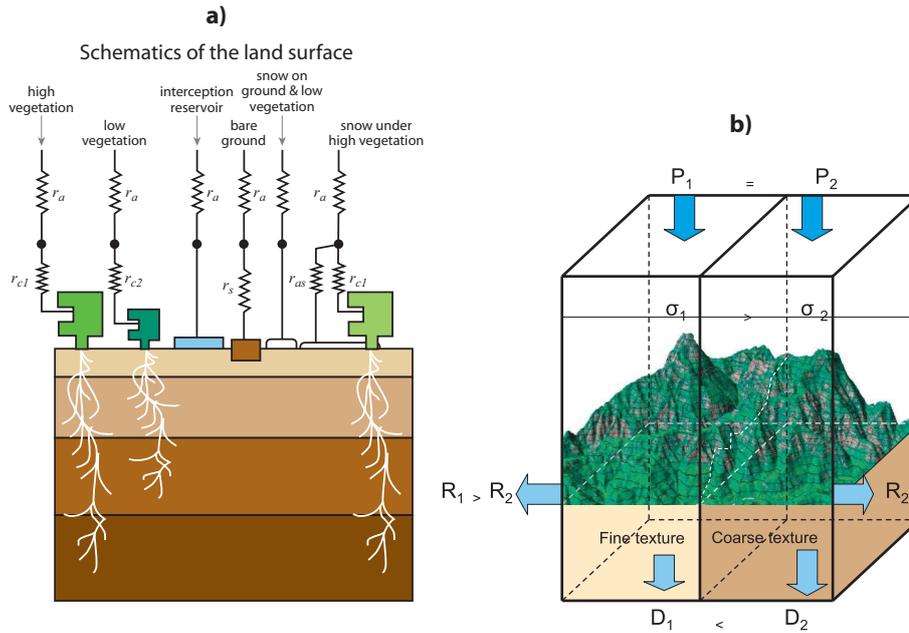
$T_0$	= 273.16 K
$T_{\text{ice}}$	= 250.16 K
$T_{\text{melt}}$	= 0° C
$\bar{w}$	area-mean generalized vertical velocity
$w$	terminal fall speed of hydrometeors
$w_{\text{up}}$	updraught velocity
$\alpha$	fraction of condensate held as liquid water
$\delta a_{\text{bl}}$	rate of increase of cloud area by boundary-layer processes
$\delta a_{\text{conv}}$	rate of increase of cloud area by convective processes
$\delta a_{\text{strat}}$	rate of increase of cloud area by stratiform condensation processes
$\delta a_{\text{evap}}$	rate of decrease of cloud area due to evaporation
$\rho$	density of moist air ( $\text{kg m}^{-3}$ )
$\rho_l$	density of cloud water ( $\text{kg m}^{-3}$ )
$\rho_{\text{rain}}$	density of rain in air ( $\text{kg m}^{-3}$ )

# Chapter 8

## Surface parametrization

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**Figure 8.1** Schematic representation of the structure of (a) TESSEL land-surface scheme and (b) spatial structure added in HTESSEL (for a given precipitation  $P_1 = P_2$  the scheme distributes the water as surface runoff and drainage with functional dependencies on orography and soil texture respectively).

## 8.1 INTRODUCTION

The parametrization scheme described in this chapter represents the surface fluxes of energy and water and, where appropriate, corresponding sub-surface quantities. Fig. 8.1 summarizes the main features of the land part of the model; the main surface scheme parameterization structures are provided by the Tiled ECMWF Scheme for Surface Exchanges over Land (Van den Hurk *et al.*, 2000; Viterbo and Beljaars, 1995; Viterbo *et al.*, 1999) or TESSEL scheme. At the interface between the surface and the atmosphere, each grid-box is divided into fractions (tiles), with up to six fractions over land (bare ground, low and high vegetation, intercepted water, shaded and exposed snow) and up to two fractions over sea and freshwater bodies (open and frozen water). Each fraction has its own properties defining separate heat and water fluxes used in an energy balance equation solved for the tile skin temperature. Special attention is devoted to the different physical mechanisms limiting evaporation of bare ground and vegetated surfaces. A revised land surface hydrology (hereafter referred as HTESSEL, Balsamo *et al.*, 2009) has been introduced to address shortcomings of the previous land surface scheme version, specifically the lack of surface runoff and the choice of a global uniform soil texture. New infiltration and runoff schemes are introduced with a dependency on the soil texture and standard deviation of orography. The snow-pack is treated taking into account its thermal insulation properties and a more realistic representation of density, the interception of liquid rain and a revise the albedo and metamorphism aging processes (Dutra *et al.*, 2010a and Balsamo *et al.*, 2011b).

A new formulation to represent inland water bodies both for resolved lakes and sub-grid coastal water in liquid and frozen state is introduced with a dedicated new water tile.

Over land, the skin temperature is in thermal contact with a four-layer soil or, if there is snow present, a single layer snow mantle overlying the soil. The snow temperature varies due to the combined effect of top energy fluxes, basal heat flux and the melt energy. The soil heat budget follows a Fourier diffusion law, modified to take into account the thermal effects of soil water phase changes. The energy equation is solved with a net ground heat flux as the top boundary condition and a zero-flux at the bottom.

Snowfall is collected in the snow mantle, which in turn is depleted by snowmelt, contributing to surface runoff and soil infiltration, and evaporation. A fraction of the rainfall is collected by an interception layer, where the remaining fraction (throughfall) is partitioned between surface runoff and infiltration.

Subsurface water fluxes are determined by Darcy’s law, used in a soil water equation solved with a four-layer discretization shared with the heat budget equation. Top boundary condition is infiltration plus surface evaporation, free drainage is assumed at the bottom; each layer has an additional sink of water in the form of root extraction over vegetated areas. The seasonal evolution of the vegetation development modulates the evapotranspiration (Boussetta *et al.*, 2013a), and revised formulation of the bare soil evaporation allowed to improved the realism of soil-atmosphere water transfer over sparsely vegetated areas and deserts (Albergel *et al.*, 2012).

The land surface exchange of carbon dioxide (CO<sub>2</sub>) which represents an essential component of the green-house-gases global budget has been added (Boussetta *et al.*, 2013b) to enable the interaction with atmospheric CO<sub>2</sub> concentration in environmental forecasting applications. The land CO<sub>2</sub> responds to meteorological and climate disturbances, modulated by the natural biomes diversity and their stress conditions. The main photosynthesis processes fixing carbon dioxide into biomass and releasing carbon dioxide via land biogenic processes are parametrized, while land carbon stocks are made dependent on land-use. The carbon parametrization is modular to the HTESSEL water cycle parametrization and it introduces the capability of interacting with the atmospheric CO<sub>2</sub> concentrations for global monitoring and prediction purposes. We refer to the CO<sub>2</sub>-enabled land surface scheme version as CHTESSEL.

Finally, open water points classified as lakes and any subgrid water area are treated by a dedicated mixed layer model (Dutra *et al.*, 2010b, Balsamo *et al.*, 2012, Balsamo, 2013). Open ocean points have a specified surface temperature driven by initial and boundary conditions that in ocean uncoupled simulations are driven by the ocean model tendency. Sea-ice occupies a fraction of the grid box specified by a sea-ice cover, while lakes and coastal ice is treated by dedicated prognostic variables in the lake model. The sea ice prognostic temperatures are evolving in the forecast following the heat budget of a four-layer ice model in thermal contact with an underlying ocean at freezing temperature. The lakes and coastal ice temperatures are evolving following a single layer ice scheme with an underlying water at freezing temperature.

## 8.2 TILES AND SURFACE FLUXES

### 8.2.1 Tile and vegetation characteristics

Grid-box surface fluxes are calculated separately for the different subgrid surface fractions (or “tiles”), leading to a separate solution of the surface energy balance equation and skin temperature for each of these tiles. This is an analogue of the “mosaic” approach of Koster and Suarez (1992). Note that the tiles at the interface soil-atmosphere are in energy and hydrological contact with one single atmospheric profile above and one single soil profile below. Each grid box is divided into eight fractions: two vegetated fractions (high and low vegetation without snow), one bare soil fraction, three snow/ice fractions (snow on bare ground/low vegetation, high vegetation with snow beneath, and sea-ice, respectively), and two water fractions (interception reservoir, ocean/lakes). The tile for “high vegetation with snow beneath” is a combined tile with a separate energy balance and evaporation model for the high vegetation and the underlying snow. A mixture of land and water (ocean/inland water) tiles is not allowed, i.e. a grid box is either 100% land or 100% sea.

In each grid box two vegetation types are present: a high and a low vegetation type. An external climate database, based on the Global Land Cover Characteristics (GLCC) data that has been derived using one year of Advanced Very High Resolution Radiometer (AVHRR) data and ancillary information (Loveland *et al.*, 2000; <http://edcdaac.usgs.gov/glcc/glcc.html>; see also Chapter 11). The nominal resolution is 1 km. The data used provides for each pixel a biome classification based on the Biosphere-Atmosphere Transfer Scheme (BATS) model (Dickinson *et al.*, 1993), and four parameters have been derived for each grid box: dominant vegetation type,  $T_H$  and  $T_L$ , and the area fraction,  $A_H$  and  $A_L$ , for each of the high- and low-vegetation components, respectively.

The coverage  $C_i$  for the tile  $i$  depends on the type and relative area of low and high vegetation, and the presence of snow and intercepted water. In the absence of snow and interception, the vegetation coverage of high ( $c_H$ ) and low ( $c_L$ ) vegetation are calculated as  $A_H c_{veg}(T_H)$  and  $A_L c_{veg}(T_L)$ , respectively, with  $c_{veg}$

**Table 8.1** *Vegetation types and parameter values (see text). H/L refer to the distinction between high and low vegetation.*

Index	Vegetation type	H/L	$r_{s,\min}$ ( $\text{sm}^{-1}$ )	$c_{\text{veg}}$	$g_{\text{D}}$ ( $\text{hPa}^{-1}$ )	$a_r$	$b_r$
1	Crops, mixed farming	L	100	0.90	0	5.558	2.614
2	Short grass	L	100	0.85	0	10.739	2.608
3	Evergreen needleleaf trees	H	250	0.90	0.03	6.706	2.175
4	Deciduous needleleaf trees	H	250	0.90	0.03	7.066	1.953
5	Deciduous broadleaf trees	H	175	0.90	0.03	5.990	1.955
6	Evergreen broadleaf trees	H	240	0.99	0.03	7.344	1.303
7	Tall grass	L	100	0.70	0	8.235	1.627
8	Desert	–	250	0	0	4.372	0.978
9	Tundra	L	80	0.50	0	8.992	8.992
10	Irrigated crops	L	180	0.90	0	5.558	2.614
11	Semidesert	L	150	0.10	0	4.372	0.978
12	Ice caps and glaciers	–	–	–	–	–	–
13	Bogs and marshes	L	240	0.60	0	7.344	1.303
14	Inland water	–	–	–	–	–	–
15	Ocean	–	–	–	–	–	–
16	Evergreen shrubs	L	225	0.50	0	6.326	1.567
17	Deciduous shrubs	L	225	0.50	0	6.326	1.567
18	Mixed forest/woodland	H	250	0.90	0.03	4.453	1.631
19	Interrupted forest	H	175	0.90	0.03	4.453	1.631
20	Water and land mixtures	L	150	0.60	0	–	–

a vegetation type dependent coverage (see [Table 8.1](#)). The bare ground fraction  $c_{\text{B}}$  is the residual.

$$\begin{aligned}
 c_{\text{H}} &= A_{\text{H}}c_{\text{veg}}(T_{\text{H}}) \\
 c_{\text{L}} &= A_{\text{L}}c_{\text{veg}}(T_{\text{L}}) \\
 c_{\text{B}} &= (1 - c_{\text{H}} - c_{\text{L}})
 \end{aligned}
 \tag{8.1}$$

Each vegetation type is characterized by a series of (fixed) parameters as detailed in [Table 8.1](#).

- (i) A minimum canopy resistance,  $r_{s,\min}$ .
- (ii) A leaf area index,  $LAI$ .
- (iii) A vegetation coverage,  $c_{\text{veg}}$ .
- (iv) A coefficient,  $g_{\text{D}}$ , for the dependence of the canopy resistance,  $r_{\text{c}}$ , on water vapour pressure deficit.
- (v) The root distribution over the soil layers, specified by an exponential profile involving attenuation coefficients,  $a_r$ , and  $b_r$ .

The numerical values for the parameters of [Table 8.1](#) are based both on experiments conducted as described in [Van den Hurk \*et al.\* \(2000\)](#) and on literature review, in particular [Mahfouf \*et al.\* \(1995\)](#), [Manzi and Planton \(1994\)](#), [Giard and Bazile \(2000\)](#), [Dorman and Sellers \(1989\)](#), [Bonan \(1994\)](#), [Pitman \*et al.\* \(1991\)](#), and [Zeng \*et al.\* \(1998a\)](#). The values for minimum canopy resistance have been revised for crops, needle-leaf forest and short-grass in [Boussetta \*et al.\* \(2013b\)](#).

The presence of snow and intercepted water dynamically modifies the coverage fractions. The coverage of snow,  $c_{\text{sn}}$ , has been revised to be linearly dependent from the snow depth  $D_{\text{sn}}$  (units m), therefore taking into account both the snow mass  $S$  (units  $\text{kg m}^{-2}$ ) and the snow density  $\rho_{\text{sn}}$  (units  $\text{kg m}^{-3}$ ), to account for the hysteresis characterizing snow accumulation and melting processes.

The interception reservoir fraction,  $c_1$ , is given by  $W_1/W_{1\text{m}}$ , with  $W_{1\text{m}}$ , the maximum value for the intercepted water in the grid box, defined from the leaf area index contributions from the high and low vegetation tiles. The water contents of the interception reservoir,  $W_1$  (units m), and  $S$  are prognostic quantities in the model. Snow cover is assumed to be overlying vegetation and bare ground with the same

fraction. The interception reservoir occupies an identical fraction of all snow-free tiles.

$$\begin{aligned}
 c_{\text{sn}} &= \min\left(1, \frac{S/\rho_{\text{sn}}}{D_{\text{cr}}}\right) \\
 W_{1\text{m}} &= W_{1\text{max}}[c_{\text{B}} + c_{\text{H}} \cdot LAI(T_{\text{H}}) + c_{\text{L}} \cdot LAI(T_{\text{L}})] \\
 c_1 &= \min\left(1, \frac{W_1}{W_{1\text{m}}}\right)
 \end{aligned} \tag{8.2}$$

In the expressions above the minimum snow depth that ensures complete coverage of the grid box is  $D_{\text{cr}} = 0.10$  m and the maximum water over a single layer of leaves or over bare ground is  $W_{1\text{max}} = 0.0002$  m. The leaf area index  $LAI$ , is prescribed from a MODIS satellite-based dataset as detailed in [Boussetta et al. \(2013b\)](#). The full set of fractional tile coverages is given by (8.3) and (8.4), where the indexing of the tiles is detailed in [Table 8.2](#). Since a mixture of land and ocean tiles is not allowed, a grid box is either 100% water (open water and ice, with ice fraction  $c_i$ ):

$$\begin{aligned}
 C_1 &= 1 - c_i \\
 C_2 &= c_i \\
 C_i &= 0, i \in [3, N_{\text{T}}]
 \end{aligned} \tag{8.3}$$

or 100% land (tiles 3 to  $N_{\text{T}}$ , where  $N_{\text{T}} = 8$  is the number of tiles):

$$\begin{aligned}
 C_1 &= C_2 = 0 \\
 C_3 &= (1 - c_{\text{sn}}) \cdot c_1 \\
 C_4 &= (1 - c_{\text{sn}}) \cdot (1 - c_1) \cdot c_{\text{L}} \\
 C_5 &= c_{\text{sn}} \cdot (1 - c_{\text{H}}) \\
 C_6 &= (1 - c_{\text{sn}}) \cdot (1 - c_1) \cdot c_{\text{H}} \\
 C_7 &= c_{\text{sn}} \cdot c_{\text{H}} \\
 C_8 &= (1 - c_{\text{sn}}) \cdot (1 - c_1) \cdot (1 - c_{\text{L}} - c_{\text{H}})
 \end{aligned} \tag{8.4}$$

Apart from the fractional gridbox coverage, each tile has a couple of additional parameters (see [Table 8.2](#)).

- (i) The skin conductivity,  $\Lambda_{\text{sk}}$ , provides the thermal connection between the skin level and the soil or snow deck. For high vegetation,  $\Lambda_{\text{sk}}$ , is different for a stable and unstable stratification of the temperature gradient between the skin level and the upper soil or snow layer. This difference is considered to represent the asymmetric coupling between the ground surface and the tree canopy layer: an effective convective transport within the tree trunk space for unstable conditions, and a limited turbulent exchange for stable stratification ([Bosveld et al., 1999](#)).
- (ii) A small fraction  $f_{R_s}$  of net short-wave radiation that is transmitted directly to the top soil or snow layer. The remaining fraction of the short-wave radiation ( $1 - f_{R_s}$ ) is absorbed by the skin layer.

Finally, the surface albedo,  $\alpha_i$ , is similar for all land tiles within a grid box except for those covered with snow (see the snow scheme description below). The climate database provides the snow-free background albedo on a monthly basis. Long-wave emissivity,  $\varepsilon$ , outside the window region is equal to 0.99 for all tiles; emissivity in the window region is tile dependent and varies between 0.93 and 0.98 (see [Table 2.5](#) in [Section 2.8.5](#) for more details). The remaining surface characteristics (roughness length for momentum,  $z_{0\text{m}}$ , and heat,  $z_{0\text{h}}$ ) are similar for all land tiles within a grid box and specified in the climate database ([Chapter 11](#)).

### 8.2.2 Surface momentum flux

The roughness lengths for momentum and heat are prescribed as constant values depending on each vegetation type following [Mahfouf et al. \(1995\)](#) since cycle 31r1 as used in ERA-Interim re-analysis. In Cy37r3 some of these values were revised based on a study by [Sandu et al. \(2011\)](#). This study emphasized that the model generally tends to overestimate the 10-m winds compared to routine observations and that the forecast errors depend on the vegetation type because the representation of the 10-m wind speed is

**Table 8.2** The resistance scheme describes the way of coupling with the atmosphere:  $P$ =Potential denotes atmospheric resistance only;  $R$ =Resistance denotes aerodynamic resistance in series with a canopy or soil resistance; In presence of snow on high vegetation  $R_s$ =Canopy and snow resistance a canopy resistance for the vegetation and an extra aerodynamic coupling to the snow surface are used (see Figs 8.1–8.2 and Subsection 8.2.3).

Type	Vegetation type	$\Lambda_{sk}$ unstable ( $\text{Wm}^{-2}\text{K}^{-1}$ )	$\Lambda_{sk}$ stable ( $\text{Wm}^{-2}\text{K}^{-1}$ )	$f_{R_s}$ –	Scheme –
1	Crops, mixed farming	10.0	10.0	0.05	$R$
2	Short grass	10.0	10.0	0.05	$R$
3	Evergreen needleleaf	40.0	15.0	0.03	$R, R_s$
4	Deciduous needleleaf	40.0	15.0	0.03	$R, R_s$
5	Deciduous broadleaf	40.0	15.0	0.03	$R, R_s$
6	Evergreen broadleaf	40.0	15.0	0.035	$R, R_s$
7	Tall grass	10.0	15.0	0.05	$R$
8	Desert	15.0	15.0	0.	$R$
9	Tundra	10.0	10.0	0.05	$R$
10	Irrigated crops	10.0	10.0	0.05	$R$
11	Semidesert	10.0	10.0	0.05	$R$
12	Ice caps and glaciers	58.0	58.	0.	$P$
13	Bogs and marshes	10.0	10.0	0.05	$R$
14	Inland water	$\infty$	$\infty$	0.	$P$
15	Ocean	$\infty$	$\infty$	0.	$P$
16	Evergreen shrubs	10.0	10.0	0.05	$R$
17	Deciduous shrubs	10.0	10.0	0.05	$R$
18	Mixed forest- Wood	40.0	15.0	0.03	$R, R_s$
19	Interrupted forest	40.0	15.0	0.03	$R, R_s$
20	Water -land mixtures	$\infty$	$\infty$	0.	$P$

mainly controlled by the value of the momentum roughness length associated with each vegetation type. This parameter is difficult to determine from observations and the values used so far in the ECMWF model were empirically chosen as 20% and 10% of the vegetation height for forests and respectively for the other vegetation types (Mahfouf *et al.*, 1995). The overestimation of the near-surface wind speed for most of the vegetation types suggests that the values used for the momentum roughness length were too low. They were therefore revised based on theoretical considerations and synop observations of wind speed at 10 m. The basic idea was to search, for each vegetation type, for a new value of the momentum roughness length for which the mean 10-m wind speed forecast error with respect to synop observations drops to zero. This calibration showed that the momentum roughness length values should be increased for nine and decreased for one of the 18 vegetation types characterizing land areas. The newly derived values for ten of the vegetation types were introduced in CY37r3 (Table 8.3). As the roughness length for momentum was on average increased, the roughness length for heat was decreased in order to account for terrain heterogeneity (Table 8.3).

The wet skin tile obtains values that are weighted between low and high vegetation according to their fractional cover. The exposed snow tile is set to table entry 12 (ice caps and glaciers), the snow under high vegetation tile has the roughness length of the high vegetation and the bare soil tile is set to the roughness length of table entry 8 (desert). The model uses the roughness lengths for the individual tiles and aggregates the fluxes. For postprocessing also an aggregated roughness length field is computed by tile averaging the neutral transfer coefficients and backing out the aggregated roughness lengths  $z_{oma}$  and  $z_{oha}$ .

$$\frac{1}{(\ln 10/z_{oma})^2} = \sum_i \frac{Fr_i}{(\ln 10/z_{omi})^2} \quad (8.5)$$

$$\frac{1}{(\ln 10/z_{oha})^2} = \sum_i \frac{Fr_i}{(\ln 10/z_{ohi})^2}$$

The result with the T1279 model is shown in Figs 11.17–11.18 for 1 August 2012.

**Table 8.3** *Roughness lengths for momentum and heat associated with high and low vegetation types.*

Index	Vegetation type	H/L veg	$z_{0m}$	$z_{0h}$
1	Crops, mixed farming	L	0.25	$0.25 \cdot 10^{-2}$
2	Short grass	L	0.2	$0.2 \cdot 10^{-2}$
3	Evergreen needleleaf trees	H	2.0	2.0
4	Deciduous needleleaf trees	H	2.0	2.0
5	Deciduous broadleaf trees	H	2.0	2.0
6	Evergreen broadleaf trees	H	2.0	2.0
7	Tall grass	L	0.47	$0.47 \cdot 10^{-2}$
8	Desert	–	0.013	$0.013 \cdot 10^{-2}$
9	Tundra	L	0.034	$0.034 \cdot 10^{-2}$
10	Irrigated crops	L	0.5	$0.5 \cdot 10^{-2}$
11	Semidesert	L	0.17	$0.17 \cdot 10^{-2}$
12	Ice caps and glaciers	–	$1.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-4}$
13	Bogs and marshes	L	0.83	$0.83 \cdot 10^{-2}$
14	Inland water	–	–	–
15	Ocean	–	–	–
16	Evergreen shrubs	L	0.100	$0.1 \cdot 10^{-2}$
17	Deciduous shrubs	L	0.25	$0.25 \cdot 10^{-2}$
18	Mixed forest/woodland	H	2.0	2.0
19	Interrupted forest	H	1.1	1.1
20	Water and land mixtures	L	–	–

### 8.2.3 Surface heat and evaporation fluxes

A resistance parametrization is used to calculate the turbulent fluxes. Momentum exchange is parametrized with the same roughness length for all tiles, but with a different stability correction for each tile. The resistance scheme for water vapour and heat exchanges is different for different tiles (see Fig. 8.2). For ocean, sea ice and snow on low vegetation, the turbulent fluxes of heat and water vapour are given by

$$H_i = \rho_a c_p |U_L| C_{H,i} (T_L + g z_L / c_p - T_{sk,i}) \quad (8.6)$$

$$E_i = \rho_a |U_L| C_{H,i} [q_L - q_{sat}(T_{sk,i})] \quad (8.7)$$

with  $\rho_a$  the air density,  $c_p$  the heat capacity of moist air,  $g$  the acceleration of gravity,  $|U_L|$ ,  $T_L$ ,  $q_L$ ,  $z_L$  the wind speed, temperature, humidity and height of the lowest atmospheric model level, and  $C_{H,i}$  the turbulent exchange coefficient, that varies from tile to tile because of different atmospheric stabilities. See Chapter 3 for a description of the exchange coefficients where different roughness lengths for heat and momentum are assumed and a Monin–Obukhov formulation is adopted for the stability dependence.

For high and low vegetation, an additional canopy resistance  $r_c$  is added with

$$E_i = \frac{\rho_a}{r_a + r_c} [q_L - q_{sat}(T_{sk,i})] \quad (8.8)$$

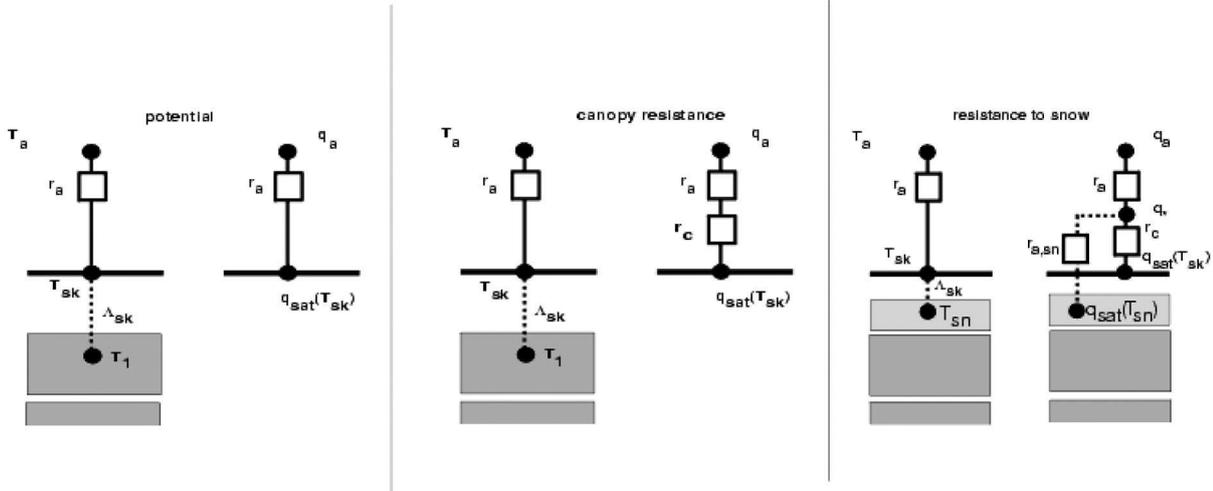
with  $r_a = (|U_L| C_{H,i})^{-1}$  and  $i$  indicating the high or low vegetation tiles.  $r_c$  is a function of downward short-wave radiation  $R_s$ , leaf area index  $LAI$ , average unfrozen root soil water  $\bar{\theta}$ , atmospheric water vapour deficit  $D_a$  and a minimum stomatal resistance  $r_{s,min}$ , following Jarvis (1976) given by

$$r_c = \frac{r_{s,min}}{LAI} f_1(R_s) f_2(\bar{\theta}) f_3(D_a) \quad (8.9)$$

$f_1$  is a hyperbolic function of downward short-wave radiation only so that

$$\frac{1}{f_1(R_s)} = \min \left[ 1, \frac{b R_s + c}{a(b R_s + 1)} \right] \quad (8.10)$$

where  $a = 0.81$ ,  $b = 0.004 \text{ W}^{-1} \text{ m}^2$  and  $c = 0.05$ .



**Figure 8.2** Resistance scheme for three categories of coupling. Potential refers to ocean, sea ice and snow on low vegetation; (Canopy) resistance to dry low and dry high vegetation, bare soil, and interception reservoir when potential evaporation exceeds the maximum reservoir content; Resistance to snow to snow under high vegetation.

**Table 8.4** Root distribution per vegetation type (in %) over the four layers. Vegetation indexes refer to Table 8.1.

Vegetation index	1	2	3	4	5	6	7	8	9	10	11	13	16	17	18	19
Layer 1	24	35	26	26	24	25	27	100	47	24	17	25	23	23	19	19
Layer 2	41	38	39	38	38	34	27	0	45	41	31	34	36	36	35	35
Layer 3	31	23	29	29	31	27	27	0	8	31	33	27	30	30	36	36
Layer 4	4	4	6	7	7	14	9	0	0	4	19	11	11	11	10	10

Function  $f_2$  is defined as

$$\frac{1}{f_2(\bar{\theta})} = \begin{cases} 0 & \bar{\theta} < \theta_{\text{pwp}} \\ \frac{\bar{\theta} - \theta_{\text{pwp}}}{\theta_{\text{cap}} - \theta_{\text{pwp}}} & \theta_{\text{pwp}} \leq \bar{\theta} \leq \theta_{\text{cap}} \\ 1 & \bar{\theta} > \theta_{\text{cap}} \end{cases} \quad (8.11)$$

where the soil moisture at permanent wilting point and at field capacity,  $\theta_{\text{pwp}}$  and  $\theta_{\text{cap}}$ , respectively, are defined in Table 8.9.  $\bar{\theta}$  is a weighted average of the unfrozen soil water given by

$$\bar{\theta} = \sum_{k=1}^4 R_k \max[f_{\text{liq},k} \theta_k, \theta_{\text{pwp}}] \quad (8.12)$$

where  $R_k$  is the fraction of roots in layer  $k$  and the fraction of unfrozen soil water,  $f_{\text{liq},k} = 1 - f_{\text{fr}}(T_k)$ , is a parametrized function of the soil temperature of layer  $k$ ,  $T_k$ , as specified in Subsection 8.5.2. Table 8.1 lists the coefficients  $a_r$  and  $b_r$  which are used to calculate the root fraction  $R_k$  according to Zeng *et al.* (1998a):

$$R_k = 0.5[\exp(-a_r z_{k-1/2}) + \exp(-b_r z_{k-1/2}) - \exp(-a_r z_{k+1/2}) - \exp(-b_r z_{k+1/2})] \quad (8.13)$$

where  $z_{k+1/2}$  is the depth of the bottom of layer  $k$  (in m;  $z_{1/2} = 0$  m). Contributions from levels exceeding the column depth are added to the deepest soil layer in order to ensure that  $\sum R_k = 1$ . Table 8.4 lists the distribution of the roots over the four soil layers.

A dependence on atmospheric humidity deficit ( $D_a = e_{\text{sat}}(T_L) - e_L$ , with  $e$  the vapour pressure) is included according to

$$\frac{1}{f_3(D_a)} = \exp(-g_D D_a) \quad (8.14)$$

where  $g_D$  depends on the vegetation type (Table 8.1), and is non-zero for high vegetation only.

Evaporation from the interception reservoir is given by (8.7) only when the amount of water in the interception reservoir,  $W_1$ , is sufficient to sustain potential evaporation during the entire time step  $\Delta t$ . If  $W_1$  is limited, an additional resistance  $r_1$ , analogue to  $r_c$  in (8.8), is introduced.  $r_1$  is calculated from the potential evaporation of the previous time step. Note that this type of flux-limiter is a time-step dependent feature of the model numerics.

The evaporation from non-vegetated areas responds to a different physical mechanism compared to densely vegetated areas. Over bare soil the vaporisation of water in the soil pores takes place in a thin layer close to the surface-atmosphere interface as a direct effect of incoming solar radiation providing the latent heat requirements 2.5106 J/kg. Atmospheric conditions (e.g. air temperature, humidity, wind velocity and radiation) as well as soil conditions (e.g. water content, roughness length) all play a role in modulating the evaporation processes.

The relationship between soil moisture and bare soil evaporation is generally parametrized in land surface models and several formulations have been compared in the study by (Mahfouf and Noilhan, 1991). They found that in all cases the bare soil evaporation formulation stops when the soil is completely dry (for soil moisture close to zero). In the TESSEL scheme (Van den Hurk *et al.*, 2000; Viterbo and Beljaars, 1995; Viterbo *et al.*, 1999) the soil-moisture-evaporation link was assumed to be varying linearly between the wilting point and the field capacity values for soil moisture. With the introduction of a tiling approach, the same stress function was applied to both vegetated and non-vegetated tiles, neglecting the fact that wilting point is a soil moisture threshold that applies uniquely to vegetated areas. A justification for this approach is that the soil underneath the surface tiles is still a unique column for both vegetated and non-vegetated fractions. In the most recent version of the HTESSEL scheme (Balsamo *et al.*, 2011a), included in IFS Cy36r4 and operational since 10 November 2010), the formulation of the bare soil evaporation has been revisited to allow a smooth transition between vegetated and non-vegetated areas and to realign the formulation of bare ground evaporation with studies in the literature.

The soil evaporation resistance,  $r_{\text{soil}}$ , is

$$r_{\text{soil}} = r_{\text{soil,min}} f_{2b}(f_{\text{liq}}\theta_1) \quad (8.15)$$

with  $r_{\text{soil,min}} = 50 \text{ s m}^{-1}$ . By this parametrization, evaporation from bare ground is treated similar to a single leaved canopy with a minimum resistance  $r_{\text{soil,min}}$ , extracting water from the upper soil layer only, and not experiencing any additional stress due to limited radiation or dry air. In case of a vegetated tile the canopy resistance is formulated according to Jarvis (1976) and the soil moisture inhibition function (see (8.8)), which depends on the root-zone soil wetness, normalized between the wilting point and the field capacity, for non-vegetated tiles is uniquely dependent on the soil moisture of the first soil layer and a minimum soil resistance.

In the new formulation the  $f_{2b}$  for bare ground is calculated as

$$\frac{1}{f_{2b}(\theta_1)} = \frac{\theta_1 - \theta_{\text{min}}}{\theta_{\text{cap}} - \theta_{\text{min}}} \quad (8.16)$$

where  $\theta_1$  is the unfrozen soil water in the first soil layer and  $\theta_{\text{min}}$  is a vegetation-weighted average of the wilting point and a residual soil moisture content ( $\theta_{\text{res}}$  which is a function of the soil textural class, as defined in Table 8.7). The weights are given by the vegetation cover  $veg$  (Van den Hurk *et al.*, 2000) over the gridbox, so that

$$\theta_{\text{min}} = veg\theta_{\text{pwp}} + (1 - veg)\theta_{\text{res}} \quad (8.17)$$

This modification of the soil moisture inhibition function allows a much lower level of soil moisture to be reached in dry areas as a consequence of direct bare ground evaporation under strong insulation. Also results are more consistent with the soil moisture values observed, for instance, over deserts.

A special treatment is included in the calculation of evaporation over high vegetation with snow underneath (see Fig. 8.2). Evaporation takes place from both the canopy component in the tile ( $E_{\text{veg},7}$ ) and from the snow lying under the vegetation. The canopy evaporation uses a canopy resistance and saturation specific humidity at the canopy skin temperature, while the snow evaporation  $E_{\text{sn},7}$  is parametrized with an additional constant aerodynamic resistance  $r_{\text{a,sn}}$  and saturation specific humidity at snow temperature  $T_{\text{sn}}$ . The evaporation from tile 7 is the combination of the canopy transpiration and the snow evaporation so that

$$E_7 = \rho_a \frac{q_L - q_*}{r_a} = \rho_a \frac{q_* - q_{\text{sat}}(T_{\text{sn}})}{r_{\text{a,sn}}} + \rho_a \frac{q_* - q_{\text{sat}}(T_{\text{sk}})}{r_c} \quad (8.18)$$

where  $q_*$  is the humidity at the connection point of the three resistances (Fig. 8.2). After elimination of  $q_*$ ,  $E_7$  can be rewritten as

$$E_7 = \rho_a \frac{q_L - q_{\text{sat}}(T_{\text{sk}})}{r_a + r_c + r_c \frac{r_a}{r_{\text{a,sn}}}} + \rho_a \frac{q_L - q_{\text{sat}}(T_{\text{sn}})}{r_a + r_{\text{a,sn}} + r_{\text{a,sn}} \frac{r_a}{r_c}} \quad (8.19)$$

The first term in the equation above is interpreted as  $E_{\text{veg},7}$  and is treated in the standard way (i.e., implicit in the tile skin temperature). The second term is interpreted as evaporation from snow ( $E_{\text{sn},7}$ ) and is handled explicitly. The values of  $r_{\text{a,sn}}$  depend on the stability of the subcanopy layer and are functions of  $\Lambda_{\text{a,u}}$  and  $\Lambda_{\text{a,s}}$  (see Table 8.2);  $r_{\text{a,sn}} = 67 \text{ s m}^{-1}$  and  $r_{\text{a,sn}} = 220 \text{ s m}^{-1}$  for an unstable and stable subcanopy layer, respectively. In spring, the latent heat flux of that tile,  $L_v E_{\text{veg},7} + L_s E_{\text{sn},7}$  will be dominated by snow evaporation since the frozen soil under the snow deck will lead to very large values of  $r_c$ .

The grid box total sensible and latent heat fluxes are expressed as an area weighted average:

$$H = \sum_{i=1}^8 C_i H_i \quad (8.20)$$

$$E = \sum_{i=1}^8 C_i E_i \quad (8.21)$$

with  $H_i$  given by (8.6), and  $E_i$  by (8.7) for ocean, sea-ice and snow on low vegetation, (8.8) for dry high and low vegetation, the interception reservoir (with  $r_c$  replaced by  $r_1$ ) and for bare soil (with  $r_c$  replaced by  $r_{\text{soil}}$ ) and (8.19) for high vegetation with underlying snow.

### 8.3 SURFACE ENERGY BALANCE AND LAND COUPLING

A skin temperature  $T_{\text{sk}}$  forms the interface between the soil and the atmosphere. As detailed in Section 3.6, it is calculated for each grid box tiles separately, by scanning the surface energy balance solver over the eight tiles, assuming a complete coverage of the specific tile. For a single tile, this procedure is very similar to the derivation of the Penman–Monteith equation in which the skin temperature is eliminated from the surface energy balance equation. The numerical approach used in HTESSEL has the advantage that the feedback of skin temperature on net radiation and ground heat flux is included (see Section 3.6). The input radiation and reference atmospheric temperature ( $T_L$ ), specific humidity ( $q_L$ ) and wind speed ( $U_L$ ) are identical for each tile. The surface fluxes “seen” by the atmosphere are calculated as an area-weighted average over the tiles (see (8.20) and (8.21)). For the high vegetation with snow underneath, the skin temperature is that of the high vegetation; the temperature of the underlying snow is calculated separately.

The energy balance equation solved for each tile takes into account partial absorption of net short-wave radiation,  $1 - f_{\text{Rs},i}$ , in the skin layer (see Table 8.2). The remaining energy is directly passed to the soil or snow so that

$$(1 - f_{\text{Rs},i})(1 - \alpha_i)R_s + \varepsilon(R_T - \sigma T_{\text{sk},i}^4) + H_i + L_{v,s}E_i = \Lambda_{\text{sk},i}(T_{\text{sk},i} - T_1) \quad (8.22)$$

where  $i$  denotes the tile index,  $R_s$  and  $R_T$  are downward short-wave radiation and long-wave radiation, respectively,  $\sigma$  is the Stefan–Boltzman constant,  $T_1$  the temperature of the upper soil or snow layer,  $H_i$

the sensible heat flux, and  $L_{v,s}E_i$  the latent heat flux from the skin layer, and  $\Lambda_{sk,i}$ , the skin conductivity for tile  $i$ . Latent heat of evaporation,  $L_v$ , is used for all evaporation terms except snow evaporation, while  $L_s$ , the latent heat of sublimation, is used for evaporation of snow (i.e., tile 5 and the contribution  $E_{sn,7}$  from tile 7, defined by (8.19)).

The tiled surface is thermally coupled to the snow deck, when present, and to a single soil profile. The net flux into the soil is a weighted average of the flux from each tile.

The solution of (8.22) is performed inside the code for turbulent exchanges in the atmosphere (Chapter 3). The atmospheric vertical diffusion equations yield a tridiagonal system of equations, with the coupling to the skin temperature given by the matrix row corresponding to the lowest model level. The first step for the solution of the system of equations, an LU decomposition, is followed by the solution of (8.22) before back-substitution. Details of the computations can be found in Chapter 3.

## 8.4 SNOW

The snow scheme represents an additional “layer” on top of the upper soil layer, with an independent, prognostic, thermal and mass contents. The snow pack is represented by a single snow temperature,  $T_{sn}$  and the snow mass per unit area (snow mass for short)  $S$ . The net energy flux at the top of the snow pack,  $G_{sn}^T$ , is the residual of the skin energy balance from the snow covered tiles and the snow evaporation from the tile with high vegetation over snow (see (8.18)). The basal heat flux,  $G_{sn}^B$ , is given by a resistance formulation modified in case of melting. The absorbed energy is used to change the snow temperature or melt the snow, when  $T_{sn}$  exceeds the melting point. Liquid water within the snow pack is diagnosed from snow temperature, snow mass and snow density. Liquid water coexists in the snowpack leading to internal phase changes (freezing/melting), and rainfall can be intercepted.

The heat capacity of the snow deck is a function of its depth, and snow density, and is modified in the presence of liquid water. The snow thermal conductivity changes with changing snow density. Snow density changes due to overburden and thermal metamorphisms (Anderson, 1976; Boone and Etchevers, 2001) and compaction related to melt water retained in the snowpack, adapted from Lynch-Stieglitz (1994). The snow albedo changes exponentially with snow age. For snow on low vegetation it ranges between 0.50 for old snow and 0.85 for fresh snow. The albedo for high vegetation with snow underneath depends on vegetation type adapted from Moody *et al.* (2007). The details of the snow scheme and its validation can be found in Dutra *et al.* (2009) and Dutra *et al.* (2010a).

### 8.4.1 Snow mass and energy budget

The snow energy budget in the presence of snow liquid water reads as

$$\begin{aligned}
 (\rho C)_{sn} D_{sn} \frac{\partial T_{sn}}{\partial t} &= R_{sn}^N + L_s E_{sn} + H_{sn} - G_{sn}^B - L_f M_{sn} - Q_{sn}^{INT} \\
 Q_{sn}^{INT} &= L_f M_{sn}^{INT} = L_f \frac{\partial S_1}{\partial t}
 \end{aligned}
 \tag{8.23}$$

where  $(\rho C)_{sn}$  is the snow volumetric heat capacity ( $\text{J m}^{-3}\text{K}^{-1}$ ),  $D_{sn}$  is the snowpack depth (m),  $T_{sn}$  is the snow temperature (K), and the energy fluxes  $R_{sn}^N$ ,  $H_{sn}$  and  $G_{sn}^B$  are the net radiation (shortwave and longwave), sensible heat flux and basal heat flux ( $\text{W m}^{-2}$ ), respectively. The mass fluxes  $E_{sn}$  and  $M_{sn}$  are the snow sublimation and melting ( $\text{kg m}^{-2}\text{s}^{-1}$ ), respectively, that are associated with the latent heat of sublimation  $L_s$  and fusion  $L_f$  ( $\text{J kg}^{-1}$ ). The superscript *INT* denotes internal phase changes, where  $Q_{sn}^{INT}$  is the heat change associated with internal phase changes, and  $S_1$  the snow liquid water content (SLW) ( $\text{kg m}^{-2}$ ). Without loss of generality, it can be assumed that for the grid box characteristic of NWP models, the following expression is valid

$$S_1 = S_1(T_{sn}, S) \approx f(T_{sn}) S_1^c(S, \rho_{sn}) \tag{8.24}$$

where  $S_1^c$  ( $\text{kg m}^{-2}$ ) is the snow liquid water capacity,  $S$  is the sum of snow and water in the snowpack (also referred as SWE) and  $\rho_{sn}$  is the snow density ( $\text{kg m}^{-3}$ ). The snow temperature function is prescribed in

an analytical form – following a similar approach described by [Viterbo \*et al.\* \(1999\)](#) for soil phase changes

$$f(T_{\text{sn}}) = \begin{cases} 0 & T_{\text{sn}} < T_f - d/2 \\ 1 + \sin\left\{\frac{\pi(T_{\text{sn}} - T_f)}{d}\right\} & T_{\text{sn}} \geq T_f - d/2 \end{cases} \quad (8.25)$$

where  $T_f$  is the triple-point temperature for water and  $d$  is a characteristic temperature difference, with respect to  $T_f$ , limiting the phase change regime. In the numerical implementation,  $d = 4$  K was chosen. Snow liquid water capacity is approximated as a function of  $S$  and  $\rho_{\text{sn}}$ , following [Anderson \(1976\)](#)

$$S_1^c = S[r_{1,\text{min}} + (r_{1,\text{max}} - r_{1,\text{min}}) \max(0, \rho_{\text{sn},l} - \rho_{\text{sn}})/\rho_{\text{sn},l}] \quad (8.26)$$

with the constants  $r_{1,\text{min}} = 0.03$ ,  $r_{1,\text{max}} = 0.1$  and  $\rho_{\text{sn},l} = 200 \text{ kg m}^{-3}$ . This equation is a simple parametrization of a very complex phenomenon and has been used recently in other snow schemes, for example, [Boone and Etchevers \(2001\)](#).

Combining (8.23) and (8.24) results in a modified snow energy budget equation,

$$\left[ (\rho C)_{\text{sn}} D_{\text{sn}} + L_f S c_1 \frac{\partial f(T_{\text{sn}})}{\partial T_{\text{sn}}} \right] \frac{\partial T_{\text{sn}}}{\partial t} = R_{\text{sn}}^N + L_s E_{\text{sn}} + H_{\text{sn}} - G_{\text{sn}}^B - L_f M_{\text{sn}} \quad (8.27)$$

with one extra term in the lhs of the equation, that can be interpreted as an additional snow heat capacity – or heat capacity barrier. In compacted snowpacks, the representation of SLW as a diagnostic increases the snow heat capacity by a factor of up to five. This increase acts as a heat barrier near  $T_f$ , representing the increased snow temperature inertial due to freeze-melt events. The snow heat capacity in (8.23), (8.27) is approximated as

$$(\rho C)_{\text{sn}} \approx \frac{(\rho C)_i}{\rho_i} \rho_{\text{sn}} \quad (8.28)$$

This diagnostic approach for SLW also allows the representation of rainfall interception. The snow mass balance reads as

$$\frac{\partial S}{\partial t} = F + c_{\text{sn}} F_1 + c_{\text{sn}} E_{\text{sn}} - R_{\text{sn}}, \quad (8.29)$$

where  $F$ ,  $F_1$ , and  $R_{\text{sn}}$  are the mass fluxes of snowfall, rainfall and runoff ( $\text{kg m}^{-2} \text{ s}^{-1}$ ) and  $c_{\text{sn}}$  is the snow cover fraction. Rainfall is considered to reach the snowpack at  $T_f$ , and latent heat released by the freezing of the intercepted rainfall, if  $T_{\text{sn}} < T_f$ , is also accounted in the energy-balance solution. Runoff is defined as the rate at which liquid water leaves the snowpack and parametrized as follows:

$$R_{\text{sn}} = c_{\text{sn}} M_{\text{sn}} + \max\left(c_{\text{sn}} F_1 - \frac{S_1^c (1 - f(T_{\text{sn}}))}{\Delta t}, 0\right) \quad (8.30)$$

Liquid water is generated by melting ( $M_{\text{sn}}$ ) and by rainfall interception ( $F_1$ ). When snow liquid water content exceeds the snow liquid water capacity (defined in (8.25)) runoff is generated.

In this section, all fluxes are per unit area and apply only to the snow area (i.e. tile 5 and 7). The snow mass  $S$  applies to the entire grid square, as well as the snowfall flux from the atmospheric model. As a general rule, all quantities with subscript  $\text{sn}$  refer to the snow area. Snow evaporation (8.27, 8.29) is defined as

$$c_{\text{sn}} E_{\text{sn}} = c_5 E_5 + c_7 E_{\text{sn},7} \quad (8.31)$$

Snow mass and snow depth are related by

$$D_{\text{sn}} = \frac{1}{\rho_{\text{sn}} c_{\text{sn}}} S \quad (8.32)$$

where  $D_{\text{sn}}$  is snow depth for the snow-covered area (units m; note that  $D_{\text{sn}}$  is not a grid-averaged quantity) and  $\rho_{\text{sn}}$  is the snow density (units  $\text{kg m}^{-3}$ ).

### 8.4.2 Prognostic snow density and albedo

Snow density is assumed to be constant with depth, and the rate of density change is parametrized as

$$\frac{1}{\rho_{\text{sn}}} \frac{\partial \rho_{\text{sn}}}{\partial t} = \frac{\sigma_{\text{sn}}}{\eta_{\text{sn}}(T_{\text{sn}}, \rho_{\text{sn}})} + \xi_{\text{sn}}(T_{\text{sn}}, \rho_{\text{sn}}) + \frac{\max(0, Q_{\text{sn}}^{\text{INT}})}{L_f(S - S_l)} \quad (8.33)$$

where the first two terms in (8.33) represent overburden and thermal metamorphism (Anderson, 1976; Boone and Etchevers, 2001), respectively, and the last term represents the compaction related to meltwater retained in the snowpack, adapted from Lynch-Stieglitz (1994). In the overburden term (first term on the rhs of (8.33)),  $\sigma_{\text{sn}}$  and  $\eta_{\text{sn}}$  are the pressure of the overlying snow (Pa) and snow viscosity (Pa s), respectively. Melted water retained in the snowpack leads to a decrease of snow depth while keeping  $S$  constant. In snowfall conditions a weighted average is taken between the current snow density and the density of snowfall

$$\rho_{\text{sn}}^* = \frac{S\rho_{\text{sn}}^t + \Delta t F \rho_{\text{new}}}{S + \Delta t F} \quad (8.34)$$

where  $\rho_{\text{sn}}^*$  is an updated snow density. Snowfall density  $\rho_{\text{new}}$  is given by an expression from CROCUS (Brun *et al.*, 1989, 1992) where fresh snow density is a function of near surface air temperature and wind speed

$$\rho_{\text{new}} = a_{\text{sn}} + b_{\text{sn}}(T_{\text{air}} - T_f) + c_{\text{sn}}(V_a)^{1/2} \quad (8.35)$$

where  $T_{\text{air}}$  and  $V_a$  are the near surface air temperature (K) and wind speed ( $\text{ms}^{-1}$ ), respectively. The coefficients are  $a_{\text{sn}} = 109 \text{ kg m}^{-3}$ ,  $b_{\text{sn}} = 6 \text{ kg m}^{-3}\text{K}^{-1}$ , and  $c_{\text{sn}} = 26 \text{ kg m}^{-7/2}\text{s}^{1/2}$ . Snow density is constrained to be between 50 to  $450 \text{ kg m}^{-3}$ .

The snow viscosity in (8.33) is formulated following Anderson (1976)

$$\eta_{\text{sn}} = \eta_0 \exp\left(a_\eta(T_f - T_{\text{sn}}) + b_\eta \rho_{\text{sn}}\right) \quad (8.36)$$

where  $\eta_0 = 3.7 \times 10^7 \text{ (Pa s)}$ ,  $a_\eta = 8.1 \times 10^{-2} \text{ K}^{-1}$  and  $b_\eta = 1.8 \times 10^{-2} \text{ m}^3\text{kg}^{-1}$ . The pressure of the overlying snow is given by  $\sigma_{\text{sn}} = \frac{1}{2}Sg$ , where  $g$  is the standard gravity ( $\text{m}^2\text{s}^{-2}$ ).

The thermal metamorphism (second term in the rhs of (8.33)) is parametrized as

$$\xi_{\text{sn}} = a_\xi \exp\left[-b_\xi(T_f - T_{\text{sn}}) - c_\xi \max(0, \rho_{\text{sn}} - \rho_\xi)\right] \quad (8.37)$$

using the constant values of Anderson (1976):  $a_\xi = 2.8 \times 10^{-6} \text{ s}^{-1}$ ,  $b_\xi = 4.2 \times 10^{-2}$ ,  $c_\xi = 460 \text{ m}^3 \text{ kg}^{-1}$ , and  $\rho_\xi = 150 \text{ kg m}^{-3}$ .

Snow albedo in exposed areas evolves according to the formulation of Baker *et al.* (1990), Versegny (1991), Douville *et al.* (1995) and Dutra *et al.* (2010a). For melting and non melting-conditions:

$$\alpha_{\text{sn}}^{t+1} = \begin{cases} \alpha_{\text{sn}}^t - \tau_a \Delta t / \tau_1 & M_{\text{sn}} = 0 \\ (\alpha_{\text{sn}}^t - \alpha_{\text{min}}) \exp(-\tau_f \Delta t / \tau_1) + \alpha_{\text{min}} & M_{\text{sn}} > 0 \text{ or } T_{\text{sn}} \geq T_f - d/2 \end{cases} \quad (8.38)$$

where  $\tau_a = 0.008$ , which will decrease the albedo by 0.1 in 12.5 days,  $\alpha_{\text{min}} = 0.5$  and  $\alpha_{\text{max}} = 0.85$ . The timescales  $\tau_1 = 86400 \text{ s}$ , and  $\tau_f = 0.24$  corresponding to an e-folding time of about 4 days (see Table 8.5).

A continuous reset is applied to snow albedo after snowfall events:

$$\alpha_{\text{sn}}^{t+1} = \alpha_{\text{sn}}^t + \min\left(1, \frac{F\Delta t}{10}\right)(\alpha_{\text{max}} - \alpha_{\text{sn}}^t) \quad (8.39)$$

This formulation assumes that  $10 \text{ kg m}^{-2}$  of fresh snowfall are needed to reset the snow albedo to its maximum value ( $\alpha_{\text{max}} = 0.85$ ).

The above formulae are inadequate to describe the evolution of the surface albedo of snow cover with high vegetation. Observations suggest a dependence on forest type but, by and large, the albedo changes from a value around 0.3 just after a heavy snowfall to a value around 0.2 after a few days (see Betts and Ball, 1997

**Table 8.5** *Snow-related parameters.*

Symbol	Parameter	Value
$D_{\text{sn}}^{\text{max}}$	Maximum snow thermal depth	1.00 m
$D_{\text{cr}}$	Threshold value for grid box coverage of snow	0.1 m
$\alpha_{\text{min}}$	Minimum albedo of exposed snow	0.50
$\alpha_{\text{max}}$	Maximum albedo of exposed snow	0.85
$\lambda_{\text{i}}$	Ice heat conductivity	$2.2 \text{ W m}^{-1}\text{K}^{-1}$
$\rho_{\text{i}}$	Ice density	$920 \text{ kg m}^{-3}$
$(\rho C)_{\text{i}}$	Ice volumetric heat capacity	$2.05 \times 10^6 \text{ J m}^{-3}\text{K}^{-1}$
$\tau_{\text{a}}$	Linear coefficient for decrease of albedo of non-melting snow	0.008
$\tau_{\text{f}}$	Coefficient for exponential decrease of snow density and melting snow albedo	0.24
$\tau_{\text{l}}$	Length of day	86400 s

**Table 8.6** *Mean values of northern hemisphere broadband surface albedo (in presence of snow) aggregated by high vegetation type.*

Index	Vegetation type	Albedo
3	Evergreen needle leaf trees	0.27
4	Deciduous needle leaf trees	0.33
5	Deciduous broad leaf trees	0.31
6	Evergreen broad leaf trees	0.38
18	Mixed forest / woodland	0.29
19	Interrupted forest	0.29

and the discussion in [Viterbo and Betts, 1999](#)). This change reflects the disappearance of intercepted snow, due to melt (for sufficiently warm temperatures) or wind drift (for cold temperatures). Ways of describing those two mechanisms would involve either a separate albedo variable for the snow in the presence of high vegetation, or the introduction of an interception reservoir for snow. In the absence of any of the two, a vegetation-type-dependent albedo adapted from [Moody \*et al.\* \(2007\)](#) is used (see [Table 8.6](#)). [Moody \*et al.\* \(2007\)](#) provide a five-year (2000–2004) climatological statistic of northern hemisphere broadband (0.–5.0  $\mu\text{m}$ ) white-sky albedo for the 16 IGBP ecosystem classes when accompanied by the presence of snow on the ground.

### 8.4.3 Additional details

#### (a) Limiting of snow depth in the snow energy equation

Initial experimentation with the snow model revealed that the time evolution of snow temperature was very slow over Antarctica. The reason is rather obvious; the snow depth over Antarctica is set to a climatological value of 10 m which can respond only very slowly to the atmospheric forcing due to its large thermal inertia. In previous model versions, the properties of layer 1 were replaced by snow properties when snow was present, which kept the timescale short. A limit is put on the depth of the snow layer in the thermal budget,  $D_{\text{sn}}^{\text{max}} = 1.0 \text{ m}$ . The energy equation reads

$$\left[ (\rho C)_{\text{sn}} D_{\text{sn}}^* + L_f S_l^c \frac{\partial f(T_{\text{sn}})}{\partial T_{\text{sn}}} \right] \frac{\partial T_{\text{sn}}}{\partial t} = R_{\text{sn}}^{\text{N}} + L_s E_{\text{sn}} + H_{\text{sn}} - G_{\text{sn}}^{\text{B}} - L_f M_{\text{sn}} \quad (8.40)$$

$$D_{\text{sn}}^* = \min(D_{\text{sn}}, D_{\text{sn}}^{\text{max}})$$

A physical solution will be explored in a future scheme with a multilayer snow model, with for example four layers to represent timescales from one day to a full annual cycle.

(b) *Basal heat flux and thermal coefficients*

The heat flux at the bottom of the snow pack is written as a finite difference using

$$G_{\text{sn}}^{\text{B}} = \frac{T_{\text{sn}} - T_1}{r_{\text{sn}}} \quad (8.41)$$

where  $r_{\text{sn}}$  is the resistance between the middle of the snow pack and the middle of soil layer 1, with two components: the resistance of the lower part of the snow pack and the resistance of the top half of soil layer 1. Therefore

$$r_{\text{sn}} = 0.5 \frac{D_{\text{sn}}^*}{\lambda_{\text{sn}}} + \frac{1}{\Lambda_{\text{sk},8}} \quad (8.42)$$

where the second term is the skin layer conductivity for bare soil (tile 8), which can be seen as an approximation of  $0.5(D_1/\lambda_{\text{T}})$ . The snow thermal conductivity, is related to the ice thermal conductivity according to [Douville \*et al.\* \(1995\)](#) given by

$$\lambda_{\text{sn}} = \lambda_{\text{i}} \left( \frac{\rho_{\text{sn}}}{\rho_{\text{i}}} \right)^{1.88} \quad (8.43)$$

[Table 8.5](#) contains the numerical values of the ice density and ice heat conductivity.

(c) *Numerical solution for non-melting situations*

The net heat flux that goes into the top of the snow deck is an output of the vertical diffusion scheme

$$H_{\text{sn}}^{\text{N}} = R_{\text{sn}}^{\text{N}} - L_{\text{s}} E_{\text{sn}} - H_{\text{sn}} \quad (8.44)$$

In the absence of melting, the solution of (8.40) is done implicitly. The preliminary snow temperature, prior to the checking for melting conditions,  $T_{\text{sn}}^*$ , is given by

$$A_1 \frac{T_{\text{sn}}^* - T_{\text{sn}}^t}{\Delta t} = H_{\text{sn}}^{\text{N}} - \frac{T_{\text{sn}}^* - T_1}{r_{\text{sn}}} \quad (8.45)$$

$$A_1 = \min \left[ \frac{(\rho C)_{\text{i}}}{\rho_{\text{i}} c_{\text{sn}}} S, A_1^{\text{max}} \right] + L_{\text{f}} S_{\text{i}}^{\text{c}} \left. \frac{\partial f(T_{\text{sn}})}{\partial T_{\text{sn}}} \right|_{T_{\text{sn}}^t} \quad (8.46)$$

$$A_1^{\text{max}} = \frac{(\rho C)_{\text{i}}}{\rho_{\text{i}}} \rho_{\text{sn}} D_{\text{sn}}^{\text{max}}$$

where superscript  $t$  refers to the current time step and superscript  $*$  to the preliminary value at the next time step. The solution for  $T_{\text{sn}}^*$  is obtained from

$$T_{\text{sn}}^* \left( 1 + \frac{\Delta t}{r_{\text{sn}} A_1} \right) = T_{\text{sn}}^t + \frac{\Delta t}{A_1} \left( H_{\text{sn}}^{\text{N}} + \frac{T_1}{r_{\text{sn}}} \right) \quad (8.47)$$

The basal snow heat flux to be used as input for the thermal budget of the soil (in the snow covered fraction only) is

$$G_{\text{sn}}^{\text{B}} = \frac{T_{\text{sn}}^* - T_1}{r_{\text{sn}}} \quad (8.48)$$

Finally, a preliminary new value for the snow mass,  $S^*$ , is computed from snow fall and snow evaporation using

$$\frac{S^* - S^t}{\Delta t} = F + c_{\text{sn}} E_{\text{sn}} \quad (8.49)$$

**8.4.4 Treatment of melting**(a) *No melting occurs*

If  $T_{\text{sn}}^* < T_0$  no melting occurs and the preliminary values  $T_{\text{sn}}^*$  and  $S^*$  become the  $t + 1$  values, while the basal heat flux is given by (8.48).

(b) *Melting conditions*

If  $T_{sn}^* > T_0$ , snow melting occurs and the time step is divided in two fractions,  $\Delta t = \Delta_1 t + \Delta_2 t$ , where the first fraction,  $\Delta_1 t$  brings the temperature to  $T_0$  with no melting so that

$$\Delta_1 t = \frac{A_1(T_0 - T_{sn}^t)}{H_{sn}^N - (T_0 - T_1^t)/r_{sn}} \quad (8.50)$$

while, during the second fraction,  $\Delta_2 t$ , melting occurs with no resultant warming of the snow so that

$$\begin{aligned} T^{t+1} &= T_0 \\ Q_{sn} &= H_{sn}^N - G_{sn}^B \\ \frac{S^{t+1} - S^*}{\Delta_2 t} &= -c_{sn} M_{sn} = -c_{sn} \frac{Q_{sn}}{L_f} = -c_{sn} \frac{H_{sn}^N - G_{sn}^B}{L_f} \end{aligned} \quad (8.51)$$

If not all the snow melts, i.e., if  $S^{t+1} > 0$ , the heat flux passed to the soil is

$$G_{sn}^B = \frac{T_0 - T_1}{r_{sn}} \quad (8.52)$$

When all the snow melts, i.e., if  $S^{t+1} < 0$ , the melting time step is redefined as

$$\begin{aligned} S^{t+1} &= 0 \\ \Delta_2 t &= L_f \frac{S^*}{c_{sn}(H_{sn}^N - G_{sn}^B)} \\ \Delta_3 t &= 1 - (\Delta_1 t + \Delta_2 t) \end{aligned} \quad (8.53)$$

and the basal heat flux is redefined as

$$G_{sn}^B = \frac{\Delta_1 t + \Delta_2 t}{\Delta t} \frac{T_0 - T_1}{r_{sn}} + \frac{\Delta_3 t}{\Delta t} H_{sn}^N \quad (8.54)$$

#### 8.4.5 Numerical stability of the snow scheme

Considering the snow scheme is adopted in an explicit formulation occasional numerical instability can occur with long timesteps. This is particularly the case since the conductivity between the forest canopy and the underlying snow (skin layer conductivity for tile 7) has increased (since the IFS cycle 39r1). The snow tiles are represented in [Fig. 8.3](#).

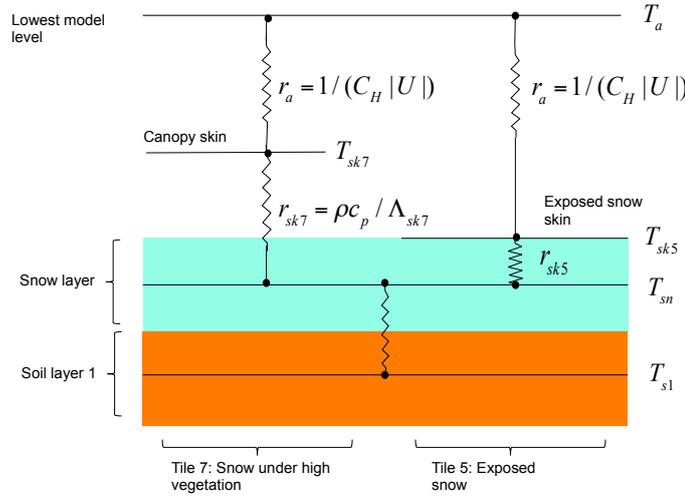
A standard solution for stability problems would imply the coupled formulation to be fully implicit. However a simpler solution that does not require the heat flux  $G$  and the snow temperature  $T_{sn}^{n+1}$  at time level  $n + 1$  was found. This involves an assumed snow temperature  $T_{sn}$  respond to heat flux  $G$  and it consider a simplified version of the snow energy equation [8.40](#) with the linear relation

$$\frac{\partial T_{sn}}{\partial t} = A_{sn} + B_{sn} G. \quad (8.55)$$

In this case the snow heat flux is approximated as

$$\begin{aligned} G &= \Lambda_{sk} [T_{sk}^{n+1} - T_{sn}^{n+1}] \\ &= \Lambda_{sk} [T_{sk}^{n+1} - T_{sn}^n - (A_{sn} + B_{sn} G) \Delta t], \\ G &= \frac{\Lambda_{sk}}{1 + \Lambda_{sk} B_{sn} \Delta t} [T_{sk}^{n+1} - T_{sn}^n - A_{sn} \Delta t]. \end{aligned} \quad (8.56)$$

The coefficients  $A_{sn}$  and  $B_{sn}$  can only be calculated correctly by the snow scheme, so a call to the snow scheme would be necessary before the surface energy balance is called involving a substantial redesign effort. A simplified estimate of  $A_{sn}$  and  $B_{sn}$  is used instead with  $A_{sn} = 0$  and that  $B_{sn} = (D(\rho C)_{sn})^{-1}$  as it was found efficient to prevent oscillatory evolution of the snow temperature. A further security is introduced in the snow temperature absolute values which are not allowed to be below 180K.



**Figure 8.3** Schematic representation of the snow tiles.

For introducing the numerical stability term is sufficient to replace  $\Lambda_{sk,i}$  in the surface energy balance equation 8.22 of tile 5 and tile 7 with the  $\Lambda_{sks,i}$  given by the formulation below.

$$\Lambda_{sks} \longrightarrow \frac{\Lambda_{sk}}{1 + \Lambda_{sk} B_{sn} \Delta t} \quad (8.57)$$

Equation (8.57) enhances the original  $\Lambda_{sk}$  to guarantee numerical stability and it mimics a fully implicit formulation provided the approximated estimate of  $A_{sn}$  and  $B_{sn}$  withstand.

## 8.5 SOIL HEAT TRANSFER

In the absence of internal phase changes, the soil heat transfer is assumed to obey the following Fourier law of diffusion

$$(\rho C)_{\text{soil}} \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left[ \lambda_T \frac{\partial T}{\partial z} \right] \quad (8.58)$$

where  $(\rho C)_{\text{soil}}$  is the volumetric soil heat capacity ( $\text{J m}^{-3} \text{K}^{-1}$ ),  $T$  is the soil temperature (units K),  $z$  is the vertical coordinate (i.e. the distance from the surface, positive downwards – units m), and  $\lambda_T$  is the thermal conductivity ( $\text{W m}^{-1} \text{K}^{-1}$ ). The above equation assumes that heat fluxes are predominantly in the vertical direction, that the effects of phase changes in the soil and the heat transfer associated with the vertical movement of water in the soil can be neglected (De Vries, 1975), and that the effects of hysteresis can be neglected (Milly, 1982).

The boundary condition at the bottom, no heat flux of energy, is an acceptable approximation provided that the total soil depth is large enough for the time-scales represented by the model or, in other words, the bottom of the soil is specified at a depth where the amplitude of the soil heat wave is a negligible fraction of its surface amplitude (see De Vries, 1975 and next Section 8.6).

### 8.5.1 Discretization and choice of parameters

For the solution of (8.58) the soil is discretized in four layers, of depths  $D_k$  ( $k = 1, 2, 3, 4$ ), the temperatures are defined at full layers ( $T_k$ ), and the heat fluxes, at half layers ( $\hat{G}_{k+1/2}$  is the heat flux, positive downwards, units  $\text{W m}^{-1}$ , at the interface between layer  $k$  and  $k + 1$ ). An energy-conserving implicit algorithm is used, leading to a tridiagonal system of equations with solution detailed in Section 8.9.

**Table 8.7** Parameters in the land-surface scheme. See [Table 8.5](#) for snow-related parameters.

Symbol	Parameter	Value
$b_I$	Interception efficiency	0.25
$D_1$	Depth of soil layer 1	0.07 m
$D_2$	Depth of soil layer 2	0.21 m
$D_3$	Depth of soil layer 3	0.72 m
$D_4$	Depth of soil layer 4	1.89 m
$F_{cv}$	Fraction of gridbox covered by convective rainfall	0.5
$k$	Heterogeneity factor for convective precipitation	0.5
$T_{f1}$	Highest temperature for existence of ice water	$T_0 + 1$
$T_{f2}$	Lowest temperature for existence of liquid water	$T_0 - 3$
$w_{1\max}$	Maximum water amount on single leaf	0.0002 m

The boundary condition at the bottom is

$$G_{4+1/2} = 0 \quad (8.59)$$

At the top, the boundary condition is the soil heat flux at the surface, computed as a weighted average over the tiles. For the snow free tiles, the flux into the soil consists of two parts. Apart from the diffusion of heat governed by  $\Lambda_{sk,i}(T_{sk,i} - T_1)$  (see (8.22)), the net short-wave radiation not absorbed by the skin layer ( $f_{Rs,i}$ ) provides energy to the soil. [Table 8.2](#) lists the values of  $\Lambda_{sk,i}$  and  $f_{Rs,i}$  for each of the tiles. For the snow tiles, the heat flux into the soil is the snow basal flux, calculated using a resistance formulation and modified in the case of partial melting (see (8.41), (8.48), (8.52), and (8.54)).

The net heat flux into the soil is given by

$$G_{1/2} = \sum_i C_i [\Lambda_{sk,i}(T_{sk,i} - T_1) + f_{Rs,i}(1 - \alpha_i)R_s] + c_s G_{sn}^B \quad (8.60)$$

where the summation scans all snow free tiles.

The volumetric soil heat capacity is assumed constant, with value  $2.19 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}$  (see [Table 8.7](#) for a list of constants used by the model). The heat conductivity,  $\lambda$ , depends on the soil-water content following [Peters-Lidard et al. \(1998\)](#) (see also [Farouki, 1986](#); [Johansen, 1975](#)) and is given by a combination of dry  $\lambda_{dry}$  and saturated  $\lambda_{sat}$  values, weighted by a factor known as the Kersten number,  $K_e$ , so that

$$\lambda = K_e(\lambda_{sat} - \lambda_{dry}) + \lambda_{dry} \quad (8.61)$$

where  $\lambda_{dry} = 0.190 \text{ W m}^{-1} \text{ K}^{-1}$  and

$$\lambda_{sat} = \lambda_{sm}^{1-\theta_{sat}} \lambda_w^\theta \quad (8.62)$$

where the heat conductivity of the soil matrix,  $\lambda_{sm} = 3.44 \text{ W m}^{-1} \text{ K}^{-1}$  and the thermal conductivity of water is  $\lambda_w = 0.57 \text{ W m}^{-1} \text{ K}^{-1}$ . Equation (8.62) represents a simplification of Peters-Lidard formulation, neglecting the changes in conductivity due to ice water and assuming the quartz content typical of a loamy soil. Finally, the Kersten number for fine soils was selected in [Peters-Lidard et al. \(1998\)](#) as

$$K_e = \log_{10} \left[ \max \left( 0.1, \frac{\theta}{\theta_{sat}} \right) \right] + 1 \quad (8.63)$$

The depths of the soil layers are chosen in an approximate geometric relation (see [Table 8.7](#)), as suggested in [Deardorff \(1978\)](#). [Warrilow et al. \(1986\)](#) have shown that four layers are enough for representing correctly all timescales from one day to one year. Using the numerical values of the heat capacity and soil depths defined in [Table 8.7](#), the amplitude and phase response of the numerical solution of (8.58) were analysed by [Viterbo and Beljaars \(1995\)](#) for typical values of soil moisture in (8.61), and for harmonic forcings at the surface with periods ranging from half a day to two years. The analysis points to an error in the numerical solution of less than 20% in amplitude and 5% in phase for forcing periods between one day and one year.

### 8.5.2 Soil-water phase changes

At high and mid latitudes the phase changes of water in the soil have an important effect on the water and energy transfer in the soil. A proper consideration of the solid phase of soil water requires modifications including, in order of importance, the following.

- (i) The thermal effects related to the latent heat of fusion/freezing (e.g. [Rouse, 1984](#)).
- (ii) Changes in the soil thermal conductivity due to the presence of ice (e.g. [Penner, 1970](#), not included in TESSEL as mentioned in the previous section).
- (iii) Suppression of transpiration in the presence of frozen ground (e.g. [Betts \*et al.\*, 1998](#)) and already described in [\(8.12\)](#).
- (iv) Soil water transfer dependent on a soil water potential including the effects of frozen water (e.g. [Lundin, 1989](#)), represented in a proxy way by [\(8.85\)](#).

The latent-heat effects are described in the following. The main impact will be to delay the soil cooling in the beginning of the cold period, and to delay the soil warming in spring, although the latter effect is less important because it occurs when the solar forcing is significant. Both effects make the soil temperatures less responsive to the atmospheric forcing and damp the amplitude of the annual soil temperature cycle. More details on the soil-freezing scheme and its impact on forecasts and the model climate are described in [Viterbo \*et al.\* \(1999\)](#).

The soil energy equation, [\(8.58\)](#), is modified in the presence of soil water phase changes as

$$(\rho C)_{\text{soil}} \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left[ \lambda_T \frac{\partial T}{\partial z} \right] + L_{\text{fus}} \rho_w \frac{\partial \theta_I}{\partial t} \quad (8.64)$$

where  $\theta_I$  is the volumetric ice-water content. Without loss of generality, for the grid squares characteristic of NWP models it can be assumed that

$$\theta_I = \theta_I(\theta, T) = f(T)\theta \quad (8.65)$$

where  $\theta$  is the total soil-water content (liquid + ice), and

$$\begin{aligned} f_{\text{fr}}(T) &= 0 & T > T_{f1} \\ 0 < f_{\text{fr}}(T) < 1 & & T_{f1} \leq T \leq T_{f2} \\ f_{\text{fr}}(T) &= 1 & T < T_{f2} \end{aligned} \quad (8.66)$$

where  $T_{f1}$  and  $T_{f2}$  are characteristic temperatures limiting the phase change regime. In reality, the values of  $T_{f1}$  and  $T_{f2}$  and the function  $f_{\text{fr}}(T)$  have complicated dependencies on soil texture and composition (see e.g. [Williams and Smith, 1989](#)), but here they are approximated in a simple way. For an idealized homogeneous, one-component soil,  $f_{\text{fr}}(T)$  would be a step-function. According to [Williams and Smith \(1989\)](#) physical reasons for having an interval over which melting/freezing is active, rather than a threshold temperature, include the following.

- (i) Adsorption, resulting from forces between the mineral parts of the soil and the water.
- (ii) Capillarity, related to the fact that the water-free surface is not plane.
- (iii) Depression of the freezing point due to the effect of dissolved salts.
- (iv) Soil heterogeneity.

To avoid an undesirable coupling between the temperature and water equations in the soil, [\(8.65\)](#) is simplified to

$$\theta_I = f_{\text{fr}}(T)\theta_f \quad (8.67)$$

where  $\theta_f$  is a constant, representing the amount of soil water that can be frozen (thawed). For simplicity,  $\theta_f = (c_H + c_L)\theta_{\text{cap}}$ . The scaling with the vegetated fractions is the simplest way of distinguishing between dry (vegetation-sparse areas, e.g. deserts) and wet (vegetated) areas. Combining [\(8.67\)](#) with [\(8.64\)](#) results in

$$\left[ (\rho C)_{\text{soil}} - L_{\text{fus}} \rho_w \frac{\partial f_{\text{fr}}}{\partial T} \right] \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left[ \lambda_T \frac{\partial T}{\partial z} \right] \quad (8.68)$$

showing that the effect of freezing can be interpreted as an additional soil heat capacity, sometimes referred in the literature as the ‘heat-capacity barrier’ around freezing; not considering the process of soil water freezing/melting can lead to very large artificial temperature changes that do not occur in nature when sufficient soil water is available.

Finally, function  $f_{fr}(T)$ , is given by

$$f_{fr}(T) = \begin{cases} 0 & T > T_{f1} \\ 0.5 \left\{ 1 - \sin \left[ \frac{\pi(T - 0.5T_{f1} - 0.5T_{f2})}{T_{f1} - T_{f2}} \right] \right\} & T_{f2} \geq T \geq T_{f1} \\ 1 & T < T_{f2} \end{cases} \quad (8.69)$$

with  $T_{f1} = T_0 + 1$ ,  $T_{f2} = T_0 - 3$ .

## 8.6 SOIL-WATER BUDGET

The vertical movement of water in the unsaturated zone of the soil matrix obeys the following equation (see [Richards \(1931\)](#), [Philip \(1957\)](#), [Hillel \(1982\)](#) and [Milly \(1982\)](#) for the conditions under which (8.70) and (8.71) are valid) for the volumetric water content  $\theta$ :

$$\rho_w \frac{\partial \theta}{\partial t} = - \frac{\partial F_w}{\partial z} + \rho_w S_\theta \quad (8.70)$$

$\rho_w$  is the water density ( $\text{kg m}^{-3}$ ),  $F_w$  is the water flux in the soil (positive downwards,  $\text{kg m}^{-2}\text{s}^{-1}$ ), and  $S_\theta$  is a volumetric sink term ( $\text{m}^3\text{m}^{-3}\text{s}^{-1}$ ), corresponding to root extraction. Using Darcy’s law,  $F_w$  can be specified as

$$F_w = \rho_w \left( \lambda \frac{\partial \theta}{\partial z} - \gamma \right) \quad (8.71)$$

$\lambda$  ( $\text{m}^2 \text{s}^{-1}$ ) and  $\gamma$  ( $\text{m s}^{-1}$ ) are the hydraulic diffusivity and hydraulic conductivity, respectively.

Replacing (8.71) in (8.70), specifying  $S_\theta = S_\theta(\theta, z)$ , and defining parametric relations for  $\lambda$  and  $\gamma$  as functions of soil water, a partial differential equation for  $\theta$  is obtained; it can be numerically integrated if the top boundary condition is precipitation minus evaporation minus surface runoff. The bottom boundary condition assumes free drainage. [Abramopoulos \*et al.\* \(1988\)](#) specified free drainage or no drainage, depending on a comparison of a specified geographical distribution of bedrock depth, with a model-derived water-table depth. For the sake of simplicity the assumption of no bedrock everywhere has been adopted.

### 8.6.1 Interception

The interception reservoir is a thin layer on top of the soil/vegetation, collecting liquid water by the interception of rain and the collection of dew, and evaporating at the potential rate. The water in the interception reservoir,  $W_1$ , obeys

$$\rho_w \frac{\partial W_1}{\partial t} = c_1 E_1 + D + I \quad (8.72)$$

where  $C_1 E_1$  is the water evaporated by the interception reservoir (or dew collection, depending on its sign),  $D$  represents the dew deposition from other tiles, and  $I$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ) is the interception—the fraction of precipitation that is collected by the interception reservoir and is later available for potential evaporation. Because the interception reservoir has a very small capacity (a maximum of the order of 1 mm, see (8.2)), it can fill up or evaporate completely in one time step; special care has to be taken in order to avoid numerical problems when integrating (8.72). In addition, since  $E_1$  is defined in the vertical diffusion code, it might impose a rate of evaporation that depletes entirely the interception layer in one time step. In order to conserve water in the atmosphere-intercepted water–soil continuum, the mismatch of evaporation of tile 3 plus dew deposition from the other tiles (which is not explicitly dealt with by the vertical diffusion) as seen by the vertical diffusion and the intercepted water has to be fed into the soil.

The equation is solved in three fractional steps: evaporation, dew deposition, and rainfall interception. The solver provides the following as outputs.

- (i) Interception layer contents at time step  $n + 1$ ,  $W_1^{n+1}$ .
- (ii) Throughfall (i.e. rainfall minus intercepted water).
- (iii) The evaporation effectively seen by the intercepted layer in each tile  $i$ .

First, the upward evaporation ( $E_1 < 0$ ) contribution is considered; because  $C_1 E_1$  depends linearly on  $W_1$  (see (8.2)), an implicit version of the evaporating part of (8.72) is obtained by linearizing  $C_1(W_1)E_1$  giving

$$\rho_w \frac{W_1^* - W_1^t}{\Delta t} = C_1(W_1^t)E_1 + \frac{E_1}{W_{1m}}(W_1^* - W_1^t) \quad (8.73)$$

where  $W_1^*$  is the new value of interception-reservoir content after the evaporation process has been taken into account. After solving for  $W_1^*$ , a non-negative value of evaporation is obtained and the evaporation seen by this fractional time step is calculated

$$\begin{aligned} W_1^1 &= \max(0, W_1^*) \\ E^1 &= \rho_w \frac{W_1^1 - W_1^t}{\Delta t} \end{aligned} \quad (8.74)$$

The dew deposition is dealt with explicitly for each non-snow tile in succession, for tiles 3, 4, 6, 7, 8, where tile 7 is also considered because in the exposed snow tile, the canopy is in direct evaporative contact with the atmosphere. When the evaporative flux is downwards ( $E_1 > 0$ )

$$\begin{aligned} W^2 &= W^1 + \min\left(W_{1m} - W^1, \frac{\Delta t}{\rho_w} c_i D_i\right) \\ D_i &= \rho_w \frac{W_{1,i}^2 - W_1^1}{\Delta t} \end{aligned} \quad (8.75)$$

where superscript 2 denotes the final value at the end of the this fractional time step.

The interception of rainfall is considered by applying the following set of equations to large-scale and convective rainfall

$$\begin{aligned} W_1^3 &= W_1^2 + \min\left(W_{1m} - W_1^2, \frac{\Delta t}{\rho_w} b_1 (c_H + c_L) R_{1s}\right) \\ T_{1s} &= R_{1s} - \rho_w \frac{W_1^3 - W_1^2}{\Delta t} \\ W_1^{t+1} &= W_1^3 + \min\left(W_{1m} - W_1^3, \frac{\Delta t}{\rho_w} b_1 (c_H + c_L) \frac{R_{cv}}{F_{cv}}\right) \\ T_{cv} &= R_{cv} - \rho_w \frac{W_1^{t+1} - W_1^3}{\Delta t} \end{aligned} \quad (8.76)$$

$R_{cv}/F_{cv}$  is a modified convective rainfall flux, computed by applying the heterogeneity assumption that convective rainfall only covers a fraction  $F_{cv} = 0.5$  of the grid box,  $b_1 = 0.25$  is a coefficient of efficiency of interception of rain. The total evaporation seen by the interception reservoir is  $D_i$  for tiles 4, 6, 7, and 8 and  $c_1 E_1 + D_i$  for tile 3.

The interception reservoir model described in this section is probably the simplest water-conserving formulation based on Rutter's original proposition (Rutter *et al.*, 1972, 1975). For more complicated formulations still based on the Rutter concept see, for instance, Mahfouf and Jacquemin (1989), Dolman and Gregory (1992) and Ridder (2001).

## 8.6.2 Soil properties

Integration of (8.70) and (8.71) requires the specification of hydraulic conductivity and diffusivity as a function of soil-water content. In the previous TESSEL the parametric relations of Clapp and Hornberger

(1978) (see also [Cosby \*et al.\*, 1984](#)) were adopted (still available as option). These are given by

$$\begin{aligned}\gamma &= \gamma_{\text{sat}} \left( \frac{\theta}{\theta_{\text{sat}}} \right)^{2b+3} \\ \lambda &= \frac{b\gamma_{\text{sat}}(-\psi_{\text{sat}})}{\theta_{\text{sat}}} \left( \frac{\theta}{\theta_{\text{sat}}} \right)^{b+2}\end{aligned}\quad (8.77)$$

where  $b$  is a non-dimensional exponent,  $\gamma_{\text{sat}}$  and  $\psi_{\text{sat}}$  are the values of the hydraulic conductivity and matric potential at saturation, respectively. A minimum value is assumed for  $\lambda$  and  $\gamma$  corresponding to permanent wilting-point water content.

[Cosby \*et al.\* \(1984\)](#) tabulate best estimates of  $b$ ,  $\gamma_{\text{sat}}$ ,  $\Psi_{\text{sat}}$  and  $\theta_{\text{sat}}$ , for the 11 soil classes of the US Department of Agriculture (USDA) soil classification, based on measurements over large samples. Since the model described here specifies only one soil type everywhere, and because the determination of the above constants is not independent of the values of  $\theta_{\text{cap}}$  and  $\theta_{\text{pwp}}$ , the following procedure is adopted.

A comprehensive review of measurements of  $\theta_{\text{cap}}$  and  $\theta_{\text{pwp}}$  may be found in [Patterson \(1990\)](#). Starting from Patterson's estimates of  $\theta_{\text{cap}}$  and  $\theta_{\text{pwp}}$  for the 11 USDA classes, a mean of the numbers corresponding to the medium-texture soils (classes 4, 5, 7, and 8, corresponding to silt loam, loam, silty clay loam and clay loam, respectively) is taken. The resulting numbers are  $\theta_{\text{cap}} = 0.323 \text{ m}^3 \text{ m}^{-3}$  and  $\theta_{\text{pwp}} = 0.171 \text{ m}^3 \text{ m}^{-3}$ . Averaging the values of [Cosby \*et al.\* \(1984\)](#) for soil moisture and soil-water conductivity at saturation for the same classes gives the numerical values  $\gamma_{\text{sat}} = 0.57 \times 10^{-6} \text{ m s}^{-1}$  and  $\theta_{\text{sat}} = 0.472 \text{ m}^3 \text{ m}^{-3}$ . The Clapp and Hornberger expression for the matric potential is

$$\psi = \psi_{\text{sat}} \left( \frac{\theta}{\theta_{\text{sat}}} \right)^{-b} \quad (8.78)$$

is used with  $\psi(\theta_{\text{pwp}}) = -153 \text{ m}$  ( $-15 \text{ bar}$ ) and  $\psi(\theta_{\text{cap}}) = -3.37 \text{ m}$  ( $-0.33 \text{ bar}$ ) (see [Hillel, 1982](#); [Jacquemin and Noilhan, 1990](#)) to find the remaining constants  $b$  and  $\psi_{\text{sat}}$ . The results are  $b = 6.04$  and  $\Psi_{\text{sat}} = -0.338 \text{ m}$ . The above process ensures a soil that has an availability corresponding to the average value of medium-texture soils, and yields a quantitative definite hydraulic meaning to  $\theta_{\text{cap}}$  and  $\theta_{\text{pwp}}$  compatible with the Van Genuchten relations (see [Table 8.8](#) for a summary of the soil constants).

The [van Genuchten \(1980\)](#) formulation used in HTESEL provides a closed-form analytical expression for the conductivity, given as a function of the pressure head,  $h$ , as

$$\gamma = \gamma_{\text{sat}} \frac{[(1 + (\alpha h)^n)^{1-1/n} - (\alpha h)^{n-1}]^2}{(1 + (\alpha h)^n)^{(1-1/n)(l+2)}} \quad (8.79)$$

where  $\alpha$ ,  $n$  and  $l$  are soil-texture dependent parameters. Pressure head  $h$  is linked to the soil moisture by the expression

$$\theta(h) = \theta_r + \frac{\theta_{\text{sat}} - \theta_r}{(1 + (\alpha h)^n)^{1-1/n}} \quad (8.80)$$

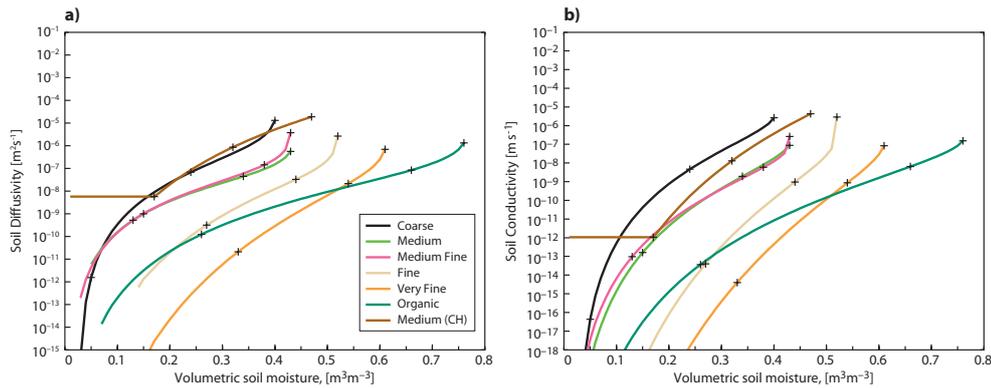
The VG scheme is recognized among soil physicists as capable of reproducing both the soil water retention and the hydraulic conductivity, and has shown good agreement with observations in intercomparison studies ([Shao and Irannejad, 1999](#)). [Table 8.8](#) lists parameter values for six soil textures for the VG scheme. HTESEL uses the dominant soil texture class for each gridpoint. This information is taken from the FAO ([FAO, 2003](#)) dataset as detailed in [Chapter 11](#). The permanent wilting point and the soil field capacity are obtained by a specified matric potential of  $\psi(\theta_{\text{pwp}}) = -15 \text{ bar}$  and  $\psi(\theta_{\text{cap}}) = -0.10 \text{ bar}$ , respectively. In [Table 8.9](#) the volumetric soil moistures associated with each soil class are shown for saturation, field capacity, wilting point, and residual moisture. Also shown is the plant available water content and the percentage of land points in each class. The last row shows the corresponding values for the single loamy soil used in the CH formulation in TESSEL. Note that the plant available soil water is greater for all the new soil classes in HTESEL. [Figure 8.4](#) shows the soil hydraulic diffusivity and conductivity for the TESSEL CH formulation and the six VG soil texture classes in HTESEL. In TESSEL those were not allowed to fall below their wilting point values. At saturation, TESSEL has the highest diffusivity and conductivity. The reduced values for fine soils in HTESEL reduces the infiltration of water and consequently the baseflow.

**Table 8.8** *Van Genuchten soil parameters.*

Texture	$\alpha$	$l$	$n$	$\gamma_{\text{sat}}$
Units	$m^{-1}$	-	-	$10^{-6}m/s$
Coarse	3.83	1.250	1.38	6.94
Medium	3.14	-2.342	1.28	1.16
Medium-Fine	0.83	-0.588	1.25	0.26
Fine	3.67	-1.977	1.10	2.87
Very Fine	2.65	2.500	1.10	1.74
Organic	1.30	0.400	1.20	0.93

**Table 8.9** *Values for the volumetric soil moisture in Van Genuchten and Clapp-Hornberger (CH, loamy; bottom row), at saturation,  $\theta_{\text{sat}}$ , field capacity,  $\theta_{\text{cap}}$ , permanent wilting point,  $\theta_{\text{pwp}}$ , and  $\theta_{\text{res}}$ , residual moisture. Last column reports the plant available soil moisture. Units are  $[m^3m^{-3}]$ .*

Texture	$\theta_{\text{sat}}$	$\theta_{\text{cap}}$	$\theta_{\text{pwp}}$	$\theta_{\text{res}}$	$\theta_{\text{cap}} - \theta_{\text{pwp}}$
Coarse	0.403	0.244	0.059	0.025	0.185
Medium	0.439	0.347	0.151	0.010	0.196
Medium-Fine	0.430	0.383	0.133	0.010	0.251
Fine	0.520	0.448	0.279	0.010	0.170
Very Fine	0.614	0.541	0.335	0.010	0.207
Organic	0.766	0.663	0.267	0.010	0.396
Loamy (CH)	0.472	0.323	0.171	-	0.151

**Figure 8.4** *Hydraulic properties of TESSEL and HTESSEL: (a) Diffusivity and (b) conductivity. The (+) symbols on the curves highlight (from high to low values) saturation, field capacity permanent wilting point.*

### 8.6.3 Runoff

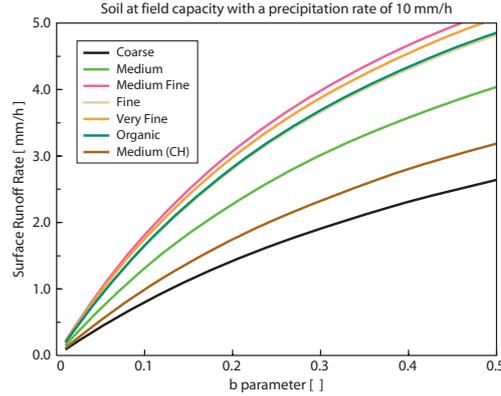
In general when the water flux at the surface exceeds the maximum infiltration rate, the excess water is put into surface runoff. A general formulation of surface runoff can be written as:

$$R = T + M - I_{\text{max}} \quad (8.81)$$

where  $I_{\text{max}}$  is the maximum infiltration rate,  $T$  the throughfall precipitation and  $M$  the snow melting. Different runoff schemes differ in the formulation of the infiltration. In TESSEL a maximum infiltration rate at the surface, defined by the maximum downward diffusion from a saturated surface was used to define the runoff term. The maximum infiltration rate  $I_{\text{max}}$  is calculated as

$$I_{\text{max}} = \rho_w \left( \frac{b_c \gamma_{\text{sat}} (-\psi_{\text{sat}})}{\theta_{\text{sat}}} \frac{\theta_{\text{sat}} - \theta_1}{z_1/2} + \gamma_{\text{sat}} \right) \quad (8.82)$$

where  $\rho_w$  is the water density, and  $z_1$  is the depth of the first soil model layer (7 cm). At typical NWP model resolutions this scheme is active only in the presence of frozen soil, when downward soil water transfer is inhibited, otherwise it hardly ever produces runoff, as shown in [Boone et al. \(2004\)](#).



**Figure 8.5** Surface runoff generation (rate mm/h) as a function of the  $b$  parameter (accounting for sub-grid effects of orography), when exposed to a precipitation rate of 10 mm/h.

In HTESSEL A variable infiltration rate, first introduced in the so-called Arno scheme by Dümenil and Todini (1992), accounts for the sub-grid variability related to orography and considers that the runoff can (for any precipitation amount and soil condition) occur on a fraction  $s$  of the grid-point area  $S$ .

$$\frac{s}{S} = 1 - \left(1 - \frac{W}{W_{\text{sat}}}\right)^b \quad b = \frac{\sigma_{\text{or}} - \sigma_{\text{min}}}{\sigma_{\text{or}} + \sigma_{\text{max}}} \quad (8.83)$$

where  $W$  and  $W_{\text{sat}}$  are vertically integrated soil water contents ( $\theta$  and  $\theta_{\text{sat}}$ ) over the first 50 cm of soil defined as an effective depth for surface runoff. Parameter  $b$  is spacially variable, depends on standard deviation of orography ( $\sigma_{\text{or}}$ ), and is allowed to vary between 0.01 and 0.5. The parameters  $\sigma_{\text{min}}$  and  $\sigma_{\text{max}}$  are set to 100 m and 1000 m respectively as in Van den Hurk and Viterbo (2003).

The surface runoff is obtained by the Hortonian runoff formulation by integrating (8.83) over the gridbox.

$$I_{\text{max}} = (W_{\text{sat}} - W) + \max \left[ 0, W_{\text{sat}} \left[ \left(1 - \frac{W}{W_{\text{sat}}}\right)^{\frac{1}{b+1}} - \left(\frac{T + M}{(b + 1)W_{\text{sat}}}\right) \right]^{b+1} \right] \quad (8.84)$$

Whenever rain or snow melt occurs, a fraction of the water is removed as surface runoff. The ratio runoff/precipitation scales with the standard deviation of orography, and therefore depends on the complexity represented in the gridbox, as well as on soil texture and soil water content via  $W$  and  $W_{\text{sat}}$ .

In Figure 8.5 the response to a 10 mm/h rain rate for the six VG soil types and for the CH case in TESSEL is shown as a function of the  $b$  parameter. At field capacity, the surface runoff may vary from roughly 1% to 50% of the rainfall (snow melting) rate, generally increasing with finer textures and orographic complexity.

#### 8.6.4 Water transport in frozen soil

Finally, the water transport is limited in the case of a partially frozen soil, by considering the effective hydraulic conductivity and diffusivity to be a weighted average of the values for total soil water and a very small value (for convenience, taken as the value of (8.77) at the permanent wilting point) for frozen water.

#### 8.6.5 Discretization and the root profile

A common soil discretization is chosen for the thermal and water soil balance for ease of interpretation of the results, proper accounting of the energy involved in freezing/melting soil water, and simplicity of the code. Equations (8.70) and (8.71) are discretized in space in a similar way to the temperature equations, i.e., soil water and root extraction defined at full layers,  $\theta_k$  and  $\rho_w S \theta_{,k}$ , and  $F_{k+1/2}$  the flux of water at the interface between layer  $k$  and  $k + 1$ . The resulting system of equations represents an implicit, water-conserving method.

For improved accuracy, the hydraulic diffusivity and conductivity are taken as (see [Mahrt and Pan, 1984](#))

$$\begin{aligned}\lambda_{k+1/2} &= (1 - f_{\text{fr}}^*)\lambda[\max(\theta_k^n, \theta_{k+1}^n)] + f_{\text{fr}}^*\lambda(\theta_{\text{pwp}}) \\ \lambda_{k+1/2} &= (1 - f_{\text{fr}}^*)\gamma[\max(\theta_k^n, \theta_{k+1}^n)] + f_{\text{fr}}^*\gamma(\theta_{\text{pwp}})\end{aligned}\quad (8.85)$$

where  $f_{\text{fr}}^* = \min[f_{\text{fr}}(\theta_k), f_{\text{fr}}(\theta_{k+1})]$ . The boundary conditions are given by

$$\begin{aligned}F_{4+1/2} &= \rho_w \gamma_4 \\ F_{1/2} &= T + M_{\text{sn}} - y_{\text{sfc}} + E_{1/2}\end{aligned}\quad (8.86)$$

The difference between throughfall  $T$  and surface runoff  $Y_{\text{sfc}}$  ( $\text{kg m}^{-2} \text{s}^{-1}$ ) is the soil infiltration at the surface:

$$\begin{aligned}T &= T_{1s} + T_{\text{cv}} \\ y_{\text{sfc}} &= \max(0, T_{1s} + M_{\text{sn}} - I_{\text{f,max}}) + \frac{\max(0, F_{\text{cv}} + T_{\text{cv}} - I_{\text{f,max}})}{F_{\text{cv}}} \\ I_{\text{f,max}} &= \rho_w \left[ \lambda_{1/2} \frac{(\theta_{\text{sat}} - \theta_1)}{0.5D_1} + \gamma_{1,2} \right]\end{aligned}\quad (8.87)$$

and  $\lambda_{1/2} = f_{\text{fr}}^*\lambda(\theta_{\text{pwp}}) + (1 - f_{\text{fr}}^*)\lambda(\theta_{\text{sat}})$ , with a similar equation for  $\gamma_{1/2}$ . The evaporation at the top of the soil layer,  $E_{1/2}$ , is computed as the sum of the evaporations of tile 8 plus the contributions necessary to conserve water with the solver of the interception layer.

- (i) Tile 3 mismatch (after the evaporated water used by the interception reservoir for the given tile is subtracted).
- (ii) When the evaporative fluxes are downward (i.e., dew deposition), the evaporation for tiles 4, 6 and the canopy evaporation of tile 7.

Root extraction is computed as sum of the transpiration contributions (expressed in  $\text{kgm}^{-2}$ ) over tiles  $i$  and soil layers  $j$  divided by the layer depth, such as

$$\rho_w S_{\theta,k} = \sum_i C_i \frac{E_i}{D_k} \frac{R_k \theta_k}{\sum_j R_j \theta_j} \quad (8.88)$$

where the sum over tiles  $i$  is done for tiles 4, 6, and 7. In case of dew deposition (i.e., tile downward evaporative flux),  $S_{\theta,k} = 0$ .

## 8.7 THE ECOSYSTEM EXCHANGES PARAMETRIZATION

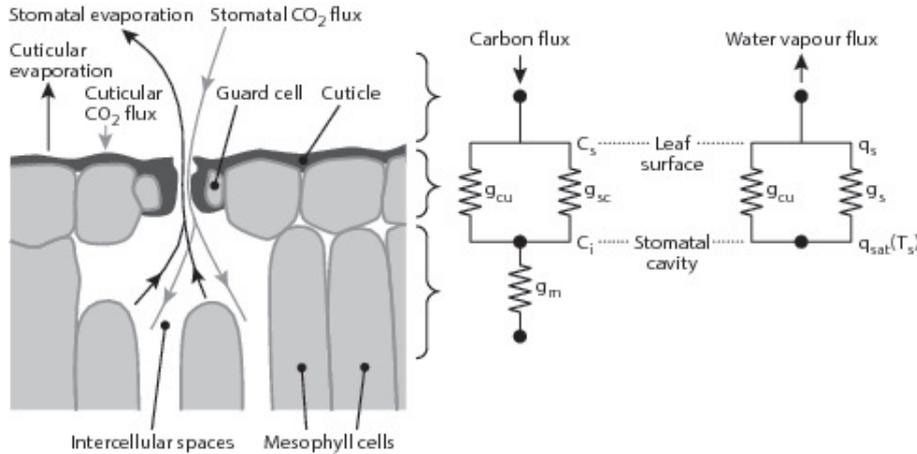
In order to obtain the net exchange of  $\text{CO}_2$  between the land surface and atmosphere by a NWP model, the Gross Primary Production (GPP) and the ecosystem respiration ( $R_{\text{eco}}$ ) needs to be represented. In CHTESSEL ([Boussetta et al., 2013b](#)) the GPP [ $\text{kg CO}_2 \text{m}^{-2} \text{s}^{-1}$ ] is parametrized following the so-called A-gs approach while  $R_{\text{eco}}$  [ $\text{kg CO}_2 \text{m}^{-2} \text{s}^{-1}$ ] is parametrized as a function of soil temperature, soil moisture, snow depth and vegetation type rather than prognostically computed. Schemes relying on prognostic land carbon "pools" (CLM-DGVM, [Levis et al., 2004](#); JULES, [Clark et al., 2011](#); ORCHIDEE, [Krinner et al., 2005](#)) are less practical for NWP purposes because they are difficult to initialize without a very long spin-up. The Net Ecosystem Exchange (NEE) of  $\text{CO}_2$  [ $\text{kg CO}_2 \text{m}^{-2} \text{s}^{-1}$ ] between the surface and atmosphere is then given by

$$NEE = GPP - R_{\text{eco}} \quad (8.89)$$

This quantity is the main observed variable in field site experiments.

### 8.7.1 The photosynthesis scheme (A-gs)

The A-gs approach is based on plant physiological considerations and describes the plant photosynthesis process and its dependence on  $\text{CO}_2$ , temperature and soil moisture ([Jacobs, 1994](#); [Jacobs et al., 1996](#); [Calvet et al., 1998](#); [Calvet et al., 2004](#)). The stomatal conductance is active for regulating both water



**Figure 8.6** Schematic representation of a leaf with the resistance analogies for carbon and water vapor, where  $g_{cu}$  is the cuticular conductance,  $g_{sc}$  the stomatal conductance to  $CO_2$ ,  $g_s$  the stomatal conductance to water vapor,  $g_m$  the mesophyll conductance,  $C_s$  and  $C_i$  the  $CO_2$  concentration at the canopy surface and inside the leaf cavity respectively,  $q_s$  and  $q_{sat}(T_s)$  the humidity at the canopy surface and the saturated humidity at the temperature  $T_s$  of the canopy surface respectively.

vapor and  $CO_2$  fluxes. The gross  $CO_2$  assimilation by the canopy  $A_g$  [ $kg\ CO_2\ m^{-2}s^{-1}$ ] is calculated using a photosynthesis module following Goudriaan *et al.* (1985). The net assimilation  $A_n$  [ $kg\ CO_2\ m^{-2}s^{-1}$ ] (i.e. the net flow of  $CO_2$  through the stomata) is  $A_g$  minus the dark respiration  $R_d$  [ $kg\ CO_2\ m^{-2}s^{-1}$ ]. Once the net assimilation is known, the stomatal conductance for  $CO_2$ ,  $g_{sc}$  [ $m\ s^{-1}$ ] can be derived by Kirchhoff's resistance/conductance analogy. It is defined as the net flow of  $CO_2$  through the stomata divided by the difference between the  $CO_2$  concentration outside the leaves  $C_s$  [ $kg\ CO_2\ m^{-3}$ ] and the concentration in the intercellular cavities  $C_i$  [ $kg\ CO_2\ m^{-3}$ ] (see Fig. 8.6):

$$g_{sc} = \frac{A_n(Env)}{C_s - C_i(Env)} \quad (8.90)$$

where the functions  $(Env)$  indicate the dependence of  $A_n$  and  $C_i$  on the different environment factors. In future version, it is planned that the stomatal conductance from the carbon module would be used for water vapor transpiration, which leads to a full coupling between carbon and water vapor fluxes.

The basics of the A-gs model are described by Jacobs (1994) and Jacobs *et al.* (1996). Further details of the current extended formulation are given in publications by Calvet *et al.* (1998) and Calvet *et al.* (2004). For the description of the canopy conductance to  $CO_2$ ,  $g_{sc}$ , a stepwise approach is followed with: (i) the definition of the temperature dependent parameters, (ii), the radiation response, (iii) the calculation of the ratio between internal and external  $CO_2$  concentration, (iv) the computation of stomatal conductance, (v) inclusion of the soil moisture response and (vi) the vertical integration over the canopy.

(i) Temperature responses

There are several parameters in the photosynthesis model that are temperature dependent, namely the compensation point, the mesophyll conductance and the maximum photosynthetic capacity. The compensation point  $\Gamma$  is defined as the  $CO_2$  concentration at which the net  $CO_2$  assimilation of a fully lit leaf becomes zero. It can be measured in a laboratory by exposing plants to a variable  $CO_2$  concentration. The mesophyll conductance  $g_m$  describes the transport of  $CO_2$  from the substomatal cavities to the mesophyll cells where the carbon is fixed. It includes the representation of physical and chemical processes. The maximum photosynthetic capacity  $A_{m,max}$  is specified as an absolute upper limit to the photosynthesis rate in full sunlight and non-limiting  $CO_2$  concentration. The temperature dependence is described with so-called  $Q_{10}$  functions, where  $Q_{10}$  represents the proportional increase of a parameter for a  $10^\circ$  increase in temperature (Berry and Raison, 1982).

For the compensation point the formulation is:

$$\Gamma(T_s) = \Gamma(25^\circ)Q_{10}^{(T_s-25)/10} \quad (8.91)$$

**Table 8.10** Parameter values as specified in the optimized CTESSEL model. The vegetation types are from the GLCC database and used in the same way as in TESSEL Van den Hurk et al. (2000)

Type	Vegetation type	$R_0$ [mgCO <sub>2</sub> m <sup>-2</sup> s <sup>-1</sup> ]	$g_m^*(25)$ [mm s <sup>-1</sup> ]	$g_c$ [mm s <sup>-1</sup> ]	$Dmax^*$ [kg kg <sup>-1</sup> ]	$Am,max(25)$ [mgCO <sub>2</sub> m <sup>-2</sup> s <sup>-1</sup> ]	$f_0^*$ [-]	$\Gamma(25)$ [ppm]
1	Crops, mixed farming	0.100	1.3	0.15	Eq.8.108	2.20	0.85	42
2	Short grass	0.080	1.3	0.20	Eq.8.108	3.00	0.65	42
3	Evergreen needleleaf	0.360	0.8	0.20	0.124	2.20	Eq.8.112	42
4	Deciduous needleleaf	0.330	0.8	0.20	0.124	2.20	Eq.8.112	42
5	Deciduous broadleaf	0.280	1.4	0.00	0.109	1.83	Eq.8.112	42
6	Evergreen broadleaf	0.270	1.1	0.25	0.124	1.83	Eq.8.112	42
7	Tall grass	0.150	2.3	0.20	Eq.8.108	1.83	0.70	2.6
8	Desert	-	-	-	-	-	-	-
9	Tundra	0.360	2.0	0.25	Eq.8.108	3.00	0.95	42
10	Irrigated crops	0.096	1.4	0.25	Eq.8.108	1.83	0.92	42
11	Semidesert	0.019	1.0	0.25	Eq.8.108	1.83	0.80	42
12	Ice caps and glaciers	-	-	-	-	-	-	-
13	Bogs and marshes	0.270	0.5	0.25	Eq.8.108	1.83	0.96	42
14	Inland water	-	-	-	-	-	-	-
15	Ocean	-	-	-	-	-	-	-
16	Evergreen shrubs	0.110	0.9	0.15	Eq.8.108	1.83	0.72	2.6
17	Deciduous shrubs	0.080	1.9	0.20	Eq.8.108	1.83	0.96	42
18	Mixed forest- Wood	0.420	1.0	0.00	0.124	2.20	Eq.8.112	42
19	Interrupted forest	0.160	0.8	0.10	0.124	2.20	Eq.8.112	42
20	Water -land mixtures	0.270	1.0	0.25	Eq.8.108	1.83	0.95	42

where  $\Gamma(25^\circ)$  is the compensation point at  $25^\circ$  C,  $Q_{10\Gamma}$  is the  $Q_{10}$ -constant and  $T_s$  is the leaf surface temperature. For  $g_m$  and  $A_{m,max}$ , the temperature dependence is further adjusted by the inhibition functions after Collatz et al. (1992):

$$g_m(T_s) = \frac{g_m(25^\circ)Q_{10g_m}^{(T_s-25)/10}}{(1 + e^{0.3(T_{1g_m}-T_s)})(1 + e^{0.3(T_s-T_{2g_m})})} \quad (8.92)$$

$$A_{m,max}(T_s) = \frac{A_{m,max}(25^\circ)Q_{10A_{m,max}}^{(T_s-25)/10}}{(1 + e^{0.3(T_{1am,max}-T_s)})(1 + e^{0.3(T_s-T_{2am,max})})} \quad (8.93)$$

where  $Q_{10g_m}$ ,  $Q_{10A_{m,max}}$ ,  $T_{1g_m}$ ,  $T_{2g_m}$ ,  $T_{1A_{m,max}}$ ,  $T_{2A_{m,max}}$ , are constants affecting the sensitivity to the plant surface temperature  $T_s$ . Parameter  $g_m(25^\circ)$  depends on soil moisture stress and will be further described in the following sections. Its unstressed value  $g_m^*(25^\circ)$  is optimized here with the help of observations. The constants in these functions are vegetation type dependent and are listed in Table 8.10 and Table 8.11.

(ii) Radiation and CO<sub>2</sub> response

For  $A_n$ , two regimes are distinguished: the radiation limiting regime and the CO<sub>2</sub> limiting regime (Goudriaan et al., 1985; Jacobs, 1994). In the radiation limiting regime with sufficient CO<sub>2</sub>,  $A_n$  is controlled by the amount of photosynthetically active radiation (PAR)  $I_a$

$$A_n = \varepsilon I_a - R_d \quad (8.94)$$

where  $R_d$  is the dark respiration and where  $\varepsilon$  is the quantum efficiency expressed according to

$$\varepsilon = \varepsilon_o \frac{C_i - \Gamma}{C_i + 2\Gamma} \quad (8.95)$$

Parameter  $\varepsilon_o$  is the maximum quantum use efficiency and  $C_s$  is the ambient CO<sub>2</sub> concentration at the leaf surface.

At high radiation intensities, net assimilation saturates at a level  $A_m$  and becomes CO<sub>2</sub> limited according to  $A_m = (C_i - \Gamma) g_m$  with  $C_i$  the CO<sub>2</sub> concentration inside the leaf cavities (see below). An absolute limit

**Table 8.11** *CTESSEL temperature response, quantum use efficiency and soil moisture stress parameters*

Parameter	Value	Notes
$Q_{10\Gamma}$	2	
$Q_{10gm}$	2	
$Q_{10A_{m,max}}$	2	
$T_{1gm}$ (°C)	5	(13 for veg. types 7 and 16)
$T_{1A_{m,max}}$ (°C)	8	(13 for veg. types 7 and 16)
$T_{2gm}$ (°C)	36	
$T_{2A_{m,max}}$ (°C)	38	
$a$	2.381	(5.323 for veg.types 7 and 16)
$b$	0.6103	(0.8923 for veg.types 7 and 16)
$D_{max}^X$ (g/kg)	300	
$\varepsilon_o$ (mg CO <sub>2</sub> /J PAR)	0.0142	(0.0117 for vegetation types 7 and 16)

to account for the maximum photosynthetic capacity of the leaves is further applied as follows

$$A_m = A_{m,max} [1 - \exp\{-g_m (C_i - \Gamma) / A_{m,max}\}] \quad (8.96)$$

The radiation and CO<sub>2</sub> limiting regimes are combined via a smooth exponential transition function

$$A_n = (A_m + R_d) \left[ 1 - \exp\left(\frac{-\varepsilon I_a}{A_m + R_d}\right) \right] - R_d \quad (8.97)$$

The autotrophic dark respiration is simply parametrized according to [Van Heemst \(1986\)](#) and includes the respiration from the leaves only

$$R_d = A_m/9 \quad (8.98)$$

The respiration from other parts of the vegetation is included in the parametrized heterotrophic respiration.

(iii) *The  $C_i/C_s$  ratio*

The CO<sub>2</sub> concentration inside the leaf cavities  $C_i$  needs to be known in order to derive the stomatal conductance from the net assimilation. Observations indicate that the ratio  $C_i/C_s$  is a rather conservative quantity for moist atmospheric conditions and that increasing humidity deficit exerts a strong stomatal control affecting this ratio. Therefore  $C_i/C_s$  is specified as a function of atmospheric moisture deficit  $D_s$  at the leaf surface.

$$\frac{C_i}{C_s} = f + (1 - f) \frac{\Gamma}{C_s} \quad (8.99)$$

where  $f$  is the coupling factor defined by:

$$f = f_o \left( 1 - \frac{D_s}{D_{max}} \right) + f_{min} \frac{D_s}{D_{max}} \quad (8.100)$$

and  $f_o$  is the value of  $f$  at  $D_s$ ,  $D_{max}$  is the maximum saturation deficit and

$$f_{min} = \frac{g_{cu}}{g_{cu} + g_m} \quad (8.101)$$

The transport of CO<sub>2</sub> is maintained in the situation where  $f = f_{min}$  through the leaf cuticle or because of imperfect closure of the stomata. This process is represented by the cuticle with conductance  $g_{cu}$ .

(iv) *Stomatal conductance*

The first computation of the stomatal conductance for CO<sub>2</sub>,  $g_{sc}^1$  is achieved by dividing net assimilation by the difference between CO<sub>2</sub> concentration in and outside the leaves. It is modified here to account for the limiting cases of very dry air and dark respiration:

$$g_{sc}^1 = \frac{A_n - A_{min} \left( \frac{D_s}{D_{max}} \frac{A_n + R_d}{A_m + R_d} \right) + R_d \left( 1 - \frac{A_n + R_d}{A_m + R_d} \right)}{C_s - C_i} \quad (8.102)$$

where  $A_{\min}$  represents the residual photosynthesis rate (at full light intensity) associated with cuticular transfers when the stomata are closed because of a high specific humidity deficit:

$$A_{\min} = g_m(C_{\min} - \Gamma) \quad (8.103)$$

In this equation,  $C_{\min}$  is the value of  $C_i$  at maximum specific humidity deficit:

$$C_{\min} = \frac{g_{cu}C_s + g_m\Gamma}{g_{cu} + g_m} \quad (8.104)$$

The diffusion of  $\text{CO}_2$  through the stomatal openings interacts with that of water vapour and therefore stomatal conductance to  $\text{CO}_2$  is corrected for this interaction by an iterative refinement:

$$g_{sc} = g_{sc}^1 + E \frac{M_a}{\rho_a M_v} \frac{C_s + C_i}{2(C_s - C_i)} \quad (8.105)$$

where  $M_v$  and  $M_a$  are molecular masses of water vapour and air respectively,  $\rho_a$  is the air density and  $E$  is the leaf transpiration based on the previous guess of the stomatal conductance:

$$E = (1.6g_{sc}^1)D_s\rho_a \quad (8.106)$$

Finally, the stomatal conductance to water vapour  $g_s$  is given by:

$$g_s = 1.6g_{sc} \quad (8.107)$$

The total conductance used by the transpiration scheme is  $g_s + g_{cu}$ , where  $g_{cu}$  is the vegetation type dependent cuticular conductance (Table 8.10).

#### (v) Soil moisture stress response

Among other possible A-gs formulations for which the soil moisture stress response is directly applied to the gross assimilation  $A_g$  (Ronda *et al.*, 2001) or the net assimilation  $A_n$  (Sala and Tenhunen, 1996), Calvet (2000) found that the soil moisture stress response is driven in a complex way through the mesophyll conductance  $g_m$ , the maximum specific humidity deficit tolerated by the vegetation  $D_{\max}$ , and the ratio  $C_i/C_s$  controlled by  $f$ . The soil moisture response behaves differently for high and low vegetation. In CHTESSEL the adopted soil moisture stress response follow the function described in Calvet (2000) and Calvet *et al.* (2004) and based on a meta-analysis of several herbaceous and woody vegetation types.

#### Low vegetation formulation:

Calvet (2000) found that the mesophyll conductance  $g_m$  and the maximum atmospheric moisture deficit  $D_{\max}$  vary with soil moisture but that they remain correlated according to:

$$\ln(g_m(25^\circ)) = a - b \ln(D_{\max}) \quad (8.108)$$

where  $g_m(25^\circ)$  in  $[\text{mm s}^{-1}]$  and  $D_{\max}$  in  $[\text{g kg}^{-1}]$

Therefore this equation is used to derive  $D_{\max}^*$  (maximum saturation deficit without soil moisture stress) from the tabulated  $g_m^*(25^\circ)$ , where superscript \* indicates optimal soil moisture conditions and  $a$  and  $b$  are tabulated empirical coefficients (Table 8.11). Then the soil moisture stress index  $f_2$  (see 8.11) is applied to find  $D_{\max}$  in stressed conditions according to a bilinear function with a breakpoint at a critical soil moisture stress index  $f_{2c}$ :

$$\begin{aligned} D_{\max} &= D_{\max}^X \frac{f_2}{f_{2c}} && \text{for } f_2 < f_{2c} \\ D_{\max} &= D_{\max}^X + (D_{\max}^* - D_{\max}^X) \frac{f_2 - f_{2c}}{1 - f_{2c}} && \text{for } f_2 \geq f_{2c} \end{aligned} \quad (8.109)$$

where  $D_{\max}^X$  is the maximum value of  $D_{\max}$  corresponding to  $f_{2c}$ . The resulting  $D_{\max}$  is substituted in (8.108) to find  $g_m(25^\circ)$ .

#### High vegetation formulation:

Observations show that  $g_m(25^\circ)$  is well correlated with the coupling factor  $f_0$  according to the following empirical expression (Calvet *et al.*, 2004):

$$\ln(g_m^*(25^\circ)) = 4.7 - 7.0f_0^* \quad (8.110)$$

where  $g_m(25^\circ)$  is in  $[\text{mm s}^{-1}]$ . In this case (8.110) is used to derive  $g_m^*(25^\circ)$  from the value  $f_0^*$  as tabulated according to vegetation type (Table 8.10). Subsequently a soil moisture stress function is applied to find  $g_m(25^\circ)$ :

$$\begin{aligned} g_m(25^\circ) &= g_m^N \frac{f_2}{f_{2c}} && \text{for } f_2 < f_{2c} \\ g_m(25^\circ) &= g_m^N + (g_m^*(25^\circ) - g_m^N) \frac{f_2 - f_{2c}}{1 - f_{2c}} && \text{for } f_2 \geq f_{2c} \end{aligned} \quad (8.111)$$

where  $g_m^N$  is the stressed value of  $g_m$  derived from Calvet *et al.* (2004) meta-analysis with the following empirical function:

$$\ln(g_m^N) = 2.8 - 7.0f_0^* \quad (8.112)$$

$g_m^N$  is in  $[\text{mm s}^{-1}]$ .

After computing  $g_m(25^\circ)$  according to (8.111), the stressed value for  $f_0$  is derived with

$$\ln(g_m(25^\circ)) = 2.8 - 7.0f_0 \quad (8.113)$$

$g_m(25^\circ)$  is in  $[\text{mm s}^{-1}]$ .

Further details on the soil stress parametrization can be found in Calvet (2000), Calvet *et al.* (2004) and Voogt *et al.* (2006).

(vi) *Vertical integration from leaf to canopy*

The net  $\text{CO}_2$  assimilation calculated at the leaf scale is upscaled to the canopy scale assuming that leaf parameters do not vary with height in the canopy, and that the attenuation of the incoming shortwave radiation in the canopy can be computed using a simple radiative extinction model. The incoming PAR above the vegetation ( $I_a(h)$ , with  $h$  the canopy height) is assumed to be 48% of the incoming shortwave radiation and then further attenuated in the canopy. The dependence of PAR on height  $z$  within the canopy is described by Roujean (1996) according to:

$$I_a(z) = I_a(h)(1 - K(z)) \quad (8.114)$$

where  $K$  is the extinction function given by:

$$K(z) = \delta(\mu_s)K_{\text{df}}(z) + (1 - \delta(\mu_s))K_{\text{dr}}(z) \quad (8.115)$$

$K_{\text{df}}(z)$  and  $K_{\text{dr}}(z)$  are the extinction coefficients of diffuse and direct light, respectively:

$$K_{\text{df}}(z) = 1 - e^{\left(\frac{-0.8bLAI(h-z)}{h}\right)}, \quad (8.116)$$

$$K_{\text{dr}}(z) = 1 - e^{\left(-\frac{G_l}{\cos(\mu_s)} \frac{bLAI(h-z)}{h}\right)} \quad (8.117)$$

where  $\mu_s$  is the solar zenith angle and  $G_l$  is a parameter that describes the distribution of leaves (a spherical angular distribution is assumed with  $G_l = 0.5$ ),  $\delta$  is the ratio of diffuse to total downward shortwave radiation at the top of the canopy,  $LAI(h-z)$  is the cumulative leaf area index above height  $z$  and  $b$  is the foliage scattering coefficient given by:

$$b = 1 - \frac{1 - \sqrt{1 - \omega}}{1 + \sqrt{1 - \omega}} \quad (8.118)$$

based on the leaf single scattering albedo  $\omega$  ( $=0.2$ ) for the solar spectrum corresponding to the PAR. Parameter  $\delta$  is given by:

$$\delta(\mu_s) = \frac{0.25}{0.25 + \cos(\mu_s)} \quad (8.119)$$

Assuming a homogeneous leaf vertical distribution, the integrated canopy net  $\text{CO}_2$  assimilation, dark respiration and conductance can be written as:

$$A_{\text{nI}} = LAI \int_0^1 A_{\text{n}} d(z/h) \quad (8.120)$$

$$R_{dI} = LAI \int_0^1 R_d d(z/h) \quad (8.121)$$

$$g_c = LAI \int_0^1 g_s d(z/h) \quad (8.122)$$

In the above equations,  $LAI$  is defined as the ratio of leaf area covering a unit of ground area ( $\text{m}^2 \text{m}^{-2}$ ). The integrations are parametrized with a three-point Gaussian quadrature method following [Goudriaan \(1986\)](#):

$$A_{nI} = LAI \sum_{i=1}^3 W_i A_n(z_i) \quad (8.123)$$

$$R_{dI} = LAI \sum_{i=1}^3 W_i R_d(z_i) \quad (8.124)$$

$$g_c = LAI \sum_{i=1}^3 W_i g_s(z_i) \quad (8.125)$$

where  $W_i$  and  $z_i$  are the Gauss weights and levels, respectively.

### 8.7.2 Ecosystem respiration and Gross Primary Production

The ecosystem respiration  $R_{eco}$  is given by two terms: the autotrophic dark respiration  $R_{dI}$  ([8.98](#)), and  $R_{soilstr}$  which represents both heterotrophic respiration from the soil and autotrophic respiration from the above and below ground structural biomass. It is parametrized following a modified formulation of [Norman \*et al.\* \(1992\)](#) as a function of soil temperature, soil moisture, snow depth and vegetation type:

$$R_{soilstr} = R_0(25) Q_{10R_0}^{\left(\frac{T_{soil}-25}{10}\right)} f_{sm} f_{sn} \quad (8.126)$$

In this equation  $f_{sn}$  and  $f_{sm}$  are snow and soil moisture attenuation functions respectively defined as:

$$f_{sn} = 1 - C_{vs}(1 - e^{-\alpha z_{snow}}) \quad (8.127)$$

$C_{vs}$  is the surface fraction covered by snow,  $\alpha$  is a constant expressing the attenuation of the soil carbon emission within the snow pack and  $z_{snow}$  is the snow depth. The soil moisture stress function for soil respiration is defined following a study by [Albergel \*et al.\* \(2010\)](#) as:

$$f_{sm} = \frac{\bar{\theta}}{\theta_{cap}} \quad (8.128)$$

$Q_{10R_0}$  represents the proportional increase of a parameter for a  $10^\circ$  increase in temperature ([Berry and Raison, 1982](#)), this case, given its variability with climate regimes,  $Q_{10R_0}$  is defined as a function of soil temperature after [McGuire \*et al.\* \(1992\)](#). The vegetation types are affecting the ecosystem respiration through a reference respiration at  $25^\circ\text{C}$ ,  $R_0(25)$ , estimated by minimizing the root mean square errors between simulated and observed  $R_{eco}$  for each vegetation type.

Finally, the gross primary production  $GPP$  and the ecosystem respiration are given by:

$$GPP = A_{nI} + R_{dI} \quad (8.129)$$

$$R_{eco} = R_{dI} + R_{soilstr}. \quad (8.130)$$

These two quantities are often derived from flux-tower NEE observations and used for evaluation of the two main  $\text{CO}_2$  processes.

### 8.7.3 Vegetation description

The state of vegetation is given by the Leaf Area Index; crucial for deriving the plant assimilation and transpiration activity. Currently a satellite observation-based climatology was considered for the representation of *LAI*. The satellite product (MOD15A2) is derived from the Moderate Resolution Imaging Spectroradiometer (MODIS) instrument on board of TERRA. It is produced daily for the land surfaces at 1 km spatial resolution from MODIS spectral reflectance with a global coverage, and synthesized on an 8-day time interval based on simultaneously retrieved maximum Fraction of Absorbed Photosynthetically Active Radiation (FAPAR) in order to remove the atmospheric noise (Myneni *et al.*, 1992). The collection 5 of the product (released in 2008 available from February 2000 to present) is used. To derive the climatological time series, 9 years of data (2000–2008) were re-projected from the sinusoidal to a geographic regular latitude/longitude projection, spatially averaged to 1/12th degree resolution, then temporally smoothed, monthly averaged (Jarlan *et al.*, 2008) and finally interpolated to the IFS reduced Gaussian grid. The MODIS LAI products were analyzed and validated in previous studies (Garrigues *et al.*, 2008; MODIS Land team <http://landval.gsfc.nasa.gov/>, Weiss *et al.*, 2012). After a positive assessment within the IFS system (Boussetta *et al.*, 2013b), this product was adopted by ECMWF for operational use. This climatology is used within HTESSEL to drive the surface carbon exchange module.

The land use classification follows from the Global Land Cover Characteristics (GLCC) data (Loveland *et al.*, 2000) according to the Biosphere-Atmosphere Transfer Scheme (BATS) classification to assign dominant high and low vegetation types and associated parameters within each grid box (as detailed in Table 8.10, and Table 8.11).

## 8.8 LAKES AND OTHER INLAND WATER BODIES

The representation of inland water bodies (lakes, reservoirs, rivers and coastal waters) is important in order to account for the thermal inertia effects and specific albedo and roughness characteristics associated to open water and to account for phase change during freezing/melting. This is simulated by the Fresh-water Lake model FLake (Mironov *et al.*, 2010), which was chosen to be included in the IFS for its intermediate complexity, particularly adapted for numerical weather prediction and climate applications. Moreover FLake benefits from a large research community efforts, contributing to validation and development (see <http://www.flake.igb-berlin.de/papers.shtml>).

The FLake implementation in the HTESSEL surface scheme has been introduced in subsequent steps, with ad-hoc simplifications that are necessary at global scale due to the lack of detailed knowledge of the lake conditions, i.e. turbidity or presence of lake bottom sediments. A freezing parameterization and specific ice prognostic variables with their energy and mass balance allows FLake to be used in all climate conditions. At present there is no account for a water balance equation and lake depth and surface area (or fractional cover) are kept constant in time.

The offline implementation and surface testing at global scale is presented in Dutra *et al.* (2010b). Balsamo *et al.* (2010) performed a sensitivity study on the lake depth parameter, which is most directly related to lake thermal inertia. The ancillary conditions such as global lake cover and bathymetry and lake initial conditions are described in Chapter 11. The forecast sensitivity of FLake in the IFS is evaluated in Balsamo *et al.* (2012), while the preparation of a retrospective lake reanalysis for initial conditions together with the coupled data assimilation impact assessment is provided in Balsamo (2013).

Previously to the introduction of FLAKE only grid-scale lakes (lake fraction larger than 50%) were considered in the IFS and treated similarly to ocean-points. In these grid points, the surface temperature was prescribed, and evolved daily during integration according to climatological temperatures (derived from ERA-40), without diurnal changes of surface temperature and inter-annual variability. With the activation of FLAKE both partial lake cover within a grid-box and fully resolved lakes are modelled prognostically and evolve with the same timestep and spatial grid resolution used to the atmospheric and land surface processes in the IFS.

A new tile (tile 9) was created in HTESSEL to represent the lakes interface with the atmosphere. The FLake model has been extensively modified removing components that were already part of HTESSEL

scheme such as the turbulent fluxes calculation in the tiling scheme, and the omission of parameterization for the treatment of lake bottom sediments for which global dataset are not available. The treatment of snow cover is omitted although the HTESSEL snow scheme can be coupled to the lake tile over resolved lakes allowing a consistent representation of snow cover over frozen lakes (Dutra *et al.*, 2010b).

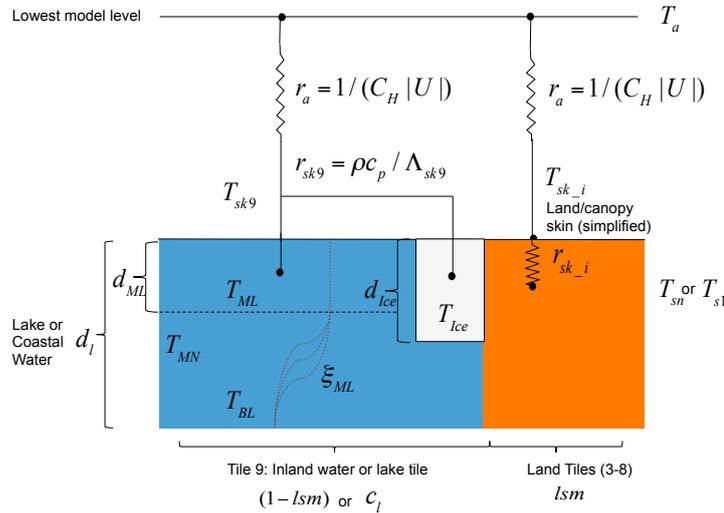


Figure 8.7 Schematic representation of the lake model.

### 8.8.1 Lake prognostic and ancillary variables

FLake meets the requirements for physical parameterizations in weather forecast systems, in terms of both physical realism and numerical efficiency. Mironov *et al.* (2010) presented a brief description of FLake, in its implementation into a limited-area atmospheric model. FLake adopts a similar concept to force-restore (Deardorff, 1978) in land surface models, since it has intermediate features between purely bulk models and multi-layer one-dimensional models.

The vertical temperature profile specified in FLake consists of a top mixed layer, with uniform distribution of temperature, and a thermocline with its upper boundary located at the mixed layer bottom, and the lower boundary at the lake bottom. The profile in the thermocline is parameterized according to the self-similarity concept, which has been supported by many theoretical and observational studies (see Mironov, 2008 for a discussion).

In the IFS implementation of FLake the surface fluxes of heat, moisture and momentum are computed by the HTESSEL routines. A new lake tile (tile 9) has been introduced to enable FLake to run on any single surface grid-point in the surface scheme, independently from the lake fractional cover. This is desirable to ease the interoperability of lake output at diverse spatial resolutions where the ratio of resolved/unresolved lakes and coastal water varies.

The prognostic variables included in FLake are: mixed-layer temperature  $T_{ML}$ , mixed-layer depth  $d_{ML}$ , bottom temperature  $T_{BL}$ , mean temperature of the water column  $T_{MN}$ , shape factor  $\xi_{ML}$  (with respect to the temperature profile in the thermocline), temperature at the ice upper surface  $T_{Ice}$ , and ice thickness  $d_{Ice}$ , which correspond to the schematic representation of Fig. 8.7.

The ancillary variables prescribed to characterize the lakes are the lake depth  $d_l$  and the lake cover  $c_l$  reported in Chapter 11.

### 8.8.2 Lake temperature and self-similarity

The self-similarity assumption leads to a significant simplification in comparison with multi-layer one-dimensional models, while still retaining a more realistic physics than bulk models. In particular, the unrealistic assumption of fully mixed temperature profile in a lake, applied in many bulk models, can be avoided. The mixed layer depth is calculated taking into account both wind and thermally driven mixing (Mironov, 2008), that follows from the volumetric heating of the water column by solar radiation. The structure of the stratified layer between the upper mixed-layer and the lake bottom, the thermocline, reposes on the concept of self-similarity (assumed shape) of a temperature-depth curve. The same concept is used to describe the temperature structure of the lake ice. Considering fresh water density as

$$\rho_w = \rho_r \left[ 1 - \frac{1}{2} \alpha_{tl} (T_{MN} - T_{MNr})^2 \right] \quad (8.131)$$

where  $\rho_w$  is the water density,  $\rho_r = 999.98 \text{ kg m}^{-3}$  is the maximum density of fresh water at the temperature  $T_{MNr} = 277.13 \text{ K}$ , and  $\alpha_{tl} = 1.6509 \text{ K}^{-2}$  is an empirical coefficient. According to 8.131, the thermal expansion coefficient  $\alpha_{tl}$  and a buoyancy parameter  $\beta$  depend on the water temperature according to:

$$\beta(T_l) = g \alpha_{T_l} = g a_T (T_l(z) - T_r) \quad (8.132)$$

where  $g$  is the acceleration due to gravity. The following two-layer parametric representation of the evolving temperature profile  $T_l(z)$  is adopted

$$T_l(z) = \begin{cases} T_{ML} & \text{if } 0 < z \leq d_{ML} \\ T_{ML} - (T_{ML} - T_{BL}) \Phi_T(\zeta) & \text{if } d_{ML} < z \leq d_l \end{cases} \quad (8.133)$$

as a function of depth  $z$ , where  $T_{ML}$  is the temperature of the upper mixed layer of depth  $d_{ML}$ .  $T_{BL}$  is the temperature at the bottom of the water body ( $z = d_l$ ), and  $\Phi_T = [T_{ML} - T_l(z, t)] / [T_{ML} - T_{BL}]$  is a dimensionless function of dimensionless depth  $\zeta = [z / d_{ML}(t)] / [d_l / d_{ML}(t)]$  that satisfies the boundary conditions  $\Phi_T(0) = 0$  and  $\Phi_T(1) = 1$ .

The mean temperature of the water column  $T_{MN}$  is obtained integrating 8.133, and relates  $d_{ML}$ ,  $d_l$ ,  $T_{ML}$ ,  $T_{BL}$  as

$$T_{MN} = T_{ML} - \xi_{ML} \left( 1 - \frac{d_{ML}}{d_l} \right) (T_{ML} - T_{BL}) \quad (8.134)$$

where  $\xi_{ML}$  is the shape factor with respect to the temperature profile in the thermocline.

The equation for the mean temperature of the water column is obtained by integrating one-dimensional heat transfer equation over  $z$  from 0 to  $d_l$ , and it reads as

$$d_l \frac{dT_{MN}}{dt} = \frac{1}{\rho_w c_w} \left[ G_s + I_s - G_{BL} - I(d_l) \right] \quad (8.135)$$

where  $c_w = 4.2 \cdot 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$  is the specific heat of water,  $G_s$  and  $I_s$  are the values at the surface of the vertical heat flux and of the heat flux due to solar radiation, respectively, and  $G_{BL}$  is the heat flux through the lake bottom. The radiation heat flux  $I$  that penetrates into the water is the surface value of the incident solar radiation flux from the atmosphere multiplied by  $1 - \alpha_w$ , where  $\alpha_w$  is the albedo of the water surface with respect to solar radiation. The surface flux  $G_s$  is a sum of the sensible and latent heat fluxes and the net heat flux due to long-wave radiation at the air-water interface. The equation of heat budget of the mixed layer reads as

$$d_{ML} \frac{dT_{ML}}{dt} = \frac{1}{\rho_w c_w} \left[ G_s + I_s - G_{ML} - I(d_{ML}) \right] \quad (8.136)$$

where  $G_{ML}$  is the heat flux at the bottom of the mixed layer, and assumes two variants for stationary/retreating or deepening mixed layers as detailed in Mironov *et al.* (2010). In the case of the mixed-layer being stationary or retreating,  $d d_{ML}/dt \leq 0$ , the bottom temperature is

$$\frac{dT_{BL}}{dt} = 0 \quad (8.137)$$

In the case of the mixed-layer deepening,  $d d_{ML}/dt > 0$ , the following equation is used:

$$\begin{aligned} & \frac{1}{2} (d_l - d_{ML})^2 \frac{dT_{ML}}{dt} - \frac{d}{dt} \left[ C_P (d_l - d_{ML})^2 (T_{ML} - T_{BL}) \right] = \\ & = \frac{1}{\rho_w c_w} \left[ C_G (d_l - d_{ML}) (G_{ML} - G_{BL}) + (d_l - d_{ML}) I_{ML} - I_{ML_{BL}} \right] \end{aligned} \quad (8.138)$$

Where  $C_P$  and  $C_G$  are dimensionless parameters. If  $d_{ML} = d_l$  both  $T_{ML}$  and  $T_{BL}$  are equal to  $T_{MN}$  and the water column profile is isothermal. The depth of a stably or neutrally stratified wind-mixed layer is determined from:

$$\frac{d d_{ML}}{dt} = \frac{d_{EQ} - d_{ML}}{d_{EQ}} C_{rh} u^* \quad (8.139)$$

where  $u^*$  is the surface friction velocity,  $C_{rh} = 0.03$  is a dimensionless constant, and  $d_{EQ}$  is a mixed-layer depth in radiative equilibrium (Mironov *et al.*, 2010).

### 8.8.3 Lake freezing and melting

The parameterisation of lake freezing is essential for treating inland water bodies in global models as mid-latitude and high-altitude lakes are often subject to extensive frozen periods especially when characterized by a shallow depth.

In the current implementation the snow accumulation over ice is not allowed and the existence of fractional ice condition on a give grid-point is not contemplated. The approach used to describe the temperature structure of the lake ice is conceptually similar to the approach used to describe the temperature structure of the lake thermocline. The following parametric representation of the evolving temperature profile within the ice is adopted (cf. 8.133):

$$T_{ice} = T_f - (T_f - T_{sk,i}) \Phi_{ice}(\zeta_i) \text{ if } -d_{ice} < z \leq 0 \quad (8.140)$$

where  $z$  is the vertical co-ordinate (positive downward) with the origin at the ice-water interface,  $d_{ice}(t)$  is the ice thickness,  $T_f = 273.15 \text{ K}$  is the fresh-water freezing point, and  $T_{sk,i}(t)$  is the temperature at the ice upper surface. The dimensionless universal function  $\Phi_{ice} = [T_f - T_{ice}(z, t)]/[T_f - T_{sk,i}]$  of dimensionless depth  $\zeta_i = z/d_{ice}(t)$  satisfies the boundary conditions  $\Phi_{ice}(0) = 0$  and  $\Phi_{ice}(1) = 1$ . The equation of the heat budget of the ice layer is

$$\begin{aligned} & \frac{d}{dt} \{ \rho_{ice} c_i d_{ice} [T_f - C_i (T_f - T_{ice})] \} - \rho_{ice} c_i T_{ice} \frac{d d_{ice}}{dt} = \\ & = G_s + I_s - I(0) + \kappa_i \frac{T_f - T_{ice}}{d_{ice}} \Phi'_{ice}(0) \end{aligned} \quad (8.141)$$

where  $\rho_{ice} = 9.1 \cdot 10^2 \text{ kg m}^{-3}$ ,  $c_i = 2.1 \cdot 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$  and  $\kappa_i = 2.29 \text{ J m}^{-1} \text{ s}^{-1} \text{ K}^{-1}$  are the ice mean property for density, specific heat and heat conductivity, respectively.  $G_s$  and  $I_s$  are the values of  $G$  and  $I$ , respectively, at the ice upper surface  $z = d_{ice}(t)$ , and  $\Phi'_{ice}(0) = d\Phi_{ice}/d\zeta_i$  at  $\zeta_i = 0$ . The radiation heat flux  $I_s$  that penetrates into the ice medium is the surface value of the incident solar radiation flux from the atmosphere multiplied by  $1 - \alpha_i$ , where  $\alpha_i$  is the ice surface albedo with respect to solar radiation and  $C_i$  dimensionless parameter representing the shape factor with respect to the temperature profile within the ice.

Equation 8.141 serves to determine  $T_{ice}$  when this temperature is below the freezing point  $T_f$ , i.e. when no melting at the ice upper surface takes place. During the ice melting from above,  $T_{ice}$  remains equal to  $T_f$ . During the ice growth or ice melting from below (these occur as  $T_{ice} < T_f$ ), the ice thickness is computed as:

$$L_f \frac{d(\rho_{ice} d_{ice})}{dt} = G_w + \kappa_{ice} \frac{T_f - T_{ice}}{d_{ice}} \Phi'_{ice}(0) \quad (8.142)$$

where  $L_f = 3.3 \cdot 10^5 \text{ J kg}^{-1}$  is the latent heat of fusion, and  $G_w$  is the heat flux in the near-surface water layer just beneath the ice. If the right-hand side of 8.142 is negative (this may occur due to a negative  $G_w$ ), ice ablation takes place. During the ice melting from above, the following equation is used:

$$L_f \frac{d(\rho_{ice} d_{ice})}{dt} = G_w - (G_s + I_s) + I(0) \quad (8.143)$$

that holds as the atmosphere heats the ice upper surface and  $T_{ice}$  is equal to  $T_f$ . The evolution of the temperature profile beneath the ice is described as follows. The temperature at the ice-water interface is fixed at the freezing point,  $T_{ML} = T_f$ . The mean temperature of the water column is computed from 8.142, where  $G_s$  and  $I_s$  are replaced with  $G_w$  and  $I(0)$ , respectively. If the bottom temperature is less than the temperature of maximum density,  $\rho$ , the mixed-layer depth and the shape factor with respect to the temperature profile in the thermocline are kept unchanged,  $d \, d_{ML}/dt = 0$  and  $\xi/dt = 0$ , and the bottom temperature is computed from 8.134. If the entire water column appears to be mixed at the moment of freezing, i.e.  $d_{ML} = d_l$  and  $T_{ML} = T_{BL}$ , the mixed layer depth is reset to zero,  $d_{ML} = 0$ , and the shape factor is reset to its minimum value,  $\xi_{ML} = \xi_{min}$ .

As the bottom temperature reaches the temperature of maximum density, its further increase is prevented and is kept constant equal to  $\rho_r$ . If  $d_{ML} > 0$ , the shape factor  $\xi_{ML}$  is kept unchanged, and the mixed-layer depth is computed from 8.134. As the mixed-layer depth approaches zero, 8.134 is used to compute the shape factor  $\xi_{ML}$  that in this regime increases towards its maximum value,  $\xi_{ML} = \xi_{max}$  (the values of  $\xi_{min} = 0.5$  and  $\xi_{max} = 0.8$  are used). If  $d_{ML} = 0$ , the heat flux from water to ice is estimated from

$$G_w = -\kappa_w \frac{T_b - T_{ice}}{d_l} \max[1, \Phi_w(0)] \quad (8.144)$$

where  $\kappa_w = 5.46 \cdot 10^{-1} \text{ J m}^{-1} \text{ s}^{-1} \text{ K}^{-1}$  is the molecular heat conductivity of water, and  $\Phi'_w(0) = d\Phi_w/d\zeta$  at  $\zeta = 0$ . Note that as soon as a mixed layer is formed ( $d_{ML} > 0$ ) the above water flux is neglected ( $G_w = 0$ ).

The shape factor with respect to the temperature profile in the thermocline is computed from

$$\frac{d\xi_{ML}}{dt} = \text{sign}(d \, d_{ML}/dt) \frac{\xi_{max} - \xi_{min}}{t_{rc}} \quad (8.145)$$

where  $\text{sign}$  is the sign function [ $\text{sign}(x) = x > 0$ ]. The shape factor  $\xi_{ML}$  evolves towards its maximum value during the mixed-layer deepening, and towards its minimum value during the mixed-layer stationary state or retreat. The adjustment occurs on a relaxation time scale  $t_{rc}$  estimated using the buoyancy frequency in the thermocline (Mironov *et al.*, 2010).

The exponential approximation of the decay law for the flux of solar radiation is used:

$$I = I_s \sum_{k=1}^n a_k e^{-\gamma_k(z+d_{ice})} \quad (8.146)$$

where  $I_s$  is the surface value of the incident solar radiation heat flux multiplied by  $1 - \alpha$  with  $\alpha$  being the albedo of the water surface or of the ice surface with respect to solar radiation,  $n$  is the number of wavelength bands,  $\alpha_k$  are fractions of the total radiation flux for different wavelength bands, and  $a_k$  are attenuation coefficients for different bands. The attenuation coefficients are piece-wise constant functions of  $z$ , i.e. they have different values for water and ice but remain constant within each media. The following parameterization of the ice surface albedo with respect to solar radiation is adopted:

$$\alpha_i = \alpha_i^{max} - (\alpha_i^{max} - \alpha_i^{min}) e^{-C_\alpha(T_f - T_{ice})/T_f} \quad (8.147)$$

where  $\alpha_i^{max} = 0.7$  and  $\alpha_i^{min} = 0.4$  are maximum and minimum values of the ice albedo, respectively, and  $C_\alpha = 95.6$  is a fitting coefficient. The presence of snow over lake ice and its seasonal changes are parametrized in 8.147 as a function of  $T_{ice}$ . During the melting season, the ice surface temperature is close to the fresh-water freezing point. The presence of wet snow, puddles, melt-water ponds is again parametrized implicitly and results in a decrease of the area-averaged surface albedo. The water surface albedo with respect to solar radiation,  $\alpha_w = 0.07$ , is assumed to be constant.

## 8.9 SEA ICE

Points that are not defined as land or lakes (i.e., a grid point with land cover less or equal 0.5 and lake cover is not dominant) are sea/ocean points and can have two tiles fractions, open water and ice. A surface analysis defines the ice fraction,  $c_I$ , and the temperature of the open water fraction; both quantities are kept constant during the forecast. No distinction is made between surface and skin temperature for the open water fraction (see Table 8.2).

The ice fraction is modelled as an ice slab, with open water underneath and a skin temperature for the thermal contact with the atmosphere. The main caveats in the sea ice parametrization are as follows.

- (i) Fixed depth of the slab, which can be relaxed once there is a reliable data set to specify its geographic distribution.
- (ii) Fixed fraction, which is a reasonable assumption for a 10-day forecast period, and avoids the need for the momentum balance of the ice and its complex rheology (see, e.g., Flato and Hibler III, 1992) and the definition of the ocean currents.
- (iii) No snow accumulation on top of the ice (although one of the main effects of snow, i.e., a markedly different surface albedo, is partially emulated by the prescribed seasonal albedo in Table 2.6).

The ice heat transfer is assumed to obey the following Fourier law of diffusion

$$(\rho C)_I \frac{\partial T_I}{\partial t} = \frac{\partial}{\partial z} \left[ \lambda_I \frac{\partial T_I}{\partial z} \right] \quad (8.148)$$

where  $(\rho C)_I = 1.88 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}$  is the volumetric ice heat capacity,  $T_I$  is the ice temperature,  $\lambda_I = 2.03 \text{ W m}^{-1} \text{ K}^{-1}$  and is the ice thermal conductivity. The boundary condition at the bottom is the temperature of the frozen water,  $T_{fr} = T_0 - 1.7$  and the top boundary condition is the net heat flux at the surface, obtained from the solution of the ice skin thermal budget.

Equation (8.148) is solved with the ice discretized in four layers, with the depth of the top three layers as in the soil model and the depth of the bottom layer defined as

$$D_{I,4} = D_I - \sum_{j=1}^3 D_{I,j} \quad (8.149)$$

and the total depth of the ice slab,  $D_I$ , is prescribed as 1.5 m. In order to ensure a constant ice fraction, the solution of the ice thermal budget is capped to the ice melting temperature,  $T_{m1} = T_0$  at all levels. the details of the numerical discretization can be found in Section 8.11.

## 8.10 OCEAN BOUNDARY CONDITIONS FOR TEMPERATURE AND SPECIFIC HUMIDITY

In the operational system, the sea surface temperature (SST) is specified from an analysis provided by OSTIA and kept constant during the 10-day forecast. This analysis is a blend of satellite retrievals and in situ observations from ships. The idea is to have a detailed horizontal distribution from satellite and to anchor this temperature fields to the rather sparse ship observations. It means that that the analyzed SST fields are calibrated as if they are ship observations and therefore they represent bulk SST fields (i.e. measured a few metres deep)

The ocean skin temperature is not always the same as the bulk SST. A very shallow layer (less than 1 mm thick) is cooler because of the turbulent and long wave radiative heat loss to the atmosphere which has to be compensated for by the inefficient molecular transport in the water skin. Solar radiation has only a small effect on the cool skin because the solar absorption in such a thin layer is small. However, at low winds, solar radiation can create a so-called warm layer with a depth of a few metres.

Parametrizations of three different near surface ocean effects are included in the code: 1. the cool skin, 2. the warm layer and 3. salinity effects on the saturation specific humidity at the surface. These parametrizations can be controlled through namelist NAEPHY. The namelist parameters are LEOCCO (default:TRUE), LEOCWA (default:TRUE), and LEOCSA (default:TRUE), for the cool skin, the warm layer and salinity effects respectively. So, all three effects are activated in this cycle (cool skin and warm layer are activated since Cy35r1). Details of the cool skin and warm layer parametrizations are given in [Beljaars \(1997\)](#) and [Zeng and Beljaars \(2005\)](#).

### 8.10.1 The cool skin

The cool ocean skin is the result of heat loss to the atmosphere which is balanced by thermal conduction in the quasi-laminar sublayer near the water surface. Scaling arguments for the skin layer lead to the following expression for the temperature difference over the skin layer (cf. [Fairall \*et al.\*, 1996](#))

$$T_{\text{sk}} - T_{-\delta} = \frac{\delta}{\rho_w c_w k_w} (Q + R_s f_s) \quad (8.150)$$

$$\text{with } Q = H + \lambda E + LW \quad (8.151)$$

where  $T_{\text{sk}}$  is the skin temperature,  $T_{-\delta}$  is the temperature below the cool skin,  $R_s$  is the net solar radiation at the surface,  $f_s$  is the fraction of solar radiation absorbed in the skin,  $H$  is the sensible heat flux,  $\lambda E$  is the latent heat flux,  $LW$  is the net long wave radiation at the surface,  $\rho_w$  ( $=1025 \text{ kgm}^{-3}$ ) is the density of sea water (at the surface),  $c_w$  ( $=4190 \text{ Jkg}^{-1}\text{K}^{-1}$ ) is the volumetric heat capacity of water, and  $k_w$  ( $=0.6 \text{ Wm}^{-1}\text{K}^{-1}$ ) is the molecular thermal conductivity of water. The fraction of solar absorbed radiation is given by

$$f_s = 0.065 + 11\delta - \frac{6.6 \cdot 10^{-5}}{\delta} \left(1 - e^{-\delta/0.0008}\right) \quad (8.152)$$

$$(8.153)$$

The thickness of the skin layer  $\delta$  is ([Fairall \*et al.\*, 1996](#))

$$\delta = 6 \left[ 1 + \left( \frac{-16g\alpha_w\nu_w^3}{u_{*w}^4 k_w^2 \rho_w c_w} (Q + R_s f_s) \right)^{3/4} \right]^{-1/3} \quad (8.154)$$

where  $g$  is the acceleration of gravity,  $\alpha_w = \max(10^{-5}, 10^{-5}(T_{-d} - 273))$  is the thermal expansion coefficient of water and  $\nu_w$  ( $=1.0 \cdot 10^{-6} \text{ m}^2\text{s}^{-1}$ ) is the kinematic viscosity.

### 8.10.2 The warm layer

The near ocean warm layer is caused by solar absorption in the top few meters of the ocean during day time. This warm layer can develop when the wind mixing is not strong enough to prevent a stable layer to build up. The result is a diurnal cycle in the surface temperature which is commonly observed by satellite, but not seen in routine bulk SST observations from ships. The warm layer is typically a few meters deep. Although wind mixing erodes the warm later at night, in very low wind conditions a residual warm layer may survive until the next day, and therefore a prognostic variable is needed. The model variable  $T_{\text{sk}}$  (which is diagnostic over land) is used as a prognostic variable over the ocean.

In the IFS a simple bulk formulation is used based on similarity temperature profiles. It results in the following differential equation for the difference between the temperature just below the cool skin (less than a millimetres deep)  $T_{-\delta}$  and the ocean bulk temperature a few metres deep  $T_{-d}$

$$\frac{\partial(T_{-\delta} - T_{-d})}{\partial t} = \frac{Q + R_s - R(-d)}{d\rho_w c_w \nu / (\nu + 1)} - \frac{(\nu + 1)k u_{*w}}{d\phi_t(d/L)} (T_{-\delta} - T_{-d}) \quad (8.155)$$

where  $d$  ( $=3$  m) is the depth scale of the warm layer,  $\nu$  ( $=0.3$ ) is the profile shape parameter and  $\phi_t(d/L)$  is the stability function with  $L$  for the Obukhov length. The solar radiation at depth  $-d$  is

$$R(-d) = R_s \sum_{i=1}^3 a_i e^{-db_i} \quad (8.156)$$

with  $(a_1, a_2, a_3) = (0.28, 0.27, 0.45)$  and  $(b_1, b_2, b_3) = (71.5, 2.8, 0.06 \text{ m}^{-1})$ . The stability function is

$$\begin{aligned} \phi_t(-z/L) &= 1 + 5 \frac{-z}{L} \quad \text{for } \frac{-z}{L} \geq 0 \\ &= (1 - 16 \frac{-z}{L})^{-1/2} \quad \text{for } \frac{-z}{L} < 0 \end{aligned} \quad (8.157)$$

The Obukhov length is

$$L = \rho_w c_w u_{*w}^3 / (k F_d) \quad (8.158)$$

The buoyancy flux  $F_d$  is

$$\begin{aligned} F_d &= g \alpha_w [Q + R_s - R(-d)] \quad \text{for } (T_{-\delta} - T_{-d}) \leq 0 \\ &= \left( \frac{\nu g \alpha_w}{5d} \right)^{1/2} \rho_w c_w u_{*w}^2 (T_{-\delta} - T_{-d})^{1/2} \quad \text{for } (T_{-\delta} - T_{-d}) > 0 \end{aligned} \quad (8.159)$$

Equation (8.155) is integrated in time with a fully implicit scheme using  $(T_{-\delta} - T_{-d})$  as the prognostic variable. Every time step, the differences  $(T_{-\delta} - T_{-d})$  from (8.155) and the difference  $(T_{\text{sk}} - T_{-\delta})$  from (8.151) are added to the ocean bulk temperature to obtain the ocean skin temperature  $T_{\text{sk}}$ . With the schemes switched off (by default), the differences are zero and the skin temperature is equal to the bulk SST.

### 8.10.3 Salinity effect on $q_s$

Many models use the saturation specific humidity at ocean surface temperature as boundary condition for humidity. However, salinity reduces the saturation value and a reasonable approximation for a salinity of 34 parts per thousand is (Sverdrup *et al.*, 1942)

$$q_s = 0.98 q_{\text{sat}}(T_{\text{sk}}) \quad (8.160)$$

The 2% difference due to salinity may look a small effect, but it should be seen as a fraction of the air-sea specific humidity difference, which is typically 15% in relative humidity. So a 2% change in saturation value at the surface is equivalent to a change of  $2/0.15=13\%$  in air-sea transfer (see Zeng *et al.*, 1998b for an intercom-parison of schemes).

## 8.11 NUMERICAL SOLUTION OF THE SURFACE EQUATIONS

### 8.11.1 Recap of the analytical equations

The water budget ((8.70) and (8.71) with boundary conditions given by (8.86)), the soil energy budget ((8.58) with boundary conditions given by (8.59) and (8.60)) and the ice energy budget (8.148) can be rewritten in a generalised form as

$$\frac{\partial \Psi}{\partial t} = \frac{1}{C} \frac{\partial}{\partial z} \left( \lambda \frac{\partial \Psi}{\partial z} - \gamma \right) + S_\Psi \quad (8.161)$$

The meaning of the different variables in each individual equations is summarized Table 8.12, together with the respective upper and lower boundary conditions,  $F_\Psi$ .

### 8.11.2 Implicit numerical solution

Equation (8.161) is time discretized using

$$\frac{\Psi^{t+1} - \Psi^t}{\Delta t} = \frac{1}{C} \frac{\partial}{\partial z} \left( \lambda \frac{\partial \hat{\Psi}}{\partial z} - \gamma \right) + S_\Psi \quad (8.162)$$

**Table 8.12** Variables in the generalized soil/ice temperature and water equation.

Equation	$\Psi$	$C$	$\lambda$	$\gamma$	$S_\Psi$	UBC	LBC
Soil moisture	$\theta$	1	$\lambda_\theta$	$\gamma_\theta$	$S_\theta$	$F_\theta = If - c_8 E_8$	$F_\theta = \gamma_\theta$
Soil temperature	$T$	$(\rho C)_{\text{eff}}$	$\lambda_T$	0	0	$F_T = H^N$	$F_T = 0$
Ice temperature	$T_I$	$(\rho C)_I$	$\lambda_I$	0	0	$F_I = H^N$	$F_{N_s+1} = T_{0,I}$

UBC and LBC stand for upper and lower boundary condition, respectively

where

$$\hat{\Psi} = \alpha_{\text{impl}} \Psi^{t+1} + (1 - \alpha_{\text{impl}}) \Psi^t \quad (8.163)$$

and the semi-implicit coefficient,  $\alpha_{\text{impl}} = 1$ . If the prognostic variable  $\Psi$  is defined at full levels and the fluxes  $F_\Psi$  are defined at half-levels (the interface between layers), (8.162) can be discretized in space to give

$$\begin{aligned} \frac{\hat{\Psi} - \Psi^t}{\alpha_{\text{impl}}} &= \frac{\Delta t}{C_k} \left( \frac{\lambda_{k-1/2}(\hat{\Psi}_{k-1} - \hat{\Psi}_k)}{\Delta z_k \Delta z_{k-1/2}} - \frac{\lambda_{k-1/2}(\hat{\Psi}_k - \hat{\Psi}_{k+1})}{\Delta z_k \Delta z_{k+1/2}} + \frac{\gamma_{k-1/2} - \gamma_{k+1/2}}{\Delta z_k} \right) + \Delta t S_{\Psi,k} & k = 2, \dots, N_s - 1 \\ \frac{\hat{\Psi} - \Psi^t}{\alpha_{\text{impl}}} &= \frac{\Delta t}{C_k} \left( \frac{F_\Psi^T}{\Delta z_k} - \frac{\lambda_{k-1/2}(\hat{\Psi}_k - \hat{\Psi}_{k+1})}{\Delta z_k \Delta z_{k+1/2}} + \frac{\gamma_{k-1/2} - \gamma_{k+1/2}}{\Delta z_k} \right) + \Delta t S_{\Psi,k} & k = 1 \\ \frac{\hat{\Psi} - \Psi^t}{\alpha_{\text{impl}}} &= \frac{\Delta t}{C_k} \left( \frac{\lambda_{k-1/2}(\hat{\Psi}_{k-1} - \hat{\Psi}_k)}{\Delta z_k \Delta z_{k-1/2}} - \underbrace{\frac{\lambda_{k-1/2}(\hat{\Psi}_k - \hat{\Psi}_{k+1})}{\Delta z_k \Delta z_{k+1/2}}}_I + \frac{\gamma_{k-1/2} - \gamma_{k+1/2}}{\Delta z_k} \right) + \Delta t S_{\Psi,k} & k = N_s \end{aligned} \quad (8.164)$$

where the horizontal brace means that the term exists only for the ice temperature equation (because of the bottom temperature boundary condition for ice) and  $\Delta z_k$ ,  $z_k$ ,  $z_{k-1/2}$ , and  $z_{k+1/2}$  represent the thickness of layer  $k$ , and the depths of its centre, the top and the bottom interface, respectively:

$$\begin{aligned} \Delta z_k &= D_k = z_{k+1/2} - z_{k-1/2} \\ \Delta z_{k+1/2} &= z_{k+1} - z_k \end{aligned} \quad (8.165)$$

Equation (8.164) leads to a triadiagonal system of equations

$$\begin{aligned} \frac{\hat{\Psi}_{k-1}}{\alpha_{\text{impl}}} \left( \frac{\hat{\lambda}_{k-1/2}}{C_k \Delta z_k} \right) + \frac{\hat{\Psi}_k}{\alpha_{\text{impl}}} \left( 1 + \frac{\hat{\lambda}_{k-1/2}}{C_k \Delta z_k} + \frac{\hat{\lambda}_{k+1/2}}{C_k \Delta z_k} \right) - \frac{\hat{\Psi}_{k+1}}{\alpha_{\text{impl}}} \left( \frac{\hat{\lambda}_{k+1/2}}{C_k \Delta z_k} \right) \\ = \left( \frac{\Psi_k^t}{\alpha_{\text{impl}}} + \Delta t \left( \frac{\gamma_{k-1/2} - \gamma_{k+1/2}}{C_k \Delta z_k} \right) + \Delta t S_{\Psi,k} \right) & k = 2, \dots, N_s - 1 \\ \frac{\hat{\Psi}_k}{\alpha_{\text{impl}}} \left( 1 + \frac{\hat{\lambda}_{k+1/2}}{C_k \Delta z_k} \right) - \frac{\hat{\Psi}_{k+1}}{\alpha_{\text{impl}}} \left( \frac{\hat{\lambda}_{k+1/2}}{C_k \Delta z_k} \right) - \frac{F_\Psi^T}{C_k \Delta z_k} \\ = \frac{\Psi_k^t}{\alpha_{\text{impl}}} + \Delta t \left( \frac{\gamma_{k-1/2} - \gamma_{k+1/2}}{C_k \Delta z_k} \right) + \Delta t S_{\Psi,k} & k = 1 \\ \frac{\hat{\Psi}_{k-1}}{\alpha_{\text{impl}}} \left( \frac{\hat{\lambda}_{k-1/2}}{C_k \Delta z_k} \right) - \frac{\hat{\Psi}_k}{\alpha_{\text{impl}}} \left( 1 + \frac{\hat{\lambda}_{k-1/2}}{C_k \Delta z_k} + \frac{\hat{\lambda}_{k+1/2}}{C_k \Delta z_k} \right) - \underbrace{\frac{\hat{\Psi}_{k+1}}{\alpha_{\text{impl}}} \left( \frac{\hat{\lambda}_{k+1/2}}{C_k \Delta z_k} \right)}_I \\ = \frac{\Psi_k^t}{\alpha_{\text{impl}}} + \Delta t \left( \frac{\gamma_{k-1/2} - \gamma_{k+1/2}}{C_k \Delta z_k} \right) + \Delta t S_{\Psi,k} & k = N_s \end{aligned} \quad (8.166)$$

with the generalized modified diffusivities,  $\hat{\lambda}_{k-1/2}$ , defined as

$$\begin{aligned} \hat{\lambda}_{k-1/2} &= \frac{\Delta t \alpha_{\text{impl}} \lambda_{k-1/2}}{\Delta z_{k-1/2}} \\ I \begin{cases} \Delta z_{N_s+1/2} & = D_{N_s}/2 \\ \hat{\Psi}_{N_s+1} & = T_{0,I} \end{cases} \end{aligned} \quad (8.167)$$

where  $D_{N_s}$  is the depth of the deepest soil layer. The discretization above conserves water (energy) and is linearly stable. The coefficients  $\lambda$  and  $\gamma$  are a function of variable at the current time step,  $\Psi^n$ .

## 8.12 CODE

The surface code is fully externalized and it communicates with the rest of the code via interfaces. The architecture of the surface code is organized in modules which contain the parametrizations and normally belong to the surface library only (internal routines).

- **ROUTINE.MOD.F90**. The module which contains the routine's code **ROUTINE** with the parametrization.

Routines which need to be called from anywhere outside the surface code are duplicated as externals. For a given external routine there are associated:

- **ROUTINE.h**. The routine's interface which is needed in the outside routine to call the external surface routine.
- **ROUTINE.F90**. The external routine which uses the routine's module **ROUTINE\_CTL\_MOD** and contains the call to the surface routine **ROUTINE\_CTL**.
- **ROUTINE\_CTL\_MOD.F90**. The module which contains the routine's code **ROUTINE\_CTL** with the parametrization.

The external routines, for which the above structure apply, are identified hereafter by **\*ROUTINE\***. This structure allows for separately compile and run the surface code (e.g. with prescribed atmospheric forcing). The access to surface parameters and fields is done by dedicated routines (**\*SURF\_INQ\*** allows for enquiry mode for scalars, and **\*SURFBC\*** for a given set of surface fields). The surface parametrization computations are shared between the vertical diffusion routine (the routine **\*SURFEXCDRIVER\*** called by **VDFMAIN**, see [Chapter 3](#)) and the main surface routine, **\*SURFTSTP\***. In **\*SURFEXCDRIVER\***, the tile fluxes and skin temperatures are computed: After the elimination part of the tridiagonal system of equations is computed, the energy budget for each tile is computed before back-substitution.

At the start of the model integration, the **\*SUSURF\*** setup routine is called to initialize modules specific to the surface code:

- **SUSCST**. Setup general constants.
- **SUSTHF**. Setup thermodynamic function constants.
- **SUSRAD**. Setup radiation constants.
- **SUSSOIL**. Setup soil constants.
- **SUSVEG**. Setup vegetation constants.
- **SUCOTWO**. Setup surface ecosystem exchange constants.
- **SUVEXC**. Setup surface exchange coefficients constants.
- **SUVEXCS**. Setup static stability constants.
- **SUSFLAKE**. Setup static mixed-layer model constants.

The main subroutine of the surface code (**\*SURFTSTP\***) is called from **CALLPAR**, with: (a) values of the surface prognostic equations at time step  $n$ , convective and large-scale rainfall and snowfall, tile evaporation, sensible and latent heat fluxes, and temperatures, net surface long-wave flux, tile net short-wave flux as inputs; and (b) tendencies for the surface prognostic variables, plus a comprehensive set of diagnostic arrays as outputs. **\*SURFTSTP\*** does a sequence of computations and subroutine calls:

- **SRFENE**. Computes soil energy in each layer, considering vegetation and snow effects.
- **SRFSN\_LWIMP**. Solution of the snow energy and water budget and computation of the next time step density and albedo fields. Inputs: snow depth, temperature, density and albedo at the current time step, soil temperature, short-wave and long-wave radiation fluxes, snowfall, and tile fluxes. Outputs: snow depth, temperature, density and albedo at the next time step, meltwater flux, and basal heat flux. The formulation include a diagnostic treatment of liquid water reservoir in the snow-pack.

- **SRFSN**. Same as **SRFSN\_LWIMP** but using the former TESSEL snow formulation (available as an option).
- **SRFRCG**. Computes apparent soil heat capacity, i.e. including effects of soil freezing. Inputs: soil temperature and vegetation covers. Output is volumetric heat capacity.
- **SRFT**. Solution of the soil heat budget. Inputs: Soil temperature, soil moisture, long-wave radiative flux, snow basal heat flux, volumetric heat capacity, tile evaporation, sensible heat flux and short-wave radiative flux. Output: Soil temperature at the next time step. First the modified heat diffusivity, the soil energy per unit area and the right-hand side of the system of equations are computed. The generalized surface tridiagonal solver, **SRFWDIF**, is called to solve for the semi-implicit variable,  $\hat{T}/\alpha$ . The soil temperatures for the next time step are computed at the end.
- **SRFI**. Solution of the ice heat budget. Inputs: Ice temperature, long-wave radiative flux, tile evaporation, sensible heat flux and short-wave radiative flux. Output: Ice temperature at the next time step. First the modified heat diffusivity, the ice energy per unit area and the right-hand side of the system of equations are computed. The generalized surface tridiagonal solver, **SRFWDIF**, is called to solve for the semi-implicit variable,  $\hat{T}_I/\alpha$ . The ice temperatures for the next time step are computed at the end.
- **SRFWL**. Solution of the interception layer water budget. Inputs: Interception layer contents, low and high vegetation water cover, maximum capacity of the interception layer, convective and large-scale rainfall, snow evaporation of shaded snow tile, and tile evaporation. Outputs: Interception layer at next time step, convective and large-scale throughfall and tile evaporation collected (or depleting) the interception layer.
- **SRFWEXC\_VG**. First part of the computation of the soil water budget (i.e., computation of the coefficients of the tridiagonal system of equations for  $\hat{\theta}$ ). This includes the partitioning of transpiration into root extraction at the different layers and soil hydraulic coefficients including the effect of frozen water. Inputs: Soil moisture and temperature, convective and large-scale throughfall, snowmelt, tile evaporation, tile evaporation collected (or depleting) the interception layer, and snow evaporation of the shaded snow tile. Outputs: Modified diffusivity for water, right-hand side of the tridiagonal system, and layer depths. Soil properties are defined according to HTESEL scheme
- **SRFWEXC**. Same as **SRFWEXC\_VG** but using the former TESSEL soil properties formulation (available as an option).
- **SRFWDIF**. Generalized surface tridiagonal solver. Inputs: Values of  $\psi$  at the current time step, generalized modified diffusivities, soil energy (or water) per unit area, and right-hand side of equations. Output:  $\hat{\psi}/\alpha$ . The routine computes the coefficients on the left-hand side of the equations and solves the equations using LU-decomposition and back substitution in one downward scan and one upward scan.
- **SRFWINC**. Computation of next time step soil water. Inputs:  $\hat{\theta}/\alpha$  and current time step soil water. Output: next time step soil water.
- **SRFWNG**. Bounded-value operator for intercepted water (limited to non-negative values and values below or equal the maximum contents of the interception layer) and soil water (limited to non-negative values and values below or equal saturation). The “soil column” is scanned from top to bottom and the amount of water needed to satisfy physical limits in each layer are borrowed from the layer below. The water exchanged in this way is accounted for as runoff. Inputs: next time step intercepted water and soil water. Output: Bounded values of the same quantities.
- **FLAKE\_DRIVER**. This is a communication routine between HTESEL and a FLake routines. It assigns the FLake variables at the previous time step to their input values given by the driving model calls a routine **FLAKE\_RADFLUX** to compute the radiation fluxes, and **FLAKE\_ENE** that returns the updated FLake variables to the driving model. The **FLAKE\_DRIVER** does not contain any Flake physics. It only serves as a convenient means to organize calls of Flake routines.
- **SRFVEGEVOL**. Performs the time evolution of vegetation parameters at solar midnight in the case of interactive vegetation .

In **★SURFEXCDRIVER★** a set of routines relevant for the vertical diffusion code are called. These are listed below and discussed in full detail in [Chapter 3](#):

- **VUPDZO**. Update of roughness lengths for heat and momentum over ocean and setup over land according to vegetation types.

- **VSURF**. Definition of bare soil resistance, low and high canopy resistances.
  - **COTWORESTRESS**. Compute the net assimilation of CO<sub>2</sub> and the canopy conductance with the A-gs model.
- **SRFCOTWO**. Post-processing of the CO<sub>2</sub> fluxes.
- **VEXCS**. Computation of aerodynamical part of exchange coefficients for heat and moisture, including stability computations.
- **VEVAP**. Computation of evapotranspiration for each tile.
- **VSFLX**. Surface fluxes for each tile, defined at time *t*.
- **\*SURFSEB\***. Computation of surface energy balance and skin temperature for each tile. This is called by **\*SURFEXCDRIVER\*** only the first time-step, otherwise called by the vertical diffusion (VDFDIFH, see Chapter 3).
- **\*SURFPP\***. Computation of quantities at the end of vertical diffusion, including post-processed weather elements and gustiness.
- **VOSKIN**. Computation of warm/cold skin effects over the ocean (called by **\*SURFPP\***).



## Chapter 9

# Methane oxidation

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## 9.1 INTRODUCTION

A study of stratospheric humidity in analyses and multi-year simulations has shown that the ECMWF system prior to 1999 was capable of producing a broadly realistic distribution of water vapour at, and immediately above, the tropopause, and that the slow upward transfer of water vapour in the tropical stratosphere could be captured quite reasonably given sufficiently fine vertical resolution in the model (Simmons *et al.*, 1999). However, values of water vapour in the tropical upper stratosphere, and throughout much of the extratropical stratosphere, were too low. This deficiency has now been remedied by the introduction of a simple parametrization of the upper-stratospheric moisture source due to methane oxidation. A sink representing photolysis in the mesosphere is also included. The scheme was derived as a simplification of an approach adopted by Peter Stott and Anne Pardaens at the Department of Meteorology, University of Edinburgh, notes on which and helpful references were supplied by Bob Harwood.

## 9.2 METHANE OXIDATION

Methane is produced by natural and anthropogenic sources at the earth's surface, and is well-mixed in the troposphere. Its volume mixing ratio is currently around 1.7 ppmv. It is carried upwards in the tropical stratosphere and decreases in relative density (due to oxidation) to values of around 0.2–0.4 ppmv around the stratopause. Mean stratospheric descent at higher latitudes results in relatively low values of methane at these latitudes in the middle and lower stratosphere.

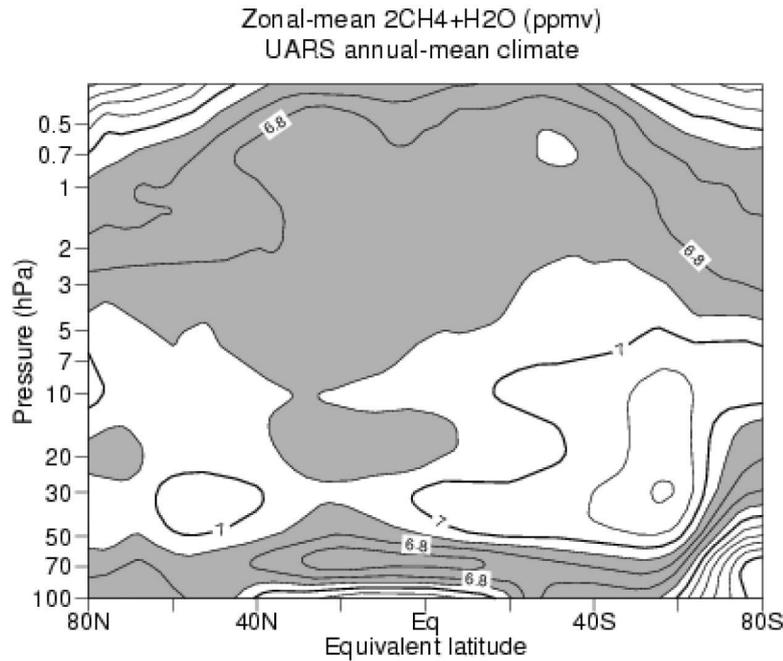
Brasseur and Solomon (1984) provide an account of the chemistry of carbon compounds in the stratosphere and mesosphere. The long chain of reactions starting from methane ( $\text{CH}_4$ ) ends with the production of water vapour ( $\text{H}_2\text{O}$ ) and molecular hydrogen ( $\text{H}_2$ ) in the stratosphere and mesosphere. This occurs such that the sum

$$2[\text{CH}_4] + [\text{H}_2\text{O}] + [\text{H}_2]$$

is approximately uniformly distributed in the absence of precipitation, where  $[\ ]$  denotes a volume mixing ratio. Le Texier *et al.* (1988) provide calculations of the relative amounts of  $\text{H}_2\text{O}$  and  $\text{H}_2$ , showing that the predominant production is that of water vapour in the vicinity of the stratopause. They indicate, however, that  $\text{H}_2$  production in the mesosphere, and relatively strong descent in winter and early spring at high latitudes, may result in the upper stratosphere being relatively dry in these seasons and latitudes.

There is, nevertheless, good observational evidence that over much of the stratosphere the quantity

$$2[\text{CH}_4] + [\text{H}_2\text{O}]$$



**Figure 9.1** Annual-mean distribution of the sum of twice the volume mixing ratio of methane and of the mixing ratio of water vapour (ppmv) as a function of pressure and potential vorticity (expressed as equivalent latitude), derived from UARS (HALOE, supplemented by CLAES and MLS) data analysed by *Randel et al. (1998)*. The contour interval is 0.1 ppmv, and shading denotes the range 6.6–6.9 ppmv.

is quite uniformly distributed with a value somewhat over 6 ppmv. *Jones et al. (1986)* provide evidence for this from the LIMS and SAMS instruments on the Nimbus 7 satellite launched in 1978, and a particularly clear demonstration is given by *Bithell et al. (1994)* based on HALOE data from the UARS satellite. In a pressure-latitude section at about the austral spring equinox, *Bithell et al.* show the result to fail significantly only below 10 hPa in the high-latitude southern hemisphere due, presumably, to condensation at the very cold temperatures in the Antarctic polar vortex.

Prior to cycle 25r1 of the IFS, the parametrization used the value 6 ppmv for the sum  $2[\text{CH}_4] + [\text{H}_2\text{O}]$ . This version was used in production of the ERA-40 reanalyses, which have been found to be generally drier in the stratosphere than the climatology derived by *Randel et al. (1998)* from UARS measurements. From cycle 25r1 onwards, the parametrization uses the value 6.8 ppmv, based on *Randel et al.*'s data as presented in [Fig. 9.1](#).

### 9.3 THE PARAMETRIZATION

#### 9.3.1 Methane oxidation

We assume that the volume mixing ratio of water vapour  $[\text{H}_2\text{O}]$  increases at a rate

$$2k_1[\text{CH}_4] \tag{9.1}$$

We further assume that

$$2[\text{CH}_4] = 6.8 \text{ ppmv} - [\text{H}_2\text{O}] \tag{9.2}$$

The rate of increase of volume mixing ratio of water vapour (in ppmv) is thus

$$k_1(6.8 - [\text{H}_2\text{O}]) \tag{9.3}$$

In terms of specific humidity,  $q$ , the source is

$$k_1(Q - q) \tag{9.4}$$

where (having divided by  $1.6 \times 10^6$  to convert from volume mixing ratio in ppmv to specific humidity) the parameter  $Q$  has the value  $4.25 \times 10^{-6}$ , or 4.25 mg/kg.

The rate  $k_1$  could be determined, for example, from a two-dimensional model with comprehensive chemistry, as in the scheme developed at Edinburgh University. However, in this first scheme for use at ECMWF we prescribe a simple analytical form for  $k_1$  which varies only with pressure.

The photochemical life time of water vapour is of the order of 100 days near the stratopause, 2000 days at 10 hPa, and effectively infinite at the tropopause (Brasseur and Solomon, 1984). A prescription of  $k_1$  that gives a reasonable profile up to the stratopause is provided by

$$k_1 = \frac{1}{86400\tau_1} \quad (9.5)$$

where  $k_1$  is given in  $\text{s}^{-1}$  and the timescale,  $\tau_1$ , in days, is given in terms of pressure,  $p$ , in Pa, by

$$\tau_1 = \begin{cases} 100 & p \leq 50 \\ 100 \left[ 1 + \alpha_1 \frac{\{\ln(p/50)\}^4}{\ln(10000/p)} \right] & 50 < p < 10000 \\ \infty & p \geq 10000 \end{cases} \quad (9.6)$$

where we define

$$\alpha_1 = \frac{19 \ln 10}{(\ln 20)^4} \quad (9.7)$$

to give a time-scale of 2,000 days at the 10 hPa level.

This parametrization moistens rising air in the tropical stratosphere. This air will earlier have been freeze-dried near the tropopause, where specific humidities can locally fall well below 1 mg/kg. Specific humidities approaching the value  $Q$  will be reached near the stratopause. Descent near the poles will bring down air with specific humidity close to  $Q$ . Expression (9.4) will then yield a source term that is weaker in polar than in tropical latitudes, so reasonable results may be obtained without imposing a latitudinal variation of  $k_1$ . (Strictly,  $k_1$  should vanish in the polar night, where photodissociation does not produce the excited oxygen  $\text{O}(^1\text{D})$ , which in turn produces the OH radical, these two species being intimately involved in the production of water vapour from methane.)

### 9.3.2 Photolysis in the mesosphere

For model versions with an uppermost level at 0.1 hPa, or lower, there is no strong need to include the sink of water vapour that occurs in the mesosphere and above due to photolysis. However, for completeness we include a simple representation of this effect, modifying the source term (9.4) by adding a decay term  $-k_2q$  above a height of about 60 km. The full source/sink term becomes

$$k_1(Q - q) - k_2q \quad (9.8)$$

As for  $k_1$  we take  $k_2$  independent of latitude with parameters chosen to match the vertical profile of photochemical lifetime presented by Brasseur and Solomon (1984). Specifically,

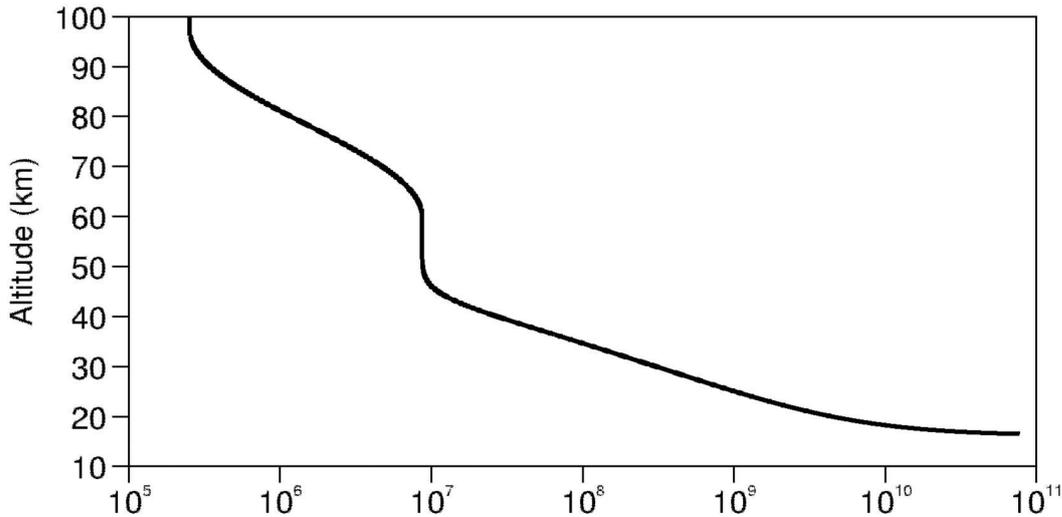
$$k_2 = \frac{1}{86400\tau_2} \quad (9.9)$$

with

$$\tau_2 = \begin{cases} 3 & p \leq 0.1 \\ \left[ \exp \left\{ \alpha_2 - 0.5(\ln 100 + \alpha_2) \left( 1 + \cos \frac{\pi \ln(p/20)}{\ln 0.005} \right) \right\} - 0.01 \right]^{-1} & 0.1 < p < 20 \\ \infty & p \geq 20 \end{cases} \quad (9.10)$$

and

$$\alpha_2 = \ln \left( \frac{1}{3} + 0.01 \right) \quad (9.11)$$



**Figure 9.2** Combined photochemical lifetime,  $(k_1 + k_2)^{-1}$ , as a function of altitude for the analytical specification given by Equations (9.5) to (9.7) and (9.9) to (9.11).

The vertical profile of the photochemical lifetime of the combined scheme,  $(k_1 + k_2)^{-1}$ , is shown in Fig. 9.2, in which we have converted to height as a vertical coordinate assuming an isothermal atmosphere with a temperature of 240 K. Comparison of this profile with that for  $\text{H}_2\text{O}$  shown in Fig. 5.21 of Brasseur and Solomon (1984) indicates reasonable agreement.

#### 9.4 CODE

The calculations for methane oxidation and photolysis of water vapour are performed in subroutine **METHOX**.

This routine calculates the tendency of water vapour due to methane oxidation and due to photolysis following (9.8). The order of the calculations is as follows.

- (i) Find time-scale for methane oxidation following (9.6).
- (ii) Solve first part of (9.8).
- (iii) Find time-scale for water vapour photolysis following (9.10).
- (iv) Solve second part of (9.8).

The setup of the constants used in **METHOX** is performed in **SUMETHOX** which is called from **SUPHEC**. The constants are kept in module **YOEMETH**. The controlling switch for the methane oxidation is **LEMETHOX** which is part of namelist **NAEPHY**.

## Chapter 10

# Ozone chemistry parametrization

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#### 10.1 Introduction

#### 10.2 The ECMWF ozone parametrization

### 10.1 INTRODUCTION

Ozone is fully integrated into the ECMWF forecast model and analysis system as an additional three-dimensional model and analysis variable similar to humidity. The forecast model includes a prognostic equation for the ozone mass mixing ratio (kg/kg) given by

$$\frac{dO_3}{dt} = R_{O_3} \quad (10.1)$$

where  $R_{O_3}$  is a parametrization of sources and sinks of ozone. Without such a source/sink parametrization the ozone distribution would drift to unrealistic values in integrations longer than a few weeks. The source/sink parametrization must maintain a realistic ozone distribution over several years of integration, without reducing the dynamic variability of ozone. In addition, we would like the parametrization to be able to create an Antarctic ozone hole when the conditions are right.

### 10.2 THE ECMWF OZONE PARAMETRIZATION

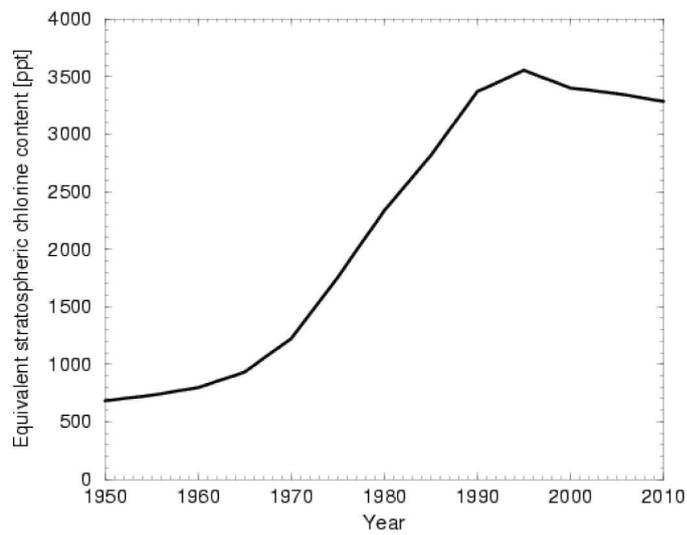
The parametrization used in the ECMWF model is an updated version of [Cariolle and Déqué \(1986\)](#), which has been used in the ARPEGE climate model at Météo-France. This parametrization assumes that chemical changes in ozone can be described by a linear relaxation towards a photochemical equilibrium. It is mainly a stratospheric parametrization. The relaxation rates and the equilibrium values have been determined from a photochemical model, including a representation of the heterogeneous ozone hole chemistry. The updated version of the parametrization (with coefficients provided by Pascal Simon, Météo-France) is

$$R_{O_3} = c_0 + c_1(O_3 - \overline{O_3}) + c_2(T - \overline{T}) + c_3(O_3^\uparrow - \overline{O_3^\uparrow}) + c_4(\text{Cl}_{\text{EQ}})^2 O_3 \quad (10.2)$$

where

$$O_3^\uparrow(p) = - \int_{p^0}^p \frac{O_3(p')}{g} dp' \quad (10.3)$$

Here ( $i = 0, \dots, 4$ ) are the relaxation rates and  $\overline{T}$ ,  $\overline{O_3}$  and  $\overline{O_3^\uparrow}$  are photochemical equilibrium values, all functions of latitude, pressure, and month.  $\text{Cl}_{\text{EQ}}$  is the equivalent chlorine content of the stratosphere for the actual year, and is the only parameter that varies from year to year (see [Fig. 10.1](#)). The heterogeneous part is only turned on below a threshold temperature of 195 K. The coefficients for the ozone parametrization ( $c_0, c_1, c_2, c_3, c_4, \overline{O_3}, \overline{T}, \overline{O_3^\uparrow}$ ) have been developed by Météo-France. Their version 2.3 is used ([Cariolle and Teyssède, 2007](#)).



**Figure 10.1** Equivalent chlorine content of the stratosphere in ppt for the heterogeneous chemistry part of the ozone source/sink parametrization (provided by Pascal Simon, Météo-France).

# Chapter 11

## Climatological data

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- 11.2 Data sets and conversion to high resolution GRIB files**
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  - 11.2.2 Surface elevation data at 30 arc seconds
  - 11.2.3 Global Land Cover Characteristics (GLCC)
  - 11.2.4 Seasonal leaf area index (LAI)
  - 11.2.5 Albedo
- 11.3 Topographic data at target resolution**
  - 11.3.1 Smoothing operator
  - 11.3.2 Mean orography
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### 11.1 INTRODUCTION

The ECMWF model uses a series of climate fields, that do not depend on initial condition or forecast step. Examples are orography, vegetation type/cover, surface albedo and leaf area index (LAI). Most fields are constant; surface albedo and LAI are specified for 12 months to describe the seasonal cycle. Dependent on the origin, the data comes at different resolutions and different projections. The processing of the data is done offline with a series of programs that are part of the so-called “climate package”. All the fields are processed in two or more steps. The first step is to convert the original data to a GRIB file on a regular lat/long grid at a resolution of 30x30 arc seconds (30'') with  $43200 \times 21600$  grid points (about 1 km at the equator). This applies to high resolution data. For data at coarser resolution than 30'', a conversion to GRIB is performed, but the resolution of the original data is kept.

In the second processing step the GRIB data is smoothed and interpolated to the IFS model grid. The climate files are created for all resolutions that are supported by IFS. The current cycle uses a linear reduced Gaussian grid. However, to cater for experimentation and future potential model implementations, also cubic grids are created. A cubic grid has twice the resolution of the linear grid for the same spectral truncation. In the following the individual data sources and processing procedures are discussed in detail.

## 11.2 DATA SETS AND CONVERSION TO HIGH RESOLUTION GRIB FILES

### 11.2.1 Land-sea mask at 30 arc seconds

The main data source is GLOBCOVER (<http://dup.esrin.esa.int/prjs/prjs68.php>), which is a vegetation data set on a regular 10 arc second lat/long grid (about 300 m) available from 85°N to 60°S. The GLOBCOVER “water body” type is used to indicate water pixels. The 300 m pixels are aggregated to the 30” grid and coded as a land fraction. Subsequently, the RAMP2 elevation data (see next section) is used to fill in or replace Antarctica. The latter data only allows for mask information i.e. the land cover is 0 or 1. It is assumed that there is no land north of 85°N.

### 11.2.2 Surface elevation data at 30 arc seconds

The model orography in the IFS is based on five orographic data sets:

- SRTM30 is the 30” version of the Shuttle Radar Topography Mission and is the primary data set for the IFS ([ftp://topex.ucsd.edu/pub/srtm30\\_plus/README.V6.0.txt](ftp://topex.ucsd.edu/pub/srtm30_plus/README.V6.0.txt)). However, it exists only between 60°N and 60°S. The 30” resolution version of SRTM is provided as an average of the original 30m resolution observations over 30” × 30” areas. Although not used in the IFS, SRTM30 is available as a global product through the use of GTOPO30 data (see: <http://edcwww.cr.usgs.gov/landdaac/gtopo30/gtopo30.html>) north of 60°N and south of 60°S.
- GLOBE data (<http://www.ngdc.noaa.gov/mgg/topo/globe.html>) replaces the areas North of 60°N in SRTM30. GLOBE data is used rather than GTOPO30 because it is believed to be of better quality.
- Antarctic RAMP2 (Radarsat Antarctic Mapping Project DEM Version 2, [http://nsidc.org/data/docs/daac/nsidc0082\\_ramp\\_dem\\_v2.gd.html](http://nsidc.org/data/docs/daac/nsidc0082_ramp_dem_v2.gd.html)) provided as raw binary data on a 1km grid in Polar Stereographic coordinates is used south of 60°S.
- BPRC for Greenland (Byrd Polar Research Center, Ohio State University, [http://bprc.osu.edu/rsl/greenland\\_data/dem/index.html](http://bprc.osu.edu/rsl/greenland_data/dem/index.html)) provided as raw binary data on a 1km grid in Polar Stereographic coordinates.
- Iceland Digital Elevation Model (DEM) by IMO (Icelandic Meteorological Service) and NLSI (National Land Survey of Iceland) provided as raw binary data on a 0.5 km grid in Lambert Conformal Conic coordinates.

To obtain a combined global data set on a regular lat/long grid of 30” resolution, the following processing is applied:

- (i) Both SRTM30 and GLOBE are already at the desired 30” grid, so they are simply merged by replacing in SRTM30 the points North of 60°N by the GLOBE points. The ocean is not identified in a particular way and is coded with elevation 0.
- (ii) The Greenland (BPRC), Iceland, and Antarctica (RAMP2) data are combined and projected on the global 30” grid. The adopted procedure is to transform the positions of the 30” latitude/longitude grid into the map coordinates of each of the three DEMs where then a nearest neighbour interpolation is applied. Because the void data surrounding the orography in the three DEMs overlapped with some orography of SRTM30/GLOBE, a modification (cutting/fitting) of the raw data was necessary before interpolation. Also, for consistency with the other DEMs, the Iceland DEM is bi-linearly interpolated from the 0.5 km to the 1 km grid in the map coordinates before inclusion.
- (iii) In the merged SRTM30/GLOBE data, all the existing points of the combined RAMP2/BPRC/Iceland data set are used to replace the SRTM30/GLOBE points.

### 11.2.3 Global Land Cover Characteristics (GLCC)

The Global Land Cover Characteristics (GLCC version 1.2) data set has been derived from 1 year of Advanced Very High Resolution Radiometer (AVHRR) data, digital elevation models, eco-regions and map data. The nominal resolution is 1 km, and the data comes on a Goode Homolosine global projection.

**Table 11.1** *Land use classification according to BATS.*

Index	Vegetation type	H/L veg
1	Crops, Mixed Farming	L
2	Short Grass	L
3	Evergreen Needleleaf Trees	H
4	Deciduous Needleleaf Trees	H
5	Deciduous Broadleaf Trees	H
6	Evergreen Broadleaf Trees	H
7	Tall Grass	L
8	Desert	-
9	Tundra	L
10	Irrigated Crops	L
11	Semidesert	L
12	Ice Caps and Glaciers	-
13	Bogs and Marshes	L
14	Inland Water	-
15	Ocean	-
16	Evergreen Shrubs	L
17	Deciduous Shrubs	L
18	Mixed Forest/woodland	H
19	Interrupted Forest	H
20	Water and Land Mixtures	L

The data base provides for each pixel a biome classification based on several of the popular classifications, including BATS, SiB and SiB2. The BATS classification has been adopted for the IFS. For the current model cycle it was decided not to change yet to version 2.0 and therefore the old processing was kept. The original data was converted to an intermediate resolution of 2'30". The derived 2'30" data set contains the fractional cover of all 20 BATS classes (see Table 11.1). However, all 20 GLCC.2.0 classes are also individually available on a 30" grid for future development.

#### 11.2.4 Seasonal leaf area index (LAI)

Monthly climatology of LAI is based on the MODIS MOD15A2 LAI product, available in ASCII format ([https://lpdaac.usgs.gov/products/modis\\_products\\_table/mod15a2](https://lpdaac.usgs.gov/products/modis_products_table/mod15a2), and <ftp://firecenter.umd.edu/pub/MODIS/Mirror/MOD15A2.005>). Processing involves the following steps (see also Boussetta *et al.*, 2011):

- (i) MODIS software is used to re-project the MODIS tiles (1200 × 1200 km) from the ISIN grid to a regular 30" lat/long grid.
- (ii) The 30" data is further averaged to a 5' lat/long grid using the MODIS quality flag.
- (iii) Temporal smoothing is achieved by averaging three 8-day composites of each month centred on the 15<sup>th</sup> of the month.
- (iv) Disaggregating LAI into High and Low components. This requires the use of TVH (high vegetation type), TVL (low vegetation type), CVH (high vegetation cover) and CVL (low vegetation cover) under the assumption of a "perfect" land use dataset. To avoid strong differences from the previous operational system which results in forecast scores deterioration owing to compensating errors from the atmospheric part, the MODIS LAI seasonality signal was added to the lookup table derived LAI:

$$\begin{aligned}
 LAI_{\text{table}} &= C_H LAI_{H,\text{table}} + C_L LAI_{L,\text{table}} \\
 \Delta LAI &= LAI_{\text{obs}} - LAI_{\text{table}} \\
 LAI_H &= LAI_{H,\text{table}} + C_H \Delta LAI \\
 LAI_L &= LAI_{L,\text{table}} + C_L \Delta LAI
 \end{aligned}
 \tag{11.1}$$

where  $LAI_{\text{obs}}$  is  $LAI$  from MODIS, and  $LAI_{\text{H}}/LAI_{\text{L}}$  are the fields that will be used by the model as  $LAI$  climatology for high and low vegetation respectively.  $LAI_{\text{H,table}}$  and  $LAI_{\text{L,table}}$  are the model tabulated values relating  $LAI$  to vegetation type.

### 11.2.5 Albedo

Snow free surface albedo over land is derived from a 5-year MODIS climatology ([http://www.umb.edu/spectralmass/terra\\_aqua\\_modis/v006/v004](http://www.umb.edu/spectralmass/terra_aqua_modis/v006/v004); Schaaf *et al.*, 2002). The data is provided on a  $0.05^\circ$  regular lat/long grid in the form of weighting parameters associated with the RossThickLiSparseReciprocal BRDF model that best describes the anisotropy of each pixel. The three parameters ( $f_{\text{iso}}$ ,  $f_{\text{vol}}$ ,  $f_{\text{geo}}$ ) are used for the UV-visible ( $0.3\text{-}0.7\ \mu\text{m}$ ) and the near-infrared ( $0.7\text{-}5.0\ \mu\text{m}$ ). They can be used to construct a direct albedo  $A_{\text{dir}}(\theta, \lambda)$  at solar zenith angle  $\theta$  and diffuse albedo  $A_{\text{dif}}(\lambda)$  with the following formulae:

$$\begin{aligned}
 A_{\text{dir}}(\theta, \lambda) = & f_{\text{iso}}(\lambda)(g_{0\text{iso}} + g_{1\text{iso}}\theta^2 + g_{2\text{iso}}\theta^3) \\
 & + f_{\text{vol}}(\lambda)(g_{0\text{vol}} + g_{1\text{vol}}\theta^2 + g_{2\text{vol}}\theta^3) \\
 & + f_{\text{geo}}(\lambda)(g_{0\text{geo}} + g_{1\text{geo}}\theta^2 + g_{2\text{geo}}\theta^3)
 \end{aligned} \tag{11.2}$$

$$\begin{aligned}
 g_{0\text{iso}} &= 1.0, & g_{1\text{iso}} &= 0.0, & g_{2\text{iso}} &= 0.0 \\
 g_{0\text{vol}} &= -0.007574, & g_{1\text{vol}} &= -0.070987, & g_{2\text{vol}} &= 0.307588 \\
 g_{0\text{geo}} &= -1.284909, & g_{1\text{geo}} &= -0.166314, & g_{2\text{geo}} &= 0.041840
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{dif}}(\lambda) = & f_{\text{iso}}(\lambda)g_{\text{iso}} \\
 & + f_{\text{vol}}(\lambda)g_{\text{vol}} \\
 & + f_{\text{geo}}(\lambda)g_{\text{geo}}
 \end{aligned} \tag{11.3}$$

$$\begin{aligned}
 g_{\text{iso}} &= 0.0 \\
 g_{\text{vol}} &= 0.189184 \\
 g_{\text{geo}} &= -1.377622
 \end{aligned}$$

In the climate software these formulae are used to construct monthly climatology at  $0.05^\circ$  resolution of four albedo fields: UV-visible-direct, UV-visible-diffuse, near-infrared-direct, and near-infrared-diffuse. A solar zenith angle  $\theta = 0$  is used to generate these fields.

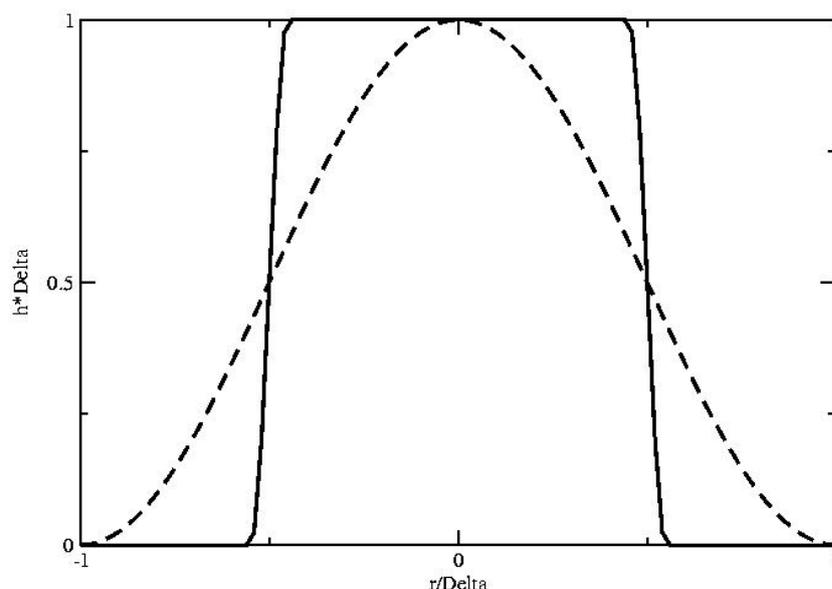
## 11.3 TOPOGRAPHIC DATA AT TARGET RESOLUTION

### 11.3.1 Smoothing operator

To avoid spectral aliasing in the process of going from high resolution  $30''$  data to model resolution, and for filtering, a general smoothing operator is used in grid point space. The smoothing scale is specified in km rather than degrees, so the filtering scale is uniform over the globe. The smoothing operator is a top hat function with smooth edges (see Fig. 11.1):

$$\begin{aligned}
 h(r) &= \frac{1}{\Delta}, & \text{for } |r| < \Delta/2 - \delta \\
 h(r) &= \frac{1}{2\Delta} + \frac{1}{2\Delta} \cos \pi(r - \Delta/2 + \delta)/2\delta, & \text{for } \Delta/2 - \delta < |r| < \Delta/2 + \delta \\
 h(r) &= 0, & \text{for } |r| > \Delta/2 + \delta
 \end{aligned} \tag{11.4}$$

The edges reduce the amplitude of the side lobes in the spectral domain. The filter is applied by convoluting the input field in two dimensions with  $h(r)$ , where  $r$  is the radial distance. Parameter  $\Delta$  is the width of the filter and  $\delta$  is the width of the edge. Sardeshmukh and Hoskins (1984) show that a rotation symmetric smoothing is equivalent to filtering of the total wavenumber. The effect of this operation is equivalent to multiplying the spectrum by a filter function  $H(k)$  where  $H$  is the square of the Fourier transform of  $h(r)$ . For all smoothing operations  $\delta = 1$  km is selected unless otherwise stated.



**Figure 11.1** Smoothing operators for  $\delta = \Delta/20$  (solid), and  $\delta = \Delta/2$  (dashed).

### 11.3.2 Mean orography

Orography, or geopotential height, is derived from the 30'' data in a few steps:

1. The smoothing operator with a *Pre-Filter-Scale* of 5 km is applied to the 30'' orography.
2. The smoothed 30'' data is interpolated to a 5 km grid (reduced Gaussian grid with  $NGL = 4000$ ). For target resolutions with 4000 grid points or more between the south and north poles ( $NGL \geq 4000$ ), the pre-smoothing and interpolation scale of 5 km is reduced to 2 km.
3. The 5 km (or 2 km) resolution field is smoothed with the *Filter-Scale* listed in Table 11.2 dependent on the selected target resolution, e.g. at T1279, the linear grid has  $NGL = 1280$  and a *Filter-Scale* of 16000 m.
4. The resulting field is interpolated to target resolution, e.g.  $NGL = 1280$  for a T1279 linear grid.
5. The grid-point orography is transformed to spectral space at target resolution, e.g. T1279.
6. The orographic spectrum is tapered by multiplication with  $1/[1 + 4\{l(l+1)\}^8\{l_{\max}(l_{\max}+1)\}^{-8}]$ , where  $l$  is the wave number index and  $l_{\max}$  is the maximum wave number of the target resolution (e.g. 1279). This is equivalent to smoothing of the grid point field with a  $5\nabla^{16}$  operator. The exponent determines the sharpness of the tapering and the factor 5 determines the filtering strength.
7. The spectral orography is transformed back to grid point space.

Although the processing and filtering is designed to achieve a compromise between sharpness of the orography and limited rippling (Gibbs phenomenon), the spectral representation still shows some rippling near steep orography. Fig. 11.2 shows the orography at T1279 resolution.

Orography for the future cubic grid is handled slightly differently. A T7999 spectral orography is derived first from the high resolution 30'' field, and the lower resolutions are created in spectral space. The cubic grid has twice as much resolution as the linear grid for the same spectral truncation (e.g. for T1279, the number of latitudes between south and north poles is  $NGL = 1280$  for the linear grid whereas  $NGL = 2560$  for the cubic grid). The processing steps are as follows:

1. The smoothing operator with a *Filter-Scale* of 2500 m is applied to the 30'' orography.
2. The field from step 1 is interpolated to a reduced Gaussian linear grid corresponding to T7999 (i.e. with  $NGL = 8000$ ).
3. To reduce the Gibbs phenomenon, a least square method is applied to constrain the curvature of the field, which is evaluated as the Laplacian of orography  $\Delta h(s)$ , where  $h$  is the terrain elevation

**Table 11.2** Resolution dependent parameters for the generation of orographic fields. NGL indicates the number of points between north and south poles.

NGL	Fact	Pre-Filter-Scale (m)	Filter-Scale (m)
160	0.001	5000	125000
256	0.001	5000	78000
320	0.001	5000	62500
400	0.001	5000	50000
512	0.001	5000	40000
640	0.001	5000	30000
800	0.001	5000	25000
1024	0.001	5000	20000
1280	0.001	5000	16000
1600	0.01	5000	14000
1920	0.02	5000	12000
2048	0.02	5000	10000
2560	0.02	5000	8000
3200	0.02	5000	6500
4000	0.03	-	5000

and  $s$  is the horizontal coordinate. The following function is minimised

$$E(h) = \int \{\tilde{h}(s) - h(s)\}^2 ds + \lambda \int \{\Delta \tilde{h}(s)\}^2 ds \quad (11.5)$$

which is equivalent in spectral space to (<http://www.ppsloan.org/publications/StupidSH36.pdf>)

$$\tilde{h}_l^m = \frac{h_l^m}{1 + \lambda \{l(l+1)\}^2 \{l_{\max}(l_{\max}+1)\}^{-2}} \quad (11.6)$$

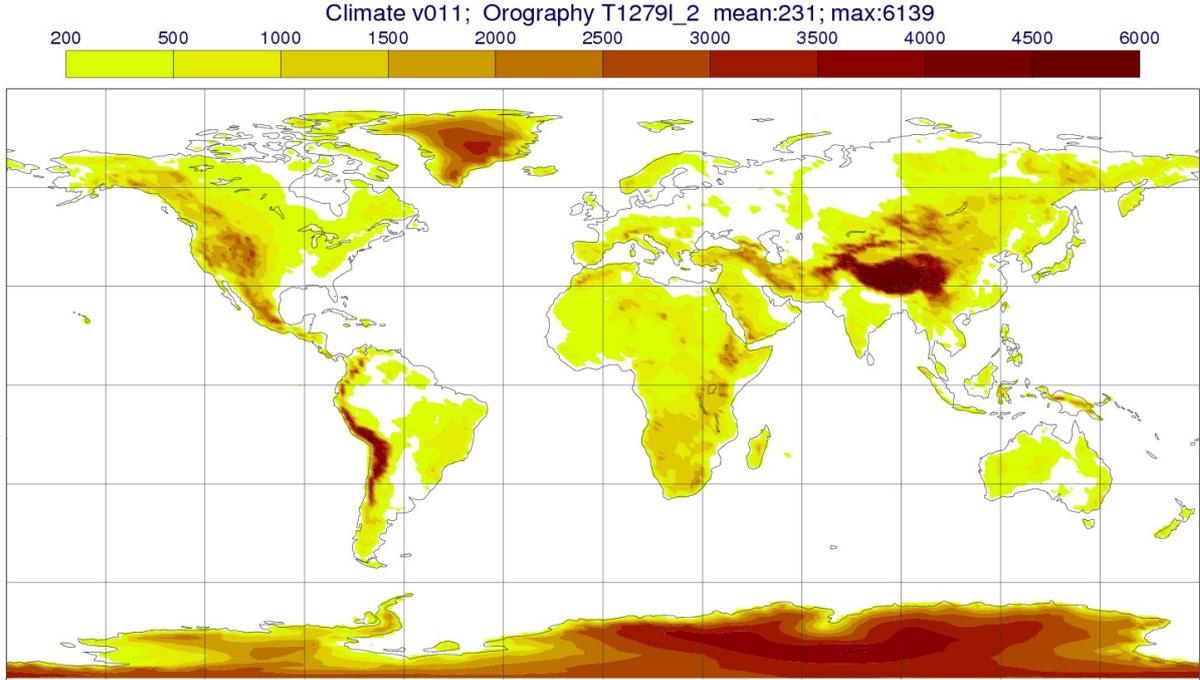
where  $h_l^m$  and  $\tilde{h}_l^m$  are the spectral coefficients of the orography before and after constraining the curvature. Parameter  $l_{\max}$  is the highest wave number (7999 in this case). In fact this is a 4th order filter with a damping factor of  $\lambda$  for the highest wave number. The damping factor  $\lambda$  is selected such that the square of the curvature is reduced by a specified factor, i.e.  $Fact = \int \{\Delta \tilde{f}(s)\}^2 ds / \int \{\Delta f(s)\}^2 ds$ . Reduction factor  $Fact$  is selected subjectively for different target resolutions (see Table 11.2).

4. The T7999 spectral orography of the previous step is truncated at target resolution.
5. To avoid minor aliasing during the time integration in the model, the spectrum is further filtered with a sharp filter to reduce the highest wave numbers. The spectrum is multiplied by  $1/[1 + 4 \{l(l+1)\}^{16} \{l_{\max}(l_{\max}+1)\}^{-16}]$ , where  $l$  is the wave number index and  $l_{\max}$  is the maximum wave number of the target resolution (e.g. 1279). This is equivalent to a smoothing of the grid point field with a  $\nabla^{32}$  operation. The exponent determines the sharpness of the tapering and the factor 4 determines the filtering strength.
6. The spectral orography is transformed back to grid point space.

### 11.3.3 Standard deviation of filtered orography

To compute the standard deviation of the small scale orography for well defined scales all over the globe (also in polar regions where the grid point spacing is much less than 1 km), the 30'' field is filtered twice with filter (11.4), shown in Fig. 11.1. The first time, the smallest scales are filtered out by using  $\Delta_1 = 2$  km and  $\delta_1 = 1$  km. The second filtering is done with  $\Delta_2 = 20$  km and  $\delta_2 = 1$  km, to isolate the larger scales. Subsequently the following processing steps are performed:

- (i) The square of the difference is computed of the two 30'' resolution filtered fields.
- (ii) The result is smoothed with the Filter-Scale (see Table 11.2 for the *Filter-Scale* at target resolution).
- (iii) The square root is calculated.



**Figure 11.2** Orography at T1279 resolution.

- (iv) For every point of the target grid, the result of (iii) is averaged over the grid square of the target grid.

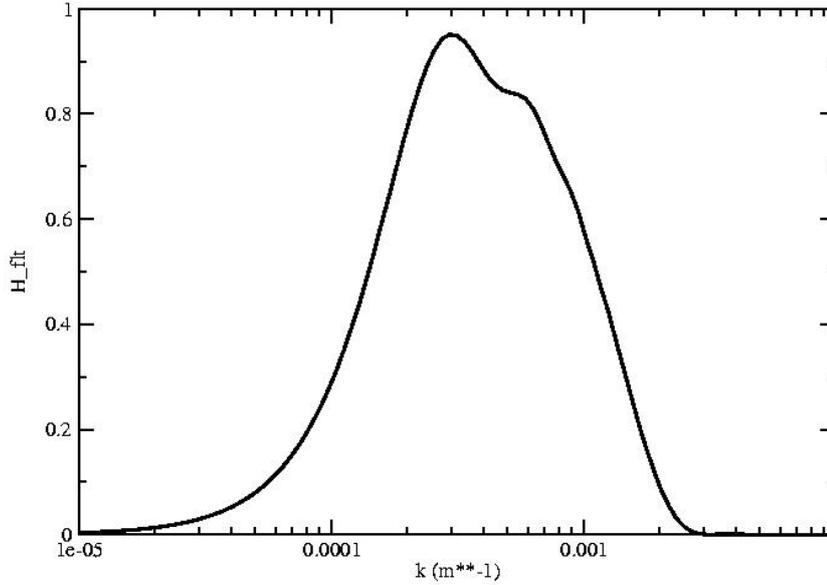
The resulting field is shown in [Fig. 11.4](#) for the T1279 model. The spectral filter for the small scale orography corresponding to the procedure described above is

$$\begin{aligned}
 H_{\text{flt}}(k) = & \frac{1}{\Delta_1^2} \left\{ \frac{\sin(k\Delta_1/2 - k\delta_1)}{k} + \frac{\sin(k\Delta_1/2 + k\delta_1)}{k} \right. \\
 & \left. + \frac{\cos(\pi/2 + k\Delta_1/2) \sin(\pi/2 + k\delta_1)}{\pi/(2\delta_1) + k} + \frac{\cos(\pi/2 - k\Delta_1/2) \sin(\pi/2 - k\delta_1)}{\pi/(2\delta_1) - k} \right\}^2 \\
 & - \frac{1}{\Delta_2^2} \left\{ \frac{\sin(k\Delta_2/2 - k\delta_2)}{k} + \frac{\sin(k\Delta_2/2 + k\delta_2)}{k} \right. \\
 & \left. + \frac{\cos(\pi/2 + k\Delta_2/2) \sin(\pi/2 + k\delta_2)}{\pi/(2\delta_2) + k} + \frac{\cos(\pi/2 - k\Delta_2/2) \sin(\pi/2 - k\delta_2)}{\pi/(2\delta_2) - k} \right\}^2
 \end{aligned} \tag{11.7}$$

The filter of (11.7) is shown in [Fig. 11.3](#). The filter has the shape of a band pass filter with the lower bound determined by  $\Delta_2$ , and the upper bound by  $\Delta_1$ . Parameters  $\delta_1$  and  $\delta_2$  control the level of overshooting. The parameter selection is based on the following ideas. First the filter should drop off quickly near  $k = 0.0012 \text{ m}^{-1}$ , because the spectrum has an aliasing tail (see [Beljaars et al., 2004b](#)). Secondly we would like to cut off below scales of 5 km because we are interested in scales smaller 5 km for TOFD. However, this leads to a very narrow filter with noisy results. Therefore we select a longer filtering scale of about 20 km. The edges of the filter defined by  $H_{\text{flt}} = 0.0005$  are  $k = 0.00014 \text{ m}^{-1}$  and  $k = 0.00112 \text{ m}^{-1}$  respectively. These wave numbers correspond to length scales (half wave length) of 22000 m and 3000 m. The advantage of having a broad filter is that resulting standard deviations will be less noisy.

With an orography spectrum  $F_o$  and the band pass filtering with (11.7), the following spectrum is obtained for the small scale orography

$$F_{\text{flt}}(k) = F_o(k)H_{\text{flt}}(k) \tag{11.8}$$



**Figure 11.3** Spectral filter corresponding to difference of two smoothing operations with:  $\Delta_1 = 2000$  m,  $\Delta_2 = 20000$  m,  $\delta_1 = 1000$  m,  $\delta_2 = 1000$  m.

The variance of the sub-grid orography as computed from the filtered fields is

$$\sigma_{\text{fit}}^2 = \int F_o(k) H_{\text{fit}}(k) dk \quad (11.9)$$

$$\approx F_o(k_{\text{fit}}) \int H_{\text{fit}}(k) dk \quad (11.10)$$

$$= F_o(k_{\text{fit}}) I_H \quad (11.11)$$

The approximation is based on the idea that the band width of the filter is small and that the spectrum of the orography does not change much over the band width of the filter. So, by computing the variance of the small scale orography  $\sigma_{\text{fit}}^2$ , an estimate is obtained of the orographic power spectrum at wavelength  $k_{\text{fit}}$ :

$$F_o(k_{\text{fit}}) = \sigma_{\text{fit}}^2 / I_H \quad (11.12)$$

For a power spectrum with exponent  $n_1$  in the range of the band filter, a filter wave number can be defined that satisfies (11.11) exactly

$$k_{\text{fit}}^{n_1} = \left\{ \int k^{n_1} H(k) dk \right\} \left\{ \int H(k) dk \right\}^{-1} \quad (11.13)$$

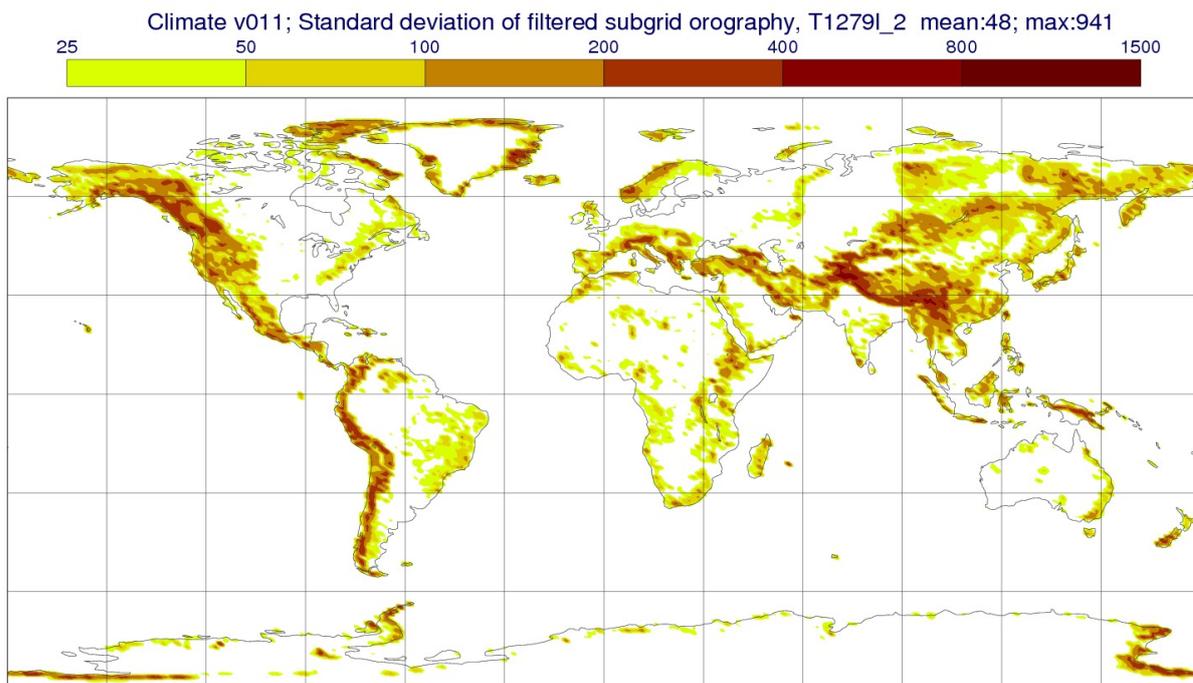
With the filter parameters  $\Delta_1 = 2000$  m,  $\delta_1 = 1000$  m,  $\Delta_2 = 20000$  m,  $\delta_2 = 1000$  m, and  $n_1 = -1.9$ , the following results are found from numerical integration:

$$I_H = 0.00102 \text{ m}^{-1}, \quad k_{\text{fit}} = 0.00035 \text{ m}^{-1} \quad (11.14)$$

These numbers are used in the parametrization scheme to account for the way the standard deviation of filtered orography was generated.

### 11.3.4 Parameters for gravity-wave and low level orographic blocking schemes

The parameters that are needed for the sub-grid orography scheme characterize the variability of the orography between horizontal scales of 5 km and target resolution for target grids with  $NGL \leq 4000$ . For grids with  $NGL > 4000$ , orographic variability with scales between 2 km and target resolution are included. The parameters are: standard deviation  $\mu_{\text{GW}}$ , anisotropy  $\gamma_{\text{GW}}$ , orientation  $\theta_{\text{GW}}$ , and slope  $\sigma_{\text{GW}}$ . They are computed as follows (see [Lott and Miller, 1997](#); [Baines and Palmer, 1990](#)).



**Figure 11.4** Standard deviation of the filtered orography for the T1279 model, to support the TOFD parametrization.

- (i) Starting point is the difference between the results of step 2 and 3 of the mean orography processing. This is a field at 5 km (or 2 km) resolution that contains the orographic variability at scales between 5 km (or 2 km) and the target resolution. These scales are considered to be the main contributors to low level blocking and gravity wave generation.
- (ii) For every point of the 5 km (or 2 km) data,  $(\partial h/\partial x)$  and  $(\partial h/\partial y)$  are computed by central differencing with help of the points to the north, south, east and west.
- (iii) Parameters  $K$ ,  $L$ ,  $M$ ,  $O$ , and  $P$  are computed for all the points of the 5 km (or 2 km) grid:

$$K = \frac{1}{2} \left\{ \left( \frac{\partial h}{\partial x} \right)^2 + \left( \frac{\partial h}{\partial y} \right)^2 \right\}$$

$$L = \frac{1}{2} \left\{ \left( \frac{\partial h}{\partial x} \right)^2 - \left( \frac{\partial h}{\partial y} \right)^2 \right\}$$

$$M = \left\{ \left( \frac{\partial h}{\partial x} \right) \left( \frac{\partial h}{\partial y} \right) \right\}$$

$$O = h^2$$

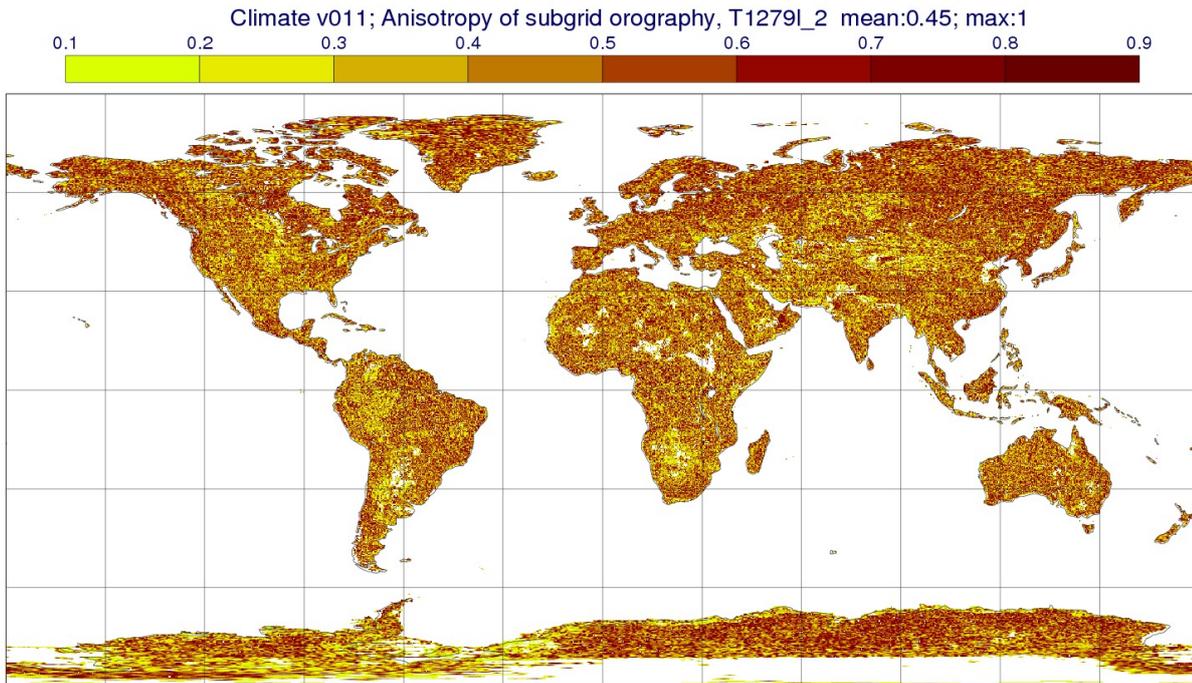
$$P = h$$

- (iv) For every point of the target grid,  $K$ ,  $L$ ,  $M$ ,  $O$  and  $P$  are aggregated by conservative interpolation of the 5 km (or 2 km) points, i.e. the fraction of every 5 km (or 2 km) grid square that overlaps with the target grid square is included with the proper weight.
- (v) Anisotropy  $\gamma_{\text{GW}}$ , orientation  $\theta_{\text{GW}}$ , and slope  $\sigma_{\text{GW}}$  are computed from  $K$ ,  $M$  and  $L$  using

$$\gamma_{\text{GW}}^2 = \frac{K - (L^2 + M^2)^{1/2}}{K + (L^2 + M^2)^{1/2}}$$

$$\theta_{\text{GW}} = \frac{1}{2} \text{atan} \frac{M}{L}$$

$$\sigma_{\text{GW}}^2 = K + \sqrt{L^2 + M^2}$$



**Figure 11.5** Anisotropy  $\gamma_{GW}$  of sub-grid orography (1 indicates isotropic, 0 means maximum anisotropy).

and the standard deviation  $\mu_{GW}$  is computed according to

$$\mu_{GW}^2 = O - P^2$$

Results are shown in [Figs 11.5, 11.6, 11.7](#) and [11.8](#).

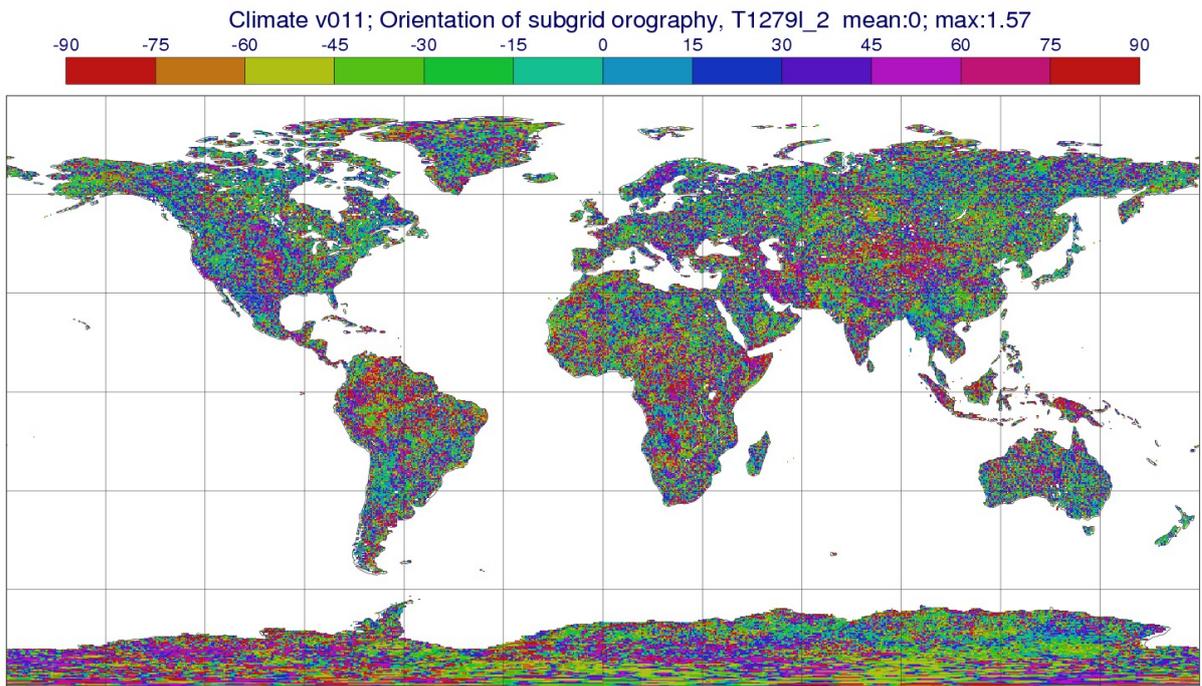
### 11.4 VEGETATION PARAMETERS

Vegetation is represented by six climatological parameters: vegetation cover of low vegetation, vegetation cover of high vegetation, low vegetation type, high vegetation type, leaf area for low vegetation and leaf area for high vegetation. The first four parameters are derived from the 2'30'' GLCC.1.2 data by aggregating over the target grid squares. GLCC.2.0 is available and prepared but not used yet. The fractional covers for low and high vegetation are obtained by combining the fractions from all the low and high vegetation types of [Table 11.1](#). The index of the dominant low and high vegetation types are also coded as climatological fields for use by the land surface scheme. The latter two fields can not be interpolated by standard procedures to another resolution. The resulting fields are shown in [Figs 11.9, 11.10, 11.11](#) and [11.12](#). [Table 11.3](#) and [Table 11.4](#) contain statistical information on the number of points in each vegetation class.

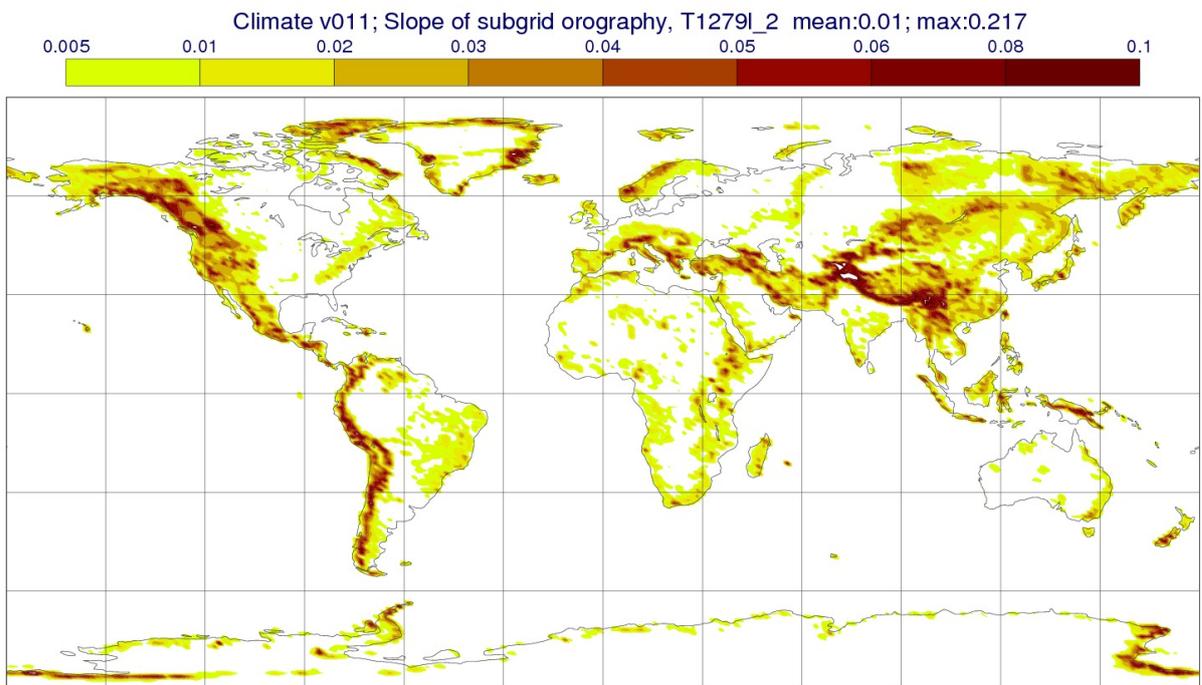
Leaf area is based on MODIS satellite data and is specified as a monthly climatology. Details are given in [Boussetta et al. \(2011\)](#) and the geographical distribution is shown for January and July in [Figs 11.13, 11.14, 11.15](#) and [11.16](#). The model interprets these fields as the climatology for the 15th of the month and does a linear interpolation between months to obtain a smooth seasonal cycle.

### 11.5 ROUGHNESS LENGTH

Roughness lengths are not prepared as climatological fields, but derived in the model from vegetation type, vegetation cover, and snow cover through a correspondence table (see the Chapter on land surface modelling). Results with the T1279 model are shown in [Figs 11.17](#) and [11.18](#) for 1 August 2012.



**Figure 11.6** Orientation  $\theta_{GW}$  of sub-grid orography. The field is in radians and has values between  $-\frac{1}{2}\pi$  and  $\frac{1}{2}\pi$ ). For clarity the plot has been converted to degrees with values between -90 and +90.



**Figure 11.7** Slope  $\sigma_{GW}$  of sub-grid orography.

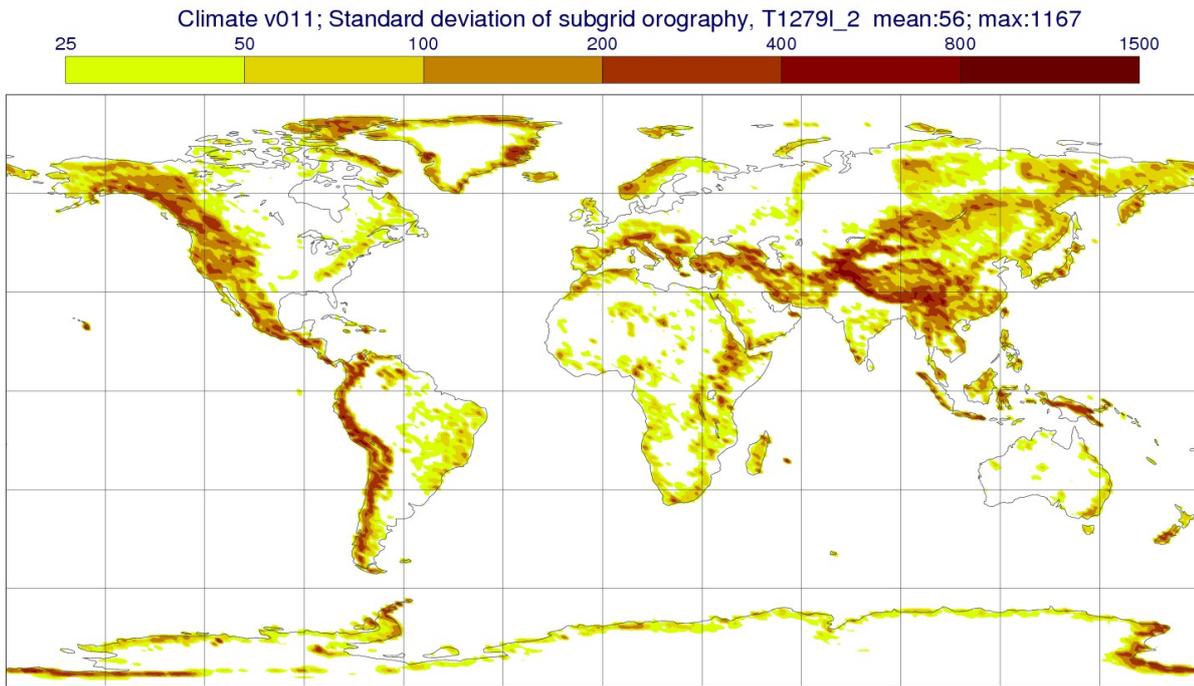


Figure 11.8 Standard deviation  $\mu_{GW}$  of sub-grid orography.

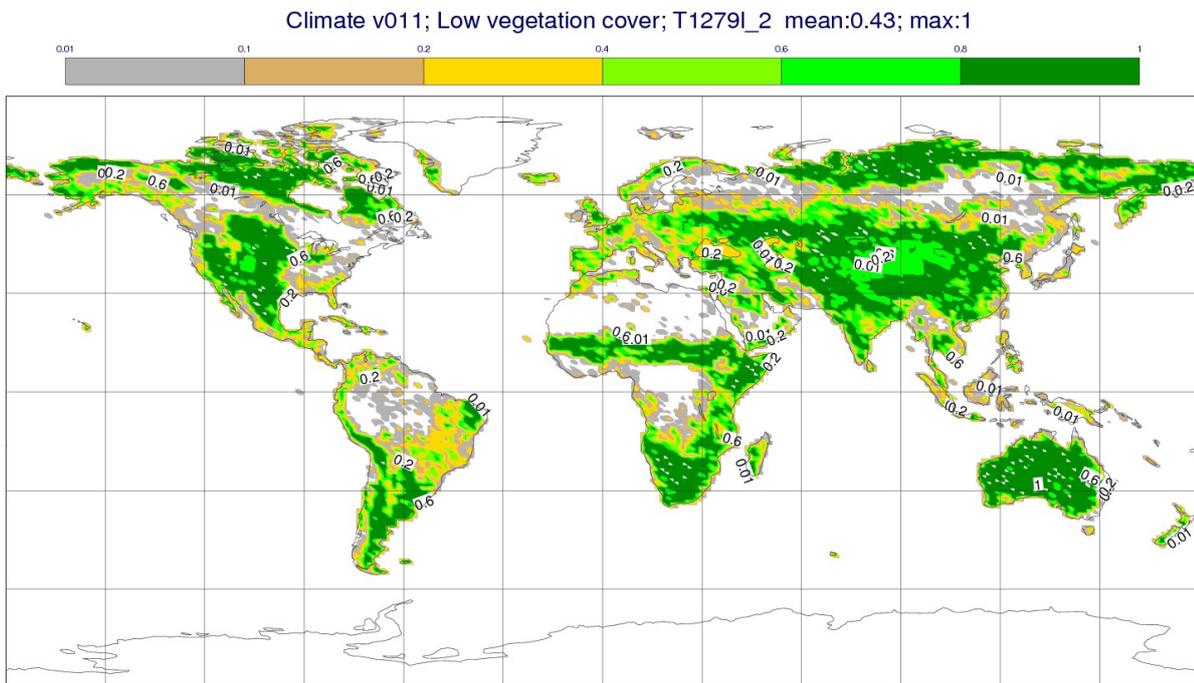


Figure 11.9 Fractional cover of low vegetation.



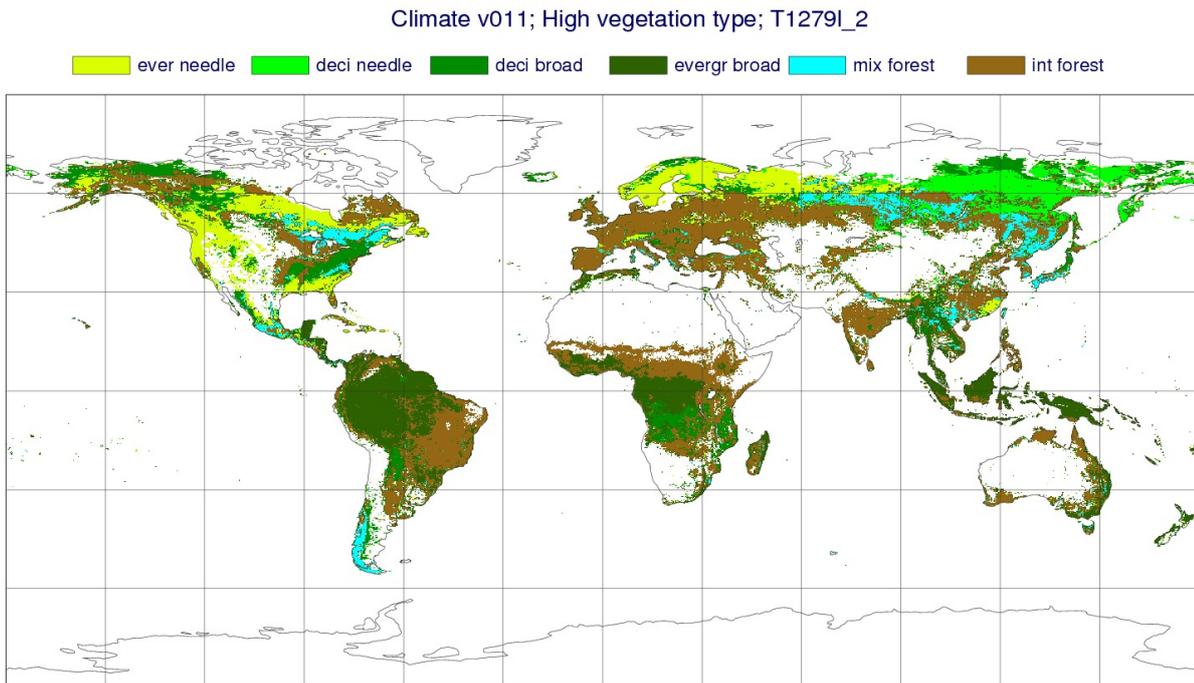


Figure 11.12 High vegetation type.

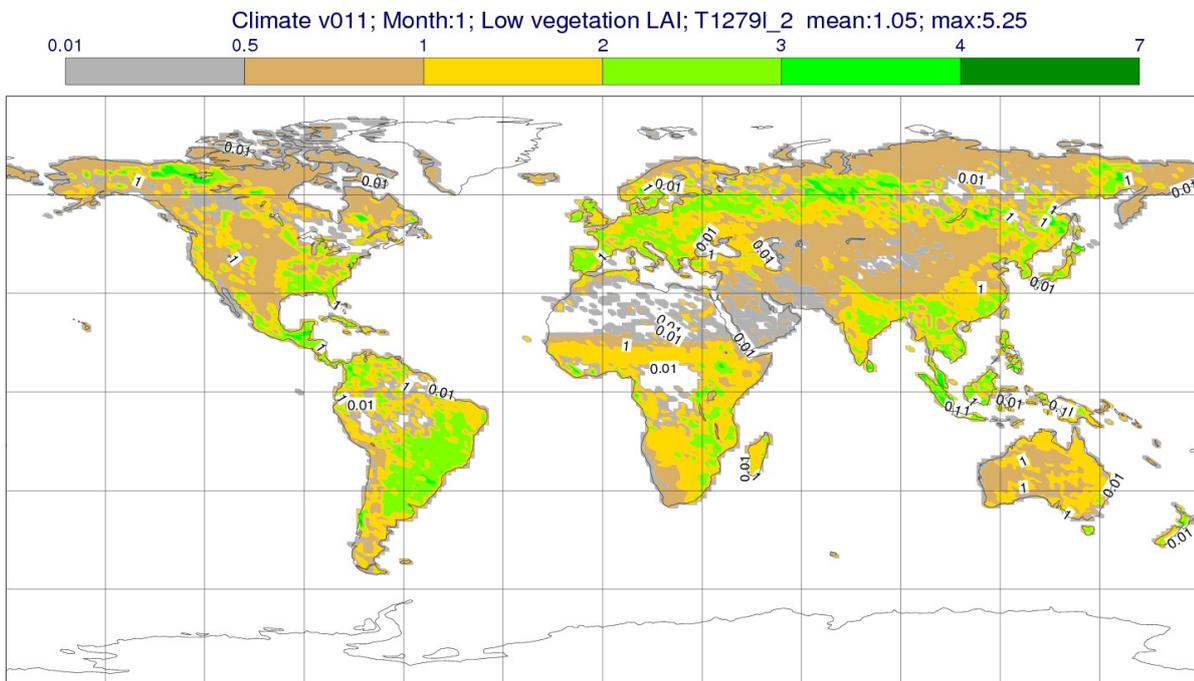


Figure 11.13 LAI climatology for low vegetation in January.

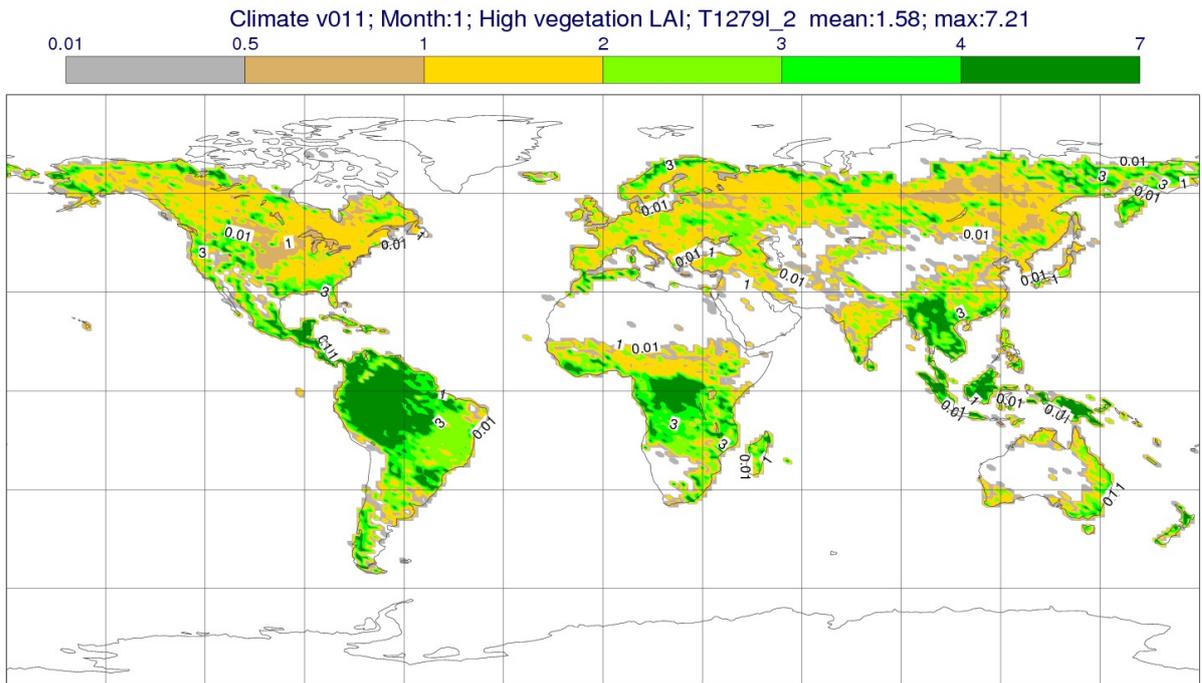


Figure 11.14 LAI climatology for high vegetation in January.

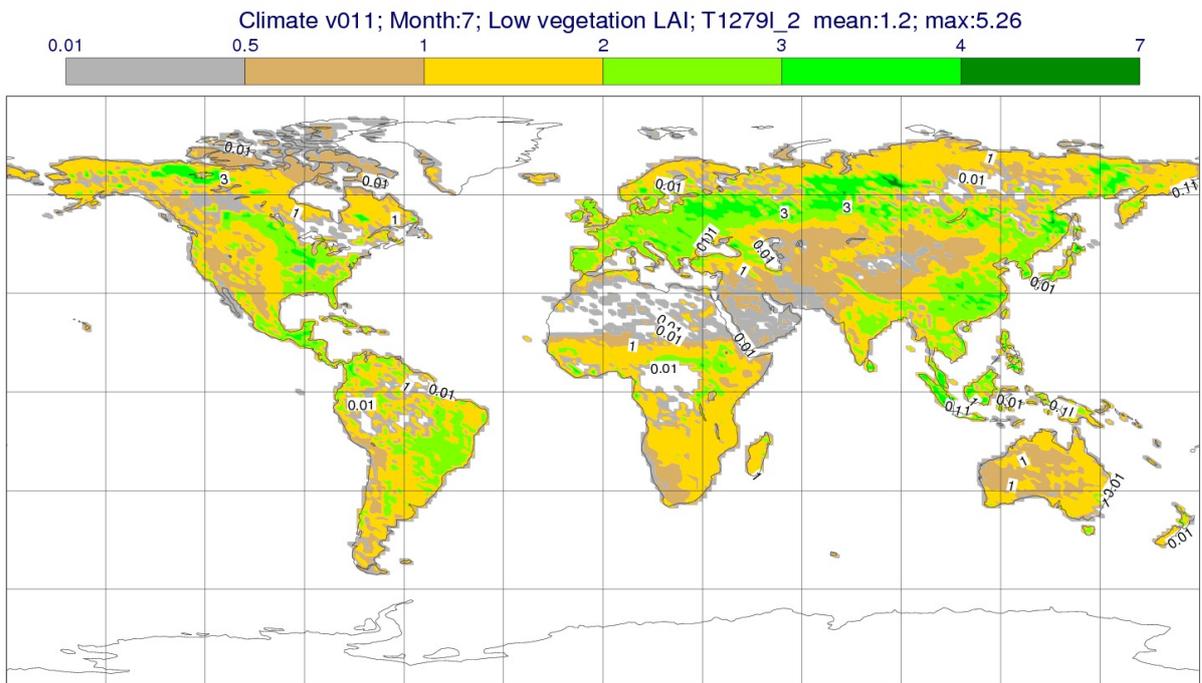


Figure 11.15 LAI climatology for low vegetation in July.

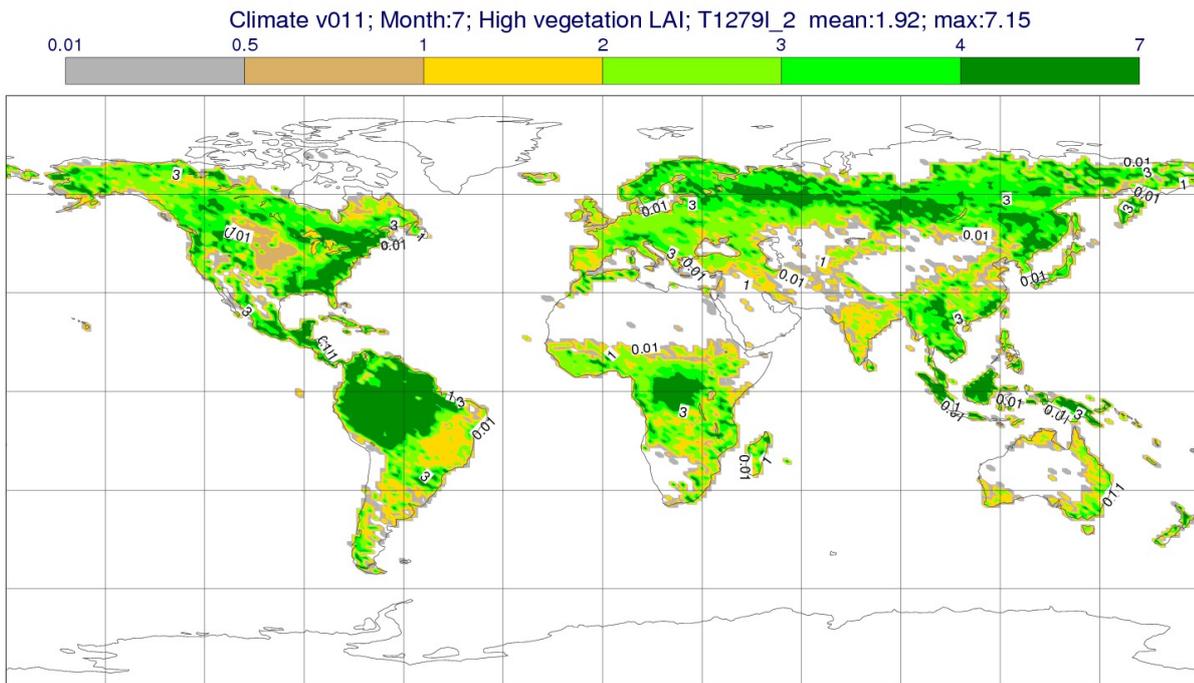


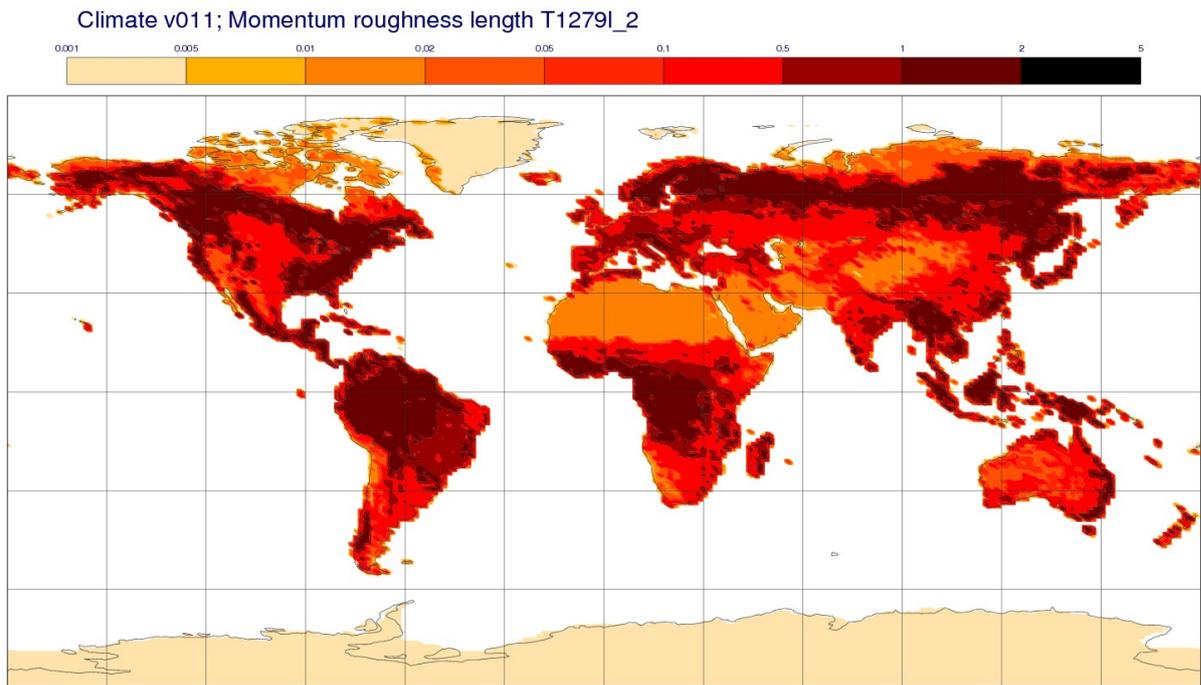
Figure 11.16 LAI climatology for high vegetation in July.

Table 11.3 Percentage of land points at T1279 for each low vegetation type.

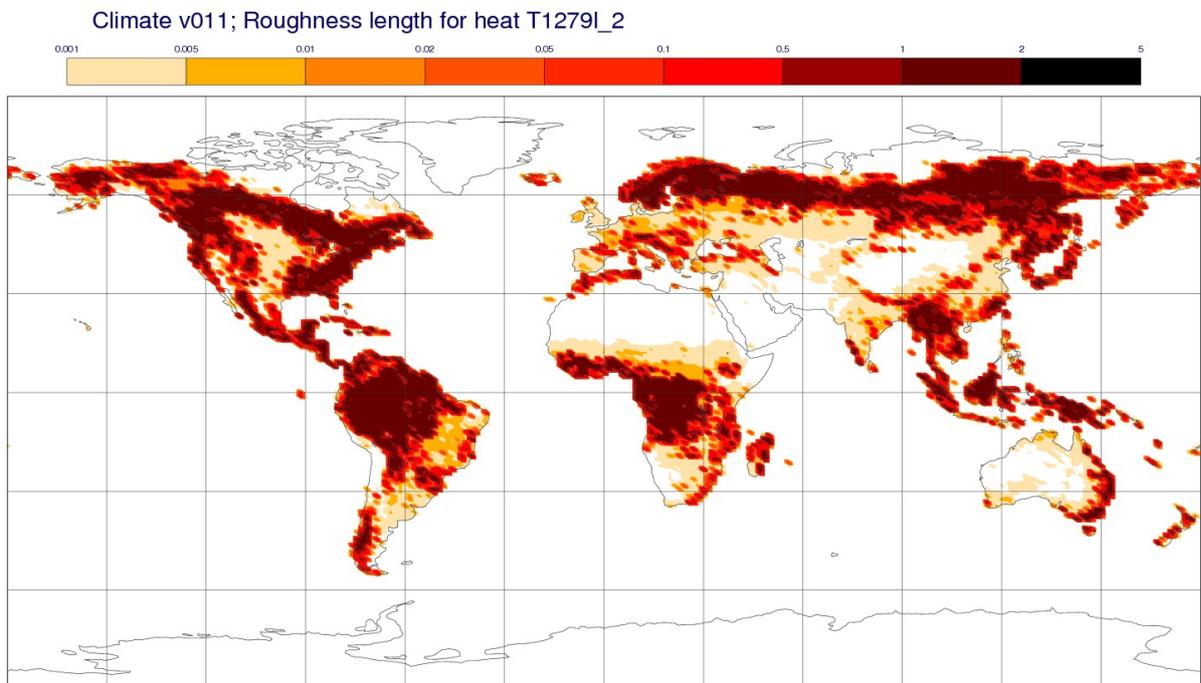
Index	Vegetation type	Percentage of land points
1	Crops, Mixed Farming	18.9
2	Short Grass	9.3
7	Tall Grass	11.7
9	Tundra	7.2
10	Irrigated Crops	3.8
11	Semidesert	12.0
13	Bogs and Marshes	1.7
16	Evergreen Shrubs	1.3
17	Deciduous Shrubs	4.0
20	Water and Land Mixtures	0
-	Remaining land points without low vegetation	29.6

Table 11.4 Percentage of land points at T1279 for each high vegetation type.

Index	Vegetation type	Percentage of land points
3	Evergreen Needleleaf Trees	6.1
4	Deciduous Needleleaf Trees	2.9
5	Deciduous Broadleaf Trees	5.8
6	Evergreen Broadleaf Trees	11.0
18	Mixed Forest/woodland	3.3
19	Interrupted Forest	25.3
-	Remaining land points without high vegetation	45.3



**Figure 11.17** Roughness length for momentum as produced by the T1279 model for 1 August 2012, using the dominant vegetation type, snow cover and correspondence [Table 8.3](#).



**Figure 11.18** Roughness length for heat as produced by the T1279 model for 1 August 2012, using the dominant vegetation type, snow cover and correspondence [Table 8.3](#).

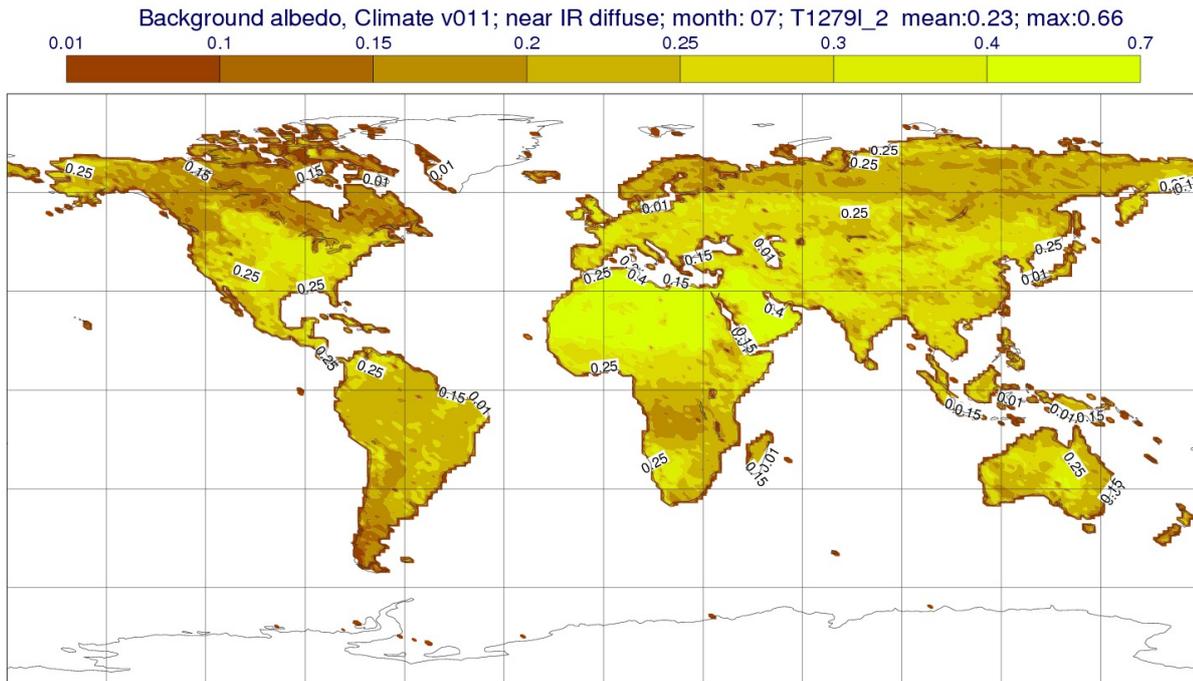


Figure 11.19 Climatological background albedo for July (near infrared, diffuse).

## 11.6 ALBEDO

Monthly albedo fields at target resolution are derived from the pre-computed high resolution  $0.05^\circ$  fields described in section 11.2.5 in the following way:

- i The resolution field is smoothed with the *Filter-Scale* listed in Table 11.2 dependent on the selected target resolution, e.g. at T1279, the linear grid has  $NGL = 1280$  and a *Filter-Scale* of 16000 m.
- ii The resulting field is interpolated to target resolution.

The fields for July are shown in Figs 11.19, 11.20, 11.21 and 11.22. To obtain a smooth evolution in time, the model does a linear interpolation between successive months, assuming that the monthly field applies to the 15th of the month. The model adapts the background albedo over water, ice and snow as documented in the chapter on radiation.

## 11.7 AEROSOLS

Five types of tropospheric aerosols are considered in the model. The geographical distributions of sea-salt, dust, organic, black carbon, sulphate aerosols have been derived from the monthly climatology of Tegen *et al.* (1997). Fig. 11.23 to Fig. 11.27 give for each of the five types the distribution for January, April, July and October. They are distributed vertically according to the profiles in Fig. 11.28. Well-mixed (vertically and horizontally) tropospheric background aerosols with an optical thickness of 0.03 and stratospheric background aerosols with an optical thickness of 0.045 are added to the previous amounts with a rate of change of optical thickness with pressure of 0.037 and 0.233 /atm respectively. The transition from troposphere to stratosphere is obtained by multiplication of the background values with  $1 - L_{\text{stratos}}$  and  $L_{\text{stratos}}$  respectively (see Fig. 11.28).

## 11.8 TRACE GASES AND OZONE

A new description of the radiatively active trace gases has been introduced in the IFS with Cy35r3 on 8 September 2009. It replaces the previous globally averaged values for  $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2\text{O}$ ,  $\text{CFCl}_3$ ,  $\text{CF}_2\text{Cl}_2$  (with possibility of defining their history since 1850) by bi-dimensional (latitude/height) climatologies

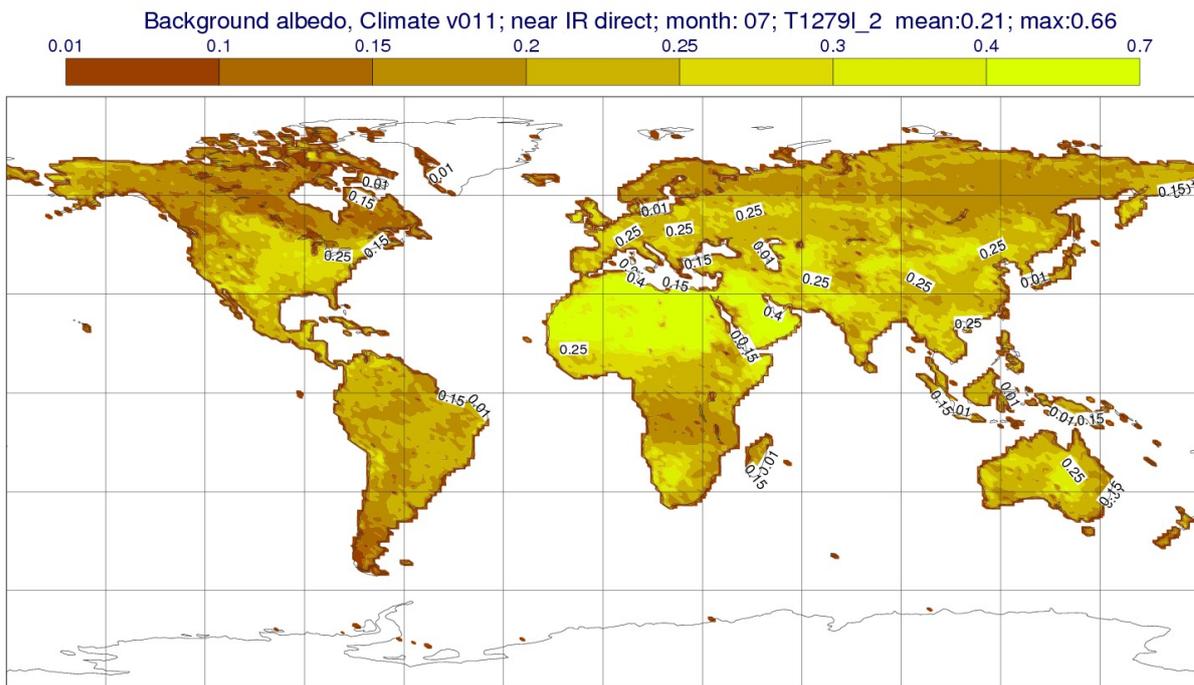


Figure 11.20 Climatological background albedo for July (near infrared, direct).

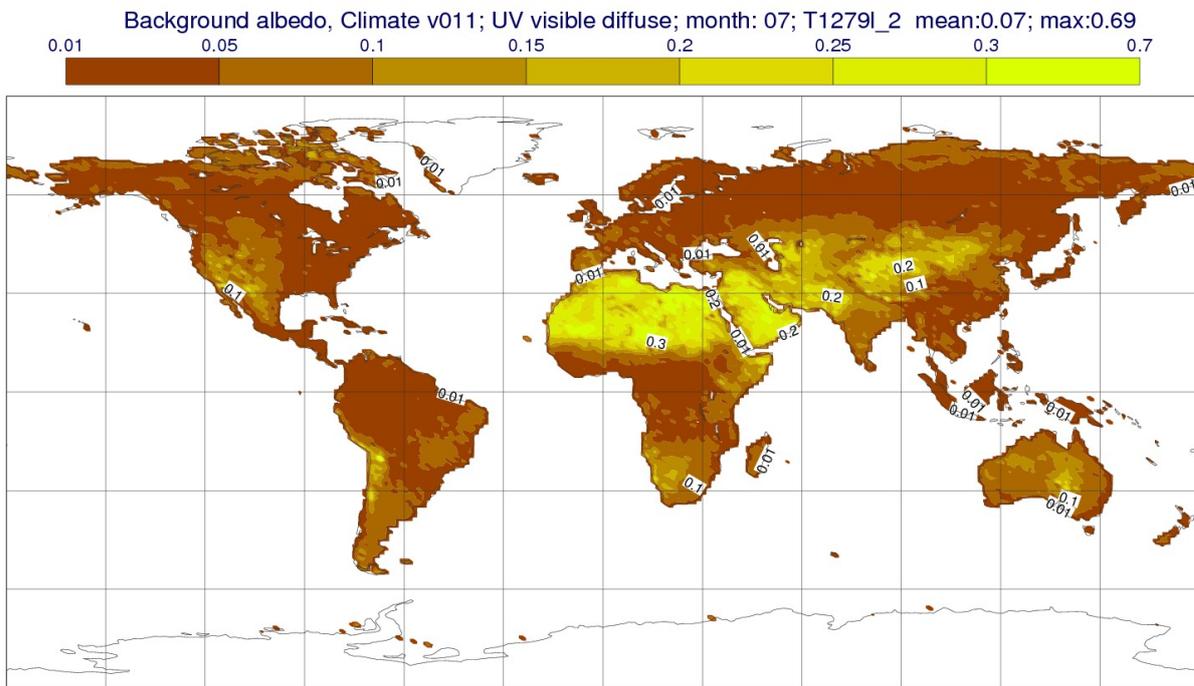


Figure 11.21 Climatological background albedo for July (UV visible, diffuse).

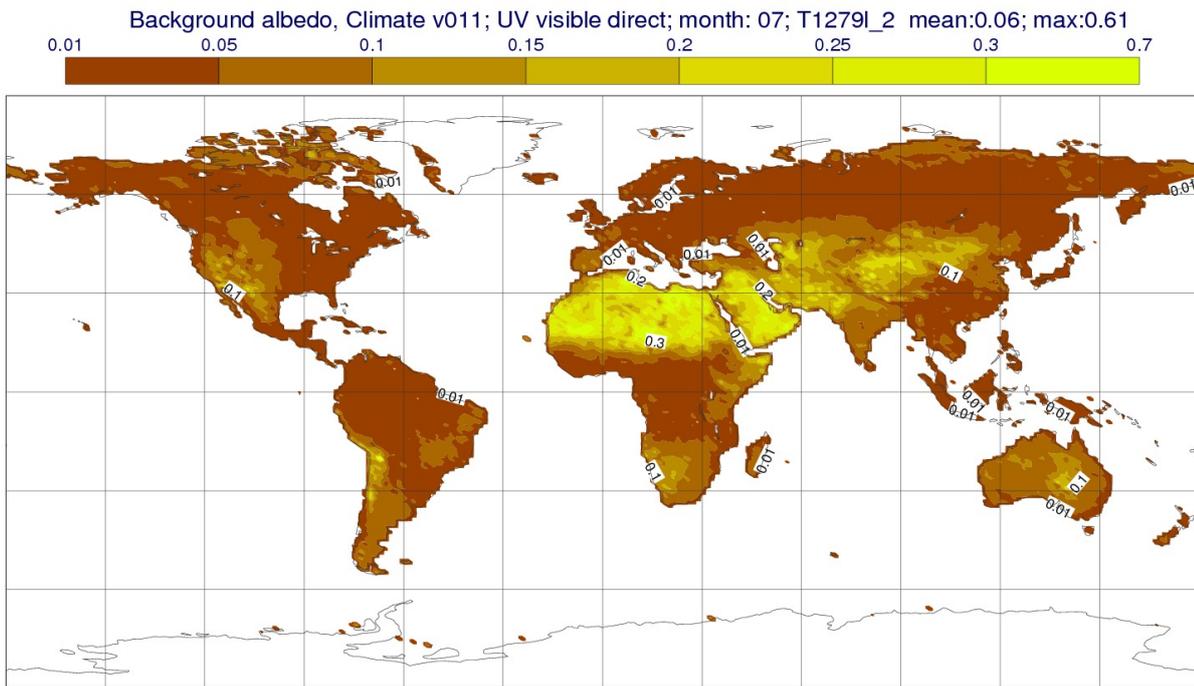


Figure 11.22 Climatological background albedo for July (UV visible, direct).

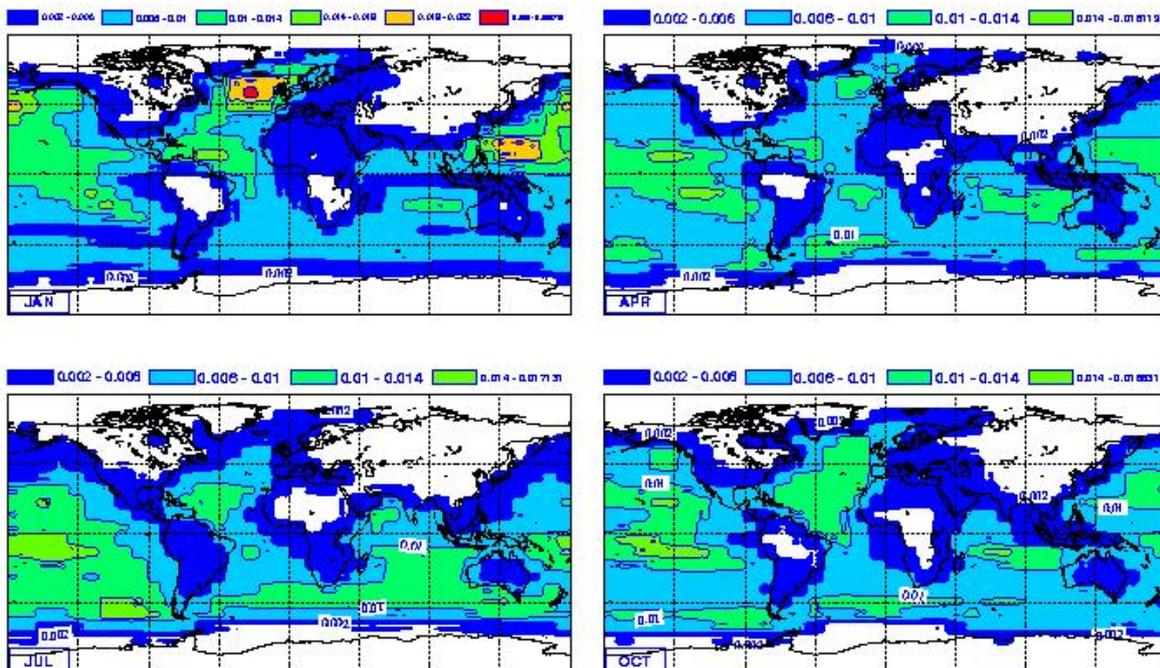


Figure 11.23 Climatological distribution of sea salt aerosols for January, April, July, and October according to Tegen et al. (1997).

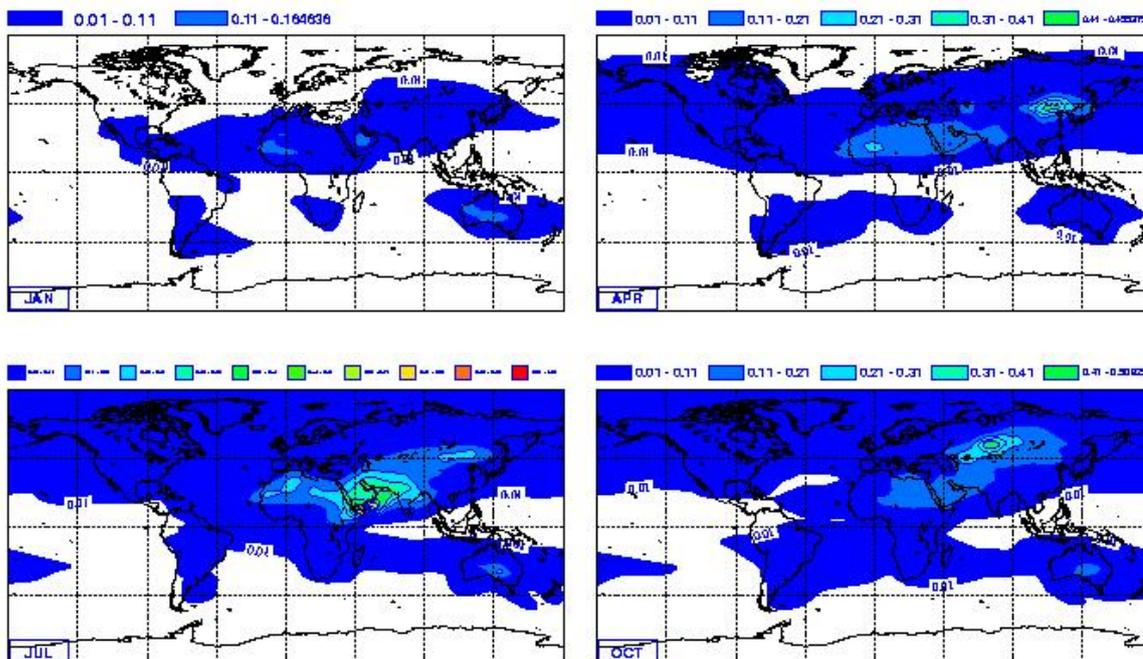


Figure 11.24 Climatological distribution of dust aerosols for January, April, July, and October according to Tegen et al. (1997).

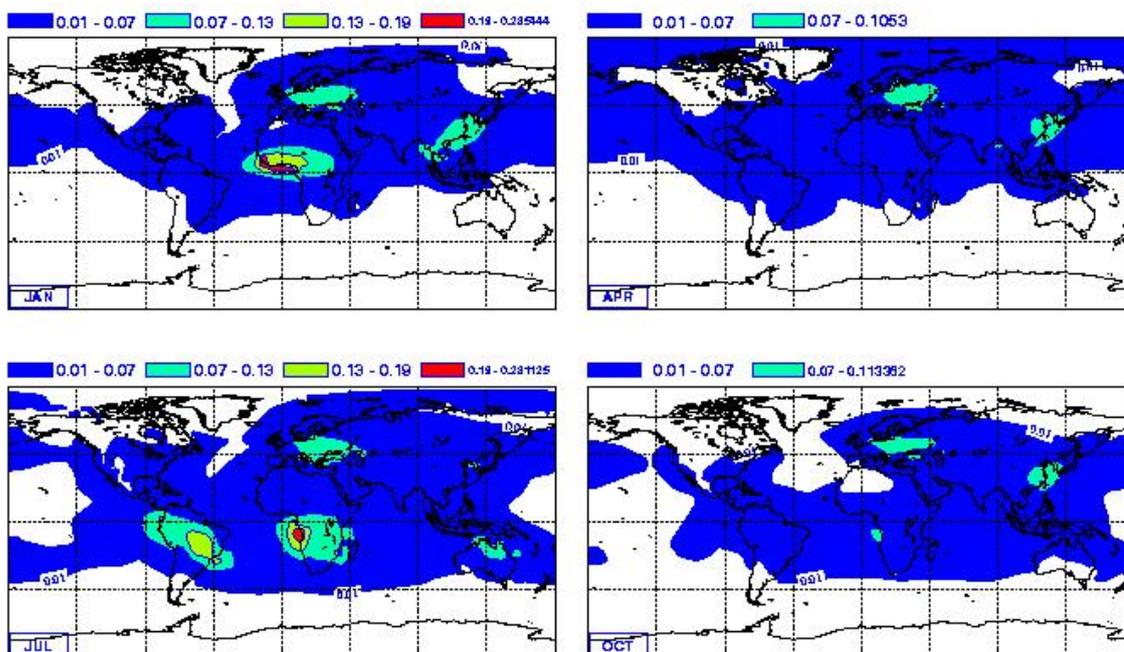


Figure 11.25 Climatological distribution of organic aerosols for January, April, July, and October according to Tegen et al. (1997).

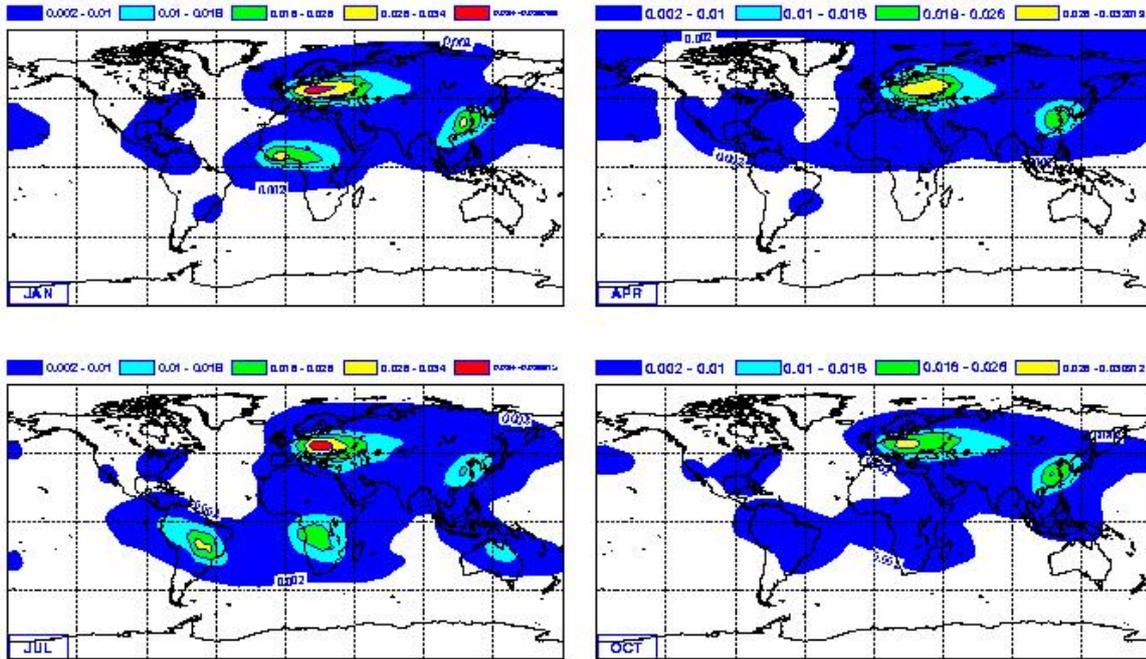


Figure 11.26 Climatological distribution of black carbon aerosols for January, April, July, and October according to Tegen et al. (1997).

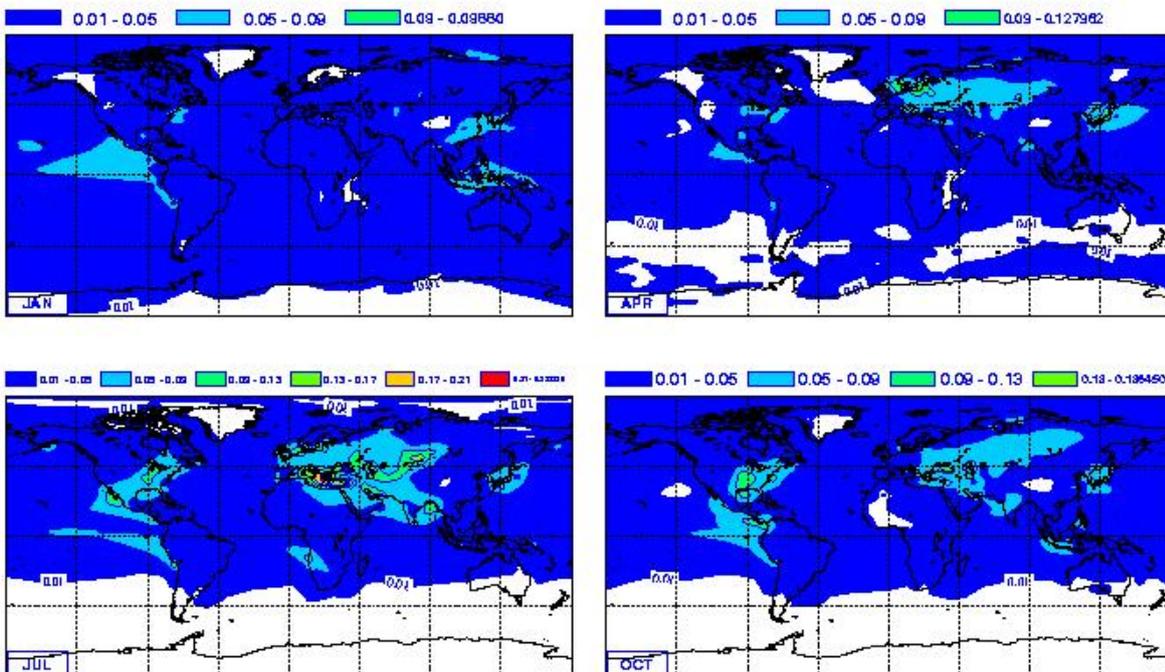
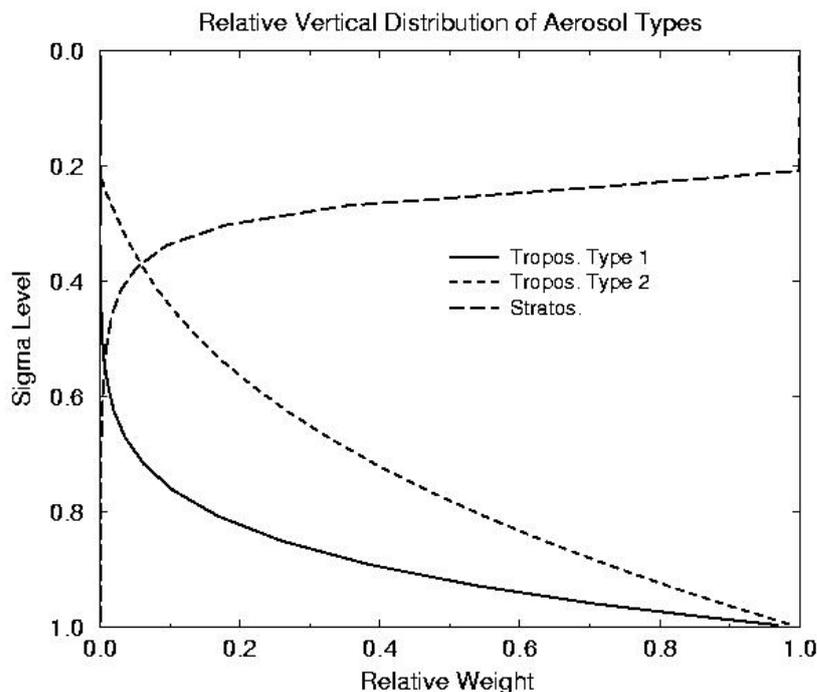


Figure 11.27 Climatological distribution of sulphate aerosols for January, April, July, and October according to Tegen et al. (1997).



**Figure 11.28 (e)** Type 1 (full line) profiles apply to maritime, continental and urban type aerosols; type 2 (short dashed line) applies to desert type; the third curve (long dashed line) represents  $L_{\text{stratos}}$  and is used to determine the transition from tropospheric to stratospheric background aerosols.

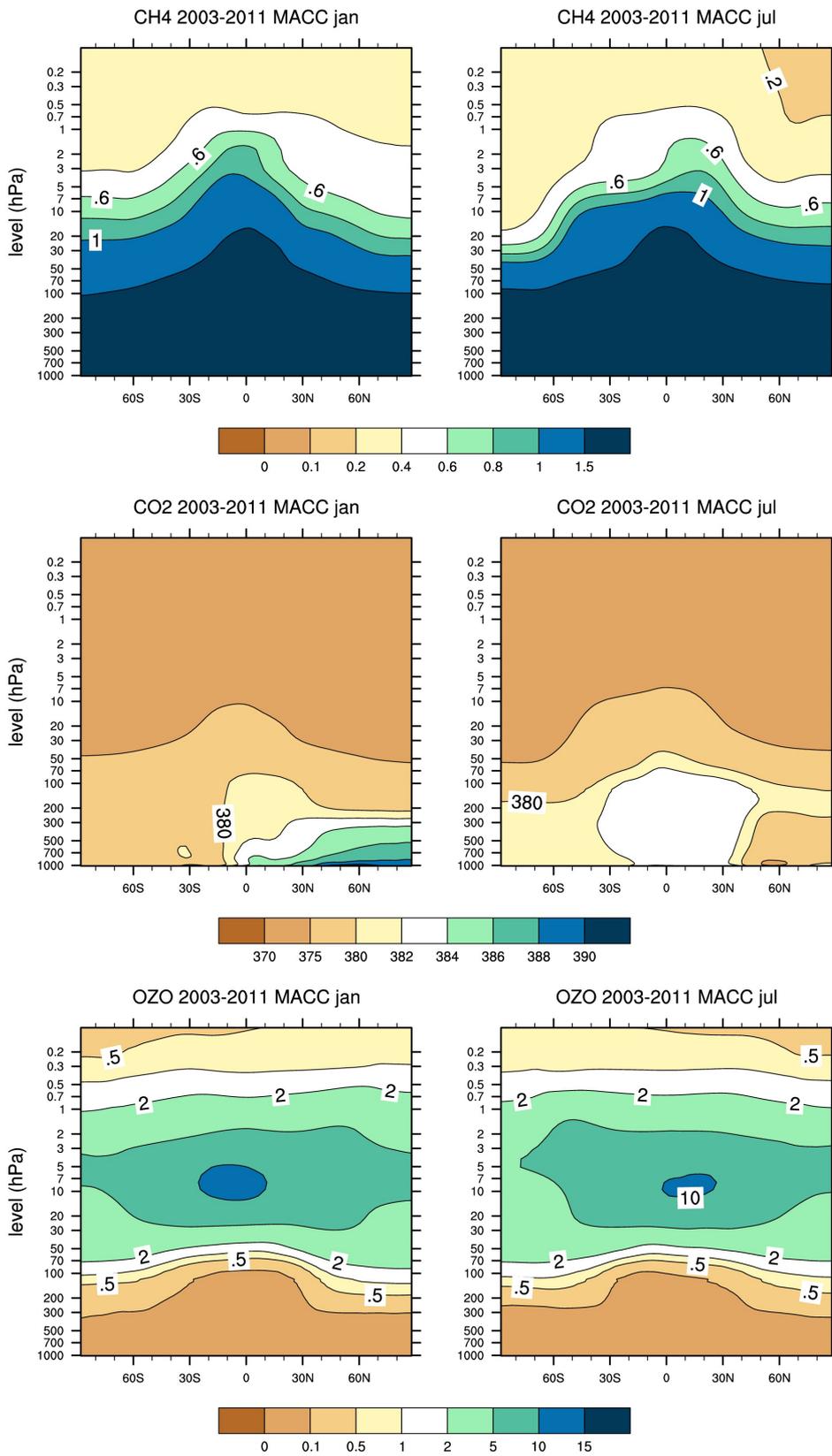
derived from either the MOBIDIC model or, for  $\text{CO}_2$ ,  $\text{CH}_4$  and  $\text{O}_3$ , from the GEMS reanalysis effort. The effect of  $\text{CHFCl}_2$  and  $\text{CCl}_4$  have also been added using their globally-defined concentrations. A discussion of the impact of these new climatologies and of further model developments (dissipation of non-orographic gravity-waves) can be found in [Bechtold \*et al.\* \(2009\)](#). The GEMS-based climatology for  $\text{CO}_2$ ,  $\text{CH}_4$  and  $\text{O}_3$  has been upgraded since Cy40r3 with a 2003–2011 climatology based on the MACC reanalysis ([Fig. 11.29](#)). The long-term evolution of  $\text{CO}_2$  and  $\text{CH}_4$  is described by yearly globally averaged volume mixing ratios over the marine surface (GISS data) and applied to the zonal-mean climatologies scaled by their global mean mixing ratio at the surface.

## 11.9 SOIL TYPE

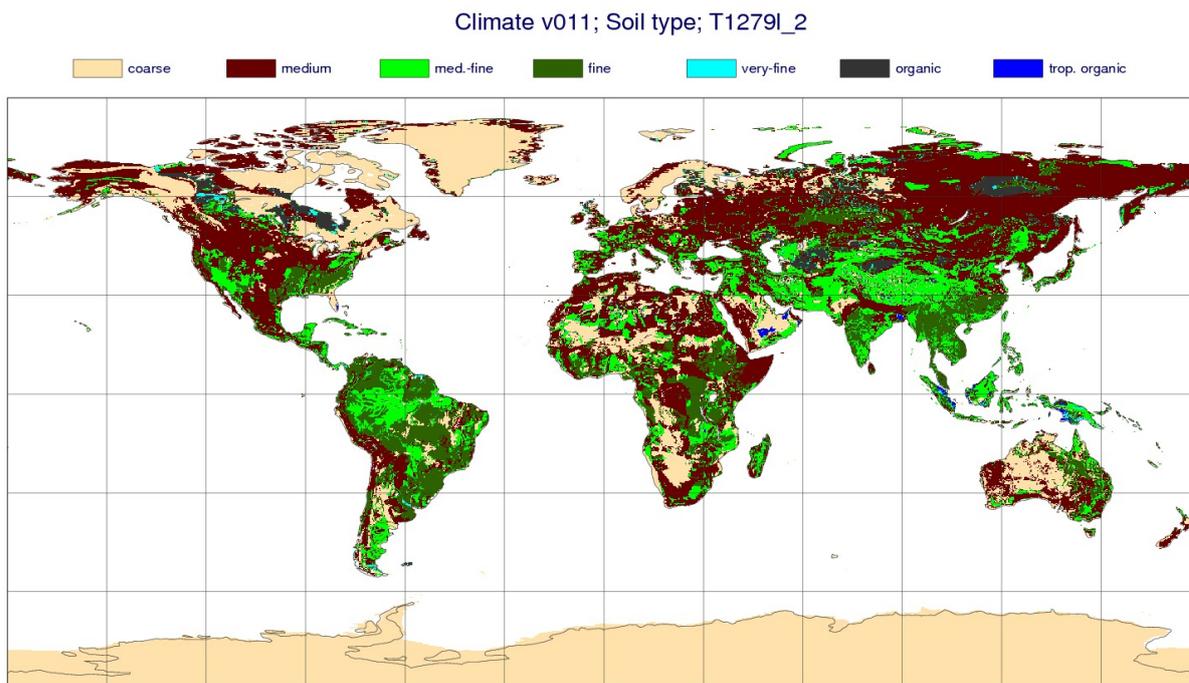
Soil types are derived from the FAO/UNESCO Digital Soil Map of the World, DSMW ([FAO, 2003](#)), which exists at a resolution of  $5' \times 5'$  (about 10 km). FAO DSMW provides the information on two levels of soil depth namely 0–30 cm and 30–100 cm. Since the root zone is most important for the water holding, the 30–100 cm layer is selected for H-TESEL. To interpolate to model target resolution the INTERPOL utility is applied. INTERPOL's interpolation option is used for resolutions with  $NGL \geq 2048$  (10 km), whereas the nearest neighbour option is chosen for  $NGL < 2048$ . In all cases, the the dominant soil type is selected, which has the advantage of preserving hydraulic properties when moving across various model resolutions ([Balsamo \*et al.\*, 2009](#)). The climate field used by the model has an index from 1 to 7 corresponding to the soil textures (see [Fig. 11.30](#)): 'coarse' (1), 'medium' (2), 'medium fine' (3), 'fine' (4), 'very fine' (5), 'organic' (6), and 'tropical organic' (7).

## 11.10 SEA SURFACE TEMPERATURE AND SEA ICE COVER

Sea surface temperature and sea ice climatology are used by the model to apply persistent SST anomalies to the ocean surface boundary condition. It is a daily climatology based on 40 years of ERA40 data. The INTERPOL utility is applied to interpolate from ERA40 to target resolution. INTERPOL also fills in the



**Figure 11.29** CO<sub>2</sub>, CH<sub>4</sub> and O<sub>3</sub> zonal mean climatology derived from the 2003-2011 MACC reanalysis. All the concentrations are in ppmv.



**Figure 11.30** Soil type classes as used in H-TESEL.

missing data for ocean points that do exist at target resolution and did not exist at ERA40 resolution. The same procedure is used for sea ice cover.

## 11.11 LAKE DATA

The lake model is supported by two climatological fields: (i) Lake mask, (ii) Lake depth.

A lake mask consistent with the land-sea mask has been produced using a flood-filling algorithm for all operational resolutions and grid types. The idea is to start from a seed somewhere in the sea of the land-sea mask, and let the flood-filling algorithm march through all sea points which are connected and mark them. What is left in the end as wet points when the ocean is filled up, are the lakes that are not connected to the ocean. Issues in this procedure are that some lakes are very close to the sea, and especially for low resolutions, the flood-filling algorithm just fills them up as well. Therefore, a number of lakes located close to the sea had to be blocked based on their coordinates in the flood-filling algorithm. Another issue was that some of the narrow fjords in Norway and Greenland as well as other regions were not completely filled up with sea points by the flood-filling algorithm (leaving the fjords as lakes that can freeze). This problem becomes more apparent the higher the resolution. A solution here was to use higher land-sea threshold than 0.5, e.g. 0.6, 0.7, 0.8, .., to distinguish sea from land in the filling algorithm, which then fills up the narrow fjords as well. A latitude-dependent land-sea threshold was finally used, with lower values in mid- and low-latitudes, but the higher values at high latitudes.

Lake depth is specified according to [Kourzeneva \(2010\)](#). To obtain a global bathymetry this data is combined with ETOPO1 as described in [Balsamo \*et al.\* \(2012\)](#). ETOPO1 is a 1 arc-minute global relief model of Earth's surface that integrates land topography and ocean bathymetry (see <http://www.ngdc.noaa.gov/mgg/global/global.html>).

## 11.12 GLACIER MASK

The glacier mask is derived from GLCC2.0 in the same way as the vegetation data is derived from GLCC1.2. Over Iceland the GLCC data set is patched with the 1 km glacier mask from the Icelandic

Meteorological Office (IMO). The use of the IMO glacier mask in the GLCC data set is documented in de Rosnay et al. (Research Memo. RD13-293, 2013).

## Chapter 12

# Basic physical constants and thermodynamic functions

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- [12.8 Mass and enthalpy conservation](#)

### 12.1 TIME

$$\begin{aligned} \text{length of the day } d &= 86400 \text{ s} \\ \text{length of the year } y_j &= 365.25 d \end{aligned}$$

Taking into account the average variation in the Sun–Earth distance, the sidereal year and sidereal day are defined as

$$\begin{aligned} \text{sidereal year } y_s &= y_j \frac{2\pi}{6,283076} \\ \text{sidereal day } d_s &= \frac{d}{1+d/d_s} \end{aligned}$$

and therefore defining the Earth's rotation rate as

$$\Omega = \frac{2\pi}{d_s}$$

### 12.2 GEOID

The Earth is supposed spherical with radius  $a$  (average value of the reference ellipsoid)

$$a = 6371229 \text{ m}$$

The gravitational acceleration  $g$  is

$$g = 9.80665 \text{ m s}^{-2}$$

from where the geopotential height  $\Phi$  is derived as  $\Phi = gz$  ( $\text{J kg}^{-1}$ ).

### 12.3 FUNDAMENTAL CONSTANTS

speed of light in vacuum	$c$	$= 299792458 \text{ m s}^{-1}$
Planck constant	$h$	$= 6.6260755 \cdot 10^{-34} \text{ J s}$
Boltzmann constant	$k$	$= 1.380658 \cdot 10^{-23} \text{ J K}^{-1}$
Avogadro number	$N$	$= 6.0221367 \cdot 10^{23} \text{ mol}^{-1}$

### 12.4 RADIATION

Stefan-Boltzmann constant	$\sigma$	$= \frac{2\pi^5 k^4}{15c^2 h^3} = 5.6705085 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
solar constant	$S_0$	$= 1370 \text{ W m}^{-2}$

### 12.5 THERMODYNAMICS, GAS CONSTANTS AND SPECIFIC AND LATENT HEATS

universal gas constant	$R$	$= Nk = 8.31451 \text{ J K}^{-1} \text{ mol}^{-1}$
dry air molar mass	$M_d$	$= 28.9644 \cdot 10^{-3} \text{ kg mol}^{-1}$
water vapor molar mass	$M_v$	$= 18.0153 \cdot 10^{-3} \text{ kg mol}^{-1}$

leading to the definition of the gas constants for dry air and water vapor

$$R_d = \frac{R}{M_d} = 287.0597 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$R_v = \frac{R}{M_v} = 461.5250 \text{ J kg}^{-1} \text{ K}^{-1}$$

The specific heats at constant pressure (first subscript p) and constant volume (first subscript v) for dry air and water vapor are

$$c_{pd} = \frac{7}{2} R_d = 1004.7090 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$c_{vd} = \frac{5}{2} R_d = 717.6493 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$c_{pv} = 4 R_v = 1846.1 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$c_{vv} = 3 R_v = 1384.575 \text{ J kg}^{-1} \text{ K}^{-1}$$

and the specific heats for water in its liquid and solid phase

$$c_l = 4.218 \cdot 10^{-3} \text{ J kg}^{-1} \text{ K}^{-1}$$

$$c_s = 2.106 \cdot 10^{-3} \text{ J kg}^{-1} \text{ K}^{-1}$$

$c_l$  and  $c_s$  are supposed constant, with the above values actually valid for the triple point temperature

$$\text{triple point temperature } T_t = 273.16 \text{ K}$$

The functions for the specific heats of vaporisation and sublimation are

$$L_v = L_v(T_t) + (c_{pv} - c_l)(T - T_t) \quad (12.1)$$

$$L_v = L_s(T_t) + (c_{pv} - c_s)(T - T_t) \quad (12.2)$$

However, the approximation chosen in the model physics is that of invariable specific latent heats together with invariable (independent of the water contribution) specific heat at constant pressure

$$L_v \equiv L_v(T_t) = L_{v0} = 2.5008 \cdot 10^6 \text{ J kg}^{-1}$$

$$L_s \equiv L_s(T_t) = L_{s0} = 2.8345 \cdot 10^6 \text{ J kg}^{-1}$$

$$c_p \equiv c_{pd} = 1004.7090 \text{ J kg}^{-1} \text{ K}^{-1}$$

as will be discussed further in the context of enthalpy conservation and mixed phase saturation adjustment.

## 12.6 THERMODYNAMICS, BASIC STATE VARIABLES

The total atmospheric pressure  $p$ , specific humidity  $q_v$ , specific masses of water in its liquid and ice phase  $q_{l,i,r,s}$ , the air density  $\rho$  and virtual temperature  $T_v$  are related through

$$p = p_d + e \quad (12.3)$$

$$q_v = \frac{\rho_v}{\rho} = \frac{\rho_v}{(\rho_d + \rho_v)} = \frac{e}{p + \epsilon^*(p - e)}; \quad \epsilon^* = \frac{R_v}{R_d} - 1 \quad (12.4)$$

$$\rho = \frac{p}{R_d T_v} \quad (12.5)$$

$$T_v = T(1 + \epsilon^* q_v - q_l - q_i - q_r - q_s) \quad (12.6)$$

where the usual notation  $p_v = e$  for the water vapor pressure has been used. In the derivation of  $T_v$  the specific volume of the condensed water phases has been neglected. Nota, the complete expression of density via  $T_v$  in (12.1-12.2) taking into account all prognostic microphysical species, is used in the dynamical prognostic equations for vector wind and  $T$ , and in the model physics when buoyancy perturbations are implied (e.g updraught kinetic energy computation in the convection). However, when air density is required in the physics it is computed using  $T_v = (1 + \epsilon^* q_v)$  only.

## 12.7 THERMODYNAMICS, SATURATION FUNCTIONS

The expressions for the water vapour saturation pressure over water and ice are derived from the Clausius-Clapeyron equation

$$\frac{d \ln e_{\text{sat}}}{dT} = \frac{L_v}{R_v T^2} \quad (12.7)$$

where the specific volume of water has been neglected. Instead of integrating 12.7 between the triple point temperature  $T_t$  and  $T$ , an empirical form is used with separate functions for saturation over water and over ice

$$e_{\text{sat},l} = e_0 \exp [17.502 (T - T_t)/(T - 32.19)] \quad \text{water} \quad (12.8)$$

$$e_{\text{sat},i} = e_0 \exp [22.587 (T - T_t)/(T + 0.7)] \quad \text{ice} \quad (12.9)$$

$$e_0 = e_{\text{sat}}(T_t) = 611.21 \quad \text{Pa} \quad (12.10)$$

Mixed-phase thermodynamics, ie when water is supposed to be present in its three forms, is represented through an interpolation as a function of temperature of the saturation functions and the latent heats between their values with respect to water and ice

$$\alpha = \left( \frac{T - T_t + \Delta T_i}{\Delta T_i} \right)^2; \quad T_t \geq T \geq T_t - \Delta T_i; \quad \Delta T_i = 23 \quad \text{K} \quad (12.11)$$

$$L = \alpha L_{v0} + (1 - \alpha) L_{s0} \quad (12.12)$$

$$e_{\text{sat}} = \alpha e_{\text{sat},l} + (1 - \alpha) e_{\text{sat},i} \quad (12.13)$$

$$q_{\text{sat}} = \frac{e_{\text{sat}}}{p + \epsilon^*(p - e_{\text{sat}})} \quad (12.14)$$

$$RH = \frac{e}{e_{\text{sat}}} \approx \frac{q_v}{q_{\text{sat}}} \quad (12.15)$$

where we have also introduced the definition of the relative humidity  $RH$ . Note that the mixed phase is supposed to occur in the temperature range  $T_t \geq T \geq T_t - \Delta T_i$  which in many cases might not be consistent with the mixed phase representation in the prognostic microphysics.

In case of supersaturation or evaporation of rain, the saturation adjustment process can then be formulated from the conservation of enthalpy

$$c_{pd}dT = L (q_v - q_{sat}(T)) \quad (12.16)$$

$$c_{pd}(T^* - T) = L \left[ q_v - \left\{ q_{sat}(T) + \frac{dq_{sat}}{dT} \Big|_T (T^* - T) + o(2) \right\} \right] \quad (12.17)$$

$$T^* = T + \frac{L}{c_{pd}} \frac{q_v - q_{sat}(T)}{1 + \frac{L}{c_p} \frac{dq_{sat}}{dT} \Big|_T} \quad (12.18)$$

$$q_v^* = q_{sat}(T^*) \quad (12.19)$$

$$q_{l,i} = q_v - q_v^* \quad (12.20)$$

$$q_l = \alpha q_{l,i}; \quad q_i = (1 - \alpha)q_{l,i} \quad (12.21)$$

where a linearisation around the initial temperature  $T$  has been applied and superscript  $*$  denotes the adjusted values. Only one iteration of the above equation set is necessary to achieve sufficient accuracy of  $O(10^{-3})$  K.

## 12.8 MASS AND ENTHALPY CONSERVATION

A conserved energy potential can be derived from the change in enthalpy of an adiabatic (better isentropic) system comprising dry air and all water species

$$\begin{aligned} (m_d c_{pd} + m_v c_{pv} + (m_l + m_r) c_l + (m_i + m_s) c_s) dT &= -L_v dm_v + (L_s - L_v) d(m_i + m_s) + V dp \\ (m_d c_{pd} + m_v c_{pv} + (m_l + m_r) c_l + (m_i + m_s) c_s) dT &= -(L_v m_v) + d((L_s - L_v)(m_i + m_s)) \\ &\quad + m_v \frac{dL_v}{dT} dT - (m_i + m_s) \frac{d(L_s - L_v)}{dT} dT + V dp \end{aligned} \quad (12.22)$$

where  $m_{d,v,r,i,s}$  are the masses of dry air and water species. Using (12.1-12.2) for the temperature dependency of the latent heats, applying the hydrostatic approximation to the last term of the rhs, dividing by the total mass and integrating one obtains

$$((1 - q_t) c_{pd} + q_t c_l) T = -L_v q_v + (L_s - L_v)(q_i + q_s) - \Phi; \quad q_t = q_v + q_l + q_i + q_s \quad (12.23)$$

The quantity therefore conserved is the moist static energy  $H$

$$H = ((1 - q_t) c_{pd} + q_t c_l) T + L_v q_v - (L_s - L_v)(q_i + q_s) + \Phi \quad (12.24)$$

The quantity conserved in the IFS is an approximation of (12.24) under the assumption of invariant  $c_p$  and latent heat that is interpolated between invariant values of  $L_v$  and  $L_s$  (12.12)

$$H = c_{pd} T + L q_v + \Phi \quad (12.25)$$

Under this approximation the conservation of the following column integrals for water mass, enthalpy (static energy) and momentum is substantially facilitated and can be made exact:

Conservation of water

$$\frac{1}{g} \int_{p_{surf}}^0 \left( \frac{dq_v}{dt} + \frac{dq_l}{dt} + \frac{dq_i}{dt} + \frac{dq_r}{dt} + \frac{dq_s}{dt} \right) dp_h = \frac{1}{g} \int_{p_{surf}}^0 \frac{dq_t}{dt} dp_h = F_{pr} - F_q \quad (12.26)$$

where tendencies are at full model levels and  $p_h$  denotes half-level pressure. Therefore, the change in total moisture is equal to the sum of surface precipitation and moisture flux ( $\text{kg water m}^{-2} \text{s}^{-1}$ ); note that the heat (negative upward) and precipitation (positive downward) fluxes have opposite signs by convention.

Conservation of enthalpy (moist static energy)

$$\frac{c_{pd}}{g} \int_{p_{surf}}^0 \frac{dT}{dt} dp_h + \frac{1}{g} \left( \int_{p_{surf}}^0 L \frac{dq_v}{dt} dp_h + (L_{s0} - L_{v0}) \int_{p_{surf}}^0 \frac{(dq_i + dq_s)_{|melt}}{dt} dp_h \right) = -F_s - L_{v0} F_q \quad (12.27)$$

where  $F_s$  and  $L_{v0} F_q$  are the surface sensible and latent heat fluxes and where subscript melt includes melting and freezing processes. Alternatively, one can formulate the enthalpy conservation as

$$\frac{c_{pd}}{g} \int_{p_{surf}}^0 \frac{dT}{dt} dp_h - \frac{1}{g} \int_{p_{surf}}^0 \left( L_{v0} \frac{dq_l}{dt} + L_{s0} \frac{dq_i}{dt} \right) dp_h + \frac{L_{v0}}{g} \int_{p_{surf}}^0 \frac{dq_r}{dt} dp_h + \frac{L_{s0}}{g} \int_{p_{surf}}^0 \frac{dq_s}{dt} dp_h = -F_s + F_{pr} \quad (12.28)$$

Finally for the conservation of zonal and meridional momentum

$$\frac{1}{g} \int_{p_{surf}}^0 \left( \frac{du}{dt} + \frac{dv}{dt} \right) dp_h = -F_{u,v} \quad (12.29)$$

where  $F_{u,v}$  is the surface stress.



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