Radiation:
Fast physics with slow consequences in an uncertain atmosphere

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ABSTRACT

This article uses radiation parameterization as a lens through which to view some general issues related to the representation of model error and uncertainty. I describe the kinds of radiation calculations required by forecasting models and consider the error sources and budgets for clear and cloudy skies. This demonstrates that, with respect to radiation, process understanding is high, benchmarks unambiguous, and uncertainty due almost entirely to limited knowledge available to the radiative transfer parameterization. I identify thee areas that may be ripe for representing uncertainty though the perturbations that can be expected are small. Still, some relevant lessons for stochastic parameterization and model uncertainty have arisen by accident from experience with two stochastic radiation parameterizations developed to reduce model error, not to represent uncertainty. Most importantly, experience has shown that perturbations introduced at the smallest temporal and spatial scales do not affect model evolution or spread/skill relationships in ensembles. These lessons have implications for the development of physically-based estimates of uncertainty.

1 Radiation as a problem in parameterization

1.1 Context

Electromagnetic radiation is the fundamental source of energy for all atmospheric motions.The equation describing the transfer of monochromatic (at a single frequency or wavelength) radiation through the atmosphere is fundamental, in that it can be derived from Maxwell's equations (Mishchenko, 2008), and is remarkably simple and unambiguous. The equation can be solved given boundary conditions and the spatial distribution of optical properties (extinction σ, phase function P, and single scattering parameter ω_0) and temperature (for computing blackbody emission). This distribution need only be known down to resolutions of a few hundred meters in cloudy skies and kilometer scales in clear skies since radiation smooths over variations smaller than this.

Weather forecasting models have fairly specific needs from a radiation transfer parameterization: heating rates in the the interior of the atmosphere and surface fluxes to compute the surface temperature evolution - that is, profiles of broadband fluxes. *Broadband* means that fluxes are integrated over the entire electromagnetic spectrum and *fluxes* (sometimes called irradiances) that only hemispheric averages of the radiation field are necessary. For the purposes of this article I'll assert that the present state-of-the-art for this problem is to use correlated-k distributions[1] (Lacis and Oinas, 1991; Fu, 1992) to do the

[1] Gas optical properties vary by orders of magnitude over very narrow wavelength intervals; k-distributions re-order the integration so it is smoother and requires many fewer quadrature points.

spectral integration and some two-stream approximation (e.g. Meador and Weaver, 1980) to the one-dimensional radiative transfer equation to compute fluxes. Radiation is often treated as two distinct calculations, one for the shortwave (wavelength λ less than about 3.7 μm, dominated by the multiple scattering of sunlight) and one for the longwave (λ longer than about 3.7 μm, dominated by emission and absorption of radiation from the earth and atmosphere). The approximation of the 3D radiative transfer equation by a 1D version is the first significant approximation; a second common simplification is to ignore scattering in the longwave calculation.

Radiation is unique among physical processes because it crucial to the long-term evolution of the atmosphere but normally pretty unimportant to the short-term behavior. Radiative heating/cooling rates are, in most circumstances, quite small relative to other terms. Radiation affects temperatures (and hence the atmospheric flow) when it acts steadily over a long time: think, for example, of the slow clear-sky cooling that drives subsidence in the subtropics, or the cloud-top radiative cooling in the boundary-layer clouds at the base of this subsidence. One practical consequence is that radiative parameterizations are frequently invoked less frequently than other physical parameterizations in atmospheric models.

The representation of radiation in dynamical models, then, is a relatively pure exercise in trading computational cost for accuracy. This is very different from the parameterization of, say, convection, which relies on far more abstract and empirical theories, and means that model error due to the radiation parameterization are primarily associated with limitations in the problem description (i.e. the inputs) rather then any uncertainty about the response of the system to a given set of conditions. Nonetheless, experience with radiation in atmospheric models offers some useful insight on the larger question of how to represent model error and uncertainty.

1.2 Clear-sky error budgets

Clear skies are not transparent; radiation still interacts with both gases and aerosol particles. But clear skies are optically homogeneous in the horizontal, both because the concentration of aerosols and gases varies slowly with location and because the "radiative smoothing scale" (Marshak et al., 1995) is large because the extinction is small. This means that the most fundamental assumptions made by radiation parameterizations – one-dimensional radiative transfer in a homogeneous medium – are good approximations in clear skies. Errors in radiative fluxes might arise from

- Knowledge of the underlying spectroscopy: how absorption by gases depends on gas amounts, temperature, pressure, etc.
- Angular discretization: the computation of fluxes using a small number (typically two) of quadrature points in polar angle.
- Spectral discretization: any errors introduced by approximating a line-by-line solution with a correlated-*k* sum.
- Other approximations including the neglect of scattering in the longwave .

The US Department of Energy's Atmospheric Radiation Measurement Program has made efforts to assess the size of these errors using "radiative closure" studies in which carefully-calibrated observations of surface radiation are compared calculations made using a carefully-observed atmosphere (see `http://www.arm.gov/data/eval/24`). Eli Mlawer is involved in this effort; he suggests that uncertainty due to spectroscopy causes a flux uncertainty of about 1 W/m^2, spectral discretization in well-tuned parameterizations about 1.5 W/m^2. Errors in the comparison with observations, in fact, are dominated by uncertainty in the characterization of the temperature and humidity structure of the atmosphere and the ability to measure broadband fluxes (Mlawer, personal communication, 2011). These errors are tiny in the context of global-mean surface radiation fluxes of 240 W/m^2.

1.3 Cloudy-sky error budgets

Error budgets for radiation parameterizations in cloudy skies are dominated by a single factor: the variability of cloud properties at scales below the filter scale of the model. This variability has two sources. One, usually called "cloud overlap," arises because a profile of cloud fraction and in-cloud optical properties implies a distribution of cloud configurations (i.e. combinations of cloudy and clear layers) within a model column. The number of combinations can, in principle, be quite large (n partially-cloud layers implies 2^n possible configurations) but is also influenced by "overlap assumptions" prescribing the correlation structure among layers.

Clouds in nature also exhibit variability in integrated properties relevant to radiation calculations (optical thickness, liquid water path) across a wide range of scales (Cahalan and Snider, 1989). This variability is almost uniformly neglected by global models. Methods to account for this variability have been proposed (see Sec. 3.1) but the more common path is to use the variability to justify the "tuning" of cloud physical properties in the computation of optical properties (see Sec. 2.1).

Calculations using cloud structures produced by fine-scale models (Barker et al., 2003) suggest that horizontal and vertical structure have comparable impacts on the radiative fluxes within domains about the size of global models.

2 Stochastic radiation algorithms in dynamical models

One approach to dealing with model error is to acknowledge that the tendencies produced by the physical parameterizations are themselves uncertain and to perturb them in some systematic fashion, either by perturbing the total tendency or by targeting specific processes. In this context it's useful to look at two stochastic radiation parameterizations even though those parameterizations were developed to reduce model error rather than represent it explicitly.

2.1 Sampling subgrid-scale variability

As I pointed out in Section 1.3 error budgets for radiation calculations in cloudy skies are dominated by the treatment of sub-grid-scale variability in cloud properties. A decade ago there were essentially three ways to treat horizontal variability:

- Analytic closure in which some particular variant of the two-stream approximation is integrated over some particular distributions of optical thickness[2] τ (Barker, 1996). This can be extended to treat multiple layers (Oreopoulos and Barker, 1999) but is inflexible.
- Rescaling of the optical properties used in the radiative transfer equation based on a description of the inhomogeneity of the medium (Cairns et al., 2000; Petty, 2002). These methods are not generally applicable to the kinds of variability present in the atmosphere.
- "Tuning" i.e. the ad-hoc reduction of optical thickness by some (normally fixed) factor, sometimes but not always justified on physical grounds (e.g Cahalan et al., 1994; Tiedtke, 1996). This has no physical basis. Nonetheless, most models today still treat this factor as a free tuning parameter.

and two ways to treat "cloud overlap"

[2] Optical thickness τ is the vertical integral of extinction σ; in clouds τ varies slowly with wavelength (which is why clouds are white).

- Analytically, by computing clear- and cloudy-sky fluxes and combining them in some way. A variety of such methods existed; all of them were incorrect when compared to benchmark simulations (Barker et al., 2003).
- Enumeration: performing radiative transfer calculations on each possible configuration within each grid cell and averaging the results (suggested by Morcrette and Fouquart (1986) and implemented by Stubenrauch et al. (1997) and Collins (2001).) This benchmark calculation is accurate but very expensive, since the number of possible configurations increases rapidly with the number of model layers.

The first stochastic radiative transfer parameterization in widespread use was aimed at finding a practical, uniform way to represent variability in cloud optical properties arising from vertical and/or horizontal variability. The Monte Carlo Independent Column Approximation (McICA, see Pincus et al., 2003) adopted the idea of sampling randomly from the distribution of possible configurations originally developed for diagnostic studies (Klein and Jakob, 1999). The domain-averaged broadband flux $\overline{F}$ for a single column with uniform optical properties is a sum over G spectral quadrature points g:

$$\overline{F}(x,y,t) = \sum_g^G w_g F_g(x,y,t). \tag{1}$$

If sub-grid-scale variability is represented with a set of S randomly-chosen samples, the domain-mean flux is the linear average of the flux computed independently in each sample:

$$\overline{F}(x,y,T) = \sum_s^S \sum_g^G w_g F_g(x,y,T). \tag{2}$$

This calculation is quite expensive because $G \sim 100$ so that even a small value of S imply thousands of individual radiation computations. McICA subverts this problem by setting $S = G$ and randomly associated each sample of the configuration space with a different quadrature point in the k-distribution,

$$\overline{F}(x,y,T) \approx \sum_g^G w_g F_g(s'_g;x,y,T) \tag{3}$$

with this association chosen randomly at each point in time and space.

McICA is stochastic because of the random association between g and s so that any given realization of Eq. 3 contains (unbiased) noise relative to Eq. 2 (though even the latter is not guaranteed to sample the distribution of possible cloud states). This noise depends on the number of sample points and on how complicated a distribution of states is implied by a model's climatology of cloud macrophysical properties but is normally modest: tests in one model (Pincus et al., 2006), for example, put the noise in surface fluxes at $O(10\ \mathrm{W/m^2})$ and noise in heating rates at a few percent for individual calculations. Noise of this magnitude does not affect the evolution of global models (Barker et al., 2008).

2.2 Sampling spectra in time

This noise introduced by McICA is limited because Eq. 3 samples the entire spectrum but experience with that algorithm suggested that dynamical models might be resilient in the face of grid-scale noise. This success inspired a more radical approach. In a cloud-scale model in which the cells are small enough that sub-grid-scale variability can be neglected, the ideal calculation is the one that captures the temporal evolution at every location in the domain, i.e. for every value of x,y and t. Recall from Sec. 1.1, though, that radiation is so computationally expensive that it is normally computed less frequently than most other physical processes in models of the atmosphere, i.e it is computed at discrete times T.

Choosing too large a radiation time step T/t can excite numerical instabilities (Pauluis and Emanuel, 2004), while choosing too small a value risks making the calculation needlessly expensive. Monte Carlo Spectral Integration (MCSI, see Pincus and Stevens, 2009) trades the spectrally dense, temporally sparse calculation for one which is dense in time but spectrally sparse:

$$\overline{F}(x,y,t) \approx \frac{G}{\tilde{G}} \sum_{g}^{\tilde{G}} w_g F_g(x,y,t) \tag{4}$$

with $\tilde{G} < G$ and the set of quadrature points used is chosen independently at each location and time step.

MCSI was originally implemented in large-eddy simulations of turbulent marine boundary layers in which even an approximation as drastic as $\tilde{G} = 1$ does not affect model evolution. Analytic insight from this simple system (Pincus and Stevens, 2009) explains this result: the noise introduced by MCSI depends on spatial and temporal scale. The noise is large at the smallest scales (where it diffuses away quickly) but small at resolved scales relative to the energy from other sources.

2.3 Two lessons from stochastic radiation parameterizations

McICA (Sec. 2.1) and MCSI (Sec. 2.2) are conceptually similar: they are approximate algorithms aimed at replacing well-posed but computationally prohibitive calculations with affordable approximate calculations that introduce random but unbiased noise in any given realization. They are designed to *reduce* model error by providing approximate solutions to the full problem (i.e. broadband calculations fully resolved in time that completely sample internal variability) rather than exact solutions to some approximate problem. This is fundamentally different from stochastic algorithms intended to *represent* model error (e.g Buizza et al., 1999), parameterization uncertainty (e.g. Tompkins and Berner, 2008; Teixeira and Reynolds, 2008), or uncertainty due to the discrete nature of a physical process (Plant and Craig, 2008; Eckermann, 2011). This distinction reflects the fact that radiation is very well understood, and that uncertainty in radiative fluxes is almost entirely due to uncertainty about the optical properties of the atmosphere used in the calculation.

McCIA and MCSI "work" in the sense that neither approximation affects model forecasts systematically (though this is not universally true for MCSI). In other words, *unbiased random noise introduced at the grid scale has no effect on the distribution of model forecasts.* This implies that ensembles using stochastic algorithms in which the noise is applied at the grid-scale will not be any broader than unperturbed ensembles. With one counter-example (Teixeira and Reynolds, 2008) I believe this has reflects the community's experience with stochastic scheme. It also explains why perturbations to physical tendencies (e.g Buizza et al., 1999) or to the circulation (Berner et al., 2009) must be correlated in space and time in order to broaden ensembles, even if there is little or no theoretical justification for those correlations. This point seems relevant to any stochastic treatment of model error or uncertainty.

3 Opportunities

I've argued so far the there radiation is such a well-understood process that there is very little room to represent error or uncertainty in this aspect of atmospheric models. In fairness, that's almost strictly true only once the problem is fully specified in terms of the distribution of optical properties, and one can argue that developing this specification from the model state is also part of the radiation parameterization process. In this section I'll briefly describe three places where there may be room for representing error or uncertainty in radiation calculations in global models: two outstanding issues with developing the problem specification and one bedrock issue regarding the problem we choose to solve.

3.1 Sub-grid-scale variability

Radiative fluxes (and so heating rates) depend non-linearly on the atmosphere's optical thickness τ. In clear skies τ varies slowly with location but clouds exhibit significant spatial variability across scales. The combination of variability and non-linearity means that the mean albedo of a domain containing clouds will always be less than the albedo implied by the domain-mean optical thickness (Cahalan et al., 1994). (This problem is more relevant in the shortwave than in the longwave since most clouds in the atmosphere are opaque in the infrared.) The amount of variability depends on the size of the domain; for domains of a few hundred km (the nominal grid size of climate models a decade ago) the bias can be several percent (Pincus et al., 1999; Oreopoulos and Davies, 1998). This explains why almost every global model "tunes" the optical thickness of clouds by reducing the liquid water content used in computing τ by some arbitrary factor (see Sec. 2.1).

Unresolved spatial variability affects other nonlinear processes, and particularly the formation of precipitation, in a similar way (Pincus and Klein, 2000; Rotstayn, 2000; Larson et al., 2001). This has sparked some interest in the use of cloud schemes that explicitly predict the probability distribution function of cloud properties within each grid cell. Assumed-PDF schemes (so called because the distribution family is normally assumed and the parameters of the distribution predicted) have been used to represent variability in boundary-layer cloudiness for several decades (Mellor, 1977; Sommeria and Deardorff, 1977). Several general versions of these schemes have been proposed for use in global models (Ricard and Royer, 1993; Tompkins, 2002; Golaz et al., 2002) but this idea has never really taken off: I'm not aware of any global model that routinely uses an assumed-PDF scheme. That's a bit of a shame, since even diagnosing a PDF of cloud condensate from (independently-predicted) cloud fraction and cloud water content and treating this variability in radiation calculations can eliminate the need for tuning (Pincus et al., 2006). I expect that there's still some low-hanging fruit down this line of thinking.

3.2 Ice optics

One-dimensional radiative transfer calculations require profiles of the atmosphere's optical properties (σ, P, and ω_0; see Sec. 1.1). These optical properties must be computed from the atmosphere's physical properties, i.e. temperature, liquid and ice water contents, aerosol loading, etc. The conversion is more-or-less straightforward with one dramatic exception: ice clouds. Cloud ice is problematic because the single scattering properties of particles depend on the particle's habit (shape) and density, and the properties of the medium must integrate over all particles. (Cloud drops are easier because they are round. Aerosol optical properties also depend on shape and chemical composition but their size limits their impact on the overall radiation budget.) Developers of radiation parameterizations are still struggling with finding appropriate geometric measures of particle habit with which to predict ice optical properties (e.g. Fu, 2007). Even the available observations can be ambiguous: the cloud probes used to obtain images of ice crystal habits do not resolve the smallest particles, and uncertainty in the shapes of these particles can lead to uncertainty in asymmetry parameter g (the first moment of P, and the one relevant for flux calculations) of $\sim 20\%$ (Um and McFarquhar, 2011); this corresponds to uncertainty in albedo of $\sim 5\%$.

Global models are a long way from predicting these details: current state-of-the-art microphysics schemes (e.g. Morrison and Gettelman, 2008) predict only the bulk properties (total mass and total number) of the ice distribution. Habit and density information are entirely missing. Thus it wouldn't be unreasonable to use observations and off-line radiative transfer calculations to estimate distributions of ice cloud optical properties based on model predictors. It may also be useful to introduce some local memory, since particle habits systematically as the ice ages.

3.3 Three-dimensional radiative transfer effects

The very first simplification made in radiation parameterizations for atmospheric models is the replacement of the three-dimensional radiative transfer equation with a one-dimensional version that allows for structure only in the vertical. Differences between these approximations are most important in the shortwave where multiple scattering can act, and in cloudy skies in which significant variability exists. Three-dimensional radiative transfer exhibits richer behavior than the 1D analog, even in overcast skies, including "smoothing" (the net transport of radiation from dense to tenuous portions of the medium, see Marshak et al., 1995) when the sun is high and "roughening" caused by the shadowing from variable cloud tops when the sun is low (Welch and Wielicki, 1984). In broken clouds the illumination of cloud sides increases reflection when the sun is low (Pincus et al., 2005).

This physics is clearly missing from radiation parameterizations in global models; what is less clear is how important this omission is. Computing three-dimensional radiative transfer is enormously expensive and the few experiments that have coupled cloud-scale models to three-dimensional radiative transfer solvers have either seen no effect on cloud evolution (Mechem et al., 2008) or have failed to demonstrate that small observed effects are statistically significant (Cole et al., 2005). To me this seems like evidence that the local heating rate anomalies produced by the one-dimensional approximation are neither large enough nor persist for long enough to affect the flow (Pincus and Stevens, 2009, c.f. Sec. 2.2). There is evidence that very simple treatments, primarily first-order corrections for the shadowing of the direct solar beam (Várnai and Davies, 1999), may have some influence on near-surface temperatures (Wapler and Mayer, 2008; Frame et al., 2009) and it might be possible to represent these effects at coarser scales.

But even if three-dimensional radiative transfer effects turn out to be important in some set of circumstances it is not at all clear how to include them in global models. These effects depend on the two-point statistics of the cloud field – how the clouds are arranged (correlated) in space – at the sub-grid-scale. Global models do not typically produce even one-point statistics at this scale (Sec. 3.1). In this context, the only way to treat 3D effects is to develop *ad hoc* estimates of cloud structure and use this structure to modify the 1D radiation calculations in some approximate way. Though the one-dimensional approximation certainly introduces error and uncertainty into radiation calculations, it's hard to see the advantage of inventing spatial structure just so it can be used to make a small perturbation to the radiation calculation.

4 Radiation and model uncertainty

One of the themes of this workshop was the quest to establish a more physical basis for treatments of model error and uncertainty than inflating parameterization tendencies or spinning the circulation up. But the talks we saw made clear that no one solution applies to every process or parameterization. Radiation is at one extreme in that our understanding of this process is deep, the errors in a well-posed problem are small, and parameterization accuracy can be objectively assessed (see, as one example, Collins et al., 2006). This suggests that that we should seek to represent uncertainty and error in the problem inputs (e.g. Sec. 3.2) rather than in the process itself. One specific implication is that varying the radiative transfer parameterization in "multi-physics" ensembles (e.g. Berner et al., 2010) is poorly founded, and certainly does not represent uncertainty.

Experience with two stochastic radiation algorithms (Sec. 2) is consistent with other experiences in introducing stochastic elements to parameterization tendencies: any reasonable amount of fully random (i.e. uncorrelated in space and time) noise does not project onto the flow. In particular, grid-scale noise neither changes the mean model trajectory nor the variance of an ensemble. The practical implication is that stochastic perturbations are only effective at improving spread/skill relationships within ensembles if they are applied with spatial and temporal patterns (Buizza et al., 1999; Berner et al., 2009). Reconciling this requirement with efforts to assign a more physical basis to parameterization uncertainty will require moving away from a column-by-column view of parameterization. This arises naturally for processes in which parameterization statistics apply over large areas and can be sampled at smaller scales (Plant and Craig, 2008) but will require deeper thinking in other circumstances.

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