Verification of Probability Forecasts

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Topics

- Verification philosophy for probability forecasts
- Measuring bias
 - Reliability diagram
- Measuring total error
 - Brier score
 - Sources of error reliability, resolution, uncertainty
- Measuring potential skill
 - Relative Operating Characteristic (ROC)

- if time...

- Measuring accuracy
 - Ranked probability score
- Measuring value
 - Relative value diagram



If the forecast was for 80% chance of rain and it rained was this a good forecast?

☐ Yes☐ No☐ Don't know



If the forecast was for 10% chance of rain and it rained was this a good forecast?





Would you dare to make a prediction of 100% probability of a tornado?





Measuring quality of probability forecasts

- An individual probabilistic forecast is neither completely correct or completely incorrect* * unless it is exactly 0% or exactly 100%
 - Need to look at a large number of forecasts and observations to evaluate:
 - Reliability can I trust the probabilities to mean what they say they mean?
 - Discrimination how well do the forecasts distinguish between events and non-events?
 - **Skill** are the forecasts better than chance or climatology?

Reliability – are the forecasts unbiased?

- Measure agreement between predicted probabilities and observed frequencies
- If the forecast system is *reliable*, then whenever the forecast probability of an event occurring is *P*, that event should occur a fraction *P* of the time.
 - For each probability category plot the frequency of observed occurrence



Interpretation of reliability diagrams



- The reliability diagram is conditioned on the forecasts (i.e., given that X was predicted, what was the outcome?)
- Gives information on the real meaning of the forecast.

Tampere (Finland) POP data

| Date 2003 | Observed rain | 24h forecast POP | 48h forecast POP |
|-----------|---------------|------------------|------------------|
| Jan 1 | no | 0.3 | 0.1 |
| Jan 2 | no | 0.1 | 0.1 |
| Jan 3 | no | 0.1 | 0.2 |
| Jan 4 | no | 0.2 | 0.2 |
| Jan 5 | no | 0.2 | 0.2 |
| | | | |
| Dec 27 | yes | 0.8 | 0.8 |
| Dec 28 | yes | 1.0 | 0.5 |
| Dec 29 | yes | 0.9 | 0.9 |
| Dec 30 | no | 0.1 | 0.3 |
| Dec 31 | no | 0.1 | 0.1 |

Tampere (Finland) 24h POP summary

| Forecast probability | # fcsts | # observed occurrences | Obs. relative frequency |
|----------------------|------------|---------------------------|----------------------------|
| 0.0 | 46 | 1 | 0.02 |
| 0.1 | 55 | 1 | 0.02 |
| 0.2 | 59 | 5 | 0.08 |
| 0.3 | 41 | 5 | 0.12 |
| 0.4 | 19 | 4 | 0.21 |
| 0.5 | 22 | 8 | 0.36 |
| 0.6 | 22 | 6 | 0.27 |
| 0.7 | 34 | 16 | 0.47 |
| 0.8 | 24 | 16 | 0.67 |
| 0.9 | 11 | 8 | 0.73 |
| 1.0 | 13 | 11 | 0.85 |
| Total | 346 | 81 Sa | 0.23 ample climatolog |



Steps for making reliability diagram

- 1. For each forecast probability category count the number of observed occurrences
- 2. Compute the observed relative frequency in each category *k*

obs. relative frequency_k = obs. occurrences_k / num. forecasts_k

- 3. Plot observed relative frequency vs forecast probability
- 4. Plot sample climatology ("no resolution" line)

sample climatology = obs. occurrences / num. forecasts

- 5. Plot "no-skill" line halfway between climatology and perfect reliability (diagonal) lines
- 6. Plot forecast frequency separately to show forecast sharpness

Tampere reliability for 48h forecasts

| Forecast probability | # fcsts | # observed occurrences | Obs. relative frequency |
|-------------------------|------------|------------------------|----------------------------|
| 0.0 | 31 | 1 | |
| 0.1 | 53 | 5 | |
| 0.2 | 67 | 7 | |
| 0.3 | 39 | 7 | |
| 0.4 | 38 | 12 | |
| 0.5 | 16 | 5 | |
| 0.6 | 26 | 8 | |
| 0.7 | 30 | 14 | |
| 0.8 | 31 | 15 | |
| 0.9 | 8 | 6 | |
| 1.0 | 7 | 6 | |



Total

Sample climatology

Tampere reliability for 48h forecasts

| Forecast probability | # fcsts | # observed occurrences | Obs. relative frequency |
|----------------------|------------|---------------------------|----------------------------|
| 0.0 | 31 | 1 | 0.03 |
| 0.1 | 53 | 5 | 0.09 |
| 0.2 | 67 | 7 | 0.10 |
| 0.3 | 39 | 7 | 0.18 |
| 0.4 | 38 | 12 | 0.32 |
| 0.5 | 16 | 5 | 0.31 |
| 0.6 | 26 | 8 | 0.31 |
| 0.7 | 30 | 14 | 0.47 |
| 0.8 | 31 | 15 | 0.48 |
| 0.9 | 8 | 6 | 0.75 |
| 1.0 | 7 | 6 | 0.86 |
| Total | 346 | 86 Sa | 0.25 ample climatolog |



Reliability diagrams in R

library(verification)
source("read_tampere_pop.r")
A <- verify(d\$obs_rain, d\$p24_rain, bins=FALSE)
attribute(A)</pre>



3rd International Verification Methods Workshop, 29 January – 2 February 2007

Brier score – what is the probability error?

Familiar mean square error measures accuracy of continuous variables

$$MSE = \frac{1}{N}\sum_{i=1}^{N} (y_i - x_i)^2$$

Brier (probability) score measures mean squared error in probability space

$$BS = \frac{1}{N} \sum_{i=1}^{N} (p_i - o_i)^2$$

 p_i = forecast probability o_i = observed occurrence (0 or 1)

Brier skill score measures *skill* relative to a reference forecast (usually climatology)

$$BSS = -\frac{BS - BS_{ref}}{BS_{ref}}$$

Components of probability error

The Brier score can be decomposed into 3 terms (for *K* probability classes and *N* samples):

$$BS = \frac{1}{N} \sum_{k=1}^{K} n_k (p_k - \overline{o}_k)^2 - \frac{1}{N} \sum_{k=1}^{K} n_k (\overline{o}_k - \overline{o})^2 + \overline{o}(1 - \overline{o})$$

reliability resolution uncertainty
es weighted (by Measures the distance Measures for the distance f

Measures weighted (by forecast frequency) error of reliability curve – indicates the degree to which forecast probabilities can be taken at face value (reliability)



Measures the distance between the observed relative frequency and climatological frequency – indicates the degree to which the forecast can separate different situations (resolution)



Measures the variability of the observations – indicates the degree to which situations are climatologically easy or difficult to predict.

Has nothing to do with forecast quality! Use the Brier skill score to overcome this problem.

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Steps for computing Brier (skill) score

- 1. For each forecast-observation pair compute the difference between the forecast probability p_i and observed occurrence o_i ,
- 2. Compute the mean squared value of these differences

$$BS = \frac{1}{N} \sum_{i=1}^{N} (p_i - o_i)^2$$

- 3. Compute the mean observed occurrence \overline{o} (sample climatology)
- 4. Compute the reference Brier score using the sample climatology as the forecast (or use long-term climatology if available)

$$BS_{ref} = \frac{1}{N} \sum_{i=1}^{N} (p_i - \overline{o})^2$$

5. Compute the skill score $BSS = -\frac{BS - BS_{ref}}{BS_{ref}}$

Brier score and components in R

```
library(verification)
source("read_tampere_pop.r")
A <- verify(d$obs_rain, d$p24_rain, bins=FALSE)
summary(A)</pre>
```

The forecasts are probabilistic, the observations are binary. Sample baseline calculated from observations. Brier Score (BS) 0.1445 = Brier Score - Baseline = 0.1793Skill Score 0.1942= Reliability 0.02536 = Resolution 0.06017 = Uncertainty 0.1793 =

Brier score for heavy rain vs all rain

H <- verify(d\$obs_heavy, d\$p24_heavy, bins=FALSE)
summary(H)</pre>

| | Heavy rain | All rain |
|------------------------|------------|----------|
| Brier Score (BS) | = 0.03746 | 0.1445 |
| Brier Score - Baseline | = 0.05446 | 0.1793 |
| Skill Score | = 0.3122 | 0.1942 |
| Reliability | = 0.003398 | 0.02536 |
| Resolution | = 0.0204 | 0.06017 |
| Uncertainty | = 0.05446 | 0.1793 |

Q: What's going on?

Brier score is sensitive to the climatological frequency of an event: the more rare an event, the easier it is to get a good BS without having any real skill .

Discrimination

Good forecasts should discriminate between events and non-events



Measuring discrimination using ROC

- Measure success using Relative Operating Characteristic (ROC)
 - Plot the hit rate against the false alarm rate using increasing probability thresholds to make the yes/no decision





ROC area – a popular summary measure

- ROC curve is independent of forecast bias is like "potential skill"
- Area under curve ("ROC area") is a useful summary measure of forecast skill
 - Perfect: ROC area = 1
 - No skill: ROC area = 0.5
 - ROC skill score

ROCS = 2 (ROC area - 0.5)

= KSS for deterministic forecast



Interpretation of ROC curves



- The ROC is conditioned on the observations (i.e., given that Y occurred, how did the forecast perform?)
- ROC is a good companion to reliability plot, which is conditioned on the forecasts (i.e., given that X was predicted, what was the outcome?)

Tampere ROC for 24h forecasts

| Forecast probability | Hits | Misses | False alarms | Corr. non- events | Hit rate | False alarm rate | |
|----------------------|------|--------|-----------------|----------------------|----------|---------------------|------------------|
| 0.0 | 81 | 0 | 265 | 0 | 1.00 | 1.00 | |
| 0.1 | 80 | 1 | 220 | 45 | 0.99 | 0.83 | |
| 0.2 | 79 | 2 | 166 | 99 | 0.98 | 0.63 | |
| 0.3 | 74 | 7 | 112 | 153 | 0.91 | 0.42 | |
| 0.4 | 69 | 12 | 76 | 189 | 0.85 | 0.29 | ate |
| 0.5 | 65 | 16 | 61 | 204 | 0.80 | 0.23 | |
| 0.6 | 57 | 24 | 47 | 218 | 0.70 | 0.18 | |
| 0.7 | 51 | 30 | 31 | 234 | 0.63 | 0.12 | ROC area=0.86 |
| 0.8 | 35 | 46 | 13 | 252 | 0.43 | 0.05 | |
| 0.9 | 19 | 62 | 5 | 260 | 0.23 | 0.02 | False alarm rate |
| 1.0 | 11 | 70 | 2 | 263 | 0.14 | 0.01 | |

Steps for making ROC diagram

- 1. For each forecast probability category count the number of hits, misses, false alarms, and correct non-events
- 2. Compute the hit rate (probability of detection) and false alarm rate (probability of false detection) in each category *k*

 $hit rate_k = hits_k / (hits_k + misses_k)$

false alarm rate_k = false alarms_k / (false alarms_k + correct non-events_k)

- 3. Plot hit rate vs false alarm rate
- 4. ROC area is the integrated area under the ROC curve

Tampere ROC for 48h forecasts



Tampere ROC for 48h forecasts

| Forecast probability | Hits | Misses | False alarms | Corr. non- events | Hit rate | False alarm rate | |
|----------------------|------|--------|-----------------|----------------------|----------|---------------------|---|
| 0.0 | 86 | 0 | 260 | 0 | 1.00 | 1.00 | |
| 0.1 | 85 | 1 | 230 | 30 | 0.99 | 0.89 | |
| 0.2 | 80 | 6 | 182 | 78 | 0.93 | 0.70 | |
| 0.3 | 73 | 13 | 122 | 138 | 0.85 | 0.47 | |
| 0.4 | 66 | 20 | 90 | 170 | 0.77 | 0.35 | rate |
| 0.5 | 54 | 32 | 64 | 196 | 0.63 | 0.25 | · 분 |
| 0.6 | 49 | 37 | 53 | 207 | 0.57 | 0.20 | |
| 0.7 | 41 | 45 | 35 | 225 | 0.48 | 0.13 | 24h ROC area=0.86 |
| 0.8 | 27 | 59 | 19 | 241 | 0.31 | 0.07 | $\begin{array}{c c} 0 & 401 \text{ ROC area=0.77} \\ 0 & 1 \end{array}$ |
| 0.9 | 12 | 74 | 3 | 257 | 0.14 | 0.01 | False alarm rate |
| 1.0 | 6 | 80 | 1 | 259 | 0.07 | 0.00 | |

ROC diagrams in R

```
library(verification)
source("read_tampere_pop.r")
A <- verify(d$obs_rain, d$p24_rain, bins=FALSE)
roc.plot(A, legend=TRUE)</pre>
```

```
roc.plot(A, CI=TRUE)
                                        0.1
roc.plot(A, binormal=TRUE,
                                        0.8
    plot="both", legend=TRUE,
    show.thres=FALSE)
                                        0.6
                                      Hit Rate
B <- verify(d$obs rain,</pre>
                                        0.4
    d$p48 rain, bins=FALSE)
roc.plot(A, plot.thres=NULL)
                                        0.2
lines.roc(B, col=2, lwd=2)
leq.txt <- c("24 h forecast",</pre>
    "48 h forecast")
```



False Alarm Rate

ROC Curve

Putting it all together...

Tampere POP forecasts





high bias (over-confident), better than climatology only for *P* near 0 or 1

Brier score

 measures probability error Brier Score (BS) = 0.1445 Brier Skill Score = 0.1942 skilled compared to climatology

ROC

measures discrimination (potential skill)



good discrimination→ good potential skill

... more probability verification ...

Ranked probability score – how accurate are the probability forecasts?

Measures the squared difference in probability space when there are multiple probability categories



Characteristics of RPS

$$RPS = \frac{1}{K-1} \sum_{k=1}^{K} (CDF_{fcst,k} - CDF_{obs,k})^{2}$$

$$CDF_{obs} - CDF_{fcst}$$

$$0$$

$$K$$

- Takes into account the ordered nature of the predicted variable (for example, temperature going from low to high values)
- Emphasizes accuracy by penalizing "near misses" less than larger errors
- Rewards sharp forecast if it is accurate
- Perfect score: 0
- RPS skill score w.r.t. climatology:

y: $RPSS = 1 - \frac{RPS}{RPS_{c \, lim}}$

Interpretation of RPS



Q: Which forecasts are skilled with respect to climatology?

Tampere 24h POP data

| Date 2003 | Observed rain (mm) | p ₁ = POP 0-0.2 mm (category 1) | p_2 = POP 0.3-4.4 mm (category 2) | p ₃ = POP 4.5+ mm (category 3) | |
|--------------|-----------------------|---|--|--|---|
| Jan 1 | 0.0 | 0.7 | 0.3 | 0.0 |] |
| Jan 2 | 0.0 | 0.9 | 0.1 | 0.0 |] |
| Jan 3 | 0.0 | 0.9 | 0.1 | 0.0 |] |
| Jan 4 | 0.0 | 0.8 | 0.2 | 0.0 | |
| Jan 5 | 0.0 | 0.8 | 0.2 | 0.0 |] |
| Jan 6 | 0.0 | 0.9 | 0.1 | 0.0 |] |
| Jan 7 | 1.1 | 0.6 | 0.4 | 0.0 | |
| Jan 8 | 0.9 | 0.3 | 0.4 | 0.3 |] |
| Jan9 | 0.0 | 0.3 | 0.4 | 0.3 |] |
| Jan 10 | 0.0 | NA | NA | NA |] |
| Jan 11 | 2.2 | NA | NA | NA |] |
| Jan 12 | 0.0 | 0.8 | 0.2 | 0.0 |] |
| Jan 13 | 1.2 | 0.8 | 0.2 | 0.0 |] |
| Jan 14 | 6.0 | 0.0 | 0.4 | 0.6 | |
| Jan 15 | 2.3 | 0.3 | 0.7 | 0.0 | |
| | | | | | |

Steps for computing RPS

- 1. For each forecast-observation pair:
 - a. Assign the observation to its appropriate category $k_{obs.}$ The cumulative density function CDF_{obs} is either 0 or 1:

$$CDF_{obs,k} = \begin{cases} 0 & k < k_{obs} \\ 1 & k \ge k_{obs} \end{cases}$$

b. From the categorical probability forecast $P = [p_1, p_2, ..., p_k]$ compute the cumulative density function for every category *k* as

$$CDF_{fcst,k} = \sum_{j=1}^{k} p_j$$

- c. Compute the RPS as $RPS = \frac{1}{K-1}\sum_{k=1}^{K} (CDF_{fcst,k} CDF_{obs,k})^2$
- 2. Average the RPS over all forecast-observation pairs

Tampere 24h POP data

$\begin{array}{l} \underline{\text{Categories}}\\ 1 &\leq 0.2 \text{ mm}\\ 2 & 0.3 - 4.4 \text{ mm}\\ 3 &\geq 4.5 \text{ mm} \end{array}$

| Date 2003 | Observed rai na(im m) | p ₁ | р ₂ | р ₃ | Observed category k_{obs} | <i>CDF_{obs,k} k</i> =1,2,3 | <i>CDF_{fcst,k} k</i> =1,2,3 | RPS |
|--------------|--|----------------|----------------|----------------|-----------------------------|---|--|-------|
| Jan 1 | 0.0 | 0.7 | 0.3 | 0.0 | 1 | 1, 1, 1 | 0.7, 1, 1 | 0.045 |
| Jan 2 | 0.0 | 0.9 | 0.1 | 0.0 | 1 | 1, 1, 1 | 0.9, 1, 1 | 0.005 |
| Jan 3 | 0.0 | 0.9 | 0.1 | 0.0 | 1 | 1, 1, 1 | 0.9, 1, 1 | 0.005 |
| Jan 4 | 0.0 | 0.8 | 0.2 | 0.0 | 1 | 1, 1, 1 | 0.8, 1, 1 | 0.020 |
| Jan 5 | 0.0 | 0.8 | 0.2 | 0.0 | 1 | 1, 1, 1 | 0.8, 1, 1 | 0.020 |
| Jan 6 | 0.0 | 0.9 | 0.1 | 0.0 | 1 | 1, 1, 1 | 0.9, 1, 1 | 0.005 |
| Jan 7 | 1.1 | 0.6 | 0.4 | 0.0 | 2 | 0, 1, 1 | 0.6, 1, 1 | 0.180 |
| Jan 8 | 0.9 | 0.3 | 0.4 | 0.3 | 2 | 0, 1, 1 | 0.3, 0.7, 1 | 0.090 |
| Jan9 | 0.0 | 0.3 | 0.4 | 0.3 | 1 | 1, 1, 1 | 0.3, 0.7, 1 | 0.290 |
| Jan 10 | 0.0 | NA | NA | NA | 1 | 1, 1, 1 | NA | NA |
| Jan 11 | 2.2 | NA | NA | NA | 2 | 0, 1, 1 | NA | NA |
| Jan 12 | 0.0 | 0.8 | 0.2 | 0.0 | 1 | 1, 1, 1 | 0.8, 1, 1 | 0.020 |
| Jan 13 | 1.2 | 0.8 | 0.2 | 0.0 | 2 | 0, 1, 1 | 0.8, 1, 1 | 0.320 |
| Jan 14 | 6.0 | 0.0 | 0.4 | 0.6 | 3 | 0, 0, 1 | 0, 0.4, 1 | 0.080 |
| Jan 15 | 2.3 | 0.3 | 0.7 | 0.0 | 2 | 0, 1, 1 | 0.3, 1, 1 | 0.045 |
| | | | | | | | | |

15-day RPS = 0.087

Tampere 48h POP data

Categories ≤ 0.2 mm 1 0.3 - 4.4 mm 2 3 ≥ 4.5 mm

| Date 2003 | Observed rain (mm) | <i>p</i> ₁ | <i>P</i> ₂ | p ₃ | Observed category <i>k_{obs}</i> | <i>CDF_{obs,k}</i> <i>k</i> =1,2,3 | CDF _{fcst,k} <i>k</i> =1,2,3 | RPS |
|--------------|-----------------------|-----------------------|-----------------------|----------------|--|---|--|-----|
| Jan 1 | 0.0 | 0.9 | 0.1 | 0.0 | | | | |
| Jan 2 | 0.0 | 0.9 | 0.1 | 0.0 | | | | |
| Jan 3 | 0.0 | 0.8 | 0.1 | 0.1 | | | | |
| Jan 4 | 0.0 | 0.8 | 0.1 | 0.1 | | | | |
| Jan 5 | 0.0 | 0.8 | 0.2 | 0.0 | | | | |
| Jan 6 | 0.0 | 0.8 | 0.2 | 0.0 | | | | |
| Jan 7 | 1.1 | 0.8 | 0.2 | 0.0 | | | | |
| Jan 8 | 0.9 | 0.7 | 0.3 | 0.0 | | | | |
| Jan9 | 0.0 | 0.4 | 0.4 | 0.2 | | | | |
| Jan 10 | 0.0 | 0.8 | 0.1 | 0.1 | | | | |
| Jan 11 | 2.2 | NA | NA | NA | | | | |
| Jan 12 | 0.0 | NA | NA | NA | | | | |
| Jan 13 | 1.2 | 0.6 | 0.4 | 0.0 | | | | |
| Jan 14 | 6.0 | 0.1 | 0.5 | 0.4 | | | | |
| Jan 15 | 2.3 | 0.2 | 0.6 | 0.2 | | | | |
| | | | | | | | | |

Tampere 48h RPS

 $\begin{array}{l} \underline{\text{Categories}}\\ 1 &\leq 0.2 \text{ mm}\\ 2 & 0.3 - 4.4 \text{ mm}\\ 3 &\geq 4.5 \text{ mm} \end{array}$

| Date 2003 | Observed rain (mm) | <i>p</i> ₁ | <i>p</i> ₂ | <i>p</i> ₃ | Observed category k_{obs} | <i>CDF_{obs,k}</i> <i>k</i> =1,2,3 | <i>CDF_{fcst,k}</i> <i>k</i> =1,2,3 | RPS |
|--------------|--------------------|-----------------------|-----------------------|-----------------------|-----------------------------|---|--|-------|
| Jan 1 | 0.0 | 0.9 | 0.1 | 0.0 | 1 | 1,1,1 | 0.9, 1, 1 | 0.005 |
| Jan 2 | 0.0 | 0.9 | 0.1 | 0.0 | 1 | 1,1,1 | 0.9, 1, 1 | 0.005 |
| Jan 3 | 0.0 | 0.8 | 0.1 | 0.1 | 1 | 1,1,1 | 0.8, 0.9, 1 | 0.025 |
| Jan 4 | 0.0 | 0.8 | 0.1 | 0.1 | 1 | 1,1,1 | 0.8, 0.9, 1 | 0.025 |
| Jan 5 | 0.0 | 0.8 | 0.2 | 0.0 | 1 | 1,1,1 | 0.8, 1, 1 | 0.020 |
| Jan 6 | 0.0 | 0.8 | 0.2 | 0.0 | 1 | 1,1,1 | 0.8, 1, 1 | 0.020 |
| Jan 7 | 1.1 | 0.8 | 0.2 | 0.0 | 2 | 0,1,1 | 0.8, 1, 1 | 0.320 |
| Jan 8 | 0.9 | 0.7 | 0.3 | 0.0 | 2 | 0,1,1 | 0.7, 1, 1 | 0.245 |
| Jan9 | 0.0 | 0.4 | 0.4 | 0.2 | 1 | 1,1,1 | 0.4, 0.8, 1 | 0.200 |
| Jan 10 | 0.0 | 0.8 | 0.1 | 0.1 | 1 | 1,1,1 | 0.8, 0.9, 1 | 0.025 |
| Jan 11 | 2.2 | NA | NA | NA | 2 | 0,1,1 | NA | NA |
| Jan 12 | 0.0 | NA | NA | NA | 1 | 1,1,1 | NA | NA |
| Jan 13 | 1.2 | 0.6 | 0.4 | 0.0 | 2 | 0,1,1 | 0.6, 1, 1 | 0.180 |
| Jan 14 | 6.0 | 0.1 | 0.5 | 0.4 | 3 | 0,0,1 | 0.1, 0.6, 1 | 0.185 |
| Jan 15 | 2.3 | 0.2 | 0.6 | 0.2 | 2 | 0,1,1 | 0.2, 0.8, 1 | 0.040 |
| | | | | | | | | |

15-day RPS =0.100

Ranked probability (skill) score in R

library(verification)

source("read_tampere_pop.r")

Make vector of observed categories

obscat <- d\$obs_norain + d\$obs_light*2 + d\$obs_heavy*3</pre>

Make Nx3 array of category probabilities

pvec <- cbind(d\$p24_norain, d\$p24_light, d\$p24_heavy)
rps(obscat, pvec)</pre>

\$rps

[1] 0.0909682

\$rpss
[1] 0.2217009

\$rps.clim
[1] 0.1168808

Continuous ranked probability score

Continuous ranked probability score (CRPS) measures the difference between the forecast and observed CDFs

$$CRPS = \int_{-\infty}^{\infty} (P_{fcst}(x) - P_{obs}(x))^2 dx$$

- Same as Brier score integrated over all possible threshold values
- Same as Mean Absolute Error for deterministic forecasts



- Advantages:
 - sensitive to whole range of values of the parameter of interest
 - does not depend on predefined classes
 - easy to interpret
 - has dimensions of the observed variable
- Rewards small spread (sharpness) if the forecast is accurate
- Perfect score: 0

Verifying individual events

Debate as to whether or not this is a good idea...

- Forecasters and other users often want to know the quality of a forecast for a particular event
- We cannot meaningfully verify a single probability forecast
 - If it rains when the PoP was 30% was that a good forecast?
- but we can compare a probability distribution to a single observation
 - Want the forecast to be accurate (close to the observed), and sharp (not too much spread)
 - This approach implicitly assumes that the weather is predictable and the uncertainty comes from the forecast system
 - Best used at short time ranges and/or large spatial scales
- Methods for individual or collections of forecasts
 - (Continuous) Ranked Probability Score
 - Wilson (MWR, 1999) score
 - Ignorance





Conveying forecast quality to users

Forecasters and other users are ~comfortable with standard verification measures for deterministic forecasts

Are there similar easy-to-understand measures for probabilistic forecasts?

| Deterministic | Probabilistic (suggestions) | Visual aid |
|------------------------|--|-------------------|
| Mean bias | Reliability term of BS $\frac{1}{N}\sum_{k=1}^{K} n_k (p_k - \overline{o}_k)^2$ | o" P _k |
| RMS error | Brier score (square root) $\sqrt{BS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (p_i - o_i)^2}$ | |
| Mean absolute error | CRPS $\int (P_{fcst}(x) - P_{obs}(x))^2 dx$ | CDF 0 x |
| Correlation | <i>R</i> ² for logistic regression | occurrence |

Relative value score

Measures the relative improvement in economic value as a function of the cost/loss ratio C/L for taking action based on a forecast as opposed to climatology

$$V = (1 - F) - \left(\frac{1 - C/L}{C/L}\right) \left(\frac{\overline{o}}{1 - \overline{o}}\right) (1 - H) \quad \text{if } C/L < \overline{o}$$
$$V = H - \left(\frac{C/L}{1 - C/L}\right) \left(\frac{1 - \overline{o}}{\overline{o}}\right) F \quad \text{if } C/L > \overline{o}$$

where *H* is the hit rate and *F* is the false alarm rate

- The relative value is a skill score of expected expense, with climatology as the reference forecast.
- Range: -∞ to 1. Perfect score: 1
- Plot V vs C/L for various probability thresholds. The envelope describes the potential value for the probabilistic forecasts.



Rank histogram (Talagrand diagram)

Measures how well the ensemble spread of the forecast represents the true variability (uncertainty) of the observations

- → Count where the verifying observation falls with respect to the ensemble forecast data, which is arranged in increasing order at each grid point.
- In an ensemble with perfect spread, each member represents an equally likely scenario, so the observation is equally likely to fall between any two members.
 - Flat ensemble spread correctly represents forecast uncertainty
 - U-shaped ensemble spread too small, many observations falling outside the extremes of the ensemble
 - Dome-shaped ensemble spread too large, too many observations falling near the center of the ensemble
 - Asymmetric ensemble contains bias
- A flat rank histogram does not necessarily indicate a skilled forecast, it only measures whether the observed probability distribution is well represented by the ensemble.



Who's using what for ensemble verification?

- WMO (ensemble NWP, site maintained by JMA)
 - Brier skill score, reliability diagram, economic value, ensemble mean & spread
 - Some operational centers (ensemble NWP) web survey in 2005

| ECMWF | BSS, reliability diagram, ROC, ROC area, econ. value, spread/skill diagram |
|------------|--|
| NCEP | RMSE and AC of ensemble mean, BSS, ROC area, rank histogram, RPSS, econ. value |
| Met Office | BSS, reliability diagram, ROC, rank histogram |
| BMRC | RMSE ensemble mean, BSS, reliability diagram, ROC, rank histogram, RPSS, econ. value |

- DEMETER (multiple coupled-model seasonal ensemble) see <u>http://www.ecmwf.int/research/demeter/d/charts/verification/</u>
 - Deterministic: anomaly correlation, mean square skill score, SD ratio
 - Probabilistic: reliability diagram, ROCS, RPSS
 - Economic value

Verifying "objects"

Significant weather events can often be viewed as 2D objects

- tropical cyclones, heavy rain events, deep low pressure centres
- objects are defined by an intensity threshold

What might the ensemble forecast look like?

- spatial probability contour maps
- distributions of object properties
 - location, size, intensity, etc.



Strategies for verifying ensemble predictions of objects

- Verify spatial probability maps
- Verify distributions of object properties
 - many samples use probabilistic measures
 - individual cases CRPS, WS, IGN
- Verify ensemble mean
 - spatially averaged forecast objects
 - generated from average object properties

Sampling issues – rare events

- Rare events are often the most interesting ones!
- Coarse model resolution may not capture intensity of experienced weather
- Difficult to verify probabilities on the "tail" of the PDF
 - Too few samples to get robust statistics, especially for reliability
 - Finite number of ensemble members may not resolve tail of forecast PDF
- Forecast calibration approaches
- Atger (*QJRMS*, 2004) approach for improving robustness of verification:
 - Fit ROC for all events (incl. rare) using bi-normal model, then relate back to reliability to get *estimated* forecast quality for under-sampled categories
 - Fitted reliability also be used instead of "raw" frequencies to calibrate ensemble



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Effects of observation errors

Observation errors add uncertainty to the verification results

- True forecast skill is unknown
 - \rightarrow An imperfect model / ensemble may score better!
- Extra dispersion of observation PDF

Effects on verification results

- RMSE overestimated
- Spread more obs outliers make ensemble look under-dispersed
 - Saetra et al (2004) compensate by adding obs error to ensemble
- Reliability poorer
- Resolution greater in BS decomposition, but ROC area poorer
- CRPS, WS, IGN poorer mean values

Can we remove the effects of observation error?

- More samples helps with reliability estimates
- Error modeling study effects of applied observation errors
- Need "gold standard" to measure actual observation errors

Not easy!

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Thank you!