How good is the tangent linear approximation for satellite observation operators?

R.W. Saunders
ECMWF, Shinfield Park, Reading, U.K.

1. INTRODUCTION

To enable the assimilation of directly observed satellite measurements (e.g. radiances, radar backscatter) in a variational analysis an observation operator (sometimes referred to as a forward model) and its adjoint must be available for each observation type. The observation operator allows a measurement to be simulated from the model fields. The current implementation of the TOVS (Tiros Operational Vertical Sounder) observation operator in the ECMWF 3D-Var assimilation system recomputes the radiative transfer (RT) for every iteration of the 3D-Var minimisation to allow the direct assimilation of radiances. This is because the tangent linear assumption for the gradient of the TOVS RT model is assumed not to be valid. The purpose of this study is to determine when the tangent linear assumption is not valid for both TOVS (i.e. HIRS+MSU radiances) and AMSU (Advanced Microwave Sounding Unit) radiances and ERS (European Remote Sensing satellite) scatterometer measurements.

2. DEFINITIONS

For variational analysis the term for describing the fit of the observations to the analysis is normally written as:

$$Jo = (H(x)-y)^T \cdot (O+F)^{-1} \cdot (H(x)-y) \tag{1}$$

where x is the atmospheric state vector which initially contains the model variables for the background (or first guess) field and ultimately the analysed field, H is the observation operator which transforms the model variables into the appropriate observed variable and y is the observation vector (e.g. a set of radiances from different TOVS channels). For TOVS $H(x)$ includes both the RT model $R(x)$ and the spatial and temporal interpolation to the observation location and time. O is the observation error covariance (including representativeness error) and F is the RT model and interpolation error covariance matrices. In order to perform the minimisation in 3D-Var the adjoint of the observation operator is also required which is recomputed for every iteration of the minimisation. However if the gradient of the RT model, $\frac{dR(x)}{dx}$, does not vary significantly in the vicinity of the first guess of x then the tangent linear assumption is valid and the adjoint of the RT model only has to be computed once during the minimisation allowing potential savings in computer time. Only the linearity of the RT model is assessed in this study, not the interpolation. The measure of linearity used, ΔR is defined as follows:

$$\Delta R = R(x_i) - R(x_{fg}) - (x_i - x_{fg}) \frac{dR(x_{fg})}{dx} \tag{2}$$

$R(x_i)$ is the radiance vector computed exactly by incrementing the profile and then running the RT model and $R(x_{fg})$ is the radiance computed from the original (i.e. first guess) profile. x_i and x_{fg} are the profile vectors for the incremented and original profiles respectively and $\frac{dR(x_{fg})}{dx}$ is the gradient of the RT model about the first guess profile. When ΔR is zero the tangent linear assumption is exact.

3. RESULTS

RT calculations were performed for a tropical and arctic profile and the tangent linear of the RT model was also computed (i.e. $\frac{dR(x_{fg})}{dx}$) for each profile. The RT model used for this study was RTTOV (*Eyre*, 1991) which predicts the transmittance for each layer of the atmosphere using a polynomial expression to predict layer transmittance $\tau(i,j)$ for channel i and level j from pressure, temperature and constituent concentration profiles. It was enhanced to allow AMSU radiances to be computed. The full matrix of the Jacobians is computed by RTTOV (i.e. $d\tau\,(i,j)/\,dx$) which gives the values for each TOVS/AMSU channel for each atmospheric variable at each level. In order to simplify the analysis only the effect of a large but realistic increment in one of the profile variables (i.e. temperature, specific humidity, ozone, surface emissivity or cloud cover/top pressure) on the computed top of atmosphere radiance was analysed. The surface emissivity was assumed to be 1 for all HIRS and MSU channels and 0.7 for all AMSU channels which are approximate values for the sea surface. MSU radiances are currently provided by NESDIS emissivity corrected so a value of 1 is appropriate whereas the AMSU radiances will not be corrected for surface emissivity. The departure from linearity ΔR is expressed as a brightness temperature difference ΔT_b for ease of interpretation.

For temperature the increment applied to both profiles was -2K at all levels including the radiative surface temperature. The response to the applied profile increments of the top of atmosphere brightness temperatures and the departures from linearity, ΔT_b, are given in Tables 1 and 2 respectively for all the TOVS and AMSU channels. All channels have a response of between 1.0-2.1 K. The exact value depends on the transmittance profile for the channel, the temperature profile (note the arctic profile has an inversion close to the surface) and the surface emissivity. For temperature increments there are no significant departures (i.e. < 0.1K) from a linear response of the RT model about the first guess profile for all channels in both the tropical and arctic atmospheres.

For water vapour the increment applied was a 50% reduction in specific humidity at all levels. For the arctic profile only HIRS channels 11 and 12 are sensitive to water vapour whereas for the tropical profile HIRS channels 4-15, 18 and 19 are all responding to the change in water vapour. Note that MSU channels 1 and 2 also have a sensitivity to water vapour due to the water vapour continuum at 50 GHz. The values of ΔT_b in Table 2 show that there is a significant departure from linearity of up to 1K for the tropical profile of the HIRS water vapour channels (11 and 12). If the water vapour absorption is weak then a linear response to changes in absorber amount is predicted whereas if the channel is in a part of the spectrum with strong water vapour absorption then a non-linear response is expected (see *Houghton et. al.*, 1986 for more details). AMSU channels 1-4 and 15-20 are all sensitive to water vapour for both the tropical and to a lesser extent the arctic profile. Note that for the channels which "see" the surface the change in brightness temperature is of opposite sign to the HIRS water vapour channels. AMSU water vapour channels (17-20) are much more non-linear (departures up to 7.5K in the tropics) due to the non-unit emissivity of the sea surface.

The response to changes in total column ozone was only computed for the HIRS channels as it is believed the microwave channels are not sensitive to ozone with the possible exception of AMSU channel 18. For a 10% increase in total column ozone, from the global mean profile, HIRS channels 1-7 all have a small response (about 0.1K or less) but channel 9 changes by more than 1K. As for water vapour the response is non-linear for the ozone sensitive channel (HIRS-9) where the absorption is strong.

The sensitivity to surface emissivity for all channels is also given in Table 1 where for HIRS and MSU the nominal emissivity of 1.0 is reduced by 0.05 whereas for AMSU the nominal emissivity of 0.7 was reduced by 0.1. This reflects the uncertainty in the emissivities for HIRS and the "emissivity corrected" MSU radiances and uncorrected AMSU radiances respectively. The sensitivity figures given in Table 1 are useful to assess if a channel can be used over land without allowing for surface emissivity. In the tropics HIRS channels 7-10 and 13, 14, 18 and 19 and MSU channels 1 and 2 are all sensitive to changes in surface emissivity and in the arctic channels 6 and 11 also start to "see" the surface. For AMSU, only channels 1-5 and 15-17 "see" the surface in the tropics but channels 18-20 also "see" the surface for arctic profiles. Unlike HIRS channel 12 care will have to be taken in using even AMSU channel 18 (the most opaque water vapour channel) over land. The response to changes in surface emissivity however is completely linear for all HIRS and AMSU channels as would be expected from theory assuming a specular emissivity model. Note the change of emissivity with windspeed and sea surface temperature is not linear and so the tangent linear assumption will not be valid for retrievals of these variables from AMSU.

Finally, for the HIRS channels only, the sensitivity to cloud top pressure and cloud amount for a tropical profile were computed together with the departure from linearity. The non-linearity of the RT model for a 50 hPa increment in cloud top pressure at 800 hPa is small (<0.1K) whereas for the same increment to a cloud top at 400 hPa the departure from linearity is almost 1K for nearly all HIRS channels. Responses to changes in cloud amount are more linear although the departures from linearity are still significant for a 400 hPa cloud top. These results apply to a cloud top which is "black" at all HIRS channel wavelengths (i.e. an optically thick cloud).

A similar approach was adopted to assess the departure from linearity of the ERS scatterometer observation operator, CMOD4, described by *Stoffelen and Anderson* (1997). CMOD4 computes the microwave backscatter which would be measured by the three antennae (fore, mid and aft) for a specified sea surface wind speed and direction, as a function of their respective incidence angles. CMOD4 is currently used for monitoring the ERS scatterometer backscatter measurements in near real time using 6 hr forecast surface wind speeds and directions at each location of a scatterometer backscatter measurement. For realistic increments in surface wind speed (i.e. -4 $m.s^{-1}$) and direction (i.e. 60°) the computed changes in backscatter values are highly non-linear as given in Table 3 where backscatter values assuming the tangent-linear approximation are compared with those accurately computed. The tangent linear approximation is clearly not valid for the scatterometer. This non-linearity of CMOD4 has to date hampered attempts to directly assimilate measured backscatter values using 3D-Var instead of retrieved ambiguous wind pairs (*Stoffelen and Anderson*, 1997).

4. CONCLUSIONS

The results presented in Table 2 indicate that for TOVS and AMSU the tangent linear approximation of the RT model, RTTOV, is valid for temperature increments to the first guess profile. Hence if TOVS/AMSU radiances are only used to influence the temperature field the RT model and its gradient need only be computed once for each profile. However if the TOVS radiances are also used to adjust the humidity or ozone fields the significant non-linearities of the forward model suggests the gradient of the RT model has to be computed several times during the minimisation. Whether the RT model should be recomputed for *every* iteration of the minimisation is less clear. A reduction in the number of times the

RT calculations are performed may allow the use of more of the TOVS radiances in the analysis. To address this point the accompanying abstract by *Derber* (1996) in this report extends this study by showing how the tangent linear approximation for the TOVS radiance assimilation in the full ECMWF 3D-Var system affects the analysed temperature and water vapour fields.

The non-linearity for the AMSU water vapour and window channels is much larger than for HIRS and shows how the reflection from the surface increases the non-linearity. This will mean a fast model will be necessary to provide a good estimate of the surface emissivity before the AMSU water vapour and window channel radiances can be used effectively in a variational analysis. The RT modelling of HIRS cloudy radiances shows the response is non-linear for high clouds in all channels. Similarly for the ERS scatterometer the observation operator is highly non-linear for changes in surface wind speed and direction.

5. REFERENCES

Derber J., 1996: The impact of non-linearities in the use of satellite sounding data. *ECMWF workshop report on Non-linear aspects of data assimilation, Reading 9-11 Sept. 1996.*

Eyre J.R., 1991: A fast radiative transfer model for satellite sounding systems. *ECMWF Research Dept. Tech. Memo. 176, March 1991.*

Houghton, J.T., 1986: The Physics of Atmospheres. *Cambridge University Press pp 45-46*

Stoffelen, A. and D. Anderson, 1997: Ambiguity removal and assimilation of scatterometer data. . *Q. J. Roy. Meteorol. Soc.,* **123** , *(Jan 1997 issue).*

Channel No.	Temperature increment (-2 K)		Humidity increment (-50%)		Ozone increment (+10%)	Surface emissivity increment (-0.05)	
HIRS	Tropical	Arctic	Tropical	Arctic	Mean	Tropic	Arctic
1	1.97	2.00	0.00	0.00	-0.03	0.00	0.00
2	1.96	2.00	0.00	0.00	-0.06	0.00	0.00
3	1.89	2.00	0.00	0.00	-0.05	0.00	0.00
4	1.56	1.83	-0.09	-0.01	0.01	0.00	0.00
5	1.51	1.78	-0.25	0.00	0.09	0.00	0.02
6	1.55	1.82	-0.55	0.00	0.12	0.01	0.14
7	1.61	1.87	-1.48	0.03	0.11	0.07	0.92
8	1.95	2.00	-1.44	0.02	0.00	1.19	2.28
9	-	-	-	-	1.30	-	-
10	1.84	1.98	-1.97	0.04	0.00	0.39	1.26
11	1.73	1.93	-3.67	-0.51	0.00	0.00	0.07
12	1.71	1.96	-4.18	-2.59	0.00	0.00	0.00
13	1.75	1.92	-0.60	0.01	0.01	0.34	0.25
14	1.79	1.94	-0.41	0.00	0.00	0.13	0.03
15	1.62	1.88	-0.13	0.00	0.00	0.03	0.00
16	1.67	1.94	0.00	0.00	0.00	0.00	0.00
17	1.87	1.94	0.00	0.00	0.00	0.00	0.00
18	1.98	2.00	-0.15	0.00	0.00	1.02	0.68
19	1.94	2.00	-0.59	0.02	0.00	0.85	0.75
MSU							
1	2.07	2.03	-0.34	0.01	0.00	6.23	5.71
2	1.79	1.94	-0.08	0.00	0.00	0.18	0.15
3	1.83	1.96	0.00	0.00	0.00	0.00	0.00
4	1.96	1.97	0.00	0.00	0.00	0.00	0.00

Channel No.	Temperature increment (-2 K)		Humidity increment (-50%)		Ozone increment (+10%)	Surface emissivity increment (-0.10)	
AMSU	Tropical	Arctic	Tropical	Arctic	Mean	Tropic	Arctic
1	1.46	1.41	8.51	0.56	-	11.27	11.61
2	1.31	1.38	3.86	0.20	-	12.95	11.44
3	1.36	1.38	3.59	0.16	-	7.26	5.79
4	1.88	1.87	0.88	0.04	-	1.76	1.37
5	1.79	1.93	0.13	0.01	-	0.32	0.23
6	1.74	1.93	0.00	0.00	-	0.01	0.00
7	1.79	1.96	0.00	0.00	-	0.00	0.00
8	1.93	1.99	0.00	0.00	-	0.00	0.00
9	1.99	1.98	0.00	0.00	-	0.00	0.00
10	1.93	1.98	0.00	0.00	-	0.00	0.00
11	1.91	1.96	0.00	0.00	-	0.00	0.00
12	1.91	1.94	0.00	0.00	-	0.00	0.00
13	1.91	1.93	0.00	0.00	-	0.00	0.00
14	1.96	1.97	0.00	0.00	-	0.00	0.00
15	1.01	1.34	14.67	0.90	-	9.09	10.90
16	1.01	1.34	14.67	0.90	-	9.09	10.90
17	1.18	1.38	23.90	3.01	-	3.53	10.63
18	1.92	1.89	-4.07	6.47	-	0.00	0.21
19	2.02	1.76	-4.71	15.34	-	0.00	1.73
20	2.08	1.47	-2.26	13.04	-	0.01	6.22

Table 1. Response of TOVS and AMSU brightness temperatures in deg K to increments of profile variables

Channel No.	Temperature increment (-2 K)		Humidity increment (-50%)		Ozone increment (+10%)
HIRS	Tropical	Arctic	Tropical	Arctic	Mean
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.3	0.0	0.0
9	-	-	-	-	0.2
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	-0.9	0.0	0.0
12	0.0	0.0	-1.0	-0.6	0.0
13	0.0	0.0	0.1	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.1	0.0	0.0
MSU					
21	0.0	0.0	0.1	0.0	0.0
22	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0

Channel No.	Temperature increment (-2 K)		Humidity increment (-50%)	
AMSU	Tropical	Arctic	Tropical	Arctic
1	0.0	0.0	0.6	0.0
2	0.0	0.0	-0.1	0.0
3	0.0	0.0	-0.2	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0
15	0.0	0.0	0.6	0.0
16	0.0	0.0	0.6	0.0
17	0.0	0.0	7.5	0.0
18	0.0	0.0	-1.2	5.3
19	0.0	0.0	-1.3	5.9
20	0.0	0.0	1.1	1.6

Table 2. Departure from linearity for TOVS and AMSU brightness temperatures in deg K for increments of profile variables.

Antenna	FG speed/dd 8 m.s^{-1}/100 ° Backscatter σ_0	Speed increment (-4 m.s^{-1}) Tangent-linear $\Delta\sigma_0$	Speed increment (-4 m.s^{-1}) Exact $\Delta\sigma_0$	Direction increment (+60°) Tangent-linear $\Delta\sigma_0$	Direction increment (+60°) Exact $\Delta\sigma_0$
Fore	-7.5 dB	1.56 dB	2.37 dB	2.35 dB	1.38 dB
Mid	0.0 dB	1.13 dB	1.75 dB	-0.24 dB	0.83 dB
Aft	-7.5 dB	2.03 dB	2.85 dB	-2.47 dB	-1.24 dB

Table 3. Predicted backscatter values, σ_0, of the ERS scatterometer in dB for each antenna and the change, $\Delta\sigma_0$ from the first guess for increments in wind speed of -4 m.s^{-1} and direction of 60° assuming the tangent-linear approximation and recomputing CMOD4 to give an exact value.