THE ECMWF SPECTRAL LIMITED AREA MODEL

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Abstract

A spectral limited area model (SLAM) is described. The basic variables of the model are split into so-called background and perturbation components which can be handled within the spectral formalism. The model uses the primitive equations with time-dependent lateral boundary conditions. A spectral representation of the horizontal fields of the prognostic variables is done by introducing a double trigonometric series. This series is composed of sine or cosine series, depending on the field, and with a π-periodicity. It treats the advection of the gravity waves in a semi-implict way. An exact interpolation procedure is described which allows to project the data on a corodinate system which is transformed under a rotation of the pole axis.

Simulations with the SLAM are discussed, showing an improvement in the sharpness of meteorological features compared with a similar forecast with the global ECMWF operational model at lower resolution.

1. INTRODUCTION

One of the ways to improve the quality of the weather forecast is to increase the model resolution. New more powerful computers will undoubtedly allow much higher resolutions than T106, used operationally at ECMWF. Tests are already well under way using T159 and even T213 resolutions which are very expensive exercises in terms of computing time and other resources. This is why a limited area model was developed at ECMWF, as it will allow very high resolution experiments at a low price. Tests with the global model have already shown the need for adapting the existing parametrizations and numerical schemes as resolutions is increased. New techniques will have to be developed to make full use of the future possibilities given by more powerful computers.

The ECMWF Limited Area model has been written in such a way that it is fully compatible with the operational model, which reduces considerably the maintenance work. Furthermore, any scheme or parametrization can be adapted from one model (SLAM) to the other (operational model) with little or no cost. The SLAM is in fact part of the code library. The user can make his choice between SLAM and operational model (OM) with a single switch.

Several attempts to design a spectral limited area model have been tried but it is only since recently that successful models have been constructed. Tatsumi wrote a model very similar to this one (Tatsumi, 1986).

In Section 2, we shall describe the coordinate system used in the SLAM as well as the data retrieval. Two types of data are required, namely the initial conditions and the boundary conditions. They both come from the ECMWF archiving system . The exact interpolation procedure used for upper air fields is briefly described.

In Section 3, the model itself is described. The equations are not discussed as they are merely the primitive equations with mapping factors to take into account the SLAM coordinate system. The model uses a vorticity-divergence formulation. This, together with the spectral method, allows a simple semi-implicit scheme that makes the model very efficient in terms of computing time. A horizontal diffusion can also be applied easily.

In Section 4, a Davies-type relaxation scheme is reviewed (Davies, 1976). A relaxation of the divergent wind is proposed instead of the relaxation of the divergence itself. This, together with relaxation of the temperature and the logarithm of the surface pressure, avoids the reflection of gravity waves against the rigid-wall condition at the boundaries.

Finally, examples of forecasts are shown in Section 5. The 48 hour simulation of hurricane Gloria on the 29th September 1985, and runs over the Pacific Ocean are discussed.

2. COORDINATE SYSTEM AND DATA RETRIEVAL

In order to allow the model to run over any area, even an area including the pole, a rotated coordinates system is used. The centre of the considered area is put over the equator of a transformed sphere. As a consequence the grid spacing is almost regular in the north/south direction throughout the domain.

This diminishes the aliasing errors created by a distorted grid and it allows a maximisation of the time step computed using the well known formula:

$$\Delta t = \frac{L}{(V+S)(NLON-1)} \sim \frac{\Delta x}{V+S}$$

where V = the maximum speed of wind

S = the speed of sound and

Δx = the smallest grid distance

L = extent of the domain

NLON = number of grid points

The interpolation from the standard to the transformed coordinate system is done spectrally for those fields having a spectral representation in the ECMWF models (i.e. temperature, vorticity, divergence, wind, humidity, log surface pressure and also surface geopotential). Spectral interpolation is attractive because there is no loss of meteorological information (for a triangular truncation of the global data). The transform can be written:

$$\tilde{S}_n^m = \sum_K M_n^{mk}(\theta) S_n^k$$

where S are the spectral coefficients in the standard coordinate system.

$\tilde{S}$ are the spectral coefficients in the rotated system.

M is a rotation matrix whose elements are Jacobi Polynomial coefficients. These are functions of the rotation angle θ which is the latitude of the centre of the considered area.

The field represented by $\tilde{S}$ is projected on to the grid point space on the LAM grid using an inverse Legendre transform.

The surface fields are unsuitable for spectral representation. They are computed on the LAM grid using a bi-linear interpolation. The data are retrieved from the ECMWF archiving system. The land/sea mask as well as the orographic variances are computed at high resolution from the US Navy data set.

3. THE MODEL

3.1 General

Interpolation of background variables

The variables of the model are temperature, humidity, vorticity, divergence and the surface pressure. The large scale information is taken into account through the so-called background variables, which are defined at regularly spaced times intervals. They are computed as described in Chapter 2 and can be the analysis, the initialized analysis or the forecast fields. Generally, they are only available every 6 hours or 12 hours depending on the type of data. They are interpolated linearly in time to make them available every time step.

$$B(t) = B(T_i) \times \frac{T_{i+1} - t}{T_{i+1} - T_i} + B\left(T_{i+1}\right) \frac{t - T_i}{T_{i+1} - T_i}$$

where T is the time at which the background is available.

t is the current time

B the background

The large scale information also consists of large scale derivatives and the wind which will be needed in the dynamics.

Two time levels of background fields are read by the model to allow for linear interpolation.

Model variables

The basic variables are defined as the sum of the background variable and a perturbation which is the departure from the large scale information. (So the basic variables are true fields and not perturbations). We define the spectral perturbations P as:

$$P = \sum_{i,j} W_{(i,j)} \left(F_{(i,j)} - B_{(i,j)}\right) . \cos m \frac{\lambda_i \pi}{\lambda_B} \cos n \frac{\theta_j \pi}{\theta_B}$$

where W is a weighting function

F the forecast field

B the Background field

λ and θ are the longitude and latitude in the transformed system of coordinates.

λ_B and θ_B are the angular extensions of the domain.

The spectral fields are expanded in terms of trigonometric functions (i.e. cosine or sine). The departures (perturbations) of the model variables are expressed in terms of cosines, except the vorticity which is expanded in a sine series in order to have the same representation of the rotational and divergent part of the wind.

$$\mu = \frac{\partial}{\partial\lambda} \nabla^{-2} \tilde{D} - \frac{\partial}{\partial\theta} \nabla^{-2} \tilde{\xi}$$

$$\sigma = \frac{\partial}{\partial\lambda} \nabla^{-2} \tilde{\xi} - \frac{\partial}{\partial\theta} \nabla^{-2} \tilde{D}$$

The input of the model consists of the initial spectral perturbations (which can be zero if no initial modifications like initialization or adjustment to a finer orography are done prior to the forecast), the background fields and the surface fields.

Modification of variables

For convenience the latitude instead of the sine of the latitude is used as north/south coordinate. As a consequence modified vorticity and divergence are used, defined by

$$\tilde{\xi} = \frac{1}{a} \left(\frac{\partial v}{\partial\lambda} - \frac{\partial U}{\partial\theta}\right)$$

$$\tilde{D} = \frac{1}{a} \left(\frac{\partial U}{\partial\lambda} + \frac{\partial v}{\partial\theta}\right)$$

where v is the meridional wind

u is the zonal wind

$U = u\cos\theta$

θ is the latitude

λ is the longitude

and a is the radius of the earth

Coriolis parameter

The Coriolis parameter is now also a function of longitude in the rotated coordinate system

$$f(\lambda, \theta) = 2\Omega \{\sin\theta \cos\theta_c + \sin\theta_c \cos\theta \cos\lambda\}$$

where λ, θ are the longitude and latitude in the transformed system
θ_c is the latitude of the centre of the limited area.

Radiation

The radiation code of the operational model was modified as it uses the physical longitudes and latitudes to calculate the solar input of energy.

3.2 The course of the model

The historical variables, split into background and perturbations (in spectral form) are reconstituted as follows (i and j are gridpoint indices):

$$H(x_i, y_j) = \sum_{n,m} P_n^m \cos m \frac{\lambda_i}{\lambda_B} \pi \cos n \frac{\theta_i}{\theta_B} \pi + B(x_i, y_j)$$

The derivatives needed in the dynamical computations are also computed from the background fields

$$\frac{\partial H}{\partial r}(x_i, y_i) = \sum_{n,m} P_n^m \frac{\partial}{\partial r} [\cos \frac{m\lambda_i \pi}{\lambda_B} \cos \frac{n\theta_i \pi}{\theta_B}] + \frac{\partial B}{\partial r}(x_i, y_i)$$

All ingredients being available, one can compute the dynamical and physical tendencies using the routines of the ECMWF global model. Mapping factors appear in the equations to take the change in the definition of vorticity and divergence into account as well as the adoption of latitude instead of sine of the latitude as north/south coordinate.

The time stepping is done using a leap-frog Asselin filtered scheme (Asselin, 1972). The Asselin filter operator can be written as:

$$\Delta_{tt} X = \varepsilon\big(X(t+\Delta t) + X(t-\Delta t)\big) + (1-2\varepsilon) X(t)$$

As the background fields are interpolated linearly in time we have $\Delta_{tt} B=0$, this implies that it does not matter if the time filtering is applied to total grid-point fields or to the deviations of the total fields from their background values. This enables the spectral limited area model to be constructed without substantial modification of the time stepping algorithms and general code structure, of the global model. The same consideration can be applied to the semi implicit computations.

As the time-stepping is done, we project the total fields in spectral space using cosine expansions.

Semi Implicit

It is now possible to apply the semi-implicit scheme for gravity wave terms. For the global model, the final equation for the spectral coefficients of D^t (eq. 2.3.40 of the ECMWF forecast model documentation) is:

$$D^{t\,m}_{n} = \left(1 + \frac{n(n+1)}{a}\,\Gamma\right)^{-1} DT'^{\,m}_{n}$$

Here the matrices to be inverted depend only on the meridional wave number n. However, for the limited area model

$$D^{t\,m}_{n} = \left(1 + \frac{\pi^2}{a^2}\left[\frac{m^2}{\lambda_B^2} + \frac{n^2}{\theta_B^2}\right]\Gamma\right)^{-1} DT'^{\,m}_{n}$$

Precomputation and storage of the inverse matrices are required for each m and n. Since even at very high resolution the maximum wave number is not exceeding 63 this is still feasible.

Diffusion

If X^m_n is a spectral coefficient of X computed prior to diffusion, then by direct analogy with the global model (section 2.4 of ECMWF documentation manual) the diffused value is:

$$\bar{X}^m_n = X^m_n \left\{ 1 + 2K\Delta t\,\frac{1}{a^4}\left(\frac{m^4}{\lambda_B^4} + \frac{n^4}{\theta_B^4}\right)\right\}^{-1}$$

This diffusion is not precisely the ∇^4 form of the global model, but differences are not significant, due to the use of rotated coordinate system.

Truncation

Since Gottlieb (1977) and Bourke et al. (1977) it is known that aliasing errors due to the folding of the spectra in non-linear products can be avoided by the lower part of the spectra. That is also true for this model and we define the maximum wave number in a given direction as:

$$M = (2 \times NLON-3)/3$$

where NLON is the number of grid points

M is the maximum wave number

In order to preserve the isotropy one first computes the zonal wave number and then for each such index the maximum meridional wavenumber N(m) by:

$$N(m) = INT \left\{ N \cdot \sqrt{1 - \frac{m^2}{M^2}} \right\}$$

where m varies from 0 to M

$N = (2 \times NLAT-3)/3$

NLAT is the number of grid points in the north/south direction.

This generates an ellipsoidal truncation, which will not give a perfect isotropy because we handle integer wave numbers only.

4. RELAXATION

Relaxation is needed to create a smooth transition between the boundary fields and the interior fields and to force inside the large scale information. A Davies-type relaxation has been chosen (Davies, 1976). It operates in grid-point space and an extra spectral transform is needed to compute the unrelaxed basic variables on the LAM grid. The background is then interpolated in time at the current time level and the relaxation operator is applied:

$$\bar{H} = H.\alpha + B.(1-\alpha) = B + \alpha(H-B)$$

GLORIA 24/09/85 INITIAL CONDITIONS
SEA LEVEL PRESSURE T106 ANALYSIS

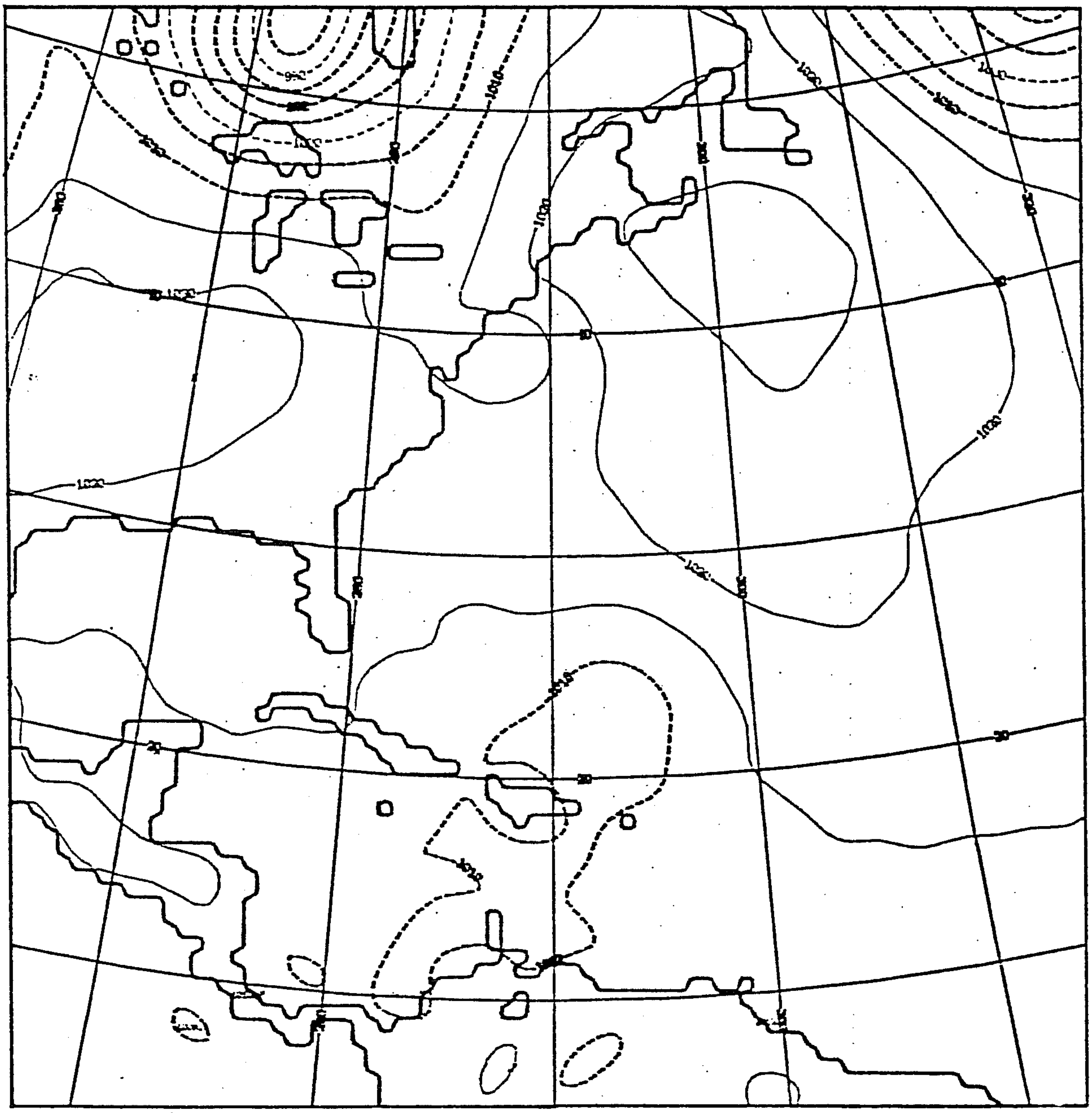


Fig. 1 Analysed sea level pressure at T106 resolution on the 24th September at 12 UTC.

where $\bar{H}$ is the relaxed field

H is the input field

B is the background containing the large scale information.

α is the relaxation parameter.

The relaxation parameter varies smoothly from 0 at the very boundary to 1 inside the domain. The relaxed field therefore coincides with the background at the boundaries and is left unchanged in the inner part of the domain.

Finally, we need to compute the perturbation in the spectral space. The grid-point perturbation is defined as:

$$P_{(x_i,y_i)} = H_{(x_i,y_i)} - B_{(x_i,y_i)}$$

which is the departure from the background. This quantity projected into spectral space will allow the computation of the fields and derivatives in the next time step.

5. NUMERICAL EXPERIMENTS

5.1 Hurricane Gloria

A good way to test the high resolution of the limited area model is to forecast the track and intensity of a hurricane. The tracks of these phenomena are highly dependent on the large scale movements. A simulation will therefore give a good indication of the accuracy of the relaxation scheme. Earlier experiments on hurricane Gloria have pointed out few deficiencies and led to the revised relaxation described in Chapter 4. Fig. 1 shows hurricane Gloria on the 24th September 1985. The hurricane is weak and situated approximately at 70° West and 20° North in the analysis. Fig. 2 shows the situation two days later. This analysis will be used as a verification although there is an uncertainty concerning the exact position of the hurricane even in the analysis. The atmospheric pressure in the eye is 999 mb which is obviously overestimated in this T106 analysis. Fig. 3 shows the sea level pressure of a 48 hour forecast using the ECMWF operational model with T106 resolution. Noise over mountainous regions should be disregarded as it comes from the rather crude extrapolation method used. The position of Gloria is too far North and too far East and the eye is no deeper than in the

GLORIA 26/09/85 VERIFICATION
SEA LEVEL PRESSURE T106 ANALYSIS

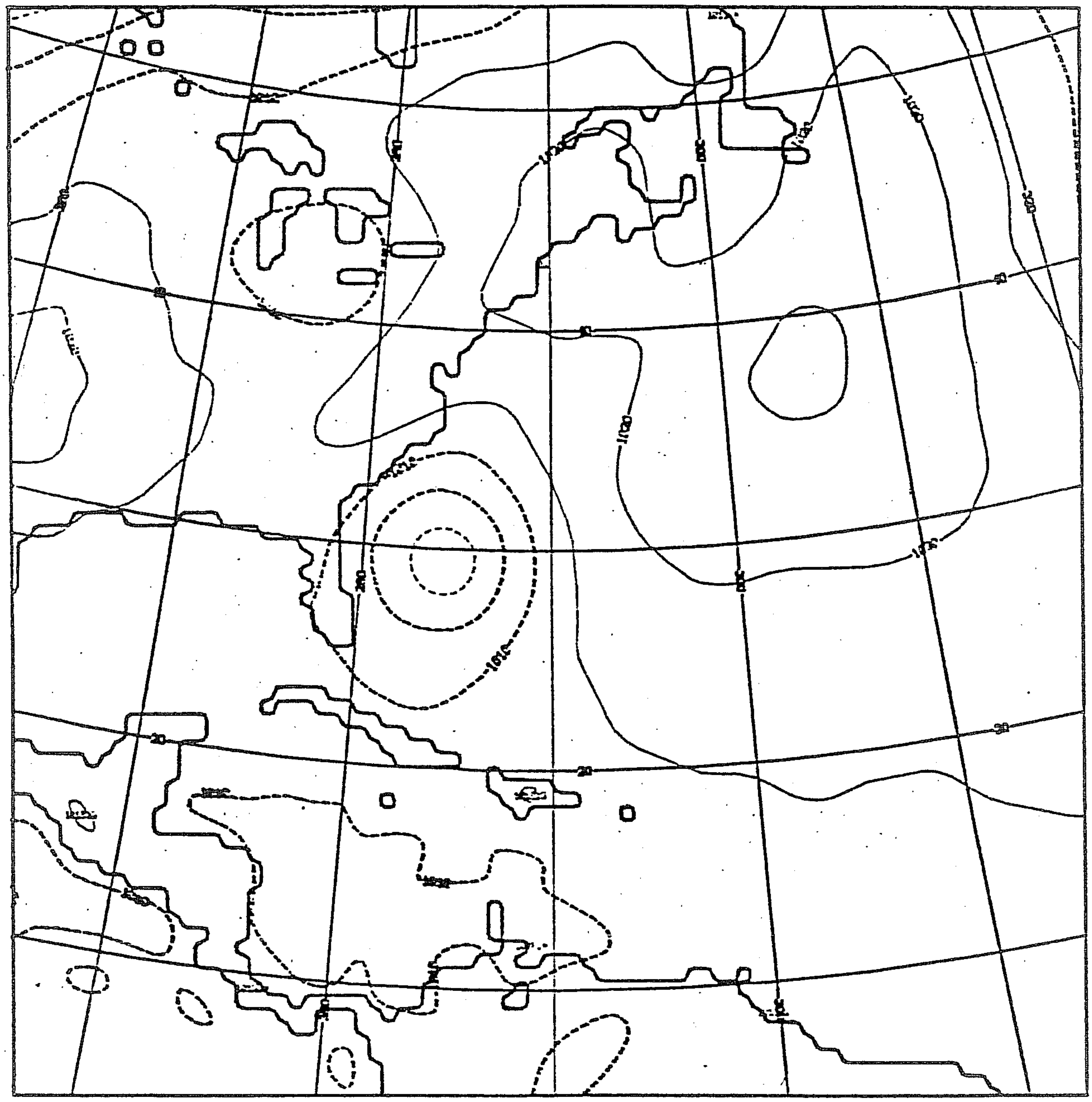


Fig. 2 Analysed sea level pressure at T106 resolution on the 26th September at 12 UTC.

GLORIA 24/09/85 + 48H FORECAST T106
SEA LEVEL PRESSURE

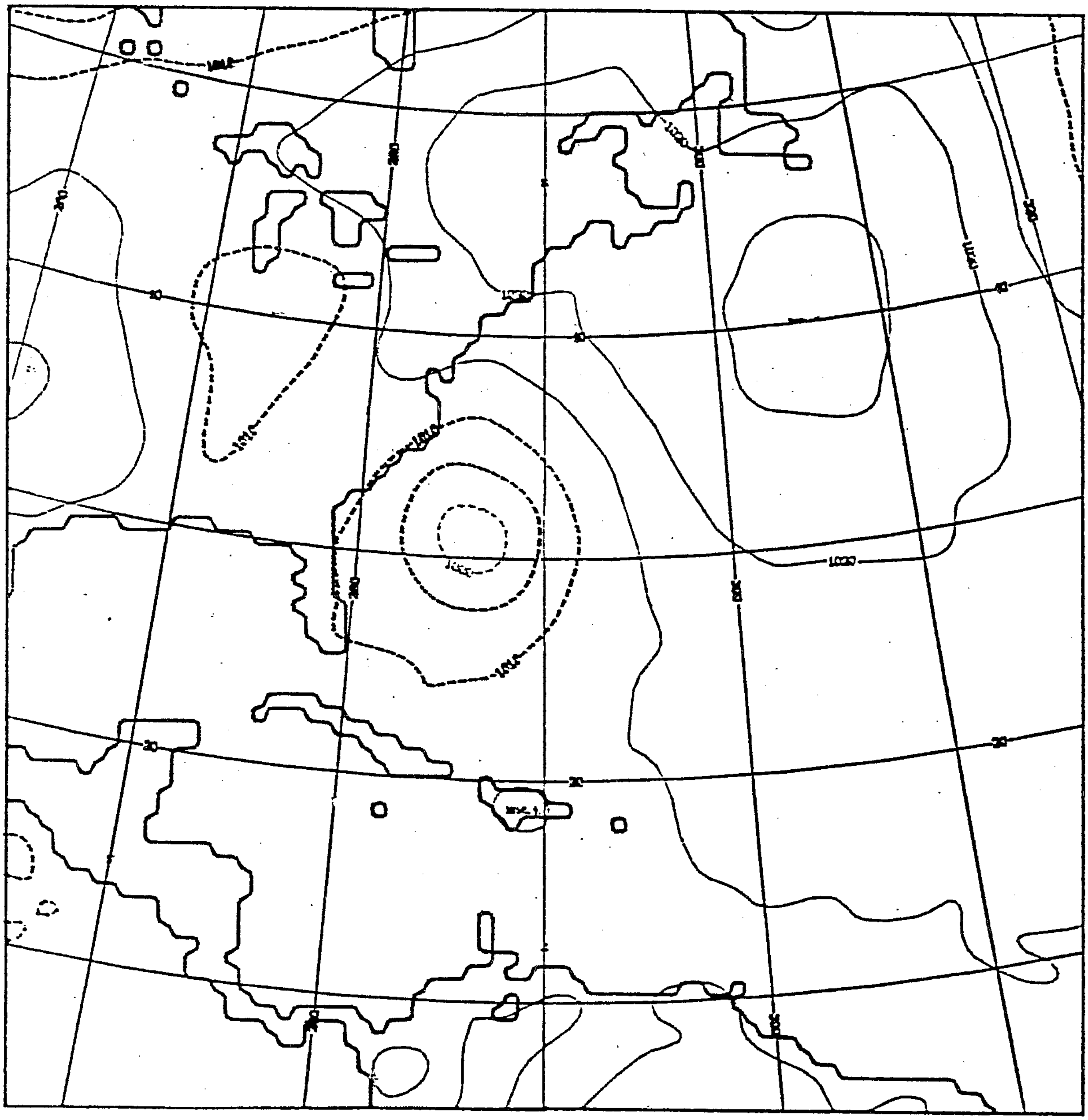


Fig. 3 Sea level pressure forecast by the T106 global model (48 hour forecast).

T106 analysis. Fig. 4 shows a 48 hour forecast of the ECMWF limited area model. The analysis has been used as background every 12 hours. The resolution is equivalent to T213 globally and the time step is 450 seconds (compared with 900 seconds in the T106 global experiment). The orography used in the LAM is still the T106 one.

The most striking effect of the increase of resolution is the deepening of the central low which goes down to a more realistic 982 mb. Also the position of the low seems a lot better than the T106 forecast. However one should keep in mind that the relaxation has been done using the analysis. Some noise seems to come from the boundaries, although some of it may be explained by the rough extrapolation to sea level. It indicates that more work is needed in the relaxation scheme.

T159 resolution experiments with the operational global model indicate also some generation of noise due to inadaptation of parametrizations and schemes at higher resolution than the operational T106.

5.2 Further experiments

Two further experiments are shown in Figs. 5 and 6. The SLAM and the OM have been run for 12 hours over the Pacific and over Europe. The SLAM resolution is equivalent to T213 and the background is a T63 resolution analysis defined every 12 hours. The initial date is the 24th September 1985 for run I and the 1st of January 1986 for run II. The T63 resolution was deliberately used for the background in order to have a large difference in resolution. The global model was run with a T106 resolution.

Run I is performed over the Pacific Ocean ie. without orography. Fronts can be observed in the domain (Fig. 5) and it is of interest to watch the front coming in at the top left corner of the domain, showing the efficiency of the relaxation scheme. This figure shows the vorticity maps for SLAM (top) and OM (bottom). One observes a sharpening of the fronts in the limited area model with respect to the global model.

Run II (Fig. 6) was performed over the European area. The presence of orography does not effect the limited area model and one again observes a sharpening of fronts.

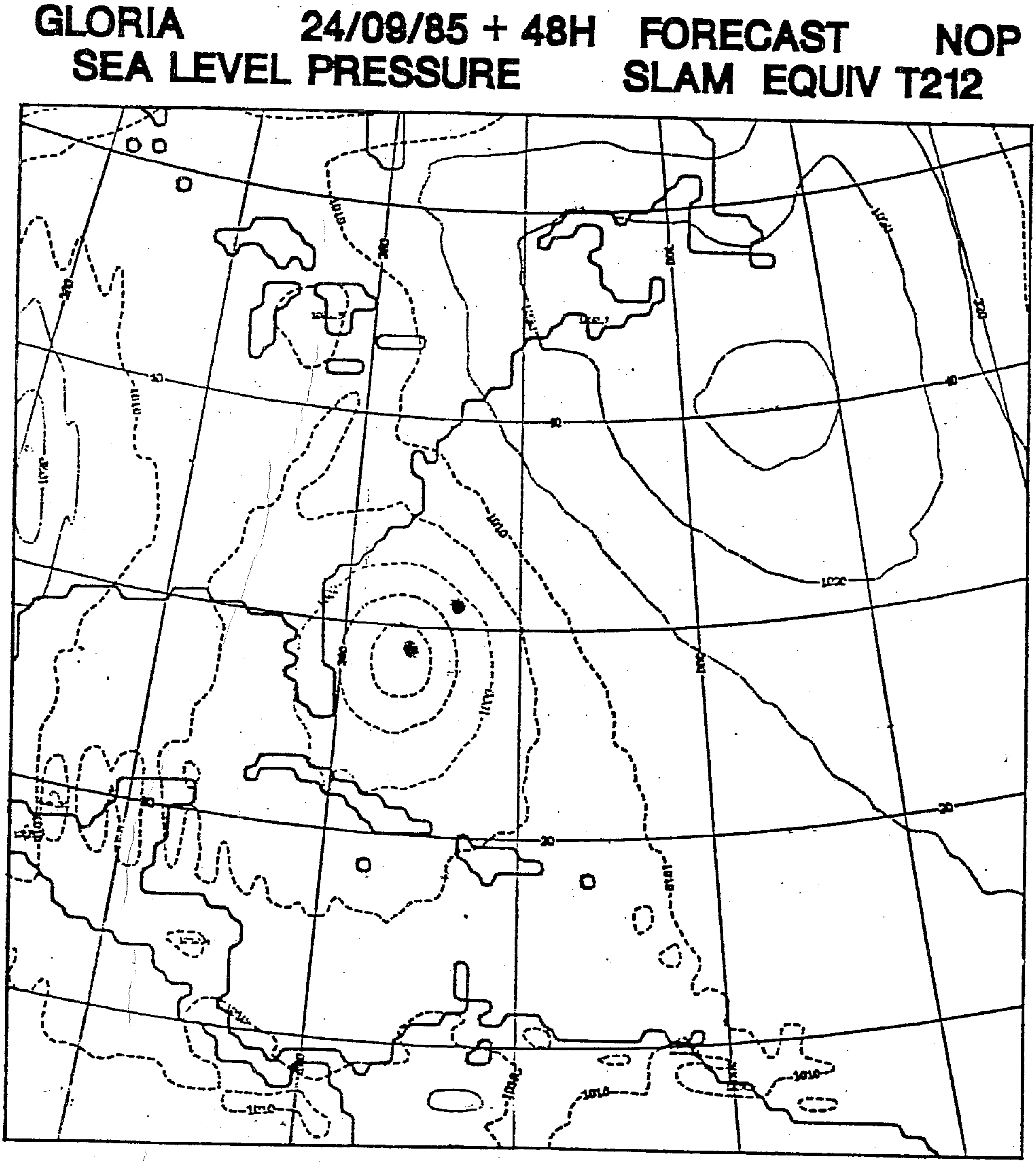
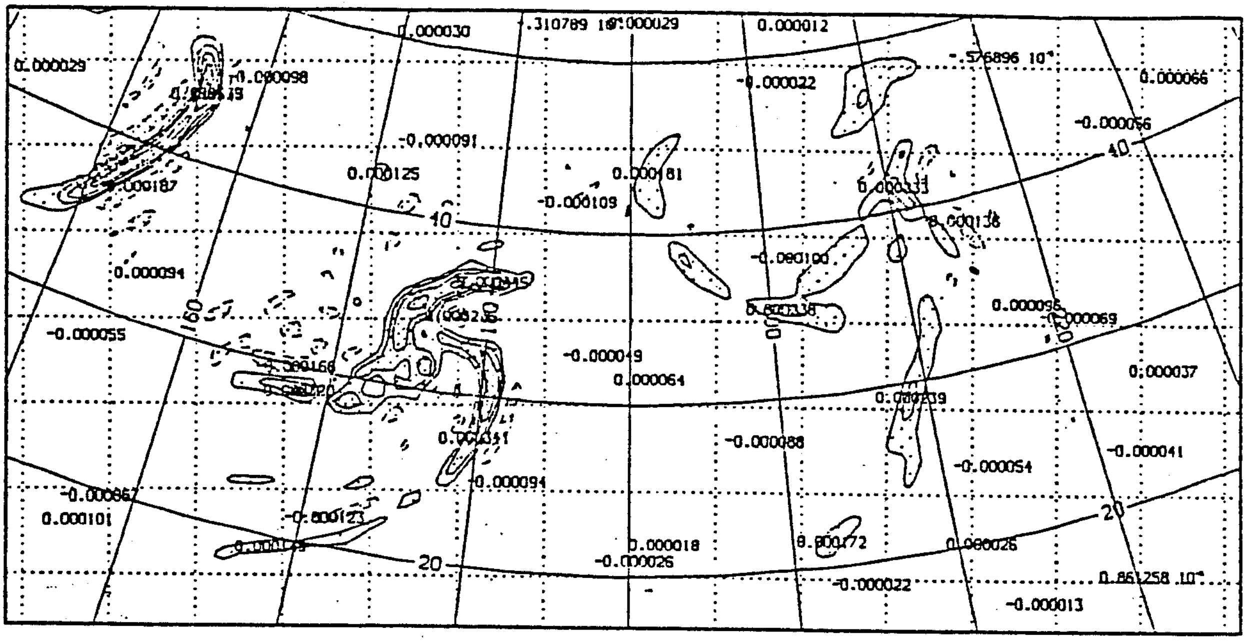


Fig. 4 Sea level pressure forecast by the T213 limited area model (48 hour forecast).

VORTICITY LEV = 16



VORTICITY T106 LEV = 16

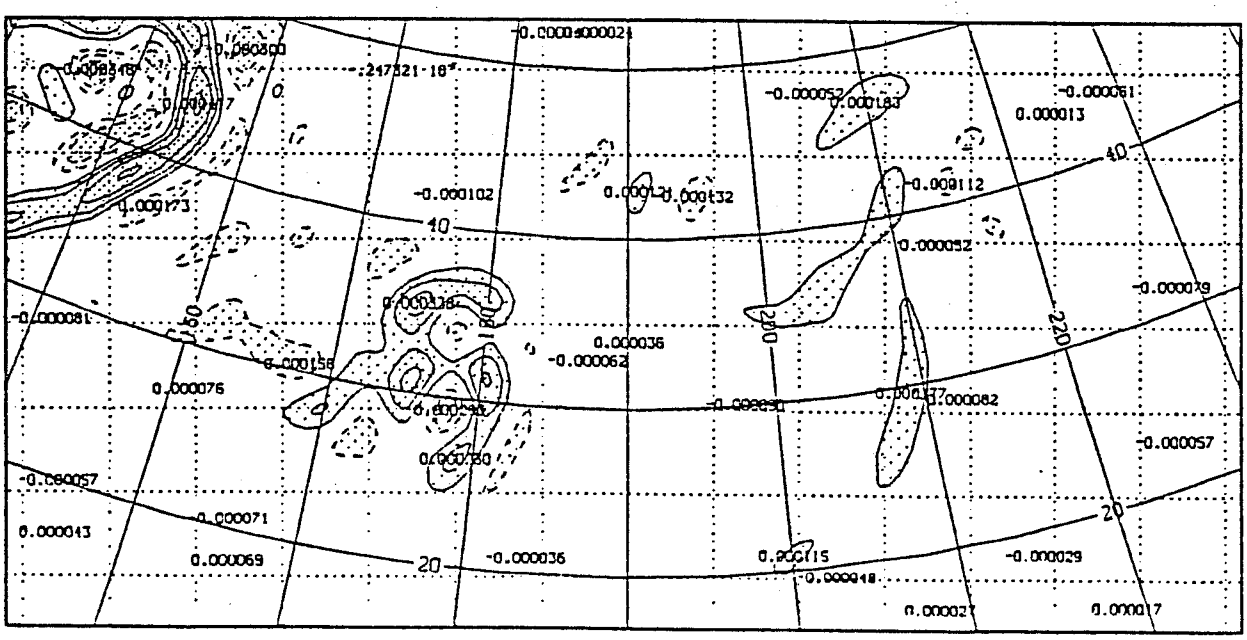
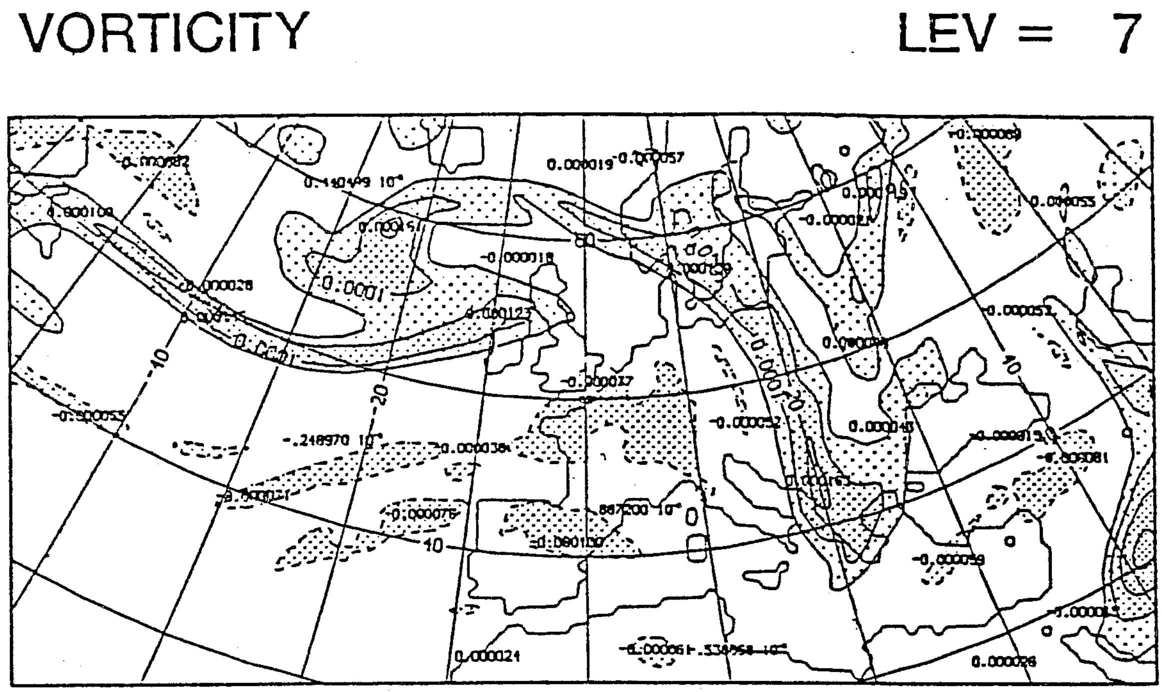


Fig. 5 Shows the results of 12 hour forecasts with the operational model (bottom map) with a T106 resolution. The top map shows the vorticity predicted by the SLAM with again a T213 resolution, 126 x 64 grid points and the dimension of the domain are 6000 x 3000 kilometres. The initialized analysis has been used as background. The domain is centred at 35°N and 170°E.



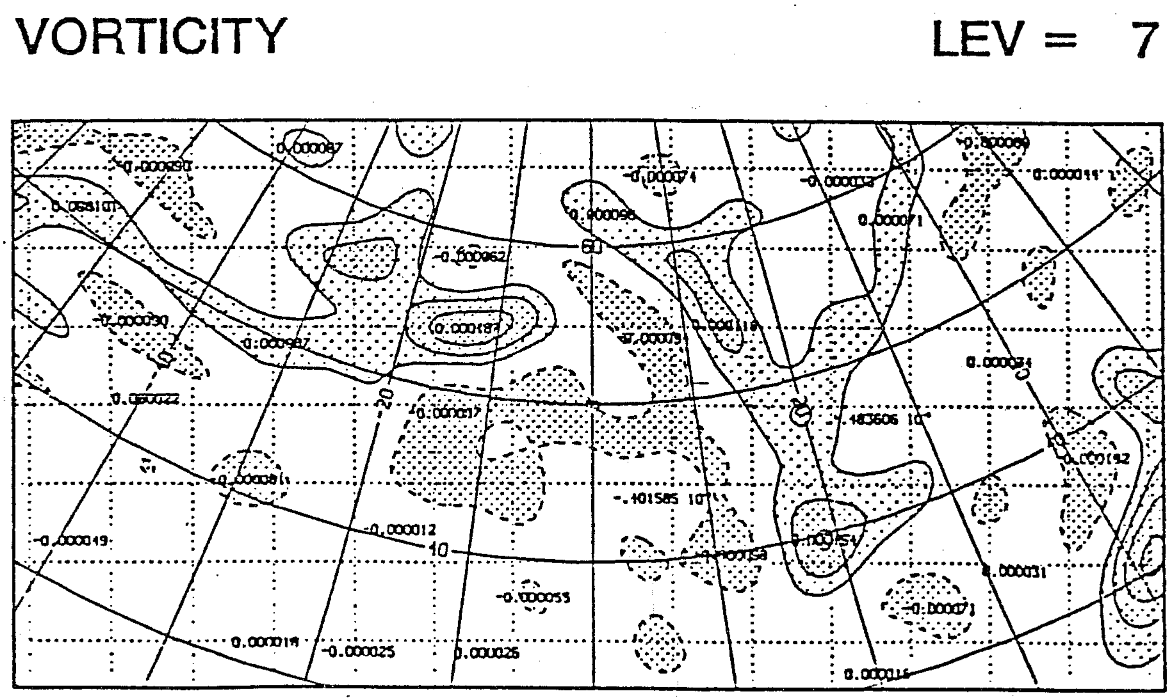


Fig. 6 Shows the results of 12 hour forecasts similar to the above except the domain is centred at 50°N and over the Greenwich meridian. Again, we see a strong sharpening of the frontal systems, inside the domain compared to the T106 forecast.

6. CONCLUDING REMARKS

The ECMWF limited area model has been designed to be as close as possible to the operational model in order to limit the maintenance work and ease the implementation of new parametrizations. It is a versatile tool to test any scheme at various resolutions and it will in particular be a useful way of testing parametrizations at resolutions that will be used in the future (i.e. T213).

Further research is required with respect to an orography and initialization.

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