BULK BOUNDARY LAYER SCHEMES

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1. INTRODUCTION

The challenge in parameterizing the atmospheric boundary layer (ABL) in large-scale models (LSM) is to find simple formulations that yet realistically describe the main characteristics of broad classes of ABL's. These formulations should have their basis in, and reflect the results of, detailed observational and modelling studies on the ABL. The ABL-parameterization as such is no instrument for ABL research but rather a result of it that comprises the conceptual insights in a clear and simple way. This preference for a simple ABL-parameterization is not only for computer-economical reasons but even more because of the complex interaction involved between various physical parameterizations in a LSM and the necessity to have a clear insight in the couplings and feedback loops. From the ABL-parameterization one should require a realistic description of: i) the surface fluxes of momentum, heat and moisture, ii) the vertical transports and the height of the ABL, iii) the behavior of typical ABL-clouds such as stratocumulus, and their interaction with the ABL-structure, iv) the coupling between the ABL-parameterization and other physical parameterizations such as cumulus convection and radiation.

In the concept of an ABL-parameterization one should realize that it is used in a LSM with a horizontal grid size much larger than the size of the largest eddies in the ABL and usually even much larger than the size of possible mesoscale features. All ABL-parameterizations are therefore formulated as one-dimensional ensemble-averaged models for which it is assumed that the conditions are statistically horizontally homogeneous on the grid-size of the LSM. Thus some averaging over a number of statistically identical energetic structures should be allowed and caution is necessary in cases where this is not warranted.

One of the pronounced features of the ABL is its highly variable depth, both in time and space. In stable conditons the depth of the ABL is typically of the order of 100-300 m, while in unstable conditions it can easily reach 1-2 km. One of the requirements for an ABL-parameterization is that it provides an accurate determination of the ABL-depth. There are a number of reasons for this. At the top of the ABL the transition between the turbulent air within the ABL and the non-turbulent free atmosphere occurs. This transition is often (at least in the unstable case) marked by sharp jump-like features in the thermodynamic and wind fields. The height of the ABL and the entrainment at the top, together with the surface fluxes, determine nominally the evolution of mean variables within the ABL. The top of the ABL usually coincides with the top of a stratocumulus cloud-deck in the upper part of the ABL. Also, cumulus clouds often have their base at the ABL-top, which therefore plays a role in the coupling of the ABL with the convection parameterization.

Considering the relevance of the ABL-height, effort is spent on formulating ABL-parameterizations that treat this height as an explicit variable. These bulk boundary-layer formulations start with vertically integrated governing equations over the ABL-depth. The latter is then an unknown explicit variable (we will not consider the oversimplification of taking the ABL-depth at some fixed height). In contrast, boundary-layer schemes with a number of levels in the ABL do not treat the ABL-depth explicitly, but it is determined implicitly through inspection of the vertical profiles of the turbulent fluxes. Usually, however, the vertical grid distances are increasing away from the surface, and the location of rather sharp features therefore has large truncation errors.

The use of bulk boundary-layer schemes only as equations for vertically integrated ABL-variables does not mean very much. Their success is dependent on whether the vertical profiles within the ABL have some general shape that depends only on a very limited number of quantities. For example, when there is enough turbulence generated in some measure, vertical mixing makes the profiles in the ABL rather flat. Thus a good approximation is that the profiles of quantities that are conserved under the mixing process are independent of height in the bulk of the ABL. This height-independence gives the opportunity to diagnose rather thin cloud layers in the upper part of the ABL which have a strong feedback on radiation and ABL-turbulence. The treatment of these thin cloud layers is quite difficult in multi-level boundary-layer schemes. It is one of the background ideas behind bulk boundary-layer schemes that there are broad enough classes of ABL's that can be characterized with vertical profiles that depend in a simple way on some overall parameters, such as e.g. stability, and that there is enough information available or to come from detailed ABL-studies to justify or refine this.

Another essential role in bulk-boundary-layer schemes is played by entrainment. Since the ABL-height is an explicit variable, there has to be an equation governing its evolution. The height of the ABL can of course change due to large-scale advection and due to large-scale convergence or divergence of mass in the ABL. There is also a dependence on the amount of deep cumulus convection that draws on the mass of the ABL and must lead to compensating shallowing of the ABL. This establishes a coupling between both the ABL and cumulus convection parameterization, which will not be discussed in this paper. In our context the most important way the ABL-depth may change is due to entrainment of mass at its top, through which the turbulent ABL is "intruding" into the non-turbulent and usually stably stratified air above, and thus its depth increases (relative to the mean vertical flow). A problem in bulk boundary-layer schemes that we will discuss is the formulation of this "entrainment equation" which in some way has to relate the energetics of the turbulence to the stability of the air into which the turbulence is entraining. The ABL is not always deepening, but can also shallow after a transition from unstable to stable conditions at the surface, or from daytime to nighttime.

In this paper we will summarize results of ABL-research on bulk formulations that are potentially useful for LSM. We will not go into the technical details of the incorporation into a particular LSM. Suarez et al. (1983) and Randall et al. (1985) discuss the incorporation extensively with special emphasis on the representation of thin cloud layers in a general circulation model. Their approach of using the top of the ABL explicitly as a coordinate surface in a GCM is quite attractive and a logical way of incorporating bulk ABL-schemes. When such an approach is used in a forecasting model additional technical problems regarding analysis and initialization have to be solved.

2. BULK EQUATIONS

The governing equations for bulk boundary-layer schemes are obtained from the primitive differential forms of the conservation equations by vertical integration over the ABL-depth h. Thus for a quantity ϕ the relevant variable is the bulk variable

$$\phi_M\,(x,y,t) = \frac{1}{h}\int_0^h \phi(x,y,z,t)dz. \tag{1}$$

The derivation of the bulk equations is quite straightforward. The real problem is the parameterization of the turbulence and entrainment which of course depends on the physical processes relevant in a particular type of ABL. We will discuss here a dry ABL (no condensational effects) and, for convenience, neglect horizontal advection. The relevant bulk equations in Boussinesq form are then:

$$h\,\frac{\partial\theta_M}{\partial t} = (\theta_{h_+} - \theta_M)(\frac{\partial h}{\partial t} - w_h) + \overline{\theta' w'}_s + F_s - F_{h_+}\ , \tag{2}$$

$$h\,\frac{\partial U_M}{\partial t} = (U_{h_+} - U_M)(\frac{\partial h}{\partial t} - w_h) + \overline{u' w'}_s + f(V_M - V_{gM}), \tag{3}$$

$$h\,\frac{\partial V_M}{\partial t} = (V_{h_+} - V_M)(\frac{\partial h}{\partial t} - w_h) + \overline{v' w'}_s - f(U_M - U_{gM}), \tag{4}$$

$$h\,\frac{\partial q_M}{\partial t} = (q_{h_+} - q_M)(\frac{\partial h}{\partial t} - w_h) + \overline{q' w'}_s\ . \tag{5}$$

Here θ is potential temperature, q specific humidity, U,V the horizontal wind, w_h the large-scale vertical velocity at the top of the ABL, F the total radiative flux, f the Coriolis parameter, and U_g,V_g the geostrophic wind. The subscript M denotes the bulk ABL-value, h_+ a value just above the ABL-top, s a surface value.

Note that these equations do not yet depend on the shape of the vertical profiles. So far they are the result of a mathematical operation. In order to step these equations for the bulk variables θ_M, q_M, U_M, V_M, h, forward in time, a number of quantities have to be provided.

a. "External" conditions: the large-scale vertical velocity w_h, the values above h (all quantities with index h_+), the boundary conditions for the earth's surface (surface temperature, humidity, roughness), geostrophic wind.

b. A method to calculate diagnostically the turbulent surface fluxes $\overline{\theta'w'}_s$, $\overline{q'w'}_s$, $\overline{u'w'}_s$, $\overline{v'w'}_s$ and also the radiative fluxes from the bulk variables and external conditions.

c. An entrainment equation for the rate of change of h, in this context for

$$w_e = \partial h/\partial t - w_h. \tag{6}$$

The conditions under a are grid-scale quantities. They are quite important for the practical implementation of the ABL-parameterization and will determine to a large extent its feasibility. In this context we will assume these quantities to be known. However, one should realize that accurate estimates of these external conditions are as important as the internal turbulence dynamics of the ABL.

The parameterization problem occurs in the calculation of the surface fluxes and in the entrainment rate. Both depend on the turbulence dynamics and will be discussed in the next sections.

3. SURFACE FLUXES

The surface fluxes of momentum, heat, and moisture have to be parameterized in terms of accessible model quantities, such as the bulk variables and surface conditions, through the following bulk transfer relations:

$$u_*^2 = C_D \, \|\vec{V}_M\|^2 \; , \tag{7}$$

$$\overline{\theta' w'_s} = C_H \, \|\vec{V}_M\| (\theta_s - \theta_M) \; , \tag{8}$$

$$\overline{q' w'_s} = C_Q \, \|\vec{V}_M\| \, (q_s - q_M) \; , \tag{9}$$

where u_*^2 denotes the magnitude of the surface stress and C_D, C_H, C_Q are bulk transfer coefficients. The angle between the surface stress and the bulk wind vector has to be determined separately and is denoted by α_M. In early ABL-parameterizations C_D, C_H, C_Q were assumed equal and assigned fixed numerical values. In more realistic ABL-parameterizations these transfer coefficients should be specified as functions of the state of the ABL. When in (7)-(9) the index M would be replaced by an index z, where z is a height within the surface layer, then the transfer coefficients could be calculated according to the well-known Monin-Obukhov similarity theory for the surface layer. In bulk models, however, the surface layer is not resolved and (7)-(9) are bulk transfer relations for the ABL as a whole. Research on these bulk transfer relations has been carried out through matching of the surface layer and outer layer similarity profiles (e.g. Arya, 1977), resulting in the following general implicit forms, that are more usual than (7)-(9):

$$\kappa \, \|\vec{V}_M\| / u_* = \{(\ln(z_o/h) + A_M)^2 + B_M^2\}^{\frac{1}{2}} \tag{10}$$

$$\tan^{-1}(\alpha_M) = B_M \, \mathrm{sign}(f)/(\ln z_o/h + A_M) \tag{11}$$

$$-\frac{\kappa u_*}{\overline{\theta' w'}_s}(\theta_M - \theta_S) = -\ln(z_o/h) - C_M \, , \tag{12}$$

$$-\frac{\kappa u_*}{\overline{q' w'}_s}(q_M - q_S) = -\ln(z_o/h) - D_M, \tag{13}$$

Here k is the von Karman constant (0.4), z_o the surface roughness length and A_M, B_M, C_M, D_M are universal functions of the similarity variables. Observations and detailed modelling studies have indicated that A_M-D_M depend most strongly on the stability parameter h/L, where L is the well-known Obukhov length scale

$$L = -u_*^3 T/(g\kappa \, \overline{\theta' w'_s}). \tag{14}$$

The dependence of A_M-D_M on other possible similarity variables, e.g. related to entrainment, baroclinity, ratio between the height h and the scale height u_*/f, is weaker.

The actual form of the universal functions A_M-D_M dependence on h/L has to come from empirical results which, however, show a large scatter. Arya (1984) summarizes some "best estimates" as follows:

$$\left.\begin{aligned} A_M &= \tfrac{1}{2}\ln(-h/L)+2.3 \\ B_M &= 0 \\ C_M &= D_M = A_M \end{aligned}\right\} \quad \text{for } -h/L > 2 \text{ (unstable)}$$

$$\left.\begin{aligned} A_M &= 2.5 - 0.96\, h/L \\ B_M &= 1.1 + 1.15\, h/L \\ C_M &= D_M = 7.0 - 3.0\, h/L \end{aligned}\right\} \quad \text{for } -h/L < -2 \text{ (stable)} \tag{15}$$

and an interpolation between these expressions for $\|h/L\| < 2$.
In these expressions, the relations for A_M and B_M in unstable conditions are relatively well tested (Garratt et al., 1982), while the others, especially in stable conditions, are not well established (Nieuwstadt, 1981; Driedonks et al., 1985).

For use in numerical models the implicit relations (15) are not very convenient since determination of the surface fluxes requires an iterative procedure. Therefore the relations are more easily rewritten in the form (7)-(9), (including a relation for α_M), and as a stability parameter it is more convenient to use the layer-averaged bulk Richardson number

$$Ri_M = \frac{g}{\theta_M} \frac{\theta_M - \theta_S}{\|\vec{V}_M\|^2} . \tag{16}$$

The bulk transfer coefficients are then expressed as functions of z_o/h and Ri_M, which are readily available in a bulk boundary-layer scheme. These transfer functions $C_D(z_o/h,\ Ri_M)$, $C_H(z_o/h,\ Ri_M)$, $C_Q(z_o/h,\ Ri_M)$, as well as $\alpha_M(z_o/h,\ Ri_M)$, can be evaluated from (15) and are represented in nomograms by Arya (1977) that can be used to fit suitable expressions.

4. THE ABL-HEIGHT

The evolution of the ABL-height h depends strongly on the turbulence within the ABL. Much attention has been given to the formulation of entrainment relations for an unstable, cloud-free ABL which have been successfully tested against observations, e.g. Driedonks (1982), and Driedonks and Tennekes (1984). For the height of the stable ABL also reasonably successful expressions have been formulated.

4.1. The unstable ABL

For an unstable ABL for which the stability parameter $-h/L \gtrsim 5$, it is generally accepted from observations that the vertical profiles, e.g. of $\theta(z)$, are almost flat and that the entrainment zone that forms the transition of the ABL to the stable air aloft is a very thin layer. This has led to the well-known representation of the ABL as a mixed layer, sketched in Figure 1. In the ABL that we consider, not only convection from the surface is producing turbulent kinetic energy (TKE). We will also have to take into account the TKE generated by wind shear. However, in this context we exclude production of TKE by some other distinct internal source as in the case when radiational processes are important, e.g. when a cloud deck is present in the ABL.

In such a well-stirred ABL there are in general three distinct mechanisms that produce turbulence and possibly entrainment:

a) convection from the surface, associated with the velocity scale w_*, defined by $w_*^3 = \frac{g}{T_s} h \overline{\theta' w'_s}$,

b) friction at the surface, associated with the friction velocity u_*, and

c) shear-generation of turbulence by wind shear at the top, associated with the velocity jump $\Delta\vec{V}$ over the entrainment zone.

Opposed to these entrainment-producing mechanisms in the ABL are the counteracting influences at the top. There, relatively warmer air has to be brought down into the ABL at the expense of turbulent kinetic energy. So the main counteracting influence on w_e is the jump in potential temperature $\Delta\theta$.

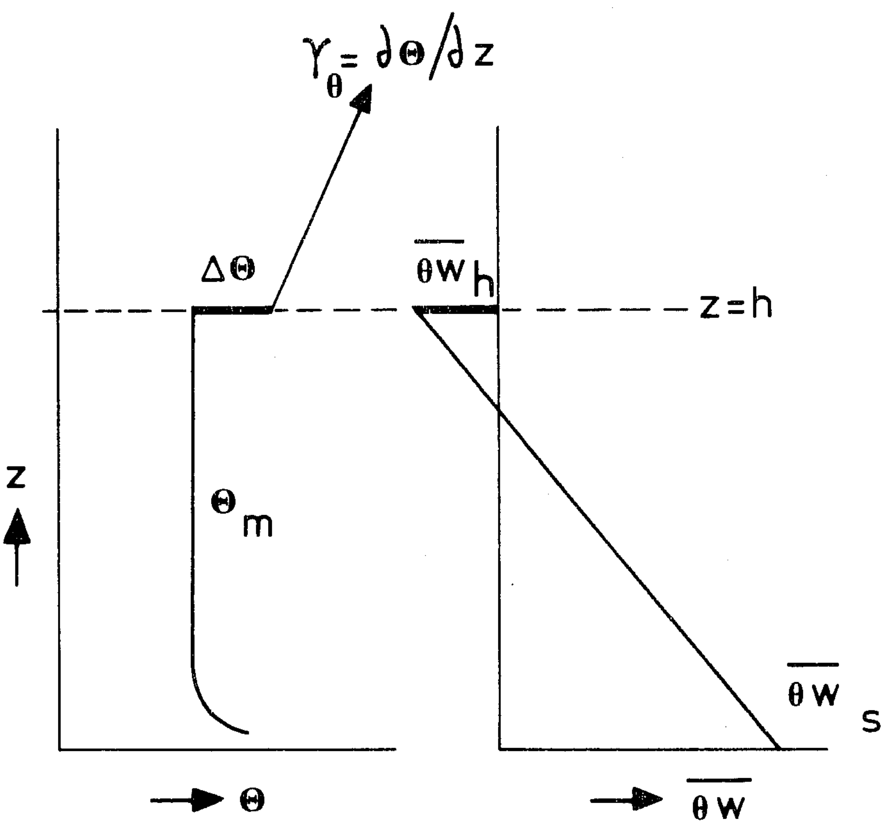

Figure 1. Schematic profiles of Θ and $\overline{\theta w}$ in a mixed layer model.

A variety of models has been developed to relate the entrainment rate w_e to the afore-mentioned production mechanisms and the opposing influences at the top. For some cases consensus has been reached, but up till now no general entrainment relation has been formulated that describes the details of all possible combinations of the three production mechanisms and that is fully tested against observations. The most recent effort is by Deardorff (1983). The main reasons for the disagreement are the difficulties encountered in the parameterization of the amount of TKE produced in the ABL that is actually available for entrainment. Since the dissipation in the ABL is large, this available energy is only a small fraction of the total production rendering parameterizations sensitive to small inaccuracies. Another problem is the lack of observations from either the laboratory or the real atmosphere that are comprehensive enough to test these parameterizations, especially in the case of shear-generated turbulence.

In this paper we will take a rather practical point of view and bias our approach toward relatively simple models for the ABL that have been tested to give satisfactory results for the evolution of h(t) and $\Theta_M(t)$ in comparison with atmospheric observations.

The entrainment equation is usually derived from consideration of the budget of TKE since entrainment goes at the expense of TKE and it is the buoyancy flux $\overline{\theta w}_h$ (or $\overline{\theta_v w}_h$) that appears in this budget. However, closure can only be achieved after parameterization of all the terms (except of course the buoyancy flux) in the TKE-budget. It is here that entrainment models start to diverge.

Following Tennekes and Driedonks (1981) we consider the TKE-budget at $z = h$. This reads

$$\underset{T_h}{\left(\frac{\partial \overline{e}}{\partial t}\right)_h} = \underset{P_h}{- \overline{u'w'_h} \left(\frac{\partial U}{\partial z}\right)_h - \overline{v'w'_h}\left(\frac{\partial V}{\partial z}\right)_h} + \underset{B_h}{\frac{g}{T} \overline{\theta' w'_h}} - \underset{F_h}{\left(\frac{\partial}{\partial z} \left(\overline{e'w} + \frac{\overline{p'w'}}{\rho}\right)\right)_h} - \underset{D_h}{\varepsilon_h} , \tag{17}$$

where $\overline{e}$ and e' are the mean and fluctuating turbulent kinetic energy and ε the dissipation rate. Other symbols are according to the usual notation. The symbolic notation refers to the temporal rate of change (T_h) of $\overline{e}$, the shear production (P_h), the buoyancy term (B_h), the flux convergence (F_h) and the dissipation (D_h).

In parameterizing the TKE-budget we have to provide scaling laws for each of the terms. Here we simply assume that all length scales are proportional to h, all turbulent velocities scale on w_m (where w_m is a representative velocity scale for the bulk of the ABL) and that $(\partial U \partial z)_h$, $(\partial V/\partial z)_h$ scale on $\Delta U/h$, $\Delta V/h$. In this way the term T_h becomes in parameterized form: $T_h = C_t \frac{w_m^2}{h} w_e$, and the dissipation as well as the flux convergence both scale on w_m^3/h. Thus we take both terms together. Furthermore we use the relations $-\overline{u'w'_h} = w_e \Delta U$, $-\overline{v'w'_h} = w_e \Delta V$, where $\Delta U = U_{h_+} - U_M$, $\Delta V = V_{h_+} - V_M$ (see eq. 2-5). With this parameterization the TKE-budget reads

$$C_t \frac{w_m^2}{h} w_e = C_M\{(\Delta U)^2 + (\Delta V)^2\} \frac{1}{h} w_e + \frac{g}{T} \overline{\theta' w'_h} + C_F \frac{w_m^3}{h} , \tag{18}$$

where C_t, C_M, C_F are constants. After some rearranging we can write:

$$\frac{w_e}{w_m} = \frac{C_F}{C_t - C_M\{(\Delta U)^2 + (\Delta V)^2\}/w_m^2 + Ri} , \qquad (19)$$

with Ri a Richardson number defined as

$$Ri = \frac{g}{T} \frac{h \ \Delta\Theta}{w_m^2} . \qquad (20)$$

For the velocity scale w_m, Tennekes and Driedonks (1981) take an interpolation between the convective velocity scale w_* and the friction velocity u_* in the form

$$w_m^3 = w_*^3 + (A'/C_F)u_*^3 . \qquad (21)$$

In eq. (18)-(21), C_t, C_M, C_F, and A' are all empirical constants, still quite a lot for such a simple model.

A few properties of this parameterization must be noted. First, if ΔU, $\Delta V \to 0$, $u_* \to 0$, and if $w_e \ll w_m$ (usually the case), then this parameterization reduces to the well-known constant heat flux ratio $\overline{\theta' w'_h} = - C_F \overline{\theta' w'_s}$. Further, when ΔU, $\Delta V \to 0$, and $Ri \to 0$, then the entrainment rate remains finite with $w_e/w_m = C_F/C_t$.

In the parameterization (18) we refrained from introducing a separate length scale Δh associated with the entrainment zone, and from estimating the local dissipation and velocity gradients with this length scale. An extensive elaboration on this issue was given by Deardorff (1983). He makes $\Delta h/h$ a function of three different Richardson numbers:

$$Ri_* = \frac{g}{T} \frac{h \ \Delta\theta}{w_*^2} ,$$

$$Ri_\tau = \frac{g}{T} \frac{h\ \Delta\Theta}{u_*^2}, \tag{22}$$

$$Ri_v = \frac{g}{T} \frac{h\ \Delta\Theta}{(\Delta U)^2 + (\Delta V)^2} .$$

His approach is equivalent to making the constants C_t, C_M, and C_F in (18) functions of these same three Richardson numbers. Deardorff derives these functions from a limited number of laboratory experiments and some suitable assumptions. Although his results are applicable to the full range of values of Ri_*, Ri_τ and Ri_v, it is also clear that the very limited amount of experimental data on which they are based still leaves a lot of uncertainties. At the moment we doubt that it will lead to better predictions in the atmospheric problem we are considering.

The entrainment equation (18) has un unattractive consequence that was pointed out by Driedonks (1982) and Manins (1982). When we neglect for a moment the C_t-term (usually small) then it appears that the entrainment rate w_e will become negative when $Ri_v < C_M$. This, of course, is unrealistic. Manins (1982) capitalized on this issue by stating that entrainment takes place <u>only</u> at the critical value $Ri_v = C_M$ and that $w_e = 0$ when $Ri_v > C_M$. Thus entrainment proceeds in jumps; whenever Ri_v becomes subcritical the ABL-height grows suddenly such that Ri_v becomes critical again. We do not consider this approach very attractive since it may lead to entrainment rates that differ from timestep to timestep by an order of magnitude. Driedonks (1982) also noted this singular behavior of eq. (18), but instead assumed tentatively that the effect of $\Delta\vec{V}$ on the entrainment could be incorporated in the other shear generation term associated with the surface friction. He omitted the C_M-term completely from (18) and instead changed the value of A' in (21) to another empirical constant A. Thus he used as entrainment relations:

$$C_t \frac{w_m^2}{h} w_e = \frac{g}{T} \overline{\theta' w'_h} + C_F \frac{w_m^3}{h} ,$$
$$w_m^3 = w_*^3 + (A/C_F) u_*^3 , \qquad (23)$$

and compared this expression with observations to estimate the constants to have the values $C_t = 1.5$, $C_F = 0.2$, $A = 5$. An example of how such a model performs in comparison with observations is given in Figure 2.

4.2 The stable ABL

The structure of the stable ABL is less well-established than that of its unstable counterpart. The turbulence in the stable ABL is not very strong and has to be maintained by mechanical production through wind shear, while buoyancy and dissipation act as sinks for turbulent kinetic energy. In the stable ABL other processes like radiative cooling can also be quite important. Especially in cases with low wind speed the turbulent fluxes are small and the cooling by longwave radiative flux divergence can be of the same order of magnitude as the cooling by the turbulent flux divergence (Garratt and Brost, 1981; André and Mahrt, 1982).

Due to the combined effects of turbulence and longwave radiation in the stable ABL, there is a difference between the height from the surface up to which there is a significant amount of turbulence (h_{turb}) and the height up to which there is a significant amount of cooling of the initial temperature profile (h_{inv}), the latter being larger. For the height of the turbulent layer several expressions have been developed from numerical and analytical models. Most of these expressions deal with the depth of the turbulent layer in a steady state, i.e. in

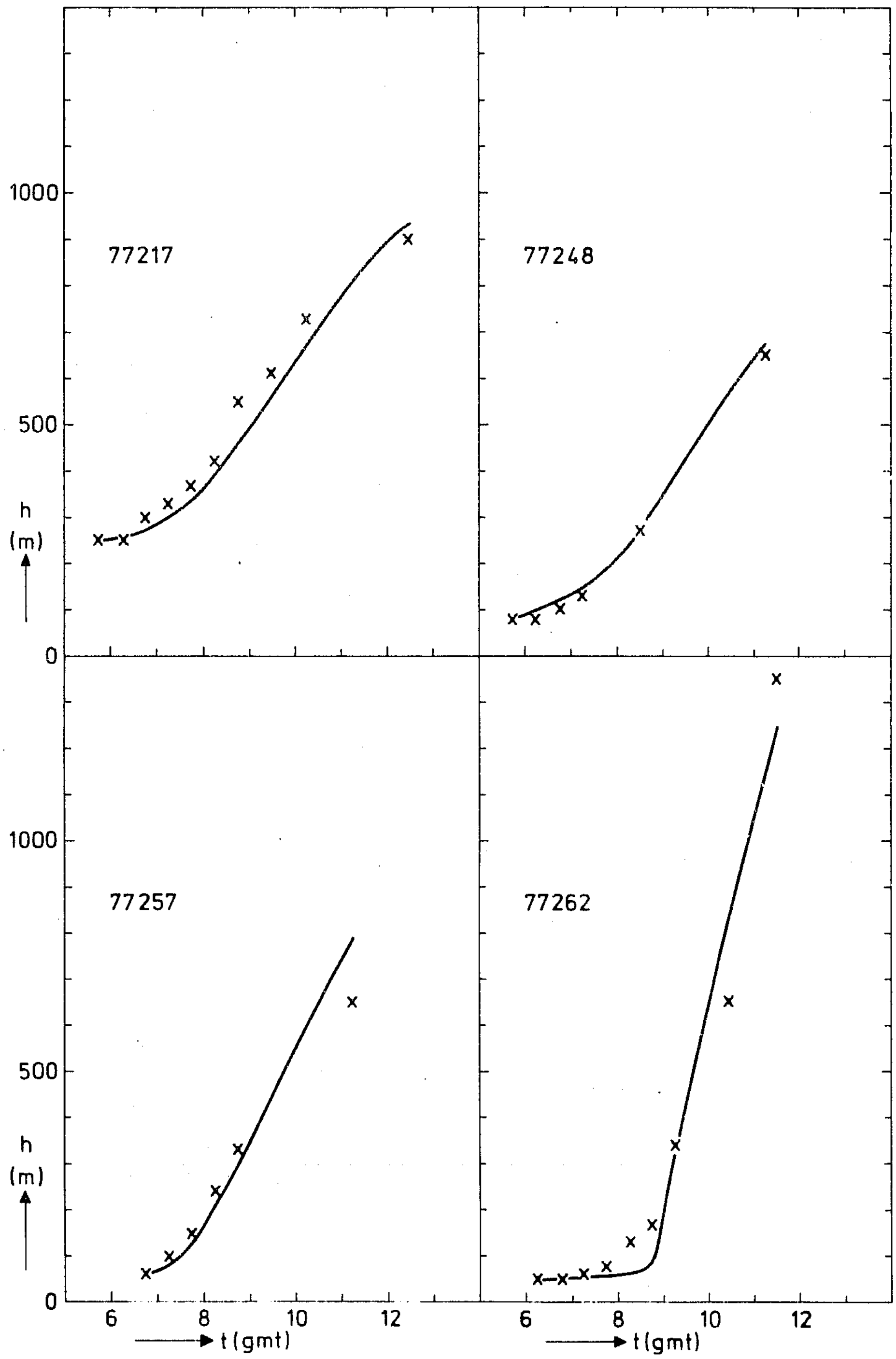

Figure 2. Observed (x) and calculated (——) values of the ABL-height h for four days (indicated by year and day number). Calculations with Eq. 23.

equilibrium with its external conditions (Nieuwstadt, 1984). Those expressions are diagnostic since they give the height in terms of variables at the same time. A well-known diagnostic relation is

$$h_{turb} = 0.4\ (u_*L\ /f)^{\frac{1}{2}}, \tag{24}$$

where L is given by (14). In general there is reasonable agreement between (24) and observations, see e.g. Nieuwstadt (1984a), for cases with $h/L > 1$. When $h/L < 1$, the ABL is close to neutral and a value of $h = 0.3\ u_*/f$ should be taken. A suitable interpolation formula between (24) and this neutral value is

$$h_{turb}/L = (0.3\ u_*/fL)/(1 + (0.3/0.4^2)h_{turb}/L). \tag{25}$$

These expressions are diagnostic relations for a steady-state stable ABL. However, the mean flow dynamics in the stable ABL react slowly and will generally not be in equilibrium with the external forcing. Therefore, several prognostic equations for the ABL-height have been developed, usually in the form of a relaxation equation in which the actual height approaches its steady-state value with a long time scale (Zeman, 1979; Nieuwstadt and Tennekes, 1981). The latter authors use the following rate equation in the stable case

$$\frac{\partial h_{turb}}{\partial t} = \frac{h_{turb} - h_{eq}}{T}, \tag{26}$$

where h_{eq} given by (24) or (25) and T a time scale given by

$$T = -(\Theta_{h_+} - \Theta_s)/\frac{\partial \theta_s}{\partial t}. \tag{27}$$

The foregoing expressions can be used to determine h_{turb}. The height of the significantly cooled layer or inversion layer (h_{inv}) is always larger. There is no well-established relation between h_{inv} and h_{turb}. From different data sets, both André and Mahrt (1982) and Driedonks et al. (1985) gave on the average a ratio $h_{inv}/h_{turb} \sim 3$, however the variation in this ratio is quite large (1-10), and it is likely to be a function of a bulk Richardson number, and the relatvie magnitude of the radiative and turbulent flux divergences. Thus in order to clarify the relation between h_{inv} and h_{turb} further studies, including detailed radiation calculations, are needed.

The shape of the vertical profiles, e.g. of Θ, in the stable ABL is not quite as simple as in the unstable case. In cases with low wind speed the gradient of Θ decreases monotonically with height and there is no such thing as a jump-like behavior of Θ near the ABL-top. The profile of Θ exhibits strong curvature and up to h_{inv} has a shape like the exponential profile as in Stull (1983). In cases with stronger wind speed the profile of Θ is often linear in the turbulent part of the stable ABL above the surface layer and has stronger curvature between h_{turb} and h_{inv} (Wetzel, 1982; Van Ulden and Holtslag, 1985). André and Mahrt (1982) introduced the overall scaled curvature of the profile of Θ between the surface and h_{inv} as $\gamma=(\Theta(h_{inv})-2\Theta(h_{inv}/2)+\Theta_s)/(\Theta(h_{inv})-\Theta_s)$ and related γ to an overall bulk Richardson number over h_{inv} (Ri_B). Although the scatter is large, the results indicate that for large values of the bulk Richardson number (> 1) there is virtually no relation with γ, which is constant at a value of about - 0.6, about the same as for an exponential profile. Thus for clear nights with weak wind the cooling below h_{inv} is dominated by radiation, which has no specific relation to the bulk Richardson number. For smaller values of Ri_B the curvature γ becomes less negative and may even become positive. Thus in these cases the turbulence exerts a stronger influence on the bulk temperature structure and the flow becomes more mixed.

From the above it is clear that bulk relations for the stable ABL are not well established. This is certainly partially due to the complicating effect of radiation. Another aspect, however, is the time scale for the evolution of the stable ABL. It is usually attempted to express the bulk relations for the calculation of the surface fluxes in the stable case (section 3) as well as the stable ABL-height and the shape of the vertical profiles of mean quantities in the form of diagnostic relations, i.e. involving only quantities that are local in time. This concept of self-similarity holds only for quantities that are in equilibrium with their boundary conditions or other governing parameters. Thus the time scale for adjustment has to be short compared to the time scale of change of external conditions. The time scale of the turbulence in the stable ABL is usually much smaller than the time scale of the mean variables (Wyngaard, 1983; Nieuwstadt, 1984b), thus the vertical profiles of turbulent quantities can be satisfactorily expressed in self-similar form. However, for the mean variables the time scale for adjustment is slow and the concept of local equilibrium or self-similarity becomes questionable, which is quite inconvenient for bulk formulations, since the shape of the vertical profiles may be an explicit function of time.

5. ENTRAINMENT EFFECTS ON THE MEAN PROFILES

In the section on the unstable ABL we used the assumption that all first-order moments of ABL-variables, such as Θ, q and a concentration of a passive scalar C are completely mixed up to z = h, so that their values become independent of z. We assumed the turbulence in the ABL to be strong enough to maintain these profiles and the conditions neither at the top nor the bottom to influence the shape. This is usually a good approximation for the Θ or Θ_v- profile in a convective ABL. However, it is often observed that other quantities, especially the specific humidity q, exhibit a distinct gradient over the ABL (André et al., 1979). This might be explained from the schematic picture in Figure 3. The air above the ABL is usually warmer but also dryer than the air within the ABL. Therefore the flux of heat at z = h will be downward, while the moisture flux is upward. Thus $\overline{\theta w}_s$ and $\overline{\theta w}_h$ have opposite sign (acting to increase the ABL-temperature from both sides) while $\overline{qw}_s$ and $\overline{qw}_h$ have the same sign, entrainment acting to decrease the ABL-humidity. Obviously this different flux behavior might have effects on the structure of the mean profiles induced by entrainment.

We therefore relax the assumption of well-mixedness a bit and allow the mean profiles to depend on z. We consider a scalar C with fluxes $\overline{cw}_s$ and $\overline{cw}_h$ at the ground at z = h respectively, and no internal sources or sinks. We assume that the <u>shape</u> of the profile of C(z,t) behaves in a self-similar way, i.e. that the time dependence occurs explicitly only in the vertically-averaged value $C_m(t)$. The existence of such a self-similar profile requires that the turbulent time scale in the ABL, $\tau_{turb} \sim h/w_*$, is small compared with external time scales imposed by the boundary conditions, i.e. by the changing surface flux $\overline{cw}_s(t)$ and the flux at the top $\overline{cw}_s(t)$, and it requires that the shape of the profile is in equilibrium with the fluxes at the boundaries.

In a self-similar shape of the profile of C in the ABL the time does not

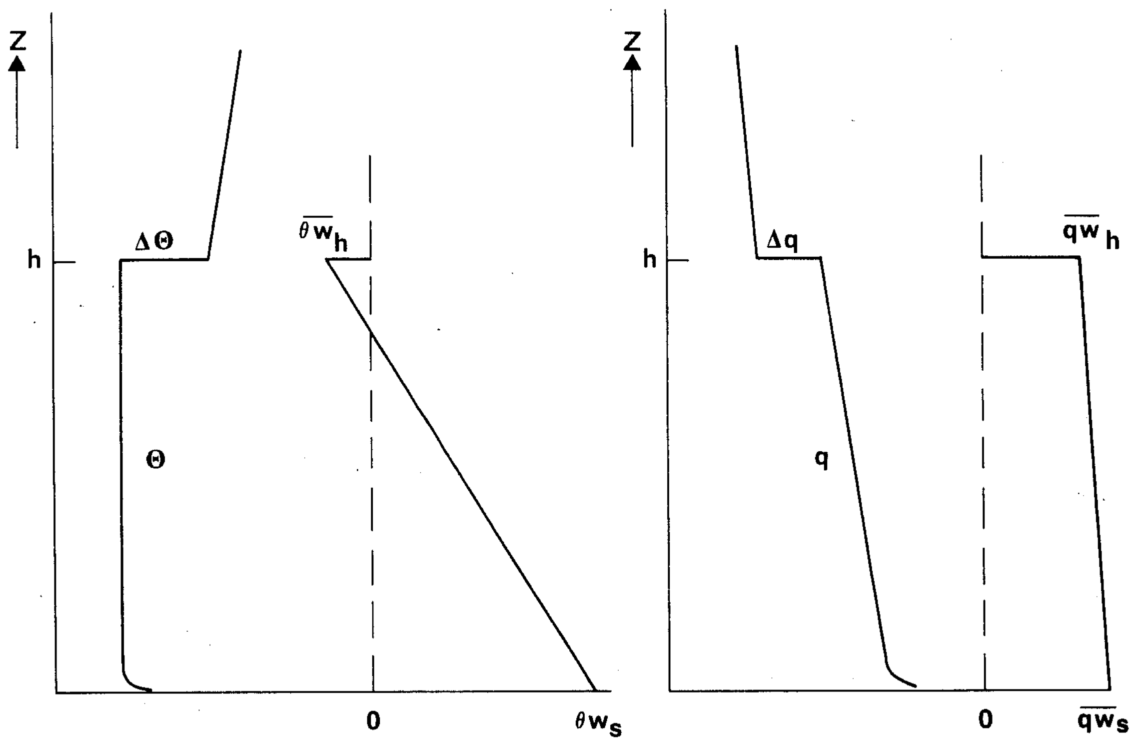

Figure 3. Schematic profile of Θ and q and their respective turbulent fluxes as observed in a convective ABL, where q is not constant with height.

occur explicitly. In non-dimensionalized form this shape will depend on z/h and on the non-dimensional boundary conditions, i.e. on $\overline{cw}_h/\overline{cw}_s$. We further assume that the ratio of the entrainment velocity to the turbulent velocity scale in the boundary layer, i.e. w_e/w_*, is very small, so that the shape of C does not depend on it. After scaling C itself with ΔC, that can be taken as $C_{h_+} - C_M$, we can then write:

$$C(z,t) = C_M(t) + \Delta C.F(z/h, \overline{cw}_h/\overline{cw}_s), \quad (28)$$

with F some universal function.

We are especially interested in the mean gradient over the ABL, which in self-similar form can be written as

$$(\partial C/\partial z)_M = \frac{\overline{cw}_s}{w_* h} G(\overline{cw}_h/\overline{cw}_s), \quad (29)$$

with G(x) some "universal" function of $x = \overline{cw}_h/\overline{cw}_s$. Driedonks and Tennekes (1984) argued that (29) should have as a tentative form

$$(\partial C/\partial z)_M = a_1 \frac{\overline{cw}_s}{w_* h} + a_2 \frac{\overline{cw}_h}{w_* h} \ , \quad (30)$$

where a_2/a_1 should be of the order 5. Note that both a_1 and a_2 will be negative. Thus we see that the entrainment-induced flux has an effect on the mean gradient in the ABL that is about 5 times larger than that of the surface flux. Wyngaard and Brost (1984) found from large-eddy simulations a ratio of the same order, but it was quite sensitive to the precise definition of z = h in their model. André et al. (1979) found about the same ratio from humidity observations, however, with a large scatter, as did André and Mahrt (1983) with laboratory data. They also found an enormous scatter and rather small correlations. The difficulties in testing a relation like (30) is partly caused by the indirect and perhaps inaccurate way to estimate $\overline{cw_h}$, but it can also be that effects that are as yet neglected are important. It may be possible that the coefficients in (30) are not constants but are functions of a set of suitable Richardson numbers.

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