ON THE PARAMETERIZATION OF OROGRAPHIC DRAG

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Abstract

A brief review of current knowledge concerning aerodynamic drag in turbulent flow over orography is presented. It is suggested that this drag is best represented in terms of the roughness length z_o and recommendations for estimating z_o are given.

Introduction

The transfer of momentum between the atmosphere and the solid earth occurs on a wide range of scales. These extend from the microscopic to the planetary scale and involve a range of physical processes. On small scales and over smooth surfaces the coupling may be due to simple "tangential" viscous forces. More usually, on scales ranging from mm to about 10km, the coupling is due to "aerodynamic" drag. This drag occurs mainly as "normal" pressure forces, it would not occur in inviscid flow but arises from the viscous dissipation occurring in the flows past surface roughness features. The limited concern of this note is the representation of the momentum coupling due to the roughness elements on scales up to perhaps 10km. Such drag forms an essential part of the driving forces in the planetary boundary layer and the influence upon the larger scale flow depends on a correct description of the whole boundary layer structure

On larger scales there are additional processes leading to momentum coupling. Orography with scales between perhaps 3km and 100km may generate gravity wave drag due to internal gravity waves. This mechanism leads to viscous dissipation other than that occurring in the boundary layer. It is believed to be an important process but is rather separate from the subject considered here. On greater planetary scales there are further mechanisms such as the radiation of Rossby waves but the actual viscous dissipation usually proceeds through the smaller scale processes involving either the boundary layer, convection or gravity waves.

Since the purpose of this note relates to the inclusion of orographic drag in large scale numerical weather prediction models it is useful to make a few points concerning this precise requirement. We can suppose that the large scale model will explicitly deal with scales greater than perhaps a few mesh spacings. The large scale orographic forcing should thus be derived by filtering the real orography of all smaller scales and the model representations of the boundary layer should include any appropriate influence of scales smaller than the mesh. On scales comparable with the mesh a statistical approach will not be valid and owing to the limitation of the finite difference representation there will be errors. The influence of the smaller scales will be complex. Part of the influence will be a mean uplift (corresponding to the zero plane displacement D noted below) arising through the failure of small scale flow to simply flow the shape of the orography. Another part will be the "aerodynamic" drag and the new boundary layer structure. Further complications might be the gravity wave generation or the rather separate way in which orography has an influence on rainfall. Rainfall is particularly complex as it can be affected a lot by the peak extent of any local orographic uplift.

In what follows we begin by considering the basic ideas and a few experimental facts. We then consider the orographic parameterization problem and various theoretical ideas. The practical rules suggested are compared with previous rules and the few available direct observations. Finally a summary and recommendations are presented.

Basic ideas and experimental data

A key feature of steady turbulent boundary layers over uniform terrain is the existence of a region in which the mean flow speed increases in proportion to the logarithm of distance from the surface. The existence of this logarithmic region is pivotal in our representation of turbulent boundary layers. The logarithmic region arises in the asymptotic limit of heights much less than the total boundary layer depth yet much greater than the scales involved in direct viscous or pressure forces with the surface. Within the logarithmic region the only relevant geometric scale is distance from the surface. For flow over a smooth wall the mean flow at a distance z from the wall is given by

$$u = \frac{u_*}{K}\left[\log\left(\frac{u_* z}{\nu}\right) + A\right] \qquad -1$$

where $u_* = \sqrt{\tau_0/\rho}$ is the square root of the surface stress τ_0 divided by density, K is Von-Karmon's constant (0·4 is the most accepted value), ν is the kinematic molecular viscosity and A a constant with a value about 2·3. If the surface is rough then it has been found (eg.Clauser 1956) that

$$u = \frac{u_*}{K}\left[\log\left(\frac{u_* z}{\nu}\right) + A - \Delta\left(\frac{h u_*}{\nu}\right)\right] \qquad -2$$

where Δ is a function of $h u_*/\nu$ and h is a height scale of the roughness features. When the Reynolds number $h u_*/\nu$ is greater than about 50 the flow is found independent of this Reynolds number and

$$\Delta = \log\left(\frac{h u_*}{\nu}\right) + B \qquad -3$$

For such so called fully rough flows the combination of 1 and 2 is usually written as

$$u = \frac{u_*}{K} \log\left(\frac{z}{z_o}\right) \qquad -4$$

Where z_o absorbs the constants A and B. In a practical flow z_o is deduced either by measurement of the velocity profile or by measurement of stress and flow at a particular height in the logarithmic region. In cases with dense arrays of surface features the height of the origin for z is uncertain and the relation

$$u = \frac{u_*}{K} \log\left(\frac{z + D}{z_o}\right) \qquad -5$$

must be considered. Here D is called the displacement height and is again determined by a fit to the velocity profile. The engineering data concerning flow over rough surfaces are extensive and range from the early work of Nikuradse (1933) to more recent studies (eg. Perry et Al. 1969) There are many aspects of these results which have a bearing on the meteorological problem of flow over orography. For flow past arrays of individual bluff bodies (called "k" type where k is used to denote the object height) the values of z_o vary in proportion to the height scale of the bodies. Flows past regular arrays of ribs or groves (called "d" type where d denotes the pipe or channel width) are more complex and z_o then depends on the boundary layer depth. The motions over the ribs involve a coherent flow, on the scale of the boundary layer, into and out of the spaces between the ribs. Such motions are not represented in theoretical models of flow over ribs and are unlikely in natural orography unless the orography is very even in structure. This class of motion will not be considered further here.

The observations of flow past fairly dense arrays of bluff "k" type roughness elements find that the value of z_o relative to the size of the object depends upon the exact shape of the roughness elements. Extreme ratios of z_o/h where h is the mean object height are about 0·2 to 0·04 and 0·1 is the value often quoted. Experiments (eg. Clauser 1956) with the fraction of surface covered by bluff roughness elements varying between about 5% and 50% show little variation in z_o. This near independence of z_o upon the density of roughness elements arises when each body lies in the wakes of other bodies. For densities greater than 50% the value of z_o reduces and the displacement height D becomes comparable with the object height. The small amounts of data on more sparse roughness elements have not led to any engineering rules. In keeping with the engineering approach for such cases the present review will consider the actual forces on the individual elements. This approach differs from the procedure adapted by Kutzbach (1961) and often used in a meteorological context.

There are many sources of information on the drag past isolated bodies in a free stream (eg. Batchelor 1967 for a text discussion). The drag on bluff bodies depend upon the Reynolds number of the flow. Only when the flow is fully turbulent at a Reynolds number $\gtrsim 10^7$ is the drag coefficient independent of Reynolds number. In meteorology this usually applies to object scales greater than a few metres. On smaller scales a Reynolds number dependence is expected. For Reynolds numbers between about 10^3 and 10^5 the drag coefficients are about twice those occurring at high ($>10^7$) Reynolds number values. Here the flow immediately around the body is laminar but the wake is turbulent. At low ($<10^2$) Reynolds numbers there is a distinct Reynolds number dependence. In meteorology we have relatively good empirical data on the small scale roughness elements which may exhibit a Reynolds number dependence. Indeed many of these roughness elements are flexible plants and this flexibility gives further dependence of the roughness length on wind speed.

At the highest Reynolds numbers ($>10^7$) the values of drag coefficient (ie. force = $\frac{1}{2}\frac{1}{\rho}U^2 \; A \; C_D$ where U the free stream speed and A the frontal area) vary with object shape. For bluff objects C_D varies between about 0·8 for rectangular objects to 0·2 for a sphere. More slender bodies show much smaller values of C_D. When bodies are not in the free stream but mounted on the surface the force on the body remains well known but there is uncertainty in the change of the frictional drag over the surrounding surface. There is evidence that the increased turbulent mixing which occurs in the wake may give an increase in the frictional drag. This seems especially likely for skew bodies which may generate powerful trailing vortices.

Determination of drag in flow over orography

The aerodynamic drag between the atmosphere and the underlying surface involves scales extending from of order mm to 10km. The influence of the smaller scale features such as vegetation can be formally represented in terms of a value of z_o and many texts summarise our empirical knowledge of the values of z_o for a variety of terrain types. Current models of the planetary boundary usually use a value of z_o to describe the surface characteristics and there is a considerable practical advantage in trying to extend this description to larger scales. On these larger scales comparable with the depth of the boundary layer there is no formal justification for the existence of a logarithmic region and a representation in terms of z_o. There is however, some evidence that it may be a reasonable approach.

Meteorological observations in complex terrain (eg. Thompson 1978) have shown a surprisingly extensive region of "logarithmic" velocity profile. With more quantitative significance are engineering observations (Perry et Al. 1969 and Buckles et Al. 1984) for objects with scale comparable with the boundary layer depth. These observations show a logarithmic velocity profile and, more important, the characteristics of the profile, z_o and u_* match the independently measured values of drag. Recent numerical simulations of flow over ridges (Newley 1985) also shows a similar result.

Forming area averages

Before embarking on a discussion of how to estimate z_o in real terrain it is useful to note the problems in averaging the surface specification over the whole of a mesh spacing in the model. The only quantity which can be correctly averaged is the surface stress. This is not generally the result of the whole boundary layer being in equilibrium with the local value of z_o. A horizontal length scale of perhaps 50 to 100kms is needed to establish a reasonable "equilibrium" stress in relation to the geostrophic wind, ie. to establish a true geostropic drag coefficient. After a change in z_o the flow is only in equilibrium with the surface up to a height of order $\ell_e \sim L k u_*/U$ where L is the upstream extent of the surface type (exact model studies suggest $\ell_e \sim L/100$ to $L/200$ depending on conditions, Rao et Al. 1974). The consequence of this adjustment is a tendency for the effective mean value of z_o to be closer to a higher value of z_o than it would be in a flow always in equilibrium. A rough rule of thumb would be to assume no horizontal variation of the velocity at a height $\ell_e \sim L/100$ and average the stress with a drag coefficient based on this height, ie.

$$\frac{1}{\log^2\left(\frac{\ell_e}{z_o}\right)} = \frac{f_1}{\log^2\left(\frac{\ell_e}{z_1}\right)} + \frac{f_2}{\log^2\left(\frac{\ell_e}{z_2}\right)} \qquad -6$$

where z_0 is the required value, and z_1 and z_2 are roughness lengths occupying fractional areas, f_1 and f_2, ℓ_e is calculated from the characteric scale of variations in z_1 and z_2. In reality there is no height at which the flow is both in equilibrium with the surface and also independent of horizontal position. The height scale ℓ_e is only the characteristic scale at which the transition from equilibrium to independence on horizontal position occurs. It follows that although the spirit of this reasoning should be correct the value of ℓ_e used in equation 6 is rather loosely defined and should be checked by a more vigorous approach.

Such a more refined approach is a numerical simulation of the flow over changing roughness lengths. Such simulations do not seem too sensitive to the turbulence closure and can be viewed with some confidence. Tables I and II show a comparison of the results of equation (6) with values obtained in a numerical simulation using a high order closure model (Nash 1980). The simulation is of flow in a periodic domain of length L_p containing a fraction δ with roughness length one hundred times that in the rest of the domain. A value of z_0 has been calculated from the mean surface stress in the numerical integration and can be considered the correct answer. The value of z_0 deduced by assuming that the boundary layer was locally in equilibrium with the surface is also given. The equilibrium result is significantly in error on scales up to of order 10kms. In contrast equation (6) with $\ell_e \sim L_p/200$ appears to give the correct bias towards the large values of z_0. This bias is even more important on short scales and Table III shows results from equation (6) for a range of values of L_p and δ.

Table 1.

Comparison of effective values of z_o arising from combining areas of different z_o. The domain length L_p is 10^3 m and the two values of z_o 1·5 and 0·015m. In equation 6 ℓ_e has taken the value 5m.

Fraction of domain with z_o large.	z_o/m Numerical result Nash (1980)	Equation 6	Equilibrium
0·3	0·50	0·48	0·15
0·5	0·82	0·81	0·35
0·7	1·20	1·12	0·69

Table II.

Comparison of effective values of z_o arising fron combining two values of z_o on different length scales. The fraction of the domain with each value of z_o is 0·5 and the values are 1·5 and 0·015m. In equation 6 ℓ_e is $\sim L_p/200$.

Length of domain L_p	z_o/m Numerical result Nash (1980)	Equation 6	Equilibrium
10^3	0·82	0·81	0·35
10^4	0·63	0·56	0·35
10^5	0·43	0·43	0·35

Table III.

Values of log $(z_0/z_1)/\log(z_2/z_1)$ arising when equation 6 with $\ell_e = L_p/200$ is used to derive z_0 from different areas of roughness z_1 and z_2 (with $z_2 = 100\, z_1$)

Fraction with z_2	0·01	0·03	0·1	0·3
length L_p/m				
10	0·47	0·60	0·75	0·87
10^2	0·30	0·49	0·68	0·85
10^3	0·09	0·21	0·46	0·74
10^4	0·04	0·10	0·28	0·59
10^5	0·02	0·07	0·21	0·50
equilibrium	0·02	0·04	0·17	0·44

<u>Dense arrays roughness elements with steep slopes</u>

When the surface is covered with many bluff obstacles whose slopes exceed 45° and which occupy at least 10% of the area, the best guide is to assume that z_0 is about 0·1 h where h is the mean height of the roughness elements. This rule has been verified in city and dense urban environments. The evidence from the engineering data is that it should also apply for appropriate large scale roughness elements. In practice there seem to be few mountainous areas with sustained slopes of 45°. In the few very extreme cases, such as regions of the Alps, it may be appropriate to use z = 0·1 h. Here we shall argue that the methods of estimating z_0 which are given below should be considered first and 0·1 h should be used as an upper bound to z_0 .

<u>Dense arrays roughness elements with moderate slopes</u>

This category of terrain is intended to include most hilly and mountainous areas. When the slopes of the terrain are less than about 45° the effective values of z_0 can be expected to be less than 0·1 h and there is a need for a refined estimate. Taylor and Gent (1974) considered a numerical model of turbulent flow over terrain and used a mixing length turbulence closure. The study was limited to slopes (for slope we shall consider values of arctangent) of terrain up to about 0·2. For a range of hill lengths and values of basic roughness length the increase in drag due to a single gaussian shaped hill was found to be $\sim 40 - 60\, u_*^2\, \theta\, h$ where u_*^2 is the undisturbed stress, h the hill height and θ the peak hill slope. Sykes (1980) considered an asymptotic analytic theory with a full 2nd order turbulence closure and suggested that in consequence of the mixing length assumption Taylor and Gent's drags were a factor of seven too large. Recently, Newley (1985) has considered a full 2nd order turbulence closure in a finite difference model. This overcomes the asymptotic limitation of Sykes's theory and for realistic parameters finds drag forces only a factor of two less than those obtained by Taylor and Gent ie. for a periodic sine wave the extra force F per wavelength is

$$F \sim 20\, u_*^2\, \theta\, h \qquad -7$$

where h is the peak-trough height and θ the peak slope. Although Newley's results are very different from those of Sykes, Newley's results show the coefficient in equation 7 slowly decreases with decreasing z_0 and it is possible that in Sykes limit $z_0 \to 0$ there might be agreement. Newley's results find confirmation in comparisons with laboratory experiments (Zilker and Hanratty 1979, Buckles et Al. 1984). Newley's model also provides estimates of drag at greater slopes. To translate the results into values of z_0 we need to use Rossby number similarity theory, ie.

$$U_g^2/u_*^2 = \frac{1}{K^2}\left(\left(\log\left(\frac{u_*}{f z_0}\right) - A\right)^2 + B^2\right) \qquad -8$$

where A and B are constant and U_g is the geostropic wind. A and B have been taken as 1·4 and 2·1 and the implied values of $C_G = u_*^2/U_g^2$ are slightly greater than values often used (eg. Ayra 1975). Table IV shows C_G as a function of z_o and U_g as given by equation (8). The present values accord with Large-Eddy simulations and the results from high order closure models. Such results probably correspond to more strictly neutral conditions than most observations. The above equation (7) for force F translates into an expression for a change in the drag coefficient, ie.

$$\frac{\Delta C_G}{C_G} \approx 6 \cdot 4\, \theta^2 \qquad -9$$

where C_G is the drag coefficient for the undisturbed surface. With an assumed basic value of $z_o \sim 0 \cdot 1$m equation (8) leads to an approximate relation for the new value of z_o. ie.

$$\log\left(\frac{z_0}{z_{01}}\right) = 6 \cdot 25 \log\left(1 + \frac{\Delta C_G}{C_G}\right) \qquad -10$$

where z_{o1} is the undisturbed value and 6·25 is approximately equal to $1/K^2$. A slope of 0·1 gives a change in z_o of 1·5 times the undisturbed value. It is thus clear that slopes less than $\sim$ 0·1 can be neglected. A slope of 0·2 gives a value of z_o of 4·1 times the undisturbed value. For slopes greater than about 0·2, Newley's (1985) results no longer follow this relation. The value of the undisturbed stress no longer forms a suitable scale for the force on the terrain and it is appropriate to consider the change in z_o (the undisturbed value is usually negligible for these slopes) in relation to height scale of the terrain. Values of z_o /h corresponding to flows with different slopes are shown in Figure 1. The

Table IV.

Values of geostrophic drag C_G and angle α of surface stress relative to geostrophic wind direction. The values shown are obtained using Rossby number similarity theory for an atmosphere with neutral static stability. The values shown are for a coriolis parameter $f = 10^{-4}$ and $U_g = 10\ ms^{-1}$. C_G is a function of $u_*/f z_0$ which to a first approximation is proportional to $U_g / f z_0$. A factor of 10 change in U_g/f is thus nearly equivalent to a factor of 10^{-1} change in z_0.

z_0/m	$C_G\ 10^3$	$\alpha\ ^\circ$
10^{-4}	0·64	7·6
10^{-3}	0·86	8·8
10^{-2}	1·20	10·5
10^{-1}	1·77	12·8
10	2·86	16·3
10^{1}	5·13	22·1
10^{2}	10·73	32·9

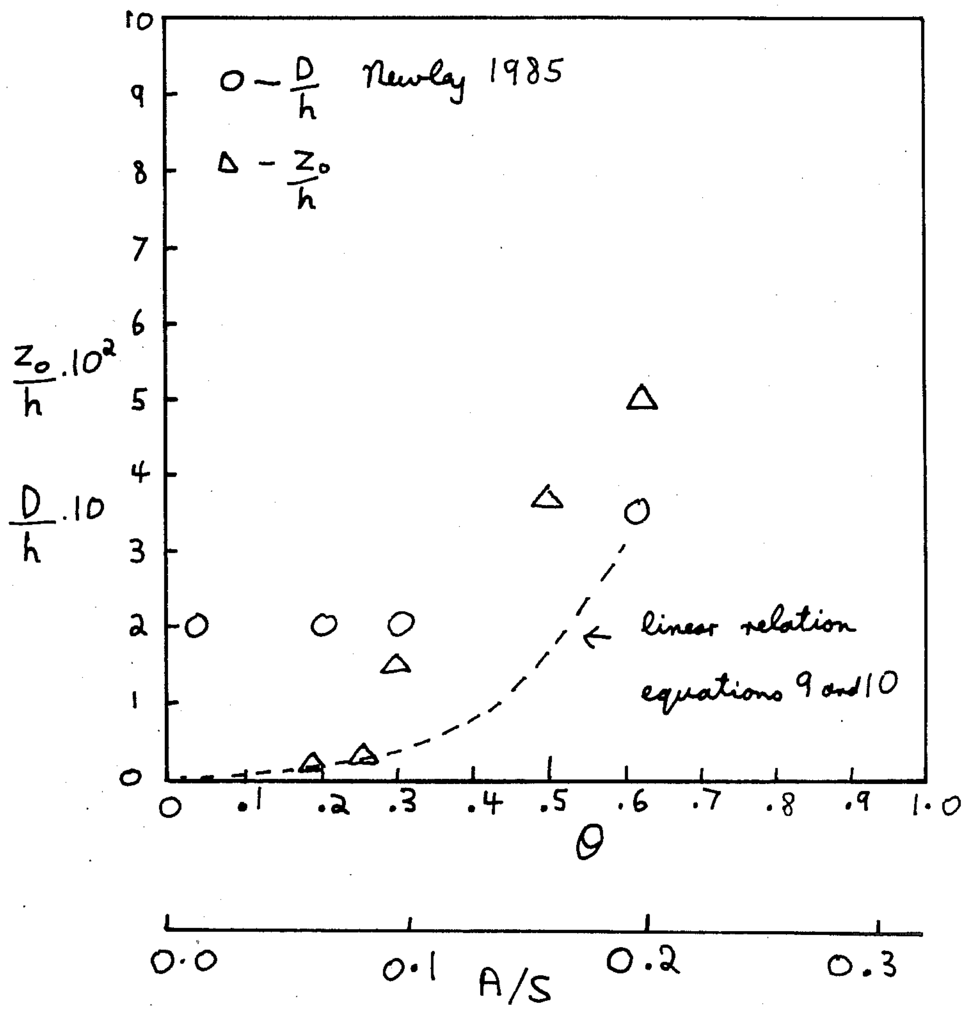

Figure 1. Values of z_o / h and D / h derived from numerical simulations of flow over sinusoidinal orography. The numerical simulations (Newley 1985) use a 2nd order closure model. The values of $\theta = \pi h / L$ where h is the peak-trough height and L the wavelength. The values of A/S = h/ L .

increase in z_0 /h with slope appears to tend towards the h/10 rule at a slope of about 1·0 (45°). The results presented are obtained with a length scale $L = 10^3$ m and a basic value of $z_0 \sim 0{\cdot}1$m. The results should not be too sensitive to the changes in L and z_0 but further work, especially with a three-dimensional model, would provide a better basis for an empirical rule to determine z_0. As shown in the next section the behaviour of z_0 /h for slopes steeper than 0·2 is in accord with simple ideas considering the object as an isolated body immersed in an equilibrium velocity profile. For the moment it thus seems adequate to use the rule proposed below.

Also shown in Figure 1 are values of the displacement height D derived from Newley's results. Again an extension to three-dimensions would be needed to provide a practical guide. It is important to note that D has been defined relative to the mean orographic height. The values of D at small slopes are subject to some inaccuracy but seem to be about 0·2 h. At larger slopes D is increased. It is interesting in the latter cases that D appears to be about 0·2 h' where h' is the height of the peaks above the regions of flow separation which occur in the valleys.

Isolated roughness elements

For a single isolated orographic feature such as an island it is possible to estimate the drag force from the usual aerodynamic rules. For a bluff feature with slopes greater than about 45° the force may be expected to be of order $\frac{1}{2} \rho C_D U^2 A$ where C_D is the body drag coefficient $\sim$ 0·2 to 0·8, U is the typical flow speed and A a frontal area of the body. Such a force can only be represented in terms of a value of z_0 on the assumption of an area in which to include it. The physical way in which the force on the feature will be incorporated into the boundary layer structure will involve the momentum deficit in the wake of the body. In an effort to provide some practical guidance to determining z_o in real cases we can only proceed if we consider a statistical distribution of isolated bodies. We may then argue that each body will be exposed to a velocity derived from the final velocity profile, ie. the velocity will be taken as

$$u\left(\frac{h}{2}\right) = \frac{u_*}{k} \log\left(\frac{h/2}{z_o}\right) \qquad -11$$

where h is the body height and z_o and u_* the final values of z_o and u_*. The force on the body is then

$$F = \frac{1}{2} \rho \, C_D \, u^2\left(\frac{h}{2}\right) A \qquad -12$$

In order to derive a relation with no singular behaviour we assume a stress u_{*1}^2 due to the undisturbed value of z_o, ie.

$$u\left(\frac{h}{2}\right) = \frac{u_{*1}}{k} \log\left(\frac{h/2}{z_{o1}}\right) \qquad -13$$

It follows that

$$S u_*^2 = \sum \frac{1}{2} C_D A \, u^2\left(\frac{h}{2}\right) + S \, C_N \, u^2\left(\frac{h}{2}\right)$$

where S is the surface area considered, $C_N = k^2/\log^2\left(\frac{h/2}{z_{o1}}\right)$ and the summation includes all bodies in the area. From these equations we obtain

$$\log^2\left(\frac{h/2}{z_o}\right) = \frac{k^2}{\sum \frac{1}{2} \frac{A}{S} C_D + C_N} \qquad -14$$

This result is simplistic but from an order of magnitude point of view should be realistic. The drag coefficient C_D will depend on the type of object. The objects in the numerical study are hardly isolated but it is useful to see the relation of (14) to the numerical results given in Figure 1. In Figure 2. curves derived from equation (14) with $C_N = 0$ are shown

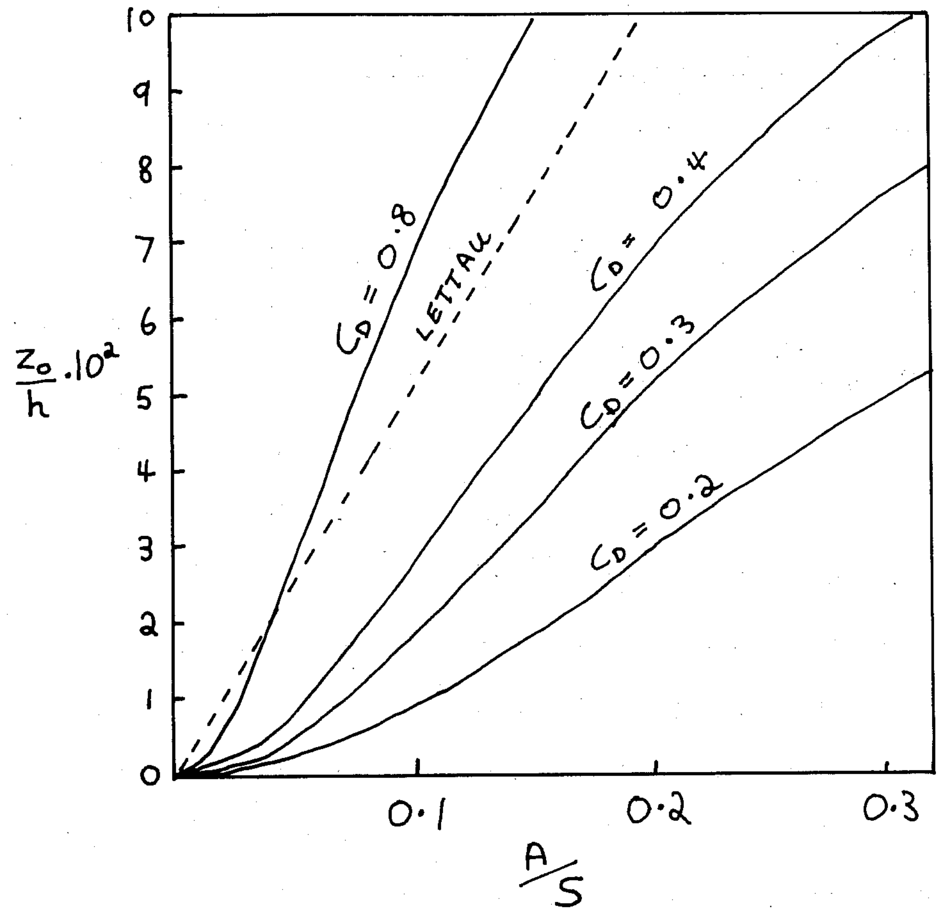

Figure 2. Values of z_o /h as a function of A/S where A is the silhouette area of the obstacles occupying a surface area S . The curve shows the relation given by equation (14) ie. $\log^2\left(\frac{h/2}{z_o}\right) = \frac{k^2}{\frac{1}{2}\frac{A}{S}C_D}$ and curves for different C_D are shown. The relation used by Lettau (1969) is also given.

for various values of C_D. The curve for $C_D \sim 0{\cdot}3$ seems to fit Newley's results for separated flows at slope angles greater than 0·3. For smaller slopes Newley's results suggest that C_D should decrease.

Comparisons with previous parameterization rules

A brief review of the estimation of z_o for flow over obstacles was given by Lettau (1969). He considered a variety of data including the experiments of Kutzbach (1961) with "bushel baskets" distributed over the surface of a frozen lake. He proposed the relationship $z_o = 0{\cdot}5\, h\, A/S$ where h is the effective obstacle height, A the silhouette area of obstacles and S the surface area. He compared this relation with a variety of data but did not consider A/S greater than 0·1 and made no allowance for the "background" values of z_o due to the "smooth" surface. Lettau noted that $z_o = h/a$ with a, a constant was not adequate for the results he considered. This is in keeping with the low values of A/S he considered and it is instructive to compare his formulae with that give by (14) with $C_N = 0$. The two relations are compared in Figure 2. In view of Lettau's empirical support it is reassuring that, except for small values of A/S equation (14) also gives z_o roughly proportional to $h\, A/S$. For values of C_D appropriate to very bluff bodies ($\sim 0{\cdot}7$) there is also quantitative agreement. At small values of A/S Lettau's formulae must fail. For it to be correct the drag coefficient on the obstacles would have to be very much greater than unity.

An alternative form of the Lettau relation is creditied to Kutzbach (1961) by Smith (1975), ie. $z_o = 0{\cdot}2\, h^2/d$ where d is the average distance between peaks separated by valleys. This relation assumes a fixed "shape" of hill and cannot be shown on Figure 2 without an assumption regarding the obstacle shape. Compared with the Lettau relation, for isolated obstacles it gives a larger value of z_o whilst for closely packed obstacles it gives a smaller value.

Observations

Observations able to provide guidance in determining values of z_o in flow over hills are very few. Fielder and Panofsky (1972) report values of z_o based on aircraft observations of vertical velocity variance. The values vary from 0·4m for plains to 1·4m for mountains. Unfortunately the paper gives no information on the exact size of the mountains. Since the data were reported to be obtained by an aircraft at a height of 75m it seems safe to assume that the mountains were rather low.

Observations of wind profiles in complex terrain (Nappo 1977, Thompson 1978) show extensive logarithic velocity profiles and give large values of z_o – 3·5m Nappo 1977 and 35m Thompson 1978. Again there are scant details of the orography but using Lettau's formulae Thompson estimates z_o = 8m. This estimate was only based on a very local 5km area and the underestimate may be due to the presence of greater orography noted to be about 5kms away.

Recent unpublished work by the Met. Office in a South Wales valley (an extension of work reported by Mason and King 1984) provided direct measurement of shear stress at 500m above 200m high ridges, which were about 2km apart. The data suggest a value of z_o of about 3m. The high order closure model of Newley (1985) has been applied to this scale of orography and also gives z_o = 3m. This orography corresponds to a slope of about 0·31 and it can be seen on Figure 1 and 2 that both the numerical results for a sine wave and equation (14) with C_D = 0·3 give the correct answer. Although Lettau's (1969) relation gives a larger value of z_o the relation provided by Smith (1975) agrees with this result. The agreement with Smith's relation, as noted above, is coincidental and due to precise value of h/L found at this location.

More observations are urgently needed to provide verification on wider ranges of scales. At present we are forced to place much trust in theory and laboratory studies.

Summary and reccommendations

This note has sought to provide guidance on the representation of "aerodynamic" drag between the atmosphere and orography on scales between metres and ten kilometres.

It is suggested that such drag can be described in terms of a roughness length z_o associated with a logarithmic velocity profile. By representing the drag through z_o allowance can be made for the usual factors including buoyancy which influence the planetary boundary layer. The direct influence of buoyancy processes upon z_o has been neglected. This is usually a good assumption for scales up to about 100m but is not usually true for scales up to 10km unless the basic wind speed is high. In particular stable stratification will suppress turbulence and give rise to internal gravity wave generation. In such cases the influence on z_o is not known and as mentioned in the introduction, gravity wave generation can perhaps be regarded as a separate but important problem.

Having decided to use z_o to represent the drag due to the smaller scale orography there are a few points which should be borne in mind in a practical application.

The first point concerns the vertical scale in the boundary layer. In order to give z_o its proper meaning this needs to be dealt with consistently, especially when z_o is large. If we assume the surface $z = s(x, y)$ represents the large scale orography of the numerical model then $s(x, y)$ can conveniently be assumed to be the height at which $u = 0$. ie. $s(x, y)$ must include <u>both</u> the large scale orography and any displacement height arising from flow over the "subgrid" scale features. The mean height of the subgrid field should be defined as zero. With this definition of z equation 4 becomes

$$u = \frac{u_*}{k} \log\left(\frac{z + z_o}{z_o}\right) \qquad -15$$

When z is comparable to z_o the use of $z + z_o$ rather than z can be important. The length scales used in any turbulence model must also be consistent and in the near surface region the mixing length should be given by

$$\ell \sim k\,(z + z_0) \qquad -16$$

The second point concerns the finite difference representation of the boundary layer and can also concern the boundary layer in absence of orography. The large values of z_0 present in flow over orography should lead to an increased drag and concomitant increase in boundary layer depth. Boundary layer models with a high vertical resolution describe this effect quite naturally but models with limited vertical resolution can fail to respond. In such coarse resolution models the large mesh spacings inhibit the representation of the increased velocity shear giving an increased mixed layer depth. The problem usually arises from the consequence of defining the Richardson number, or its equivalent, over a wide mesh spacing. The value of such a Richardson number is $\sim \Delta\rho\, \Delta z / \Delta V^2$ where $\Delta\rho$ is a "potential" density difference, Δv a velocity difference and Δz the mesh spacing. For a positive $\Delta\rho$ a large value of Δz will give large stable Richardson number. The real problem is simply a lack of resolution. A crude improvement can be obtained by replacing Δz with a fixed plausible length scale such as 100m. If too small a value of this scale is used, the Richardson number may be less than the critical value (perhaps 0·25) everwhere, so this is not a modification to be used casually. Some tests with this feature would be desirable. The basic problem is serious; if the boundary layer growth is restricted by the mesh then the effect of orography, both in terms of drag and vertical diffusion, fails to be realised.

The final point is just a reminder that it is ill conceived to try to estimate z_o other than on the assumption of a statistical representation of the orography. In other words the scales comparable with mesh scales will not be dealt with correctly. The value of z_o at each mesh point should, to be consistent with the numerical representation, be subject to the same scale of filtering as that used to derive the large scale orography. This smoothing should not be applied to the actual values of z_o but the procedure represented by equation (6) should be used to ensure an attempt to average the surface stress.

In contrast to the many restrictions and qualifications the actual recommendation can be presented quite simply:- Divide the terrain up into areas of broadly homogenous types of terrain and then estimate z_o in each such area.

1. Allocate a value of z_{ol}, based on the very small scale surface characteristics such as grass, fields, forests or rocks.

2. Estimate A/S, θ_S , and h where A is the silhouette area of the obstacles, S the surface area involved, θ_S the representative orographic slope and h the typical peak-valley height. θ_S should be taken as h/L_S where h is the peak-valley height and L_S is the horizontal scale over which most of the height change occurs. θ_S will not be used quantitatively but to determine which rule to follow. There are many possible ways of estimating A/S depending on the type of orographic data available. A simple method is to consider a number of 2-D sections with different orientations being used to obtain a statistical estimate. On each section the height differences h_{up} over whilst the height is increasing are summed.

Then
$$\frac{A}{S} = \frac{\sum h_{up}}{\sum L_{sec}}$$

-17

where L_{sec} is the section length and the summation is over all sections. The value of h_{up} will be greater the finer the representation of the map. In practice most orography has had sharp features attenuated by errosion and evaluation of h_{up} on, for example, a 1:50000 map provides an estimate of the main scales of orography. The value of h can be taken as the mean of the main height changes between distinct peaks and valleys. In towns and urban environments values of A/S due to buildings need a different approach, such as the number of buildings and mean building size.

3. If θ_S is less than 0·2 we should use linear theory and not bluff body dynamics. We use equation (9) and (10) but estimate θ as π A/S on the assumption that the orography looks similar to a sine wave, ie.

$$\log\left(z_o / z_{o1} \right) = 6{\cdot}25 \log\left(1 + 63\, A^2 / S^2\right) \qquad -18$$

4. If θ_S is greater than 0·2 we assume the bluff body relation equation (14) with C_D dependent on θ_S. On the basis of available information we suggest $C_D \sim 0{\cdot}3$ for θ_S up to 1·0 and $C_D \sim 0{\cdot}7$ for θ_S greater than 1·0. In most orography θ_S is $<$ 1·0 and the higher values of C_D is really intended for sharp bluff bodies such as buildings, ie. we use

$$\log^2\left(\frac{h/2}{z_o}\right) = \frac{k^2}{\frac{A}{S}\,\frac{1}{2}C_D + C_N} \qquad -19$$

where $C_N = k^2 / \log^2\left(h/2 \,/ z_{o1}\right)$ (and may be negligible mountainous terrain) and h is the typical peak-valley height.

5. Finally we note that z_o derived from (19) should not exceed 0·1 h.

Application of the above rules gives results which are not widely different from the values suggested by previous workers. (eg. Smith and Carson 1977). However, the precise difference may be significant and the procedures advocated here have a slightly firmer basis and more general application.

As an example of the present rules, application has been made to three specific sites corresponding to the South Downs, South Wales and the Cairngorms.

For the South downs a ten kilometre square centred at 50° 55 N 0° 45 W was selected. Here values of h are about 100m and θ_S is between 0·1 and 0·2. The value of A/S derived from various sections is $\sim$ 0·01. The basic value of z_o is harder to estimate. About one quarter of the area is woodland occurring on scales of about 1 to 2 km. Taking z_o = 1m for the woodland and z_o = 0·1m for the open areas equation (6) indicates an overall value of z_o = 0·27m. Taking $\theta = \pi$ A/S = 0·03 equation (18) gives an increase in z_o of 1·4 ie. a final value of z_o = 0·4m.

For South Wales a ten kilometre square centred at 51° 45 N, 3° 12 W was selected. This corresponds to the field site used by Mason and King (1984) and discussed above. Here $\theta_S \sim 0{\cdot}33$ so the bluff body dynamics should be considered. h is about 250m and A/S $\sim$ 0·04. Owing to the ridge-valley orientation A/S $\sim$ 0·08 for East - West flow but only the average value will be considered here. The basic surface value of z_o has been taken as 0·1m to correspond to open country with scattered trees and walls. Application of equation (19) then gives a final value of z_o = 2·0m (3·5m for across-valley flow)

For the Cairngorms a ten kilometre square centered at 57° 2 N and 3° 45 W was selected. Here θ_S is 0·45, h is $\sim$ 600m and A/S $\sim$ 0·1.

The basic value of z_o is probably around 0·1m but not very important. Application of equation (19) gives z_o = 12m.

Further work and observations are needed to provide better guidance on estimating the displacement height D. A very unrefined guess would be to set D, on a fairly local basis, to a fraction of peak to valley topographic height. Newley's (1985) results suggest that D might be 0·2 h for θ_s up to about 0·4 and then rise to about 0·5 h for $\theta_S \sim 1\cdot0$. An area average of the local values of D would then need to be formed.

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